Robotica (2009) volume 27, pp. 225–233.
doi:10.1017/S026357470800461X Printed in the United Kingdom

PID control of robot manipulators equipped with brushless DC motors

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(Received in Final Form: April 3, 2008. First published online: April 29, 2008)

SUMMARY
This paper is concerned with PID control of rigid robots equipped with brushless DC (BLDC) motors when the electric dynamics of these actuators is taken into account. We show that an adaptive PID controller yields global stability and global convergence to the desired link positions. Moreover, we also show that virtually the PID part of the controller suffices to achieve the reported global results. We present a theoretical justification for the torque control strategy, commonly used in practice to control BLDC motors. Our controller does not require the exact knowledge of neither robot nor actuator parameters.

KEYWORDS: Robot control; Brushless DC motors; PID control; Torque control.

1. Introduction

It is widely recognized at present that use of brushless DC (BLDC) motors as actuators in robotics presents a number of advantages with respect to use of brushed DC motors.[1−4] However, it is also known that control of BLDC motors is more complicated because of the nonlinear and multivariable nature of the *corresponding* model.

On the other hand, the traditional approach in robot control theory has been the design of controllers assuming that, no matter the particular motor used, the electric dynamics of actuators can be neglected. This implies that torque is assumed to be the control signal when real life tells that voltage is the actual control signal and torque is to be generated by the electric dynamics of actuators. Further, some studies have shown that neglecting the electric dynamics of actuators may result in performance degradation of the closed-loop system.[5,6] This observation has motivated lots of work on robot control taking into account the actuator electric dynamics when different electric motors are used.[1−4,7−13]

In particular, several control schemes have been presented until now for rigid robots actuated by BLDC motors when their electric dynamics is taken into account.[1−4] Hemati *et al.*[1] introduced a robust nonlinear controller which, however, requires measurement of rotor acceleration. Further, *performance* of the controller deteriorates significantly when accurate feedback measurements are unavailable since the controller is based on feedback linearization. On the other hand, Dawson *et al.*[2,3] presented adaptive nonlinear backstepping-based controllers which are robust to parameter uncertainties while ensuring global stability. However, implementation of these controllers requires rather complicated computations which, as stressed by Ortega *et al.*[13] (pp. 395, 403), increase sensibility to numerical errors. Another drawback of these schemes is their large dynamic order, i.e. up to eight parameters have to be updated.[4] Finally, Melkote and Khorrami[4] presented another adaptive nonlinear control scheme which is robust to magnetic saturation. Although only two adaptation parameters are required, this scheme requires complex computations involving some high-order terms. We recall that such a feature is also recognized by Ortega *et al.*[13] (p. 257), to produce input voltage saturation (aside from magnetic saturation) as well as noise amplification in practice. An important feature of the aforementioned controllers is that they solve the trajectory tracking control problem.

It is the belief of the authors that the mathematical complexity of the BLDC motors model has deviated attention of researchers toward the design of complicated nonlinear controllers for robots equipped by this kind of actuators. This explains why no result has been presented until now for the stability analysis of PID control for robots equipped with BLDC motors when the electric dynamics of these actuators is taken into account, even for regulation tasks. In the present paper we are concerned with the analysis and design of this control problem. We extend the controller proposed by Su *et al.*,[14] for the case when the actuator dynamics does not exist, to the case when the electric dynamics of the BLDC motor actuators is taken into account. Interesting features of controller in reference [14] are that it is composed of a linear PD controller plus an integral action driven by a simple saturation function of the position error and global asymptotic stability results are ensured.

Our contribution is explained as follows. We succeed to ensure global stability and global convergence to the desired constant link positions. This is achieved by simply adding to controller in reference [14] some adaptive terms, to cancel some key terms of the BLDC motor model, as well as linear feedback of electric current. Adaptation is required

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to ensure that our control strategy does not require the exact values of neither robot nor actuator parameters. On the other hand, taking advantage of the global character of this result, we can always render as small as desired the effect of the adaptive terms by choosing zero initial values for the estimated parameters and adaptation gains arbitrarily close to zero. This is possible because, as we show, the adaptive terms do not have any effect on the steady state response. Thus, we ensure robustness with respect to possible numerical errors and noise amplification introduced when the nonlinear high-order terms present in the adaptation law are computed. Further, this also means that we have found, for the first time, theoretical evidence suggesting that a simple linear PD controller plus a nonlinear integral action, implemented by means of the common industrial practice known as torque control,[15,16] suffice to globally control robots equipped with BLDC motors.

This paper is organized as follows. In Section 2, we present the dynamic model of rigid robots actuated by BLDC motors. Section 3 is devoted to present our main results. Some simulations are shown in Section 4 and conclusions are given in Section 5.

Finally, some remarks on notation. We use $\lambda_{\min}(A(x))$ and $\lambda_{\max}(A(x))$ to represent, respectively, the smallest and the largest eigenvalues of the symmetric positive definite matrix $A(x)$, for any $x \in \mathcal{R}^n$. Given an $x \in \mathcal{R}^n$ and a matrix $A(x)$ the norm of x is defined as $\|x\| = \sqrt{x^T x}$ and the spectral norm of $A(x)$ is defined as $\|A\| = \sqrt{\lambda_{\max}(A^T A)}$ which implies $\|A\| = \max_i |\lambda_i(A(x))|$, where $|\cdot|$ stands for the absolute value function, if $A(x)$ is a symmetric matrix. Symbol $p = (\mathrm{d}/\mathrm{d}t)$ denotes the differential operator.

2. Dynamic Model of Robots With BLDC Motors

The dynamic model of an n-degrees-of-freedom rigid robot equipped with a direct-drive BLDC motor at each joint is given as[3,17]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + F\dot{q} = [K_{T1}I_B + K_{T2}]I_a \tag{1}$$

$$L_a\dot{I}_a + R_aI_a + N_pL_bI_B\dot{q} + K_{T2}\dot{q} = V_a \tag{2}$$

$$L_b\dot{I}_b + R_bI_b - N_pL_aI_A\dot{q} = V_b \tag{3}$$

where

$$K_{T1} = N_p(L_b - L_a), \quad K_{T2} = \sqrt{\frac{3}{2}}N_pK_B, \tag{4}$$

$$V_a = [v_{a1}, v_{a2}, \ldots, v_{an}]^T \in \mathcal{R}^n,$$
$$V_b = [v_{b1}, v_{b2}, \ldots, v_{bn}]^T \in \mathcal{R}^n,$$
$$I_a = [i_{a1}, i_{a2}, \ldots, i_{an}]^T \in \mathcal{R}^n,$$
$$I_b = [i_{b1}, i_{b2}, \ldots, i_{bn}]^T \in \mathcal{R}^n,$$
$$I_A = \mathrm{diag}\{i_{a1}, i_{a2}, \ldots, i_{an}\} \in \mathcal{R}^{n\times n},$$
$$I_B = \mathrm{diag}\{i_{b1}, i_{b2}, \ldots, i_{bn}\} \in \mathcal{R}^{n\times n}.$$

