Insufficient reason in the 'new cosmological argument'

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Abstract: In a recent article in this journal,[1] Richard Gale and Alexander Pruss offer a new cosmological proof for the existence of God relying only on the Weak Principle of Sufficient Reason, W-PSR. We argue that their proof relies on applications of W-PSR that cannot be justified, and that our modal intuitions simply do not support W-PSR in the way Gale and Pruss take them to.

Cosmological arguments for God's existence typically proceed from the Principle of Sufficient Reason, one version of which states

> PSR   For all propositions p, (p and not Necessarily(p)) entails E(p)

where E(p) is true iff p has an explanation. Recently, Richard Gale and Alexander Pruss [hereafter GP] have argued that God's existence can be inferred from a weaker – and, they claim, more plausible – principle, the *Weak* Principle of Sufficient Reason. According to this principle, for any contingently true proposition, it is at least *possible* that it has an explanation, even if it does not *actually* have an explanation; that is,

> W-PSR   For all propositions p, (p and not Necessarily(p)) entails Possibly(E(p)).

By 'p has an explanation' GP mean an explanation whose *explanans* and *explanandum* are true. Thus, for any proposition p, E(p) entails p.[2]

  GP's 'new cosmological argument' then proceeds as follows:

(1) Let A be the conjunction of all contingently true propositions.
(2) The proposition A is itself contingently true, and so, from W-PSR, we may conclude Possibly(E(A)). Let w be a possible world in which E(A) is true.
(3) Since for all p, E(p) entails p, A is true in w.

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(4) If there were a proposition p true in w that is actually false, then since A would entail not p and A is true in w, both p and not p would have to be true in w – which is absurd. Thus, the actual world and w must agree on all contingent propositions.

(5) It follows immediately that w is the actual world and, thus, that A has an explanation in the actual world.

(6) All explanations are either personal explanations, that rely on an individual's agency, or scientific explanations.

(7) Proposition A cannot be given a scientific explanation, because any relevant scientific laws are themselves contained as conjuncts in the *explanandum*, and laws are not self-explaining.[3]

(8) Therefore, the only explanation A can have is that it is made true by the free agency of an individual. This individual must exist necessarily; for a contingent being cannot bring about its own existence.

Our main interest is not in the philosophy of science and religion steps of this argument, (6)–(8), but rather the logical and metaphysical assumptions underlying steps (1)–(5). In the following section, we shall prove that if the conjunction A of step (1) is indeed a well-defined proposition, as GP suppose, then Necessarily (not E(A)). Thus GP's application of W-PSR to A in step (2) is inconsistent, rendering their argument unsound. Furthermore, while GP might be able to avoid this inconsistency in their premises by replacing A by some other contingently true proposition that logically entails all contingent truths, we shall argue that no such replacement can make the argument sound. We shall then present a modification of GP's argument with consistent premises that avoids the use of a proposition logically entailing all contingent truths. However, we shall argue that the substitution instances of W-PSR used in this modified argument are unwarranted, and, hence, that W-PSR itself lacks the intuitive support GP take it to have. Thus the modified argument is also unpersuasive.

GP's argument unsound

Let the relation $p < q$ hold between propositions p and q iff $q = \mathrm{E}(p)$ or q is a non-trivial conjunction of which p is a conjunct. Let $\subset$ denote the transitive closure of the relation $<$. For convenience, we say p is a proper sub-proposition of q if $p \subset q$. If $p \subset q$ or $p = q$, we write $p \subseteq q$, and say p is a sub-proposition of q. Now let C be the collection of all contingently true propositions. GP suppose that A = (the conjunction of all $p \in C$) is a well-defined proposition. We claim that this commits them to the well-definedness of the conjunction A* = (the conjunction of all p such that $p \in C$ and not($p \subset p$)). Note that this latter conjunction is non-trivial, because it contains, e.g., the propositions 'Pittsburgh is a city' and 'Father Christmas does not exist'. Note also that A* $\subset$ A. GP might claim that C* = {$p : p \in C$

and not $(p \subset p)$} is a class, and, therefore, the proposition A* does not exist. But, if C* is a class, so is C, and the proposition A would not exist for the same reason. Perhaps GP could argue, however, that C* is not even a class, because C* can only be constructed by applying the axiom of separation to the class C, yet the axiom of separation fails for classes. But C is also obtained by applying the axiom of separation to the class of *all* propositions, and so, again, GP would have to question the existence of A itself.

