J. Plasma Physics (2008), *vol.* 74, *part* 6, *pp.* 719–723. © 2008 Cambridge University Press
doi:10.1017/S0022377808007344 Printed in the United Kingdom

Letter to the Editor

Dispersion relations for electromagnetic waves in a dense magnetized plasma

P. K. SHUKLA† and L. STENFLO‡

Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany
(ps@tp4.ruhr-uni-bochum.de)

(Received 7 April 2008, accepted 28 April 2008, first published online 6 June 2008)

Abstract. Dispersion relations for elliptically polarized extraordinary as well as linearly polarized ordinary electromagnetic waves propagating across an external magnetic field in a dense magnetoplasma are derived, taking into account the combined effects of the quantum electrodynamical (QED) field, as well as the quantum forces associated with the Bohm potential and the magnetization energy of the electrons due to the electron-1/2 spin effect. The QED (vacuum polarization) effects, which contribute to the nonlinear electron current density, modify the refractive index. Our results concern the propagation characteristics of perpendicularly propagating high-frequency electromagnetic waves in dense astrophysical objects (e.g. neutron stars and magnetars), as well as the next-generation intense laser–solid density plasma interaction experiments and quantum free-electron laser schemes.

Recently, there has been a great deal of interest, e.g. [1, 2], in investigating the properties of electromagnetic waves in plasmas incorporating quantum electrodynamical (or vacuum polarization) effects [3]. In particular, Lundin et al. [2] presented a general dispersion relation for large-amplitude circularly polarized electromagnetic (CPEM) waves [4, 5] propagating along an external magnetic field by including the vacuum polarization current [2] in the Maxwell equation. They reported some new features of the CPEM waves in dense quantum magnetoplasmas.

In this letter, we derive dispersion relations for elliptically polarized extraordinary (EX) and linearly polarized ordinary (O) electromagnetic (EM) waves propagating across an external constant magnetic field $B_{0z}\hat{\mathbf{z}}$, where B_{0z} is the strength of

† Also at School of Physics, University of KwaZulu Natal, 400 Durban, South Africa; the Nonlinear Physics Centre & Centre for Plasma Science and Astrophysics, Ruhr-Universität Bochum, D-44780 Bochum, Germany; Department of Physics, Umeå University, SE-90187 Umeå, Sweden; Max-Planck-Institut für extraterrestrische Physik, D-85741 Garching, Germany; Grupo de Lasers e Plasmas, Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico, Universidade Técnica de Lisboa, 1049-001 Lisboa, Portugal; CCLRC Centre for Fundamental Physics, Rutherford Appleton Laboratory, Chilton, Didcot, Oxon 0X11 0QX, UK; Department of Physics, Scottish Universities of Physics Alliance (SUPA), University of Strathclyde, Glasgow G 40NG, UK.

‡ Also at Department of Physics, Umeå University, SE-90187 Umeå, Sweden, and Department of Physics, Linköping University, SE-58183 Linköping, Sweden.

the external magnetic field and $\hat{\mathbf{z}}$ is a unit vector along the z-axis in a Cartesian coordinate system, in a dense magnetoplasma accounting for vacuum polarization, electron tunneling and electron-1/2 spin effects.

The propagation of the EM waves in a magnetized plasma is governed by the Faraday law

$$c\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \tag{1}$$

and the modified (by the QED effect) Maxwell equation [2, 3]

$$(1-\beta)\left(c\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t}\right) + 4\pi n_0 e\mathbf{u} = 0, \tag{2}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the wave electric and wave magnetic field vectors, respectively, c is the speed of light in vacuum, e is the magnitude of the electron charge, n_0 is the unperturbed electron number density, and $\mathbf{u}$ is the electron fluid velocity. The ions do not respond to the high-frequency EM waves, and are therefore considered as being stationary. In (2), we have used the symbol

$$\beta = \frac{2\alpha}{45\pi E_{\mathrm{c}}^2}[B_{0z}^2 + \mathbf{E}\cdot\mathbf{E}(n^2-1)], \tag{3}$$

where $\alpha = e^2/\hbar c$ is the fine structure constant, $\hbar$ is the Planck constant divided by 2π, $E_{\mathrm{c}} = m^2c^3/\hbar e$, m is the electron rest mass, and $n = kc/\omega$ is the refractive index. Here ω and $\mathbf{k}$ are the EM wave frequency and wave vector, respectively. We note that β represents the contribution of the vacuum polarization current [3] under the approximation that the wave magnetic (wave electric) field is much smaller than B_{0z} (E_{c}) and that the EM wave frequencies are smaller than the Compton frequency $mc^2/\hbar$.