Link positions are represented by $q \in \mathcal{R}^n$ *whereas* $M(q)$ is the $n \times n$ symmetric positive definite inertia matrix. $C(q,\dot{q})\dot{q}$ is the centripetal and Coriolis term, $g(q) = \frac{\partial U(q)}{\partial q}$ is the gravity effects term, where $U(q)$ is a scalar-valued function representing the potential energy, and F is an $n \times n$ constant diagonal positive definite matrix representing the viscous friction coefficients at each joint. Throughout this paper we use $\tilde{q} = q - q_d$ to represent the position error, where $q_d \in \mathcal{R}^n$ represents the constant desired link positions. We also assume that robot under study is equipped only with revolute joints.

Model (1)–(4) is obtained after a DQ (Park's) transformation is applied on the original Y-connected 3-phase model of each motor.[4,17,18] Thus, V_a and V_b represent, respectively, the DQ transformed phase voltages associated with each motor. I_a and I_b are electric currents defined correspondingly. L_a, L_b, R_a, and R_b are constant diagonal positive definite matrices representing, respectively, inductances and resistances of the DQ transformed phases of each motor. $N_p = \mathrm{diag}\{n_{p1}, \ldots, n_{pn}\}$ is a constant diagonal positive definite matrix containing the number of permanent magnet rotor pole pairs for each motor whereas K_B is a constant diagonal positive definite matrix containing electromotive force coefficients. Finally, K_{T1} and K_{T2} are diagonal torque constant matrices whereas $\tau = [K_{T1}I_B + K_{T2}]I_a$ is torque applied at robot joints.

Let $V_j = [v_{j1}, v_{j2}, v_{j3}]^T \in \mathcal{R}^3$ and $I_j = [i_{j1}, i_{j2}, i_{j3}]^T \in \mathcal{R}^3$ be, respectively, the phase voltages and currents of the Y-connected 3-phase BLDC motor placed at the jth robot joint. Application of a DQ (Park's) transformation means that (see reference [17], p. 373)

$$\begin{bmatrix}\zeta_{aj}\\ \zeta_{bj}\end{bmatrix} = \sqrt{\frac{2}{3}}\begin{bmatrix}\zeta_{j1}\cos(n_{pj}\,q_j) + \zeta_{j2}\cos\left(n_{pj}\,q_j - \frac{2\pi}{3}\right) + \zeta_{j3}\cos\left(n_{pj}\,q_j + \frac{2\pi}{3}\right)\\ \zeta_{j1}\sin(n_{pj}\,q_j) + \zeta_{j2}\sin\left(n_{pj}\,q_j - \frac{2\pi}{3}\right) + \zeta_{j3}\sin\left(n_{pj}\,q_j + \frac{2\pi}{3}\right)\end{bmatrix} \tag{5}$$

$$\begin{bmatrix}v_{j1}\\ v_{j2}\\ v_{j3}\end{bmatrix} = \sqrt{\frac{2}{3}}\begin{bmatrix}v_{aj}\cos(n_{pj}\,q_j) + v_{bj}\sin(n_{pj}\,q_j)\\ v_{aj}\cos\left(n_{pj}\,q_j - \frac{2\pi}{3}\right) + v_{bj}\sin\left(n_{pj}\,q_j - \frac{2\pi}{3}\right)\\ v_{aj}\cos\left(n_{pj}\,q_j + \frac{2\pi}{3}\right) + v_{bj}\sin\left(n_{pj}\,q_j + \frac{2\pi}{3}\right)\end{bmatrix} \tag{6}$$

where ζ stands for either v or i (voltages or currents). On the other hand, as it is by now well known, some important properties of the mechanical part (1), when all joints are

https://doi.org/10.1017/S026357470800461X Published online by Cambridge University Press

revolute, are the following:

Property 1 (See references [19], [20], [21], p. 98). Matrices $M(q)$ and $C(q, \dot{q})$ satisfy

$$\dot{q}^T \left(\frac{1}{2}\dot{M}(q) - C(q, \dot{q})\right)\dot{q} = 0, \quad \forall \dot{q} \in \mathcal{R}^n \tag{7}$$

$$\dot{M}(q) = C(q, \dot{q}) + C^T(q, \dot{q}) \tag{8}$$

$$0 < \lambda_{\min}(M(q)), \quad \lambda_{\max}(M(q)) < \beta, \quad \forall q \in \mathcal{R}^n$$

where β is a finite positive constant scalar.

Property 2 (See references [20], [22], [23], [21], pp. 101, 102). There exist positive constants k_g, k', and k_c such that for all $w, y, z, q \in \mathcal{R}^n$ we have

$$\|C(w, y)z\| \le k_c\|y\|\|z\| \tag{9}$$

$$\left\|\frac{\partial g(q)}{\partial q}\right\| < k_g, \quad \|g(w) - g(y)\| \le k_g\|w - y\| \tag{10}$$

$$\|g(q)\| \le k'. \tag{11}$$

Property 3 (See reference [23]). For any constant vector $q_d \in \mathcal{R}^n$, the function

$$U(q_d - \tilde{q}) - U(q_d) - \tilde{q}^T g(q_d) + \frac{k_g}{2}\|\tilde{q}\|^2 \tag{12}$$

is positive definite and radially unbounded with respect to $\tilde{q} \in \mathcal{R}^n$.

Finally, we list some well-known properties of the spectral norm. Let $w, y \in \mathcal{R}^n$ be two vectors and let $B(x)$ and $G(x)$ be two $n \times n$ matrices, the former being symmetric and positive definite $\forall x \in \mathcal{R}^n$, then

$$\pm y^T G(x)w \le \|y\|\|G(x)\|\|w\| \tag{13}$$

$$\pm y^T B(x)w \le \lambda_{\max}(B(x))\|y\|\|w\| \tag{14}$$

$$y^T B(x)y \le \lambda_{\max}(B(x))\|y\|^2 \tag{15}$$

$$y^T B(x)y \ge \lambda_{\min}(B(x))\|y\|^2 \tag{16}$$

$$\|G(x)B(x)\| \le \|G(x)\|\|B(x)\|. \tag{17}$$

3. Main Result

In this section we present an adaptive PID controller, inspired by Su *et al.*,[14] which achieves global stability and global convergence to the desired position. However, instead of the saturation functions introduced in that paper we prefer to use the function introduced by Arimoto *et al.*,[24] and refined by Kelly,[23] which, as we show below, has the same useful properties as reported by Su *et al.*[14] Define the following scalar potential function:

$$Cos(u) = \begin{cases} 1 - cos(u), & \text{if } |u| < \frac{\pi}{2} \\ u - (\pi/2 - 1), & \text{if } u \ge \frac{\pi}{2} \\ -u - (\pi/2 - 1), & \text{if } u \le -\frac{\pi}{2} \end{cases} \tag{18}$$

for $u \in \mathcal{R}$. The first derivative of $Cos(u)$ with respect to u can be expressed as

$$s(u) = \begin{cases} \sin(u), & \text{if } |u| < \frac{\pi}{2} \\ 1, & \text{if } u \ge \frac{\pi}{2} \\ -1, & \text{if } u \le -\frac{\pi}{2} \end{cases}. \tag{19}$$

