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A new insight into the duality between serial and parallel non-redundant and redundant manipulators

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SUMMARY

This paper describes detailed velocity kinematics and the statics of serial and parallel robots. Without resorting to the screw theory, the duality between these two types of robots is demonstrated. This duality concerns operational speeds of serial robots and operational forces of parallel robots, as well as internal speeds in redundant serial robots and internal forces in redundant parallel robots. This approach allows a deeper understanding of the duality between these two types of robots, particularly when they are redundant.

KEYWORDS: Kinetostatics; Duality; Redundancy; Parallel robots

1. INTRODUCTION

In robotic textbooks, relations describing the position and velocity kinematics and the statics of serial and parallel robots are mostly described separately.[1] Although this approach allows the use of an adapted methodology to handle their respective models, these relations can appear quite different for these two types of robots and no similarities between them can be demonstrated.

A strong duality between serial and parallel robots exists however, as stated by the work of many authors.[2–10] Although some similarities between their topologies or their position kinematics can be shown when considering particular geometries,[4,7] this duality concerns essentially the velocity kinematics of serial robots and the statics of parallel manipulators. Based on the screw theory, it is possible to demonstrate that the direct kinematic model of serial robots and the direct static model of parallel robots are dual. These models can always be written immediately even for redundant robots. But to write the inverse models, redundancy must be taken into account. In the literature, the latter models are not written with the same level of detail as in robotic textbooks where the two types of robots are however handled separately. Only non-redundant robots are considered in some references,[3,4,8,9] while in others redundancy is solved in a particular way, either by means of pseudo inverse[5] or weighted pseudo inverse methods.[10]

In this paper, a synthesis of these two approaches is proposed. Section 2 introduces the notations used further and Section 3 studiesthe notion of singularity. Relations describing the velocity kinematics and the statics are then written in an alternative compact matrix form based on relations that are always physically relevant. Serial robots are discussed in Section 4, while parallel robots are discussed in Section 5. In these sections, redundancy is handled in the most general way. Finally, the duality between these two types of robots is demonstrated in Section 6. This duality holds even for redundant robots independently of the way used to solve the redundancy. It concerns operational speeds of serial robots and operational forces of parallel robots, as well as internal speeds in redundant serial robots and internal forces in redundant parallel robots; the latter property has never been clearly demonstrated before.

It is, however, of particular value to help finding new control modes for redundant robots. In fact, several interesting control modes have been developed for redundant serial robots and, while redundant parallel robots have not yet received the same attention, numerous control modes have also been developed for grasping problems that can be considered as redundant parallel robots. The duality outlined in this paper should be used as a guide to apply the same type of solutions found for one type of redundant robots to the dual type of manipulators, leading to improved performance or functionalities for both types of robots.

2. NOTATIONS

A robot arm is composed of an articulated mechanical system linking a fixed base to a mobile end effector. This system allows to control the position and/or the orientation of the end effector with respect to the base. Two types of robots are considered, serial robots and parallel robots.

The joint space of the manipulator (or configuration space) is defined as the space in which all the actuated joint variables are represented. This space is of dimension n equal to the total number of actuators. The operational space is the space in which the situation of the end effector is represented (position and/or orientation). This space is of dimension m ($m \leq n$ and $m \leq 6$).

A robot is redundant when the dimension of the joint space n (equal to the number of actuated joints) is strictly greater than the dimension of the operational space m (equal to the end effector's number of degrees-of-freedom). The degree of redundancy is defined as $r = n - m$.

An example of redundant serial robot is illustrated on Figure 1. The four revolute joints are actuated and the degree of redundancy is equal to 1 to position and orientation of the end effector in the plane. This redundancy allows different joint configurations for the robot in the

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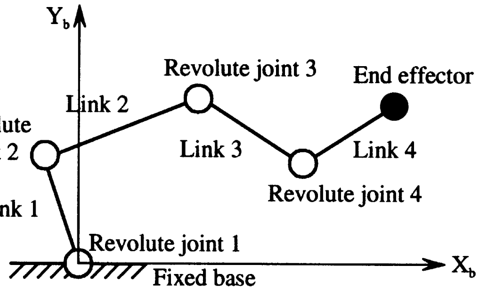

Fig. 1. Serial arm kinematically redundant.

same operational configuration. In other words, it allows the robot to perform internal movements, hence the name of kinematic redundancy.

An example of redundant parallel robot is illustrated in Figure 2. The revolute joints at the base of both chains and the prismatic joints are actuated and the degree of redundancy is equal to 1 to position and orientation of the end effector in the plane. Note that both chains situated between the base and the end effector are not kinematically redundant. Thus no internal movement can appear in any one of these chains when the end effector remains fixed. The redundancy comes from an over actuation intended to avoid parallel singularities. It allows the robot to apply internal efforts, hence the name of static redundancy.

