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The Earth's rotation and laminar pipe flow

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A pipe flow facility with a length of 32 m and a diameter of 40 mm has been designed in which a laminar flow of water can be maintained for Reynolds numbers up to 60 000. Velocity measurements taken in this facility show an asymmetric velocity profile both in the vertical as well as horizontal direction with velocities that deviate strongly from the parabolic Hagen–Poiseuille profile. The cause of this asymmetry is traced back to the influence of the Earth's rotation. This is confirmed by means of a comparison of the experimental data with the results from a perturbation solution and from a numerical computation of the full nonlinear Navier–Stokes equations. The physical background of this unforeseen result lies in the fact that a Hagen–Poiseuille flow is governed by a force equilibrium and inertia forces are everywhere negligible. This implies that the Coriolis force can be balanced only by a viscous force. So even the small Coriolis force due to the Earth's rotation causes a large velocity distortion for a case such as ours where the kinematic viscosity is small.

1. Introduction

Every student of fluid mechanics is familiar with the parabolic Hagen–Poiseuille profile for the laminar flow in a cylindrical, straight tube as one of the few exact solutions of the Navier–Stokes equations. The simplicity of this flow geometry would perhaps lead one to expect that it is straightforward to reproduce this flow in the laboratory. As with most experimental verifications of theoretical results this is not always the case, especially if the experimental facility used differs from the facilities that have been employed in previous experiments.

The setting of the present study is the flow in a 32 m long cylindrical pipe with a diameter of 0.04 m that has been designed to study the transition to turbulence. The fluid is water. It is well known that a pipe flow can be kept laminar far beyond the lowest critical Reynolds number, i.e. $Re_{crit} \simeq 2000$, if flow disturbances are kept small. In our facility we managed to keep the flow laminar to a maximum Reynolds number of $Re_{max} \sim 60\,000$ where the Reynolds number is based on the pipe diameter and mean velocity. This value could be obtained as a result of careful construction and design in which all possible causes of flow disturbances such as pipe entrance conditions, temperature differences between the fluid and the outside conditions, wall roughness, etc. were eliminated as much as possible. Despite these precautions, the observed velocities deviated strongly from the theoretical parabolic profile. The cause turned out to be the Earth's rotation. The objective of this paper is to draw attention to this perhaps unexpected phenomenon and to present the experimental observations

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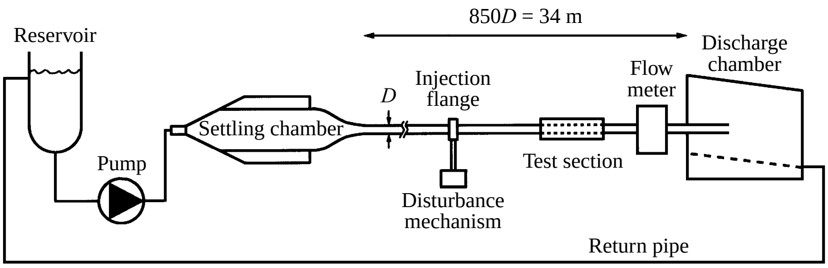

FIGURE 1. Schematic overview of the facility used for the observations of laminar pipe flow. The inner diameter of the pipe is 40 mm; the disturbance mechanism is used for transition experiments and has not been employed for the experiments discussed here. Further details of this experimental facility are explained in the text.

that we have obtained in our pipe facility, in comparison with some theory to explain the cause of this result.

The organization of the paper is as follows. In the next section the experimental facility is discussed in some detail. The following section is devoted to the governing equations and their solution and to a discussion of the non-dimensional parameters that play a role in our problem. Next we discuss the experimental data and their comparison with some theoretical and numerical results. In the final section we present some conclusions.

2. Experimental facility

The pipe-flow facility that was used for the experiments was designed especially for the purpose of studying transitional flow. This objective required that laminar flow, preferably fully developed, was to be maintained to as high a Reynolds number as possible. In this section we give a brief description of this flow facility and for further details we refer to Draad (1996).