Functions $Cos(u)$ and $s(u)$ in (18) and (19) have the following properties:

Property 4. Function $Cos(u)$ is twice continuously differentiable and $Cos(u) > 0$, $\forall u \neq 0$ whereas $Cos(u) = 0$ for $u = 0$.

Property 5. The following properties are adaptations of properties listed by Kelly[23]:

$$|u| \ge |s(u)| \ge k_a|u|, \ \forall u \in \mathcal{R} : \ |u| < \xi \tag{20}$$

$$|u| \ge |s(u)| \ge k_a\xi, \ \forall u \in \mathcal{R} : \ |u| \ge \xi \tag{21}$$

$$1 \ge (\mathrm{d}/\mathrm{d}u)s(u) \ge 0 \tag{22}$$

where $\xi = 1$ and $k_a = \sin(\xi) = 0.841$.

Property 6. There is a constant $b > 0$ such that

$$Cos(u) \ge bs^2(u) > 0, \ \forall u \neq 0 \tag{23}$$

Property 7. There is a constant $k > 0$ such that

$$u^2 \ge kCos(u) > 0, \ \forall u \neq 0 \tag{24}$$

Property 8.

$$U(q) - U(q_d) - \tilde{q}^T g(q_d) + \frac{1}{4}\tilde{q}^T[2(k_g I + \Lambda)]\tilde{q}$$
$$> a\|\tilde{q}\|^2 \ge a\|h(\tilde{q})\|^2 \tag{25}$$

$$h(\tilde{q}) = [s(\tilde{q}_1), s(\tilde{q}_2), \ldots, s(\tilde{q}_n)]^T \tag{26}$$

where $a = \frac{1}{2}\lambda_{\min}(\Lambda)$ and I, Λ are, respectively, the identity matrix and a diagonal positive definite matrix, both of them are $n \times n$ matrices.

Property 9. The following bound holds for all $\tilde{q} \in \mathcal{R}^n$:

$$\|g(q) - g(q_d)\| \le \frac{k_{h2}}{k_a}\|h(\tilde{q})\| \tag{27}$$

where k_{h2} is any number satisfying $k_{h2} \ge \frac{2k'}{s(2k'/k_g)}$.

Property 4 is obvious. Property 7 can be proven as follows. Both functions involved in (24) are zero at $u = 0$. Hence, (24) is true for $u \ge 0$ if $(\mathrm{d}/\mathrm{d}u)[u^2] \ge (\mathrm{d}/\mathrm{d}u)[kCos(u)]$, $\forall u \ge 0$. From this condition and the facts that $|u| \ge |s(u)|$ and that both functions in (24) are symmetric with respect to $u = 0$ we find that (24) is true with $k = 2$. Property 6

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is proven to hold with $b = 0.5$ proceeding similarly by considering that $(\mathrm{d}/\mathrm{d}u)[Cos(u)] \geq (\mathrm{d}/\mathrm{d}u)[bs^2(u)]$ for $u \geq 0$ if $(\mathrm{d}^2/\mathrm{d}u^2)[Cos(u)] \geq (\mathrm{d}^2/\mathrm{d}u^2)[bs^2(u)]$ for $u \geq 0$. Property 8 is readily obtained using again the fact that $|u| \geq |s(u)|$ and (12). Property 9 is proven as follows. Using Property 5 we obtain

$$\|h(\tilde{q})\| \geq \begin{cases} k_a\|\tilde{q}\|, & \text{if } \|\tilde{q}\| < \xi \\ k_a, & \text{if } \|\tilde{q}\| \geq \xi \end{cases} \tag{28}$$

$$\|h(\tilde{q})\| \leq \begin{cases} \|\tilde{q}\|, & \text{if } \|\tilde{q}\| < \xi \\ \sqrt{n}, & \text{if } \|\tilde{q}\| \geq \xi \end{cases} \tag{29}$$

$$s(\|\tilde{q}\|) \leq \frac{1}{k_a}\|h(\tilde{q})\|. \tag{30}$$

On the other hand, proceeding as in reference [21] (pp. 105–107), we find

$$\|g(q) - g(q_d)\| \leq k_{h2}s(\|\tilde{q}\|). \tag{31}$$

Finally, using (30) and (31) we obtain (27).

Proposition 1. *Consider the dynamic model* (1), (2), (3) *together with the control law*

$$V_a = -r_a I_a + (\overline{K_P} + \overline{K_I})E\,\nu - \overline{K_D}\,\vartheta - \overline{K_I}\int_0^t \varepsilon_0 h(\tilde{q}(r))\,\mathrm{d}r \tag{32}$$

$$\nu = \mathrm{diag}\left\{\frac{1}{p+e_i}\right\}[-\,\tilde{q} - K\dot{q} - \varepsilon_0\, B\, h(\tilde{q})] \tag{33}$$

$$\vartheta = \mathrm{diag}\left\{\frac{b_i p}{p+a_i}\right\}q \tag{34}$$

$$V_b = -\dot{Q}\Delta_a\widehat{\theta}_1 - \varepsilon_0 H(\tilde{q}) I_A \widehat{\theta}_2 \tag{35}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\widehat{\theta}_1\\ \widehat{\theta}_2\end{bmatrix} = \Gamma\begin{bmatrix} I_B\,\dot{Q}\,\delta_a \\ \varepsilon_0 I_B H(\tilde{q})\, I_a\end{bmatrix} \tag{36}$$

$$\begin{aligned}\delta_a &= (\,\overline{K_P} + \overline{K_I}\,)\,E\nu - \overline{K_D}\,\vartheta - \overline{K_I}\int_0^t \varepsilon_0 h(\tilde{q}(r))\,dr \\ &= [\delta_{a1}, \delta_{a2}, \ldots, \delta_{an}]^T \in \mathcal{R}^n \\ H(\tilde{q}) &= \mathrm{diag}\{s(\tilde{q}_1), s(\tilde{q}_2), \ldots, s(\tilde{q}_n)\} \in \mathcal{R}^{n\times n} \\ \Delta_a &= \mathrm{diag}\{\delta_{a1}, \delta_{a2}, \ldots, \delta_{an}\} \in \mathcal{R}^{n\times n} \\ \dot{Q} &= \mathrm{diag}\{\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_n\} \in \mathcal{R}^{n\times n}\end{aligned} \tag{37}$$