Assuming, then, that A* is a well-defined proposition, we shall now prove that Necessarily(not E(A)). Suppose, for *reductio ad absurdum*, that there is a possible world in which E(A) is true. By steps (3)–(5) of GP's argument, E(A) is actually true. But because $A \rightarrow A^*$ is a tautology, E(A*) must also be true. Since E(A*) entails A*, and A* is not true necessarily, E(A*) can only be contingently true. Thus, $E(A^*) \in C$. Moreover, E(A*) must be a proper sub-proposition of itself. For, if $not(E(A^*) \subset E(A^*))$ were true, E(A*) would have to be a conjunct of A*, and (hence) $E(A^*) \subseteq A^*$. But plainly $A^* \subset E(A^*)$; so, by transitivity, $E(A^*) \subset E(A^*)$. Thus, in any case, $E(A^*) \subset E(A^*)$. Now, because the only proper sub-propositions of E(A*) are the sub-propositions of A*, $E(A^*) \subseteq A^*$. Equality cannot hold, because A* is a non-trivial conjunction and E(A*) is not. So $E(A^*) \subset A^*$. But the only way this can be is if E(A*) is a (not necessarily proper) sub-proposition of one of the conjuncts of A*. So A* itself is a proper sub-proposition of one of the conjuncts of A*, i.e. there exists a $p \in C$ such that $not(p \subset p)$, and $A^* \subset p$. But if p is a conjunct of A* and $A^* \subset p$, transitivity requires that $p \subset p$–contradiction.

We have thus shown that if A is a well-defined proposition, Necessarily(not E(A)). Therefore, the instance of W-PSR that GP need in step (2) of their argument is false. Could GP remove this inconsistency in their premises by applying W-PSR to some other proposition A? Notice, after all, that they need not require A to be the conjunction of all contingent truths. All stage (4) of their argument requires of A is that it logically entail all contingent truths. While GP's attempt at constructing such an A fails, we have not (and do not) rule out the possibility of an alternative construction.[4] However, we shall now argue that the application of W-PSR to any such A would be entirely unwarranted.

First, consider an analogy. Imagine Ben, who is relatively ignorant of chemistry, being given a vessel and told, truthfully, that it holds one mole of hydrogen. Imagine Ben is also told that a mole is just some fixed number of molecules, although he is not told what this number is. Consider the contingent proposition $p = $ 'this vessel contains an even number of molecules'. Although Ben is entitled to assert 'for all I know, p' (an epistemic claim), he is *not* free to exercise his modal imagination and assert that Possibly(p). This is because, given that the vessel holds one mole of hydrogen, Possibly(p) is true iff Avogadro's number is even,[5] and so whether Possibly(p) is true is entirely determined by a fact about the value of Avogadro's number that Ben simply does not know. Modal intuitions and thought experiments can offer him no help in deciding whether Possibly(p) is true. Indeed,

no intuition could possibly have any authority over Ben to induce him to either assert Possibly(p) *or* Possibly(not p).

The same goes for any attempt to apply W-PSR in step (2) of GP's argument to some other contingently true proposition A that logically entails all contingent truths. For such an A, we know that either Necessarily(A → E(A)) or Necessarily(A → not E(A)); that is, A either necessarily entails its own explainability or necessarily entails its own unexplainability. But we also do *not* know which of these entailments holds; for, otherwise, GP's argument for E(A) would either be unnecessary or unsound! We are thus in precisely the same situation Ben was in with respect to his ignorance of the value of Avogadro's number. Whether or not E(A) is possible is fully determined by something we do not know: this time, whether Necessarily(A → E(A)) or Necessarily(A → not E(A)). Again, we are not free simply to exercise our modal imaginations, for they give us no authority to either assert Possibly(E(a)) *or* Possibly(not E(A)). Since there can be no warrant for endorsing even a *consistent* application of W-PSR to a contingent proposition A that logically entails all contingent truths, we conclude that the soundness of GP's argument cannot be restored.

A modified cosmological argument

We now need to consider a modified cosmological argument that shares much in common with GP's while avoiding any application of W-PSR to a contingent proposition that logically entails all contingent truths. The modified argument begins with the following simple proof that W-PSR entails PSR communicated to us by Pruss and used recently by Gale to justify PSR.[6]

(1) Assume W-PSR is true and PSR is false. Then, for some contingently true p, p and not E(p).

(2) The proposition (p and not E(p)) is also contingently true. Applying W-PSR, we have Possibly(E(p and not E(p))). Let w be a possible world in which E(p and not E(p)).

(3) Using the principle that for all A and B, E(A and B) entails E(A), E(p) is true in w.

(4) But also, since for all A and B, E(A and B) entails B, not E(p) is true in w.

(5) This is a contradiction. Therefore no such p exists, and W-PSR entails PSR.

We may now append the following to Pruss's argument, freely assuming PSR:

(6) Let S be the conjunction of all true scientific laws. (Perhaps S is just the conjunction of the postulates of quantum mechanics.)

(7) Proposition S is contingently true. By PSR, E(S) is true.

(8) All explanations are either personal or scientific.

(9)   Proposition S cannot be given a scientific explanation (for the same reasons as before).

