EXISTENCE AND ENERGY BALANCE OF THE SOLAR DYNAMO

J.H.G.M. VAN GEFFEN
Laboratory for Space Research, Sorbonnelaan 2
3584 CA Utrecht, The Netherlands
Astronomical Institute, P.O. Box 80000
3508 TA Utrecht, The Netherlands

Abstract. The idea behind the use of ensemble averaging and the finite magnetic energy method of Van Geffen and Hoyng (1992) is briefly discussed. Applying this method to the solar dynamo shows that the turbulence — an essential ingredient of traditional mean field dynamo theory — poses grave problems: the turbulence makes the magnetic field so unstable that it becomes impossible to recognize any period.

1. Introduction

The Sun's large-scale magnetic field shows a cyclic behaviour with a period of about 22 years. The origin of this magnetic cycle is not well known. The principal processes involved are turbulent convection in a spherical shell (called the convection zone) directly underneath the solar surface and differential rotation in that shell. Both these velocity fields drag the magnetic field along: they stretch, twist and spread the field lines. The combination of these processes results in what is called the solar $\alpha\Omega$-dynamo.

Models of large-scale fields $\boldsymbol{B}$ usually employ the so-called dynamo equation for the mean magnetic field $<\boldsymbol{B}>$. Traditionally it is assumed that $<\boldsymbol{B}>$ is marginally stable. Such solutions, however, face two problems:

1. The solution is strictly periodic, which means that the dynamo has an infinite phase memory, even though the underlying mechanisms are of a stochastic nature (Hoyng, 1987; 1988). This means that we can expect at best quasi-periodic solutions, which is also what the solar dynamo exhibits: successive cycles vary both in length and in strength.
2. The mean magnetic energy $<\boldsymbol{BB}>$ appears to grow without bounds, which implies that the field $\boldsymbol{B}$ itself does not remain

F. Krause et al. (eds.), The Cosmic Dynamo, 19–23.

https://doi.org/10.1017/S0074180900173802 Published online by Cambridge University Press

finite (Parker, 1979, Sect. 17.6), and this is physically unacceptable.

The omission of a non-linear feedback of the magnetic field on the fluid motions is not the main reason for these two problems (though non-linearities are ultimately needed to determine the strength of the magnetic field, but that is a separate problem). Instead, they are consequences of an improper treatment of the effects of fluctuations in the turbulence. The origin of this problem is the following. The traditionally used averaging procedures, indicated by $<\cdot>$, do not satisfy the Reynolds rules (see e.g. Krause and Rädler, 1980, Ch. 2) exactly and/or results in fluctuating terms. To account for this an extra term — with the nature of a random forcing term — appears in the dynamo equation. Most authors assume that this extra term may be ignored, but there is no manageable expression for the random forcing term to check whether it can indeed be ignored (Hoyng, 1978).

If one applies *ensemble averaging* the Reynolds rules are fulfilled exactly and no terms are omitted in the dynamo equation, so that the effects of the (stochastic) fluctuations are properly accounted for. The ensemble is to be taken literally: an infinite amount of copy systems, each one with a different realization of the turbulence. Unfortunately, ensemble averaging means that the mean field $<\boldsymbol{B}>$ cannot be identified with an observable large-scale field in any of the copy systems; the magnetic field $\boldsymbol{B}$ in a given ensemble member must therefore be described by other means (Hoyng, 1988).

2. The finite magnetic energy method

With ensemble averaging as a starting point, the *finite magnetic energy method* is developed to describe the effects of fluctuations in the turbulence properly with a linear theory. The idea behind the finite magnetic energy method is that the field $\boldsymbol{B}$ in some ensemble member remains finite if and only if the mean magnetic energy $<\boldsymbol{BB}>$ is finite. In linear mean-field theory, namely, only exponential growth or decay is possible. This means that if $<\boldsymbol{BB}>$ goes to zero, then so does $\boldsymbol{B}$, contrary to the assumption that a dynamo is present. And if $<\boldsymbol{BB}>$ increases, the magnetic energy increases indefinitely, which is unphysical. Therefore $<\boldsymbol{BB}>$ must be marginally stable. The combination of parameters that renders $<\boldsymbol{BB}>$ marginally stable is then used to solve the dynamo equation. We then find that the mean field $<\boldsymbol{B}>$ is damped. The damping time of $<\boldsymbol{B}>$ is interpreted

https://doi.org/10.1017/S0074180900173802 Published online by Cambridge University Press

as the auto-correlation time of $\boldsymbol{B}$, i.e. it is a measure of the period stability of the dynamo.

The finite magnetic energy method is described and applied to realistic models of the solar dynamo by Van Geffen and Hoyng (1992; Paper I) and Van Geffen (1992a,b; Papers II and III, respectively). This contributed paper summarizes the results obtained so far. The method yields an energy balance for the mean energy density $<B^2>/8\pi$, in which appear the relative rates of production of mean magnetic energy by vorticity and by differential rotation, and the energy that escapes from the dynamo through the surface. Helicity does not produce mean energy; it only affects the distribution of energy over the various components of $<\boldsymbol{BB}>$. The energy is transported to the surface by turbulent diffusion. Resistivity can be neglected in the solar dynamo (Paper I). The mean energy of the dynamo is only marginally stable for a fairly large value of the turbulent diffusion coefficient β ($\beta = 10^{14}\ \mathrm{cm^2\,s^{-1}}$) in the bulk of the convection zone (Paper I). Since the method is linear, it does not provide the absolute magnitude of the magnetic field. But a tentative identification of the escaping energy flux with the flux needed to heat the corona yields an estimate of the r.m.s. field strengths at which the dynamo operates.