where $A = \mathrm{diag}\{a_i\}$, $B = \mathrm{diag}\{b_i\}$, $E = \mathrm{diag}\{e_i\}$, *and* $K = E^{-1} + B$ *are* $n \times n$ *diagonal positive definite matrices,* $h(\tilde{q}) = [s(\tilde{q}_1), \ldots, s(\tilde{q}_n)]^T$ *where* $s(\cdot)$ *is defined in* (19), Γ *is an arbitrary* $2n \times 2n$ *diagonal positive definite matrix and* $\widehat{\theta}_1$, $\widehat{\theta}_2$ *are, respectively, the estimates of parameters defined as*

$$\begin{aligned}\theta_1^* &= \left[\frac{N_{p1}L_{b1}}{R_1}, \ldots, \frac{N_{pn}L_{bn}}{R_n}\right]^T \in \mathcal{R}^n \\ \theta_2^* &= \left[K_{T1_1}, \ldots, K_{T1_n}\right]^T \in \mathcal{R}^n\end{aligned} \tag{38}$$

where subindex indicates a diagonal entry of the corresponding matrix. There always exist diagonal positive definite matrices A, B, E, $\overline{K_P}$, $\overline{K_D}$, $\overline{K_I}$, r_a, and a constant scalar $\varepsilon_0 > 0$ *such that the closed-loop system has an equilibrium point where* $\tilde{q} = 0$ *which is globally stable and global convergence* $\lim_{t\to\infty} q(t) = q_d$ *is ensured.*

Remark 1. *Term* $(\overline{K_P} + \overline{K_I})E\,\nu$, used *in* (32) *and* δ_a, *represents filtering of the proportional term* $-(\overline{K_P} + \overline{K_I})\,\tilde{q}$. *Reason to introduce filtering is explained as follows. As we show in the following proof we require to use a small* $\varepsilon_0 > 0$, *hence we need large values for* $\overline{K_I}$ *in order to avoid slow integral adjustments. However this produces very large peak values of control signals because of the large gain* $(\overline{K_P} + \overline{K_I})$ *and discontinuity of* $\tilde{q}$ *when abrupt changes in* q_d *are commanded. Filtering of the proportional term results in important reduction of the peak values of control signals. On the other hand, proof of Proposition 1 can be used in a straightforward manner to prove that results in Proposition 1 still stand when term* $-(\overline{K_P} + \overline{K_I})\,\tilde{q}$ *is used instead of term* $(\overline{K_P} + \overline{K_I})E\;\nu$ *in* (32) *and* δ_a, *i.e. when variable* ν *is not used. Note that in such a case* V_a *represents a linear PD controller plus an integral action driven by a saturation function of* $\tilde{q}$. *In order to consider both possibilities of controller in Proposition 1 we refer to it as an adaptive PID controller.*

Proof of Proposition 1. Note that a realization of filter (33) is given as

$$\dot{\nu} = -E\nu - \tilde{q} - K\dot{q} - \varepsilon_0 B h(\tilde{q}). \tag{39}$$

We can write (39) as

$$\dot{\sigma} = -E\sigma - B\,\dot{q} - \varepsilon_0 B h(\tilde{q}) \tag{40}$$

by using $K = E^{-1} + B$ and defining $\sigma = \nu + E^{-1}\tilde{q}$. On the other hand, note that

$$\dot{\vartheta} = -A\vartheta + B\dot{q} \tag{41}$$

is a realization of filter (34) and define $R = R_a + r_a$ and

$$\rho = I_a - R^{-1}\delta_a. \tag{42}$$

Using these facts as well as (39) and $\sigma = \nu + E^{-1}\tilde{q}$, we can replace (32) in (2) to find

$$\begin{aligned}L_a\dot{\rho} &= -R\rho - N_p L_b I_B\dot{q} - K_{T2}\dot{q} + L_a R^{-1}(\overline{K_P} + \overline{K_I})EE\sigma \\ &\quad + L_a R^{-1}[(\overline{K_P} + \overline{K_I})EK + \overline{K_D}B]\dot{q} \\ &\quad - L_a R^{-1}\overline{K_D}A\vartheta + \varepsilon_0 L_a R^{-1}(\overline{K_I} + [\overline{K_P} + \overline{K_I}]EB)h(\tilde{q}).\end{aligned} \tag{43}$$

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On the other hand, replacing (35) in (3) we get

$$L_b \dot{I}_b = -R_b I_b + N_p L_a \dot{Q} \rho + N_p L_a \dot{Q} R^{-1} \delta_a - \dot{Q} \Delta_a \widehat{\theta}_1 - \varepsilon_0 H(\tilde{q}) I_A \widehat{\theta}_2. \quad (44)$$

Note that we can write

$$K_{T2} R^{-1} \delta_a = \delta_a^* \quad (45)$$

where $\delta_a^* = -K_P \tilde{q} - K_D \vartheta - K_I z + g(q_d) + (K_P + K_I) E\sigma$ by using $\sigma = \nu + E^{-1}\tilde{q}$ and defining

$$K_P = K_{T2} R^{-1} \overline{K_P} \quad (46)$$

$$K_D = K_{T2} R^{-1} \overline{K_D} \quad (47)$$

$$K_I = K_{T2} R^{-1} \overline{K_I} \quad (48)$$

$$z = \tilde{q} + \int_0^t \varepsilon_0 h(\tilde{q}(r))\, \mathrm{d}r + (K_I)^{-1} g(q_d). \quad (49)$$

Thus, replacing (45) in (44) we get

$$L_b \dot{I}_b = -R_b I_b + N_p L_a \dot{Q} \rho + N_p L_a \dot{Q} K_{T2}^{-1} \delta_a^* - \dot{Q} \Delta_a \tilde{\theta}_1 - \dot{Q} \Delta_a \theta_1^* - \varepsilon_0 H(\tilde{q}) I_A \tilde{\theta}_2 - \varepsilon_0 H(\tilde{q}) I_A \theta_2^* \quad (50)$$

$$\tilde{\theta} = \begin{bmatrix} \widehat{\theta}_1 - \theta_1^* \\ \widehat{\theta}_2 - \theta_2^* \end{bmatrix}.$$