A schematic overview of the pipe-flow facility is given in figure 1. The main part of the facility consists of a smooth-walled pipe constructed out of Plexiglas with an inner diameter of 40 mm, and a total length of 32 m. Christianson & Lemmon (1965) give the following expression for the length after which the centreline velocity deviates by less than 1% from the parabolic form:

$$\frac{L_{99\%}}{D} = 0.056\,Re.$$

Based on this expression we estimate that a fully developed, parabolic pipe flow can be attained in our facility only for Reynolds numbers less than 14 300. Our experiments showed that a (partly developed) laminar flow could be maintained up to $Re = 60\,000$. Such a high value could be obtained by minimizing all sources of flow disturbances. For this, much attention was given to the construction of the pipe which was made out of sections with a length of 2 m connected to each other by specially designed couplings. The pipe sections were centred based on their inner diameter and design of the couplings limited any misalignment of sections to less than 0.02 mm. To eliminate flow disturbances entering the pipe, a settling chamber was constructed in which swirl was suppressed with the help of honeycombs and other flow disturbances with a series of screens. A smooth contraction of area ratio 9 was used to damp the disturbances further. Finally, by careful insulation of the pipe and settling chamber and also by thermostatically matching the water temperature to that of the ambient flow within 0.2 °C, any influence of convection currents was minimized.

Along the pipe at any location the velocity profile can be observed by means of laser Doppler velocimetry (LDV). This is done by replacing a pipe section with a specially designed measuring section. The measuring section consists of a rectangular box positioned around the pipe and filled with water under the same pressure as in the pipe. In this box the pipe wall is replaced with a 0.19 mm thin fluorocarbon film (Teflon FEP 750A by Du Pont) with an index of refraction of 1.35, which is close to that of water (1.33). In this way refraction effects due to the curved pipe wall could be eliminated. The two-dimensional LDV equipment consists of a Dantec probe connected by optical fibres to an argon-ion laser. The probe is operated in the backscatter mode. The Doppler bursts are analysed by two Dantec Burst Spectrum Analyzers (BSA) and transformed into a two-dimensional velocity signal. We have not applied any correction to these data because we felt this not to be necessary in this particular case where only mean velocities are observed in laminar flow. To obtain an adequate data rate the flow has been seeded with latex particles which have a diameter of approximately 1 μm.

At the downstream end of the pipe, a magnetic inductive flow meter (Krohne-Altometer) is used to monitor the flow rate. The pipe ends in a discharge chamber from which the water returns into a reservoir. By means of a pump the water is then circulated back into the pipe.

In view of all the precautions that were taken in the design of this pipe facility, nothing prepared us for the observational results, given in figure 2, which show a clear deviation from the well-known parabolic profile found in Hagen–Poiseuille flow. The explanation and background of these deviations will be discussed in the following sections.

3. Governing equations and their solution

3.1. *Equations of motion and non-dimensional parameters*

The Navier–Stokes equations for the incompressible velocity field u_i in a fluid with density ρ and kinematic viscosity ν in a rotating frame of reference (see e.g. Batchelor 1967, pp. 140, 555) read

$$\frac{\mathrm{D}u_i}{\mathrm{D}t} = f_i - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu\frac{\partial^2 u_i}{\partial x_j^2}$$

in which the first term on the right-hand side denotes the Coriolis force per unit mass f_i and in which the centrifugal force has been absorbed in the pressure p. The z-coordinate is chosen along the pipe axis, the x-direction perpendicular to the pipe and the Earth's surface and the y-direction perpendicular both to the x- and z-directions as illustrated in figure 3.

For the case of a uni-directional flow vector $[0, 0, W(r)]$ the components of the Coriolis forces become

$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = -2\Omega \begin{pmatrix} \sin\alpha_L \\ \cos\alpha_L \sin\alpha_N \\ \cos\alpha_L \cos\alpha_N \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ W(r) \end{pmatrix} = -2\Omega W(r) \begin{pmatrix} \cos\alpha_L \sin\alpha_N \\ -\sin\alpha_L \\ 0 \end{pmatrix}, \tag{3.1}$$

where Ω is the angular velocity of the Earth's rotation, the angle α_L is the latitude and α_N the angle of the pipe axis with the direction of true north. From this equation