3. A solar convection zone dynamo

In Paper II the finite magnetic energy method is applied to a dynamo operating throughout the entire solar convection zone, using a constant turbulent diffusion coefficient β. The mean magnetic energy of such a convection zone dynamo is mainly generated by vorticity — i.e. by random field line stretching at small spatial scales — since it turns out that differential rotation produces only 2 to 10% of the total mean energy flux.

The r.m.s. field strength at which the dynamo operates is about 9 G at the surface and about 140 G at the base of the convection zone — see Fig. 1a. These values are too low to explain the observed field strength and orientation of active regions on the solar surface. Furthermore, it appears that the field itself is rapidly fluctuating: the damping time of the mean field $<\boldsymbol{B}>$ is only about 14 days, a fraction of the cycle period of 22 years. Hence, a convection zone dynamo shows no clear period and no well-defined large-scale field: it is merely a rapidly fluctuating small-scale field dynamo, unable to sustain the solar cycle.

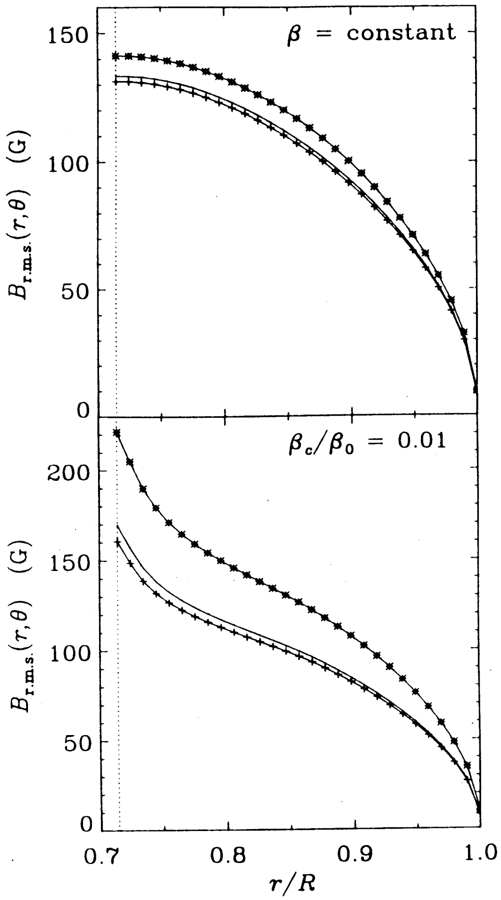

Figure 1
The r.m.s. *field strength as a function of radius at the pole (plus signs), at the equator (solid line) and at 40° latitude (asterisks), for* **a)** *a convection zone dynamo, and for* **b)** *a localized dynamo with a turbulent diffusion coefficient* β *which is at the base a factor of* 100 *smaller than in the bulk of the convection zone. [After Van Geffen (1992b).]*

4. A localized solar dynamo

It has been suggested that the magnetic field strengths required to explain the observed fluxes in active regions may be generated by a dynamo operating mainly in a thin boundary layer at the interface between the radiative core and the convection zone.

In Paper III the finite magnetic energy method is applied to such a localized solar dynamo. This is achieved by making the turbulent diffusion coefficient β decrease near the base of the convection zone. The results are as follows. Differential rotation produces now relatively more mean energy than in the convection dynamo, namely about 20%. As expected, the magnetic field is concentrated towards the base and the r.m.s. field strength increases there, whereas the r.m.s. surface value is unaffected — see Fig. 1b. But the r.m.s. field strength at the base of the convection zone has increased only to 230 to 450 G, which is still too low by a factor of 10 to 100 to explain the observed active region-phenomena. Furthermore, the damping time of the mean field is still very small: no more than about 20 days. Hence, the localized dynamo modelled in Paper III is also an aperiodic, small-scale field dynamo, incapable of sustaining the solar cycle.

5. Conclusion

From the results of the finite magnetic energy method, briefly summarized above, it appears that the solar cycle cannot be explained by conventional mean-field $\alpha\Omega$-dynamo models, regardless whether the dynamo processes mainly operate throughout the convection zone or in a boundary layer. The major difficulty with such $\alpha\Omega$-dynamos is that a *stochastic* mechanism (helical convection) is invoked to produce a *periodic* magnetic field. But the convection makes the field at the same time so unstable that fluctuations make it impossible to recognize any period. An explanation of the solar cycle may therefore require alternative theories in which traditional mean field $\alpha\Omega$-mechanisms play only a minor role, or in which differential rotation and helicity are spatially separated.

References

Hoyng, P., "Turbulent transport of magnetic fields, II. The role of fluctuations in kinematic theory," *Astron. Astrophys.* **171**, 357 (1987).

Hoyng, P., "Turbulent transport of magnetic fields, III. Stochastic excitation of global magnetic modes," *Astrophys. J.* **332**, 857 (1988).

Krause, F. and Rädler, K.-H., *Mean-Field Magnetohydrodynamics and Dynamo Theory*, Pergamon Press, Oxford (1980).

Parker, E.N., *Cosmical Magnetic Fields: their origin and their activity*, Clarendon Press, Oxford (1979).

Van Geffen, J.H.G.M., "Distribution of magnetic energy in $\alpha\Omega$-dynamos, II: a solar convection zone dynamo," *Geophys. & Astrophys. Fluid Dynamics*, in press (1992a) (=Paper II).

Van Geffen, J.H.G.M., "Distribution of magnetic energy in $\alpha\Omega$-dynamos, III: a localized solar dynamo," *Astron. Astrophys.*, submitted (1992b) (=Paper III).

Van Geffen, J.H.G.M. and Hoyng, P., "Distribution of magnetic energy in $\alpha\Omega$-dynamos, I: the method," *Geophys. & Astrophys. Fluid Dynamics*, in press (1992) (=Paper I).