On the other hand, *using* (42) we can write (1) as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + F\dot{q} = [K_{T1} I_B + K_{T2}]\rho + [K_{T1} I_B + K_{T2}] R^{-1} \delta_a. \quad (51)$$

Using (45) we can write (51) as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) - g(q_d) + F\dot{q} = [K_{T1} I_B + K_{T2}]\rho + [K_{T1} I_B + K_{T2}] K_{T2}^{-1} (K_P + K_I) E\sigma + K_{T1} I_B K_{T2}^{-1} [-K_P \tilde{q} - K_D \vartheta - K_I z + g(q_d)] - K_P \tilde{q} - K_D \vartheta - K_I z. \quad (52)$$

Thus, the closed-loop dynamics is given by (52), (50), (43), (41), (40) and

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \dot{q} + \varepsilon_0 h(\tilde{q})$$

$$\frac{\mathrm{d}\tilde{\theta}}{\mathrm{d}t} = \Gamma \begin{bmatrix} I_B \dot{Q} \delta_a \\ \varepsilon_0 I_B H(\tilde{q}) I_a \end{bmatrix}. \quad (53)$$

We stress that use of (42) and (45) allows to write I_a and δ_a, in the closed-loop dynamics, as a function of the state $(\tilde{q}, \dot{q}, z, \vartheta, \rho, \sigma, I_b, \tilde{\theta})^T$:

$$I_a = \rho + K_{T2}^{-1} \delta_a^*$$

$$\delta_a = R K_{T2}^{-1} \delta_a^*.$$

Moreover, recalling (4) and (37) we can see that I_A and Δ_a can also be written in terms of the state. This shows that (52), (50), (43), (41), (40), (53) is an autonomous closed-loop dynamics. Note that the origin is an equilibrium point of this closed-loop dynamics. Now, we proceed to study the stability of this equilibrium point. Su *et al.*[14] have shown, by means of Properties 6, 7, and 8, that the following function is positive definite and radially unbounded:

$$V_1(\tilde{q}, \dot{q}, z, \vartheta) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2}\tilde{q}^T K_P \tilde{q} + \varepsilon_0 h^T(\tilde{q}) M(q)\dot{q} + \sum_{i=1}^{n} \varepsilon_0 f_i Cos(\tilde{q}_i) + U(q) - U(q_d) - \tilde{q}^T g(q_d) + \frac{1}{2} z^T K_I z + \frac{1}{2}\vartheta^T K_D B^{-1} \vartheta \quad (54)$$

where f_i stands for the diagonal entries of matrix F, if

$$\lambda_{\min}(K_P) \geq \frac{4\varepsilon_0^2}{b\,k} \lambda_{\max}(M(q)) \quad (55)$$

$$\lambda_{\min}(K_P) > 2(k_g + \lambda_{\min}(\Lambda)) \quad (56)$$

for b, k, and Λ defined in (23)–(25). Thus, the following scalar function qualifies as a Lyapunov function candidate:

$$W(\tilde{q}, \dot{q}, z, \vartheta, \rho, \sigma, I_b, \tilde{\theta}) = V_1(\tilde{q}, \dot{q}, z, \vartheta) + V_2(\rho, \sigma) + V_3(I_b, \tilde{\theta})$$

$$V_2(\rho, \sigma) = \frac{1}{2}\rho^T L_a \rho + \frac{1}{2}\, \sigma^T K_{T2} R^{-1} B^{-1} (\overline{K_P} + \overline{K_I}) E\sigma$$

$$V_3(I_b, \tilde{\theta}) = \frac{1}{2} I_b^T\, L_b I_b + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}. \quad (57)$$

Using (4) and the diagonal nature of all the involved matrices, we have $\dot{q}^T K_{T1} I_B \rho - \rho^T N_p L_b I_B \dot{q} + I_b^T N_p L_a \dot{Q} \rho = 0$ and $\dot{q}^T K_{T1} I_B K_{T2}^{-1} \delta_a^* + I_b^T N_p L_a \dot{Q} K_{T2}^{-1} \delta_a^* = I_b^T N_p L_b \dot{Q} K_{T2}^{-1} \delta_a^*$. These facts, the diagonal property of all matrices involved in definitions given in (38) and use of (46)–(48), (7), (8), $g(q) = \frac{\partial U(q)}{\partial q}$, allow to find the following time derivative along the trajectories of dynamics (52), (50), (43), (41), (40), (53):

$$\dot{W} = -\dot{q}^T \left[F - \varepsilon_0 \frac{\partial h(\tilde{q})}{\partial \tilde{q}} M(q) \right] \dot{q} + \varepsilon_0 h^T(\tilde{q}) C^T(q,\dot{q})\dot{q} + \varepsilon_0 h^T(\tilde{q})(g(q_d) - g(q)) + \varepsilon_0 h^T(\tilde{q}) K_{T2} \rho - \varepsilon_0 h^T(\tilde{q}) K_P \tilde{q} - \varepsilon_0 h^T(\tilde{q}) K_D \vartheta - I_b^T R_b I_b - \vartheta^T K_D B^{-1} A \vartheta - \rho^T R \rho - \sigma^T B^{-1} (K_P + K_I) E E \sigma + \rho^T L_a K_{T2}^{-1} [(K_P + K_I) E K + K_D B]\dot{q} - \rho^T L_a K_{T2}^{-1} K_D A \vartheta + \varepsilon_0 \rho^T L_a K_{T2}^{-1} [(K_P + K_I) E B + K_I] h(\tilde{q}) + \rho^T L_a K_{T2}^{-1} (K_P + K_I) E E \sigma. \quad (58)$$

According to (22), $\frac{\partial h(\tilde{q})}{\partial \tilde{q}}$ is a diagonal matrix whose entries are nonnegative and smaller than or equal to 1. On the other hand, using Property 9 and taking advantage of the facts that K_P is a diagonal matrix and that $|u| \geq |s(u)|$ we can write $\varepsilon_0 h^T(\tilde{q})(g(q) - g(q_d)) + \varepsilon_0 h^T(\tilde{q}) K_P \tilde{q} \geq \varepsilon_0 [-\frac{k_{h2}}{k_a} + \lambda_{\min}(K_P)] \|h(\tilde{q})\|^2$. As proposed by Su *et al.*,[14] the following

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condition is important for our purposes:

$$-\frac{k_{h2}}{k_a} + \lambda_{\min}(K_P) \geq a + \frac{1}{2}\lambda_{\max}(K_D). \tag{59}$$