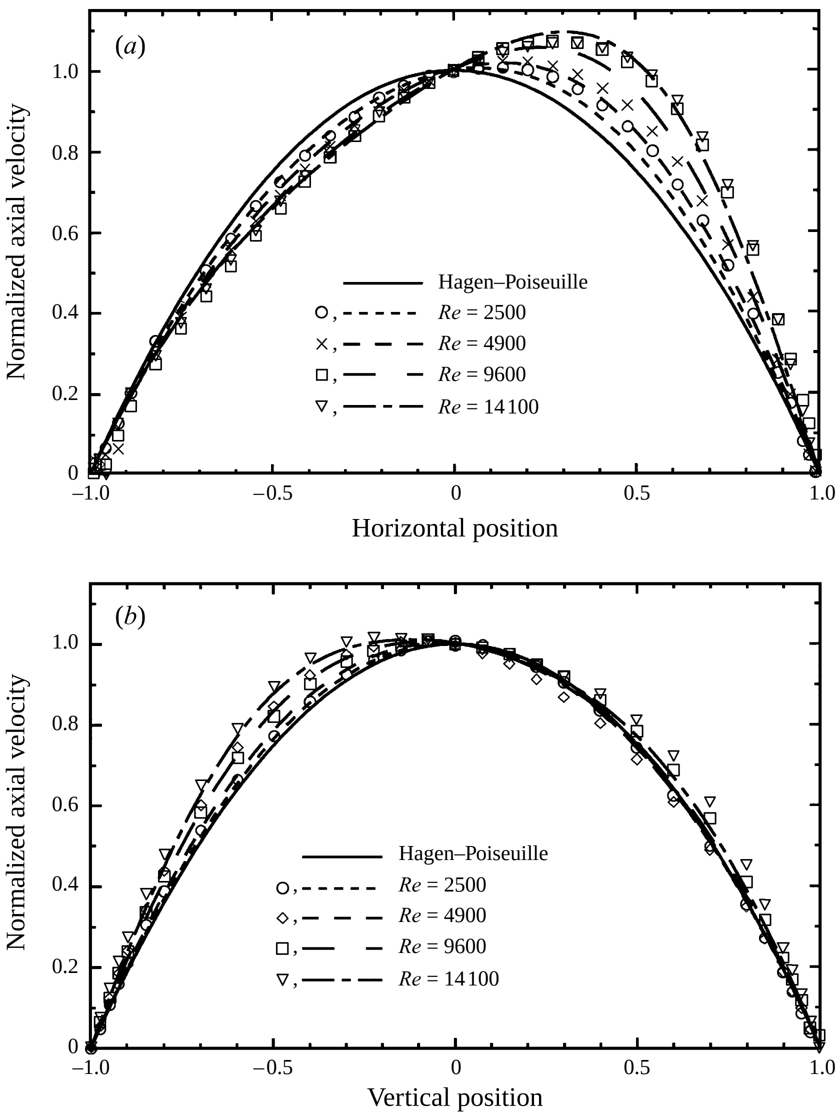

FIGURE 2. Observation (denoted by the symbols) of the velocity profile (normalized with the maximum value) in a laminar pipe flow at various Reynolds number in (*a*) a horizontal and (*b*) a vertical direction; the lines denote the results from computations with the full nonlinear Navier–Stokes equation discussed in § 4.

it follows that the Coriolis force can be avoided only when the pipe is aligned with the rotation axis of the Earth which in our case with a pipe of 32 m length and at a latitude of $\alpha_L = 52°$ north is clearly not practical. In our case $\alpha_N = 22°$. The magnitude of the Coriolis force per unit mass is usually written as $|f_i| = -2\Omega W(r) \sin\alpha$ where α is the angle between the pipe axis and the rotation axis of the Earth. For our pipe the angle α follows from angles α_L and α_N which gives $\alpha = 55°$.

With $\Omega = 2\pi/(24 \times 3600) = 7.272 \times 10^{-5}\,\mathrm{s}^{-1}$ we find for the Coriolis forces: $f_x/W(r) = -3.3 \times 10^{-5}\,\mathrm{s}^{-1}$, $f_y/W(r) = 1.1 \times 10^{-4}\,\mathrm{s}^{-1}$ and $f_z = 0\ \mathrm{m\ s}^{-2}$. We thus find that the Coriolis force has a positive component in the y-direction, i.e. to the right

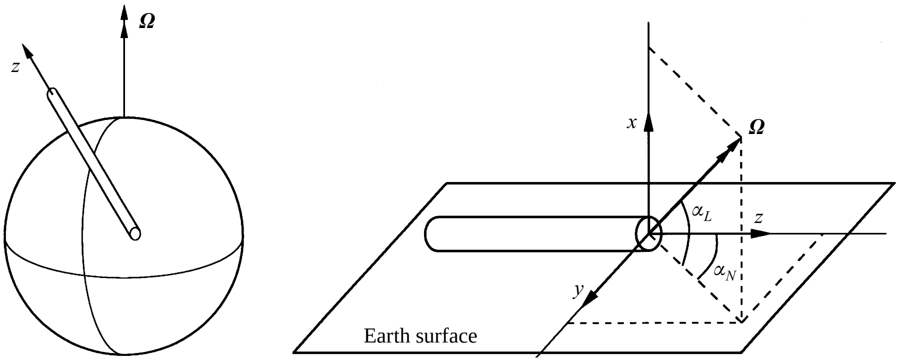

FIGURE 3. Cartesian coordinate system with the orientation of the rotation vector of the Earth.

looking downstream. Furthermore, we find a negative component in the x-direction with a magnitude of 29% of the y-component.