The electron fluid velocity $\mathbf{u}$ is determined from the equation of motion

$$m\frac{\partial \mathbf{u}}{\partial t} = -e\left(\mathbf{E} + \frac{1}{c}\mathbf{u}\times\mathbf{B}_0\right) - \frac{\nabla P_1}{n_0} + \mathbf{F}_Q, \tag{4}$$

where the electron pressure perturbation [6] for a dense quantum plasma is

$$P_1 = 2T_{\mathrm{F}}n_1, \tag{5}$$

with $T_{\mathrm{F}} = (\hbar^2/2m)(3\pi^2)^{2/3}n_0^{2/3}$ the Fermi electron temperature [7], and $n_1(\ll n_0)$ the small electron density perturbation in our non-relativistic quantum electron–ion magnetoplasma. It is obtained from the Poisson equation

$$n_1 = -(1/4\pi e)\nabla\cdot\mathbf{E}. \tag{6}$$

The quantum force acting on the electron fluid is [8, 9]

$$\mathbf{F}_Q = \frac{\hbar^2}{4mn_0}\nabla\nabla^2 n_1 - \eta(\alpha)\mu_{\mathrm{B}}\nabla B_z, \tag{7}$$

where the first and second terms on the right-hand side of (7) are associated with the quantum Bohm potential [10] and the magnetization energy of the electrons due to the electron-1/2 spin effect [11], respectively. The Langevin parameter [12] $\eta(\alpha) = 2\tanh(\alpha)$ accounts for the macroscopic magnetization of the electrons owing to the thermal motion and electron–electron collisions. Here $\alpha = \mu_{\mathrm{B}}B_0/T_F$, where $\mu_{\mathrm{B}} = e\hbar/2mc$ is the magnitude of the electron magnetic moment (Bohr magneton). The parallel (to $\hat{\mathbf{z}}$) component of the wave magnetic field is denoted by B_z.

From (1) and (2) we obtain the modified EX–EM wave equation

$$(1-\beta_x)\left(\frac{\partial^2 \mathbf{E}_\perp}{\partial t^2} - c^2\nabla^2\mathbf{E}_\perp + c^2\nabla_\perp\nabla\cdot\mathbf{E}\right) - 4\pi n_0 e\frac{\partial \mathbf{u}_\perp}{\partial t} = 0, \tag{8}$$

where $\beta_x = (2\alpha/45\pi E_c^2)[B_{0z}^2 + E_\perp^2(n^2-1)]$, $E_\perp^2 = E_x^2 + E_y^2$, $\perp$ denotes components transverse to $\hat{\mathbf{z}}$, $\mathbf{E}_\perp = (E_x, E_y, 0)$, and E_x (E_y) is the x (y) component of the wave electric field perpendicular to $\hat{\mathbf{z}}$.

In the following, we first focus on wave propagation along the x-axis. Thus, $\nabla = (\partial/\partial x, 0, 0)$. Following [8] we then obtain, from (4) and (8),

$$\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial x^2} + \Omega_p^2\right)\left[\frac{\partial^2}{\partial t^2} + \Omega_H^2 - V_F^2\left(1 - \lambda_q^2\frac{\partial^2}{\partial x^2}\right)\right]E_y$$
$$-\Omega_p^2\omega_c^2\left[1 - \eta(\alpha)\lambda_B^2\frac{\partial^2}{\partial x^2}\right]E_y = 0, \tag{9}$$

where $\Omega_H = (\Omega_p^2 + \omega_c^2)^{1/2}$, $\Omega_p^2 = \omega_p^2/(1-\beta_x)$, $\omega_p = (4\pi n_0 e^2/m)^{1/2}$, $\omega_c = eB_0/mc$, $V_F = (2T_F/m)^{1/2}$, $\lambda_B = \sqrt{\hbar/2m\omega_c}$, and $\lambda_q = \hbar/2mV_F$.