Also note that, according to (29), we can bound $\|h(x)\| \leq \sqrt{n}$, $\forall x \in \mathcal{R}^n$. Thus, from (9) we obtain that $\varepsilon_0 h(\tilde{q})^T C^T(q,\dot{q})\dot{q} \leq \varepsilon_0\|h(\tilde{q})\|\|C^T(q,\dot{q})\dot{q}\| \leq \varepsilon_0\sqrt{n}k_c\|\dot{q}\|^2$. Finally, from $(\|h(\tilde{q})\| - \|\vartheta\|)^2 \geq 0$ we obtain $\|h(\tilde{q})\|^2 + \|\vartheta\|^2 \geq 2\|h(\tilde{q})\|\,\|\vartheta\|$. Inspired by Su *et al.*[14] we can use these facts as well as (13)–(17) to find that $\dot{W}$ can be bounded as

$$\dot{W} \leq -\begin{bmatrix}\|\dot{q}\|\\ \|\vartheta\|\\ \|\rho\|\\ \|I_b\|\end{bmatrix}^T N \begin{bmatrix}\|\dot{q}\|\\ \|\vartheta\|\\ \|\rho\|\\ \|I_b\|\end{bmatrix} - \begin{bmatrix}\|h(\tilde{q})\|\\ \|\rho\|\end{bmatrix}^T \bar{N} \begin{bmatrix}\|h(\tilde{q})\|\\ \|\rho\|\end{bmatrix} - \begin{bmatrix}\|\sigma\|\\ \|\rho\|\end{bmatrix}^T \bar{M} \begin{bmatrix}\|\sigma\|\\ \|\rho\|\end{bmatrix} \tag{60}$$

where entries of matrices N and $\bar{N}$ are:

$$N_{11} = \lambda_{\min}(F) - \varepsilon_0[\lambda_{\max}(M(q)) + \sqrt{n}k_c]$$
$$N_{22} = \lambda_{\min}(K_D B^{-1} A) - \frac{\varepsilon_0}{2}\lambda_{\max}(K_D)$$
$$N_{33} = \gamma_1\lambda_{\min}(R)$$
$$N_{44} = \lambda_{\min}(R_b)$$
$$N_{12} = N_{21} = 0$$
$$N_{13} = N_{31} = -\frac{1}{2}\lambda_{\max}\left(L_a K_{T2}^{-1}[(K_P + K_I)EK + K_D B]\right)$$
$$N_{14} = N_{41} = N_{42} = N_{24} = N_{43} = N_{34} = 0$$
$$N_{23} = N_{32} = -\frac{1}{2}\lambda_{\max}\left(L_a K_{T2}^{-1} K_D A\right)$$
$$\bar{N}_{11} = a\varepsilon_0$$
$$\bar{N}_{12} = \bar{N}_{21} = -\frac{\varepsilon_0}{2}\lambda_{\max}(K_{T2}) - \frac{\varepsilon_0}{2}\lambda_{\max}\left(L_a K_{T2}^{-1}[(K_P + K_I)EB + K_I]\right)$$
$$\bar{N}_{22} = \gamma_2\lambda_{\min}(R)$$
$$\bar{M}_{11} = \lambda_{\min}(B^{-1}[K_P + K_I]EE)$$
$$\bar{M}_{12} = \bar{M}_{21} = -\frac{1}{2}\lambda_{\max}\left(L_a K_{T2}^{-1}[K_P + K_I]EE\right)$$
$$\bar{M}_{22} = \gamma_3\lambda_{\min}(R)$$

for some positive numbers such that $\gamma_1 + \gamma_2 + \gamma_3 = 1$. Matrix N is positive definite if *all of its principal minors are positive, i.e.*,

$$N_{11} > 0, \quad N_{22} > 0, \quad N_{33} > 0, \quad N_{44} > 0 \tag{61}$$
$$N_{11}N_{22}N_{33} - N_{13}N_{22}N_{31} - N_{23}N_{11}N_{32} > 0 \tag{62}$$

Condition (62) *follows by computing the third principal minor of N through cofactors of the third column.* Matrix $\bar{N}$ is positive definite if

$$\bar{N}_{11} > 0, \quad \bar{N}_{22} > 0 \tag{63}$$
$$\bar{N}_{11}\bar{N}_{22} > \bar{N}_{12}\bar{N}_{21}. \tag{64}$$

Matrix $\bar{M}$ is positive definite if

$$\bar{M}_{11} > 0, \quad \bar{M}_{22} > 0 \tag{65}$$
$$\bar{M}_{11}\bar{M}_{22} > \bar{M}_{21}\bar{M}_{12}. \tag{66}$$

According to (46)–(48), *matrices K_P, K_D, and K_I depend on R^{-1}. However, given a change on R we can adjust any of $\overline{K_P}$, $\overline{K_D}$, or $\overline{K_I}$ to maintain the desired values of any of K_P, K_D, or K_I. Hence, we can arbitrarily enlarge N_{33} by enlarging R, i.e. r_a, and this does not produce any change on any of the other entries of matrix N. Thus, we can always select N_{33} as large as necessary to render* (62) *true. Note that* all of conditions in (61)–(66) are satisfied by choosing a small $\varepsilon_0 > 0$, suitable positive definite matrices K_P, K_I, K_D, A, B, a large matrix R, i.e. a large r_a, and a large $a > 0$, i.e. by means of a large K_P. Note that R_b is a positive definite matrix. Hence, $\dot{W}$ given in (60) can always be rendered globally negative semidefinite. This, together with the global positive definiteness and radial unboundedness of W ensure stability of $(\tilde{q}, \dot{q}, z, \vartheta, \rho, \sigma, I_b, \tilde{\theta}) = (0, 0, 0, 0, 0, 0, 0, 0)$, i.e. the whole state is bounded. Convergence $q(t) \to q_d$ as $t \to \infty$ follows using standard adaptive control arguments. From (60) we can show that $h(\tilde{q})$ is square integrable. This and the fact that $\tilde{q}$ is bounded allow to conclude that $\tilde{q}$ is also square integrable. Recall that $\dot{q}$, the time derivative of $\tilde{q}$, is also bounded. Note that these properties hold globally. Thus, global convergence $q(t) \to q_d$ as $t \to \infty$ is ensured. This completes the proof of Proposition 1. Finally, it is important to say that this result is possible thanks to the realistic assumption on viscous friction at robot joints: even if a small viscous friction, F, is present it is enough to choose a small $\varepsilon_0 > 0$. This assumption is also fundamental in reference [14].