To get a first-order estimate of the effect of the Coriolis force the standard procedure is to compare this term with the other terms in the equations in terms of a non-dimensional number. Let us introduce the notation $\overline{W}$ for the mean velocity over the pipe cross-section. The ratio of the inertia to the Coriolis force then becomes

$$Ro = \frac{\overline{W}}{2\Omega \sin \alpha D} \tag{3.2}$$

which is known as the Rossby number. For our pipe with $\overline{W} = 0.25$ m s^{-1} and $D = 40$ mm the Rossby number is $Ro \simeq 52\,000$ and one would expect that rotation effects are negligible for this flow.

However, the Rossby number is not the correct parameter to estimate the influence of rotation because in a fully developed unidirectional pipe flow inertia does not play a role because the flow everywhere satisfies a force equilibrium, i.e. the pressure gradient is balanced by the viscous force. Therefore, the appropriate parameter to estimate the influence of rotation is the ratio of the viscous to the Coriolis force. This leads to the following dimensionless parameter:

$$Ek = \frac{\nu}{2\Omega D^2 \sin \alpha} \tag{3.3}$$

which is known as the Ekman number. For our case it follows that $Ek \simeq 5.2$ which suggests that the presence of the Coriolis force, which in magnitude is almost 20% of the viscous force, may significantly affect the parabolic profile.

3.2. *Perturbation analysis*

Given the discussion of the previous section the next step is to perform some analysis by assuming that the deviation from the parabolic profile due to the Coriolis force is small. Let us at the same time consider a cylindrical coordinate system with the axial direction z along the pipe axis and the tangential coordinate θ measured from the x-axis. In this coordinate system the velocity components are given by $[u_r(r,\theta), u_\theta(r,\theta), W(r) + w(r,\theta)]$ where we have assumed that the flow is stationary and fully developed, i.e. independent of time and the axial coordinate. In this notation upper-case letters stand for the zero-order parabolic Hagen–Poiseuille flow and lower-case letters for the perturbations, with u_r/W, u_θ/W and $w/W \ll 1$. Substituting these velocity components in the full equations, i.e. the continuity and Navier–Stokes equations, and truncating after the linear term, we find the following set of equations for the perturbation velocities:

$$\frac{1}{r}\frac{\partial r u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} = 0, \tag{3.4a}$$

$$0 = f_r - \frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right], \tag{3.4b}$$

$$0 = f_\theta - \frac{1}{\rho}\frac{1}{r}\frac{\partial p}{\partial \theta} + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2}\right], \tag{3.4c}$$

$$u_r\frac{\mathrm{d}W(r)}{\mathrm{d}r} = \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right], \tag{3.4d}$$

where

$$f_r = -2\Omega W(r)\left(\cos\alpha_L \sin\alpha_N \cos\theta - \sin\alpha_L \sin\theta\right),$$
$$f_\theta = -2\Omega W(r)\left(-\cos\alpha_L \sin\alpha_N \sin\theta - \sin\alpha_L \cos\theta\right).$$

These equations show clearly that the effect of the Coriolis force in the r- and θ-direction must be balanced by a viscous force. The resulting secondary flow in the radial direction then will influence the axial velocity through a balance between the inertial and the viscous force.

The set of equations (3.4c) has been analysed by Benton (1956) who shows that the maximum value of the velocity in the (r,θ)-plane is given by the expression

$$\left(u_r{}^2 + u_\theta{}^2\right)^{1/2}_{max} = \frac{\overline{W}}{192Ek} \tag{3.5}$$

which for our case, where $Ek = 5.23$, is equal to $0.000\,996\ \overline{W}$. The secondary velocity is found to be very weak indeed.

Although this secondary flow seems very weak, it can nevertheless lead to appreciable modification of the axial profile. The solution for the axial perturbation velocity given by Benton, with $\overline{d} \equiv 2r/D$, reads

$$\frac{w}{\overline{W}} = \frac{Re}{2^9 3^2 Ek}\overline{d}\left(1-\overline{d}^2\right)\left(3-3\overline{d}^2+\overline{d}^4\right)\sin\theta. \tag{3.6}$$

Since Ek is constant, $w/\overline{W}$ increases linearly with Re and this implies that the disturbance of the axial velocity profile due to the Coriolis force scales as $w \sim \overline{W}^2$. It is precisely this quadratic increase of the axial disturbance velocity which is the source of the large deformation of the axial velocity profile at high Reynolds numbers.