(10) The only explanation S can have is that it is made true by some agent's free action, which it would be absurd to suggest was a human agent's action. Consequently, S must be true by divine volition.[7]

Again, we are only interested in questioning steps (1)–(5) of this argument. For this, W-PSR needs to be examined more closely. GP claim correctly that many substitution instances of W-PSR enjoy great intuitive appeal. After all, we can imagine that there might have been some explanation for the occurrence of the Big Bang, or some joint explanation for the fact that Kevin has two cats and Rob only one, even if there are no such explanations in the actual world. But does the fact that Possibly(E(q)) is true for these and other instances of q *really* warrant the conclusion that Possibly(E(q)) is true when we make the unusual substitution $q =$ 'p and not E(p)'?[8] GP give insufficient reason for endorsing W-PSR in such generality.

In fact, W-PSR without restriction runs counter to other modal intuitions at least as deeply entrenched as those concerning possible explanations for facts about cosmology and cats. Consider events that are taken to be genuinely random, such as the flip of a coin that comes up heads, or a quantum measurement of an electron that returns a value of 'spin up'. Our intuitions suggest it is possible that the specific results of these experiments cannot be explained. Yet such an intuition contradicts PSR, and hence W-PSR (by Pruss's argument). We are thus left with two options. We could hold on to our original modal intuitions, which are entirely consistent. Alternatively, we could endorse W-PSR in full generality – in particular, extending it to cases utterly unlike those that motivated our endorsement of W-PSR in the first place. Since this second option forces us to give up other modal intuitions at least as firmly entrenched, there is simply no reason to take it.

To drive home the unreasonableness of asserting W-PSR without restriction, consider another analogy. Let the Weak Principle of Correct Belief be the claim that for any true proposition, it is possible that it be a correct belief of someone,

W-PCB   For all propositions p, p entails Possibly(CB(p)),

where CB(p) iff p is true and p is believed by somebody. Consider now the Principle of Correct Belief,

PCB   For all propositions p, p entails CB(p).

Based on a few run of the mill instances of W-PCB, it is tempting to infer that for *all* true propositions p, Possibly(CB(p)). But an argument identical to Pruss's (with 'CB' replacing 'E') shows that W-PCB entails the evidently *false* PCB. What has gone wrong is that this argument involves asserting an instance of W-PCB, viz. Possibly(CB(p and not CB(p)), going far beyond the scope of our original

intuitions. Gale makes identically the same mistake in offering Pruss's proof that W-PSR → PSR as a *justification* for PSR. Together with the fact that endorsing W-PSR requires us to abandon equally reasonable modal intuitions, we think that W-PSR must simply be denied.

Conclusion

GP suggest that the sceptic who initially accepts W-PSR, only to later revoke it once he realizes its theistic consequences, is simply being intellectually disingenuous. This is not right. The sceptic can quite correctly suggest that GP have applied W-PSR to cases too different from those that motivated him originally to assent to it. It would have been better had the sceptic originally qualified his endorsement of W-PSR with any necessary provisos. But, as with all illusions, the fact that one has been tricked shows only the ingenuity of the trickster, and not the reality of the illusion presented.[9]

Notes

1. Richard Gale and Alexander Pruss 'A new cosmological argument', *Religious Studies*, **35** (1999), 461–476.
2. Instead of 'E(p)', GP write 'There is a true proposition q such that "q explains p" is true', but this is of no consequence for what follows.
3. GP view explanation as non-agglomerative, that is, for a conjunction to have an explanation it does not suffice that each conjunct has an explanation. This blocks the objection to steps (6) and (7) that if each conjunct of A were explained by some other conjunct of A, A itself would be self-explaining without further ado.
4. But note that requiring the existence of a maximal consistent proposition is stronger than requiring the existence of a maximal consistent *set* of propositions. (For example, the propositional calculus, with infinitely many free generators, has maximal consistent sets but no maximal consistent proposition.)
5. We treat Avogadro's number as a rigid designator; i.e., we assume it has the same value in all possible worlds.
6. Richard Gale 'Why traditional cosmological arguments don't work: and a sketch of a new one that does', in Michael Peterson (ed.) *Debates in the Philosophy of Religion*, (Oxford: Blackwell, forthcoming). Since writing this paper, we have learned that Pruss's proof that W-PSR entails PSR was found independently by Graham Oppy. Oppy has used the entailment in his paper, 'On "a new cosmological argument"', *Religious Studies*, **36** (2000), 345–353, to argue, *modus tollens* (against Gale's *modus ponens*), that the sceptic should *reject* W-PSR. We agree with this conclusion (as will be seen below), but our arguments for rejecting W-PSR are somewhat different from Oppy's.
7. It no longer follows that the existence of this divine agent is necessary, but this is cold comfort to the sceptic.
8. The laws of quantum theory together with Bell's Theorem seem to provide an explanation for why certain correlations between quantum events occur and have no *causal* explanation, but this is a far cry from an explanation of why the correlations occur and have *no* explanation.
9. We have profited from discussions with John L. Bell, David M. Finkelstein, Richard Gale, Graham Oppy, Alexander Pruss, Nicholas Rescher, and Myron Silberstein.