Remark 2. *It is important to say that the adaptive part of the controller, i.e. V_b in* (35)*, has no effect on the steady state response. This can be seen from the fact that $(\tilde{q}, \dot{q}, z, \vartheta, \rho, \sigma, I_b, \tilde{\theta}) = (0, 0, 0, 0, 0, 0, 0, \theta_0)$ for any constant $\theta_0 \in \mathcal{R}^{2n}$ qualifies as an equilibrium point of the closed-loop dynamics* (52), (50), (43), (41), (40), (53). *On the other hand, note that the adaptive gain matrix Γ is any arbitrary positive definite diagonal matrix. Also note that the global character of controller in Proposition 1 allows us to choose any finite initial values for the estimated parameters. Thus, we can always choose $\widehat{\theta_1}(0) = 0$, $\widehat{\theta_2}(0) = 0$, and Γ as a diagonal matrix whose diagonal entries are arbitrarily close to zero. This ensures that V_b, in* (35)*, can always be kept as close to zero as desired to render its effect negligible. This implies robustness with respect to numerical errors and noise amplification as well as avoidance of undesired input voltage saturations which, as pointed out by Ortega et al.,*[13] *pp. 257, 395, 403, can be produced when computing the complex high-order terms appearing in the adaptive part of the controller, i.e. V_b in* (35). *Moreover, this also means that*

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virtually only the PID controller plus linear current feedback, given in (32)*, is applied. This fact, together with the following remark, means that theoretical evidence has been found, for the first time, suggesting that a simple linear PD controller plus a nonlinear integral action, implemented by means of the common practice known as torque control,*[15,16] *suffice to globally control robots equipped with BLDC motors.*

Remark 3. *In industrial practice it is common to consider that the torque applied by BLDC motors to robot joints is proportional to current. Further, the drives for those motors include some current controllers ensuring the generation of the desired torque. This is known as torque control or current control.*[16] *In the following we recall the procedure presented by Campa et al.*[15] *to implement this strategy for controlling BLDC motors under the assumption that $L_a = L_b$. In such a case torque applied by motors to robot joints is given as $\tau = K_{T2} I_a$ and torque control can be written as*

$$V_a = K_d(I^* - I_a), \tag{67}$$

where K_d is a diagonal positive definite matrix and I^* represents the value of the electric current I_a necessary to generate the desired torque τ^*, i.e.,

$$I^* = K_{T2}^{-1}\tau^*. \tag{68}$$

Additionally, $V_b = 0$ is assumed. Note that if a PID control law is used as the desired torque,

$$\tau^* = (\kappa_p + \kappa_i)\, E\nu - \kappa_d\, \vartheta - \kappa_i \int_0^t \varepsilon_0 h(\tilde{q}(r))\, dr, \tag{69}$$

then V_a given in (32) *is retrieved from* (67)–(69) *by setting*

$$r_a = K_d \tag{70}$$

$$\overline{K_P} = K_d K_{T2}^{-1}\kappa_p \tag{71}$$

$$\overline{K_D} = K_d K_{T2}^{-1}\kappa_d \tag{72}$$

$$\overline{K_I} = K_d K_{T2}^{-1}\kappa_i. \tag{73}$$

Aside from these facts, it is important to stress that our result is valid even if $L_a \neq L_b$.

Remark 4. *Conditions ensuring result in Proposition 1 are summarized in* (55), (56), (59), (61)–(66)*. It is clear that all of these conditions can be satisfied without requiring the exact knowledge of neither robot nor actuators' parameters because all of them are given as inequalities. Further, connection between K_P, K_D, K_I and $\overline{K_P}$, $\overline{K_D}$, $\overline{K_I}$, established by* (46)–(48)*, can be embedded in* (55), (56), (59), (61)–(66) *by considering the largest and the smallest values of product $K_{T2}R^{-1}$. Moreover, we stress that Proposition 1 is valid for any values of L_a and L_b, i.e., contrary to the common assumption we do not require inductance to be small.*

4. Simulation Results

In this section we present some simulation results to study performance of controller in Proposition 1. We use the numerical values of the rigid robot reported by Kelly *et al.*[21] (Ch. 5) and Campa *et al.*[25] This is a two-degrees-of-freedom rigid robot with both revolute joints moving on a vertical plane. Position $[q_1, q_2] = [0, 0]$ corresponds to configuration where both links are parallel and downwards. For simulation purposes we assume that this robot is equipped with two BLDC motors whose numerical parameters are those identified by Campa *et al.*,[15] i.e., $N_p = \text{diag}\{120, 120\}$, $R_a = R_b = \text{diag}\{1.9, 1.9\}$[Ohms], $L_b = \text{diag}\{0.00636, 0.00636\}$[Hy], $L_a = \text{diag}\{0.00672, 0.00672\}$[Hy], $K_B = \text{diag}\{0.0106, 0.0106\}$[Wb], $J = \text{diag}\{0.0025, 0.0025\}$[Kg m^2], $F = \text{diag}\{0.203, 0.203\}$[Nm/(rad/s)]. We choose all initial conditions equal to zero and desired link positions $q_d = [\pi/9, \pi/30]^T$[rad], as proposed in experiments reported by Kelly *et al.*[21] concerning position regulation. V_a in (32) is implemented following Remark 3. i.e. using the torque control strategy (67) with $K_d = r_a$ and

$$I^* = r_a^{-1}(\overline{K_P} + \overline{K_I})\, E\nu - r_a^{-1}\overline{K_D}\, \vartheta - r_a^{-1}\overline{K_I} \int_0^t \varepsilon_0 h(\tilde{q}(r))\, dr. \tag{74}$$

This means that the desired torque, τ^*, is set according to (68). Controller gains are chosen such that all of conditions (55), (56), (59), (61)–(66) are satisfied. As stressed in Remark 4, these conditions are verified to be satisfied without requiring the exact values of neither robot nor actuator parameters by considering that a 10% uncertainty exists on values of R_a, L_a, and K_{T2}. In Figs. 1–4, we present simulation results when the following controller parameters are used $\overline{K_P} = \text{diag}\{33000, 33000\}$, $\overline{K_D} = \text{diag}\{25000, 14000\}$, $\overline{K_I} = \text{diag}\{40000, 9000\}$, $A = \text{diag}\{10, 10\}$, $B = \text{diag}\{0.39, 0.39\}$, $E = \text{diag}\{13, 13\}$, $\varepsilon_0 = 0.3$, $\Gamma = \text{diag}\{1 \times 10^{-14}, 1 \times 10^{-14}, 1 \times 10^{-14}, 1 \times 10^{-14}\}$. Finally, we choose $r_a = \text{diag}\{698, 698\}$[Ohm] because in reference [15] it was found that this value (i.e. $K_d = \text{diag}\{698, 698\}$[Ohm]) is used in the actual commercial drive

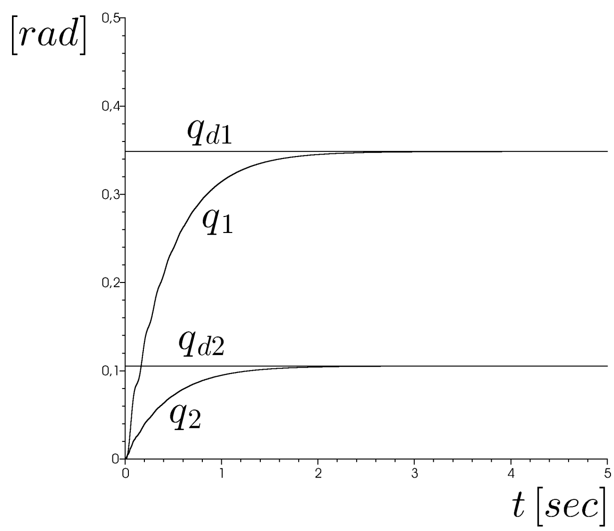

Fig. 1. Simulation results. Link positions.