To give an example we find, using the value of Ek valid for our facility, that for a Reynolds number of only $Re = 2\,000$ the axial disturbance velocity maximum is 7% of the bulk velocity and at $Re = 5\,000$ it is already 18%. In other words, for $Re = 5\,000$ the linear analysis is no longer valid. For $Re = 15\,000$ the axial disturbance velocity is 54% of the bulk velocity. Moreover, the velocity profile is nowhere near parabolic. Based on these results we may conclude that the presence of Coriolis force due to the Earth's rotation is the explanation for the observed large distortions of the parabolic velocity profile.

The question arises why in previous experiments on laminar pipe flow this effect has not been widely recognized. The majority of previous experiments on laminar pipe flow at high Reynolds numbers have been carried out with air as fluid rather

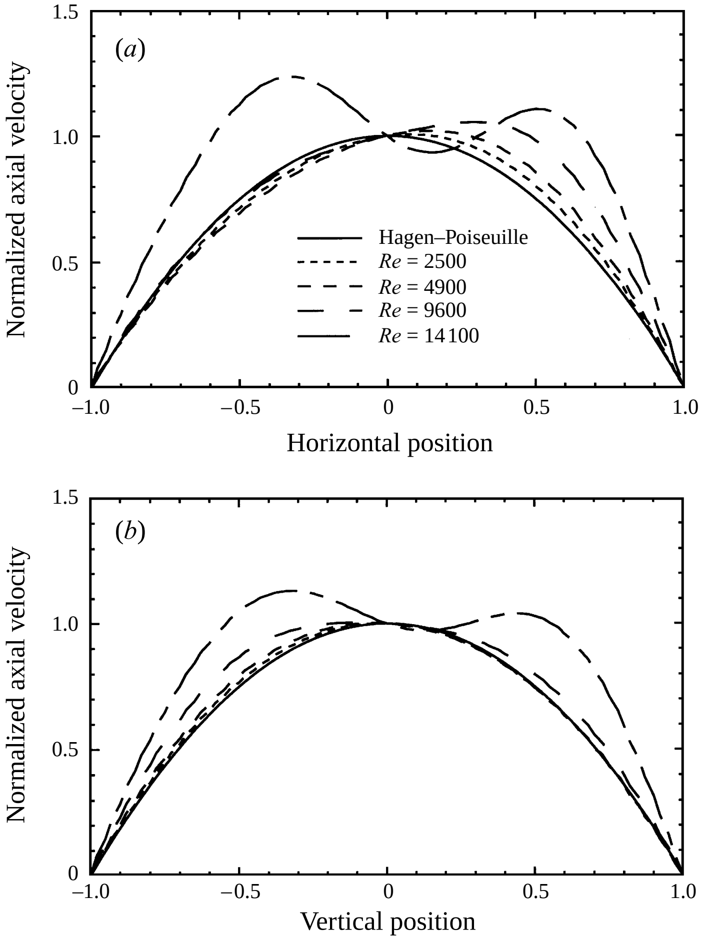

FIGURE 4. Calculated axial disturbance velocity profiles according to Berman & Mockros (1984) in (*a*) the horizontal and (*b*) the vertical direction for various Reynolds numbers.

than water. It follows directly from (3.3) that for the same rotation velocity and pipe diameter, the Ekman number in air is larger by a factor of ~ 15 than water. As the disturbance is inversely proportional to the Ekman number, the influence of the Coriolis forces becomes negligible for most such pipe flows. In other experiments, where water has been used, the pipe diameters are usually smaller. As Ek scales according to ν/D^2, the effect of the Coriolis force is much smaller for small pipe diameters. Also, the length of the pipe was in general much too short to allow fully developed flow to occur. Then the Rossby number becomes more important and we have shown above that this number is always very large.

We have seen above that for Reynolds numbers beyond $Re \simeq 5000$ the perturbation becomes larger than 10% and consequently linear analysis is no longer valid. Berman & Mockros (1984) have extended the solution to a third-order perturbation analysis. In figure 4 we show the calculated axial velocity profiles in both the horizontal and vertical directions using Berman & Mockros' solution. In comparison with the observations given in figure 2 we find that curves for $Re \leqslant 9\,600$ are very similar to the measurements. However, the curves for $Re = 14\,100$ show a dip close to the centreline that is not observed in the measurements. So, at these Reynolds numbers, even a third-order perturbation analysis is insufficient to correctly predict the axial velocity profiles of our pipe flow.