Supposing that E_y is proportional to $\exp(ikx - i\omega t)$, where k and ω are the wavenumber and the frequency, respectively, we Fourier transform (9) to obtain the dispersion relation for the EX–EM waves

$$(1-\beta_x)(n^2-1) + \frac{\omega_p^2}{\omega^2} + \frac{\omega_p^2\omega_c^2[1+\eta(\alpha)k^2\lambda_B^2]}{\omega^2[\omega^2 - \Omega_H^2 - k^2V_F^2(1+k^2\lambda_q^2)]} = 0. \tag{10}$$

Two comments are in order. First, in the absence of the QED effects we have $\beta_x = 0$ and (10) then reproduces the results of [8]. Second, in the absence of the quantum forces, (10) yields

$$(1-\beta_x)(n^2-1) + \frac{\omega_p^2(\omega^2 - \Omega_p^2)}{\omega^2(\omega^2 - \Omega_p^2 - \omega_c^2)} = 0. \tag{11}$$

Equation (11) reveals that the cut-off frequency (at which $k = 0$) in our QED plasma is obtained from

$$\omega = \pm\frac{\omega_c}{2} \pm \frac{1}{2}\left[\omega_c^2 + \frac{4\omega_p^2}{(1-\beta_{x0})}\right]^{1/2}, \tag{12}$$

where $\beta_{x0} = (2\alpha/45\pi E_c^2)(B_{0z}^2 - E_\perp^2)$.

Next, we consider the O-mode radiation [13] for which $\mathbf{E} = \hat{\mathbf{z}}E_z$ and $\mathbf{u} = \hat{\mathbf{z}}u_z$. There are no density and parallel (to $\hat{\mathbf{z}}$) magnetic field fluctuations associated with the linearly polarized O-mode radiation. Thus, from (1), (2), and (4) we have

$$\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial x^2} + \Omega_p^2\right)E_z = 0, \tag{13}$$

where $\Omega_p^2 = \omega_p^2/(1-\beta_0)$, and $\beta_0 = (2\alpha/45\pi E_c^2)[B_{0z}^2 + E_z^2(n^2-1)]$. We observe that the quantum forces do not affect the O-mode radiation.

Supposing that E_z is proportional to $\exp(ikx - i\omega t)$, we Fourier transform (13) to obtain the dispersion relation for the O-mode radiation

$$(1-\beta_0)(n^2-1) + \frac{\omega_p^2}{\omega^2} = 0. \tag{14}$$

Equation (14) reveals that the cut-off frequency for the linearly polarized O-mode radiation in our QED plasma is

$$\omega = \pm \frac{\omega_{\mathrm{p}}}{(1-\beta_{0O})^{1/2}}, \tag{15}$$

where $\beta_{0O} = (2\alpha/45\pi E_{\mathrm{c}}^2)(B_{0z}^2 - E_z^2)$.

To summarize, we have derived the general dispersion relations for high-frequency elliptically polarized extraordinary and linearly polarized ordinary electromagnetic waves in a dense quantum magnetoplasma accounting for the quantum forces (involving electron tunneling and electron-1/2 spin effects) and vacuum polarization effects. The plasma and vacuum polarization currents have been retained on an equal footing. It is found that vacuum polarization effects significantly modify the electron plasma frequency. Accordingly, the cut-off frequencies strongly depend on the external magnetic and wave electric fields. This conclusion should be of much interest for the propagation characteristics of the high-frequency EM waves in dense magnetoplasmas, such as those in magnetars [14–16], as well as in the next generation of intense laser–solid density plasma interaction experiments [1, 17–19] and free-electron laser schemes [20, 21] in which quantum vacuum and electron degeneracy can play an important role.

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