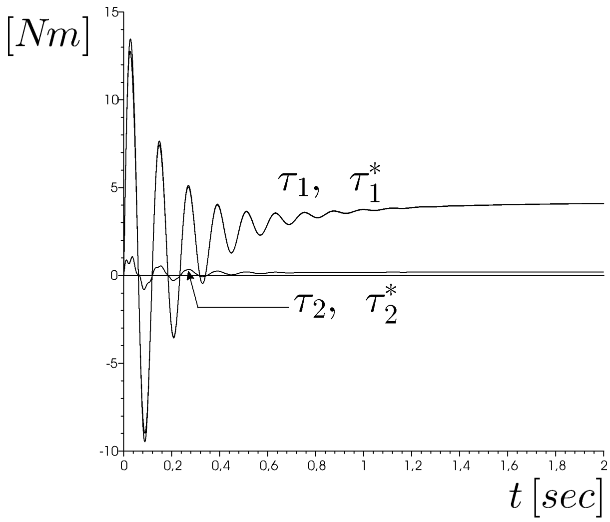

Fig. 2. Simulation results. Applied and desired torques.

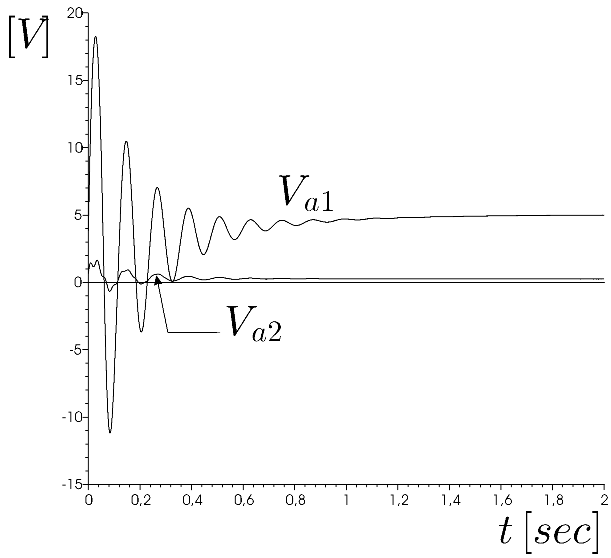

Fig. 3. Simulation results. Applied voltages.

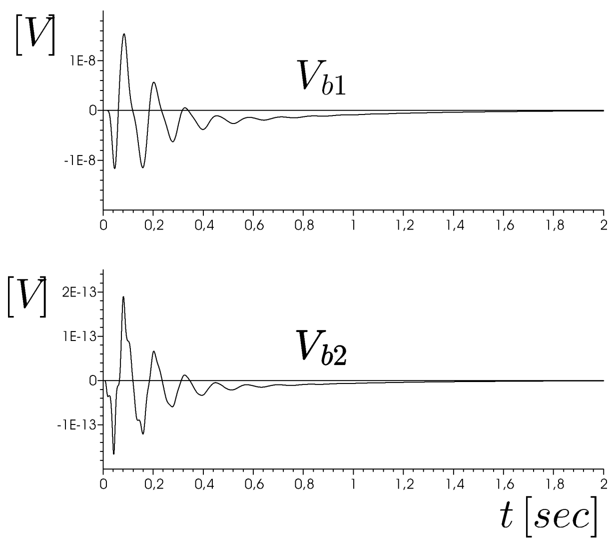

Fig. 4. Simulation results. Voltages V_{b1} and V_{b2} remain close to zero.

provided together with the BLDC motor identified in that work. We can see in Fig. 1 that fast convergence to the desired position is achieved. We stress that this time response is about the same order as time responses of controllers tested experimentally in reference [21]. It is important to say that controllers in that book are designed without taking into account the electric dynamics of actuators. Thus, our results show that conditions introduced by the electric dynamics do not impose much slower responses as it might be expected. Further, we realize from Fig. 2 that this is achieved without violating torque bounds imposed by the experimental platform reported by Campa *et al.*[25] (i.e. robot used by Kelly *et al.*[21]): 15 [Nm] and 4 [Nm] for actuator at the first and the second joint, respectively. We can also see in Fig. 2 that the actual torques applied to links $\tau = [K_{T1}I_B + K_{T2}]I_a$ converge to their desired values τ^*, given in (68), which is an important reason to introduce torque control in practice. We stress that such convergence is possible thanks to the fact that term $K_{T1}I_BI_a$ has very small values as explained by Dawson *et al.*[17] (Remark 4.2). Note, from Fig. 4, that voltages V_{b1} and V_{b2} remain very small compared with values of voltages V_{a1} and V_{a2} shown in Fig. 3. Also note that although the PID gains used may be thought to be very large, however V_{a1} and V_{a2} are not in fact very large considering that the PID gains are in the order of 10^4. This remarkable feature is due to use of the torque control strategy (67) and (74), i.e. current error $I^* - I_a$ decreases very fast, and use of filtering of the proportional term, i.e. ν.

5. Conclusions

In this paper, we present stability analysis for rigid robots actuated by BLDC motors when the electric dynamics of these actuators is taken into account. Linear feedback of electric current is instrumental for our results which is a very common strategy to control BLDC motors in practice known as torque control or current control. Although some practical justifications exist to use such strategy, this is the first time that torque control is studied in a formal stability analysis. We have also found, for the first time, theoretical evidence indicating that a PID controller suffices to globally regulate position in rigid robots actuated by BLDC motors when the electric dynamics of actuators is not neglected. Simulation results show that short-time responses are achieved which are about the same order as time responses of controllers designed without taking into account the electric dynamics of actuators. This means that taking into account this dynamics does not impose much slower responses as it might be expected. Fruther, this is accomplished without violating torque bounds imposed by the experimental platform reported in references [21] and [25] whose numerical values are used to perform our simulations. Important to obtain these features is filtering of the proportional term $-(\overline{K_P} + \overline{K_I})\tilde{q}$.

Acknowledgments

V. Santibáñez's work is supported by DGEST and CONACTY, México.

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