4. Numerical computation

We have seen in the previous section that at high Reynolds numbers the disturbance velocities become so large that linear theory and even third-order analysis can no longer be used to compute the perturbations. Therefore, for a reliable comparison with the measured velocity profiles, numerical computations of the full equations of motion are necessary. For these we have made use of a numerical code in which the full Navier–Stokes equation is solved. The code is based on a second-order finite volume discretization of the equations formulated in cylindrical coordinates and has been developed for numerical simulation of turbulent pipe flow. Further details are described by Eggels *et al.* (1994). In this case where a fully developed laminar flow is considered, only a minimum number of grid points, in our case three, is needed in the axial direction. To resolve the small secondary flow a fine grid size is needed and in our computations we have used 60 grid points in the radial direction and 120 points in the tangential direction. The grid size in the radial direction was non-uniform with the largest size near the centreline. The solution was found by integration in time from a given initial profile until a fully developed steady state is reached. A calculation takes typically 80–160 CPU hours on a Convex–3840.

Before performing calculations at high Reynolds numbers, where the parabolic flow becomes strongly distorted, the numerical code was first checked against the linear solution according to Benton (1956). For $Re = 1\,500$ and $Ek = 15.4$ the maximum axial perturbation velocity according to the linear analysis is 2% of the bulk velocity. The differences between the calculations using the full equation and the linear solution are approximately 2.5%. Thus, the inaccuracy of the numerical computation is 0.05% of the bulk velocity, which seems acceptable for our purpose. Now that we have established the suitability of the numerical code, we can compare our measurements of the axial velocity profiles with those predicted by the numerical computations.

In figure 2 the results obtained from the computations are shown as dotted and dashed lines. Although quantitative agreement between observation and experiments is far from perfect, the qualitative agreement is excellent. It is also clear that both for the observations and for the computations the asymmetry increases with the Reynolds number as predicted by the theory of the previous section. For both the horizontal and vertical profiles the shapes of the measured profiles are in good agreement with those obtained from the calculations. This is perhaps more convincingly shown in figure 5 where the difference between the measured and calculated results and the parabolic profile is illustrated. Experiment and computations seem even to agree on a subtle detail shown in this figure, where at positive values of the vertical position the perturbation is found to be negative for small Reynolds numbers but positive for large Reynolds numbers. We note that such change from a negative to a positive perturbation value is not predicted by the linear theory. The higher-order perturbation analysis of Berman & Mockros (1984) does predict such behaviour but the deviation from the parabolic profile is much too large for higher Reynolds numbers as can be seen from the 'double-maxima' curve for $Re = 14\,100$ in figure 4 when compared to the measured velocity profiles.

A possible source of the deviations between the calculations and the measurements may have been the fact that the air temperature exceeded the water temperature

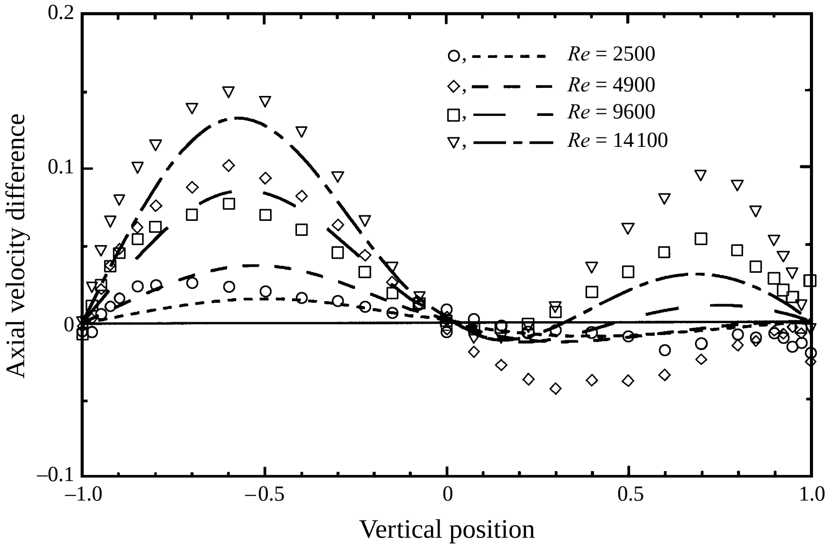

FIGURE 5. The difference between the computed or observed velocity profiles and the parabolic profile in the vertical direction for various Reynolds numbers as a function of the radial coordinate; computations are denoted by lines and observations by symbols. All results are normalized with the centreline velocity.

by 0.2 °C. Even very weak convection cells as analysed by Morton (1959) or swirl could displace or rotate the secondary flow induced by the Coriolis force. Keeping this sensitivity in mind, we feel that the comparison between the measured and the calculated axial velocity profiles presents strong proof that the asymmetry in the velocity profiles is for the largest part due to the rotation of the Earth.

As some additional information we show in figure 6(*a*) the streamfunction of the secondary circulation for $Re = 26\,500$. It can be seen that the cells are not symmetrical as predicted by linear perturbation theory, but somewhat shifted to the right, i.e. in the direction of the Coriolis force. The corresponding axial velocity contours are shown in figure 6(*b*). The axial velocity profile is clearly strongly asymmetric and the isolines are not circular in shape. Again we note that the linear perturbation solution of Benton (1956) and even the higher-order perturbation analysis of Berman & Mockros (1984) are not able to predict the axial velocity contours shown in figure 6(*b*) since the perturbation terms are of the same order of magnitude as the base flow, in other words the perturbation analyses are not valid under these flow conditions. The higher-order perturbation solution even predicts negative axial velocities.

The effect of the Coriolis force is not limited to a distortion of the axial velocity profile but it also influences the frictional drag. The (Moody-) friction factor f_M is defined as

$$f_M = \frac{D\,\Delta p}{\frac{1}{2}\rho \overline{W}^2 L}, \tag{4.1}$$

where Δp is the measured pressure drop over a pipe length L. In our experiments pressure measurements were taken at a distance of 28 m and 30.5 m from the pipe entrance from which f_M could be computed with the help of (4.1). For the Hagen–Poiseuille flow the relationship between f_M and Re is $f_M = 64/Re$. We should note

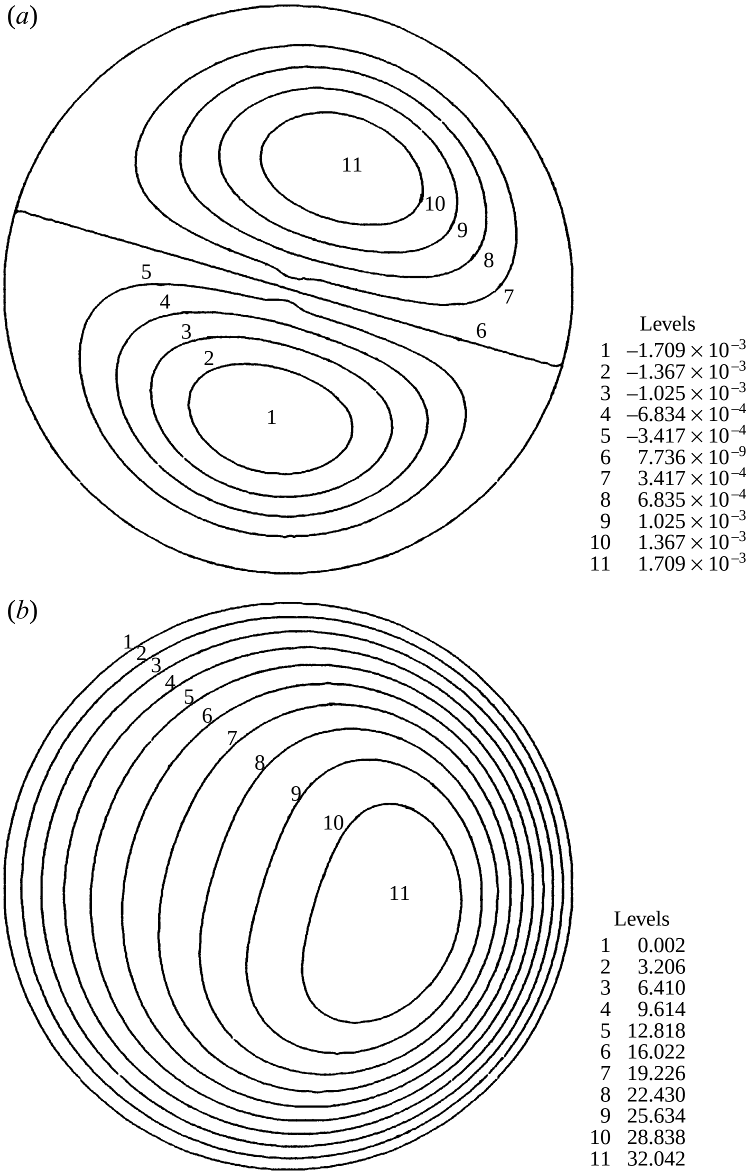

FIGURE 6. (*a*) Streamfunction of the secondary circulation and (*b*) axial velocity contours at $Re = 26\,500$.

here that this equation is valid only for fully developed pipe flow. When the entrance length before the pressure measurements is not long enough, a higher value of f_M must be expected.

The observations for f_M are plotted in figure 7 together with the results of our numerical computations and the result for Hagen–Poiseuille flow. It is seen that at high Reynolds numbers, say $Re > 10\,000$, both the observations and computations deviate from the line $f_M = 64/Re$. At still larger Reynolds numbers, i.e. at $Re > 14\,300$, the flow in the pipe facility can no longer be considered as fully developed and beyond this value the observations are indeed seen to deviate from the computations.

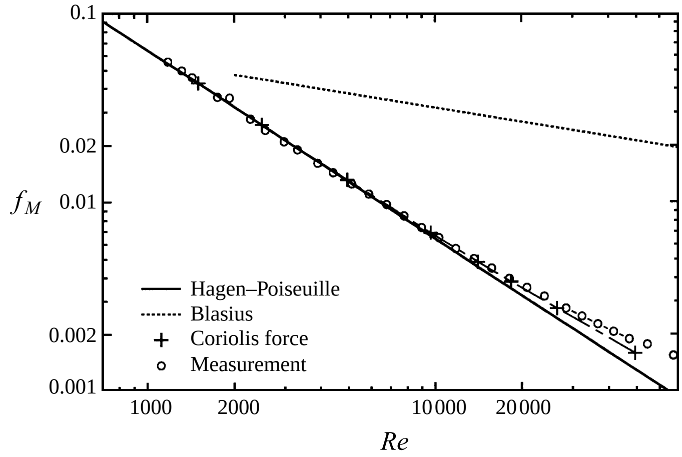

FIGURE 7. Moody-diagram, friction factor f_M vs. Reynolds number Re: results are given for the experimental setup indicated as measurement and for the numerical results including the Coriolis force indicated as Coriolis force. The lines indicated by Hagen–Poiseuille and Blasius are the friction factors for fully developed axisymmetric pipe flow for the case of laminar and turbulent flow respectively.

5. Conclusions

We have shown that the Coriolis force due to the rotation of the Earth can strongly distort the laminar velocity profile in a long pipe-flow facility which uses water as fluid. The effect is particularly pronounced at high Reynolds numbers, say above $Re = 5000$. The axial velocity profile becomes strongly asymmetric and the asymmetry increases with Re.

A linear perturbation analysis first explored by Benton (1956) and a third-order perturbation analysis by Berman & Mockros (1984) are able to predict the distortion of the velocity profile for low Reynolds numbers. However, at high Reynolds numbers, say $Re > 10\,000$, the results of these perturbation theories are no longer adequate and a numerical computation of the full nonlinear Navier–Stokes equation is needed to obtain good agreement with the experiments. Apart from a distortion of the axial velocity profile, the Coriolis force also influences the friction factor. Also here excellent agreement is found between the measurements and the numerical computations up to a value of the Reynolds number $Re \simeq 14\,300$. Beyond this value the flow in our facility can no longer be considered as fully developed.

The main point of this paper is that when studying laminar pipe flows at high Reynolds numbers, in particular when water is used as a working fluid, one should be aware of a possible effect of the Coriolis force due to the Earth's rotation. This perhaps surprising result, that a small influence such as the Earth's rotation can have such a large effect, is due to the fact that the flow in a long pipe is governed by a force equilibrium and inertia does not play a role. As kinematic viscosity is in general small it is clear that even to balance such a small force as the Coriolis force a large deviation of the velocity profile is needed. Put in other words the effect of the Coriolis force in a fully developed pipe flow scales with the Ekman number which depends on the kinematic viscosity. A consequence of this explanation is that the distortion is larger, at given Re, when the kinematic viscosity is smaller. For instance the effect is larger for water than for air.

It is perhaps superfluous to mention that, in practice, pipe flows at high Reynolds numbers, i.e. $Re > 2\,000$, are in general turbulent. In case of a fully developed turbulent pipe flow, the ν in (3.3) should be replaced by the effective (eddy) viscosity which is by definition much larger than the kinematic viscosity of the fluid. Therefore the effect reported in this study will not appear. Another way of looking at this result is by realizing that in a turbulent flow the eddy viscosity K can be estimated as $K \simeq u\ell$ where u and ℓ are a characteristic velocity and length scale, respectively. If we substitute this estimate for K into (3.3) we obtain (3.2) in terms of u and ℓ. This means that in a turbulent flow the Rossby number is the proper parameter rather than the Ekman number and this is consistent with the fact that turbulence, in particular with respect to the large scales of motion, is dominated by inertia. In all turbulent engineering flows the Rossby number based on the Earth's rotation is very large so that the effect of the Earth's rotation is negligible.

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