In this paper, only serial arms kinematically redundant or not and parallel arms statically redundant or not will be considered. We will use the same notations to describe these two types of robots. The joint variables, speeds and torques will be noted $q_{art}=[q_1\ q_2\ \dots\ q_n]$, $\dot{q}_{art}=[\dot{q}_1\ \dot{q}_2\ \dots\ \dot{q}_n]$ and $\tau_{art}=[\tau_1\ \tau_2\ \dots\ \tau_n]$, the operational variables, speeds and forces will be noted $X=[x_1\ x_2\ \dots\ x_m]$, $V=[V_1\ V_2\ \dots\ V_m]$ and $F=[F_1\ F_2\ \dots\ F_m]$.

3. KINETOSTATIC MODELS

The kinematic model is used to study speed transmissions in a manipulator. It allows to compute the operational speed from the joint speeds and *vice-versa*. The static model is used to study force transmissions in a manipulator. It allows to compute the operational forces and torques from the joint torques and *vice-versa*. Generally, these models depend on the operational configuration of the robot therefore on the joint variables. However, we will omit to mention it in order to simplify the notations.

In some configurations, the robot cannot be fully controlled. These configurations are called singular configurations or singularities. The robotics community agrees to define physically two types of singularities:

- Serial singularities: in such configurations, movements of the end effector cannot be controlled in certain directions. This restriction of the end effector's movement comes along with an internal movement in the robot. Serial singularities can appear in serial robots or in substructures of parallel robots.
- Parallel singularities: in such configurations, forces applied in certain directions on the end effector cannot be sustained by the actuators. Thus, movements of the end effector cannot be blocked in certain directions. These uncontrolled end effector's movements are associated with internal forces in the bodies of the robot. Parallel singularities can only appear in parallel robots. As they depend on the number of actuated joints, many designers over-actuate parallel robots.

Out of the singular configurations, no uncontrolled movement is possible and kinematic and static models are invertible, provided if needed that redundancy is taken into account.

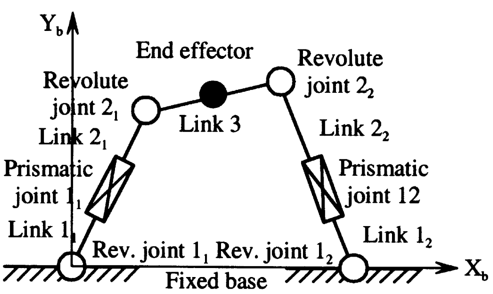

Fig. 2. Parallel arm statically redundant.

4. SERIAL ROBOTS

4.1. Statics

Serial robots are composed of an open kinematic chain without any closed loop. In such conditions, joint torques τ_{art} can always be computed from operational forces F by writing the equilibrium condition for each of the robot's body. Thus, statics are considered first and the inverse static model is written under the following form:

$$\tau_{art}=J^T_{art\_s}\,.\,F \qquad \text{with } J^T_{art\_s} \text{ an } n\times m \text{ matrix} \tag{1}$$

For a non-redundant robot (when $n=m$), system (1) has the same number of equations as unknowns. Thus, it can be inverted out of the singularities to obtain the direct static model:

$$F=G^T_{art\_s}\,.\,\tau_{art} \qquad \text{with } G^T_{art\_s}=J^{T\ -1}_{art\_s} \text{ an } n\times n \text{ matrix} \tag{2}$$

But for a redundant robot, system (1) has more equations n than unknowns m (the unknowns are the operational forces F_i). Therefore, when matrix $J^T_{art\_s}$ is full rank (equal to m), there are necessarily $r=n-m$ linear relations of compatibility between the joint torques τ_i. These relations can be written:

$$E_{art\_s}\,.\,\tau_{art}=0 \qquad \text{with } E_{art\_s} \text{ an } r\times n \text{ matrix} \tag{3}$$

The static constraint matrix E_{art_s} can be computed by writing that the r characteristic determinants of system (1) are equal to zero.

A solution for the over determined system (1) is given by relation (4) that constitutes the direct static model. The interest of the choice of $G_{art\_s}=J^T_{art\_s}\,.\,(J_{art\_s}\,.\,J^T_{art\_s})^{-1}$ is explained further.

$$F=G^T_{art\_s}\,.\,\tau_{art}$$
$$\text{with } G^T_{art\_s}=(J_{art\_s}\,.\,J^T_{art\_s})^{-1}\,.\,J_{art\_s} \text{ an } m\times n \text{ matrix} \tag{4}$$

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4.2. *Kinematics*

Kinematic models are written based on previous results. Under the assumption of perfect joints without friction, the *Principle of Virtual Work* states that the power developed by the joint torques equals the power developed by the operational forces, that is:

$$V^T . F = \dot{q}^T_{art} . \tau_{art} \quad (5)$$

Taking into account relations (1) and (5) and considering the independency of operational forces F_i, the direct kinematic model is obtained as follows:

$$V = J_{art\_s} . \dot{q}_{art} \quad (6)$$

For a non-redundant robot, the elements of τ_{art} are independent. Thus, taking into account relations (2) and (5), the inverse kinematic model is obtained as follows:

$$\dot{q}_{art} = G_{art\_s} . V \qquad \text{with } G^T_{art\_s} = J^{T}_{art\_s}{}^{-1} \text{ an } n \times n \text{ matrix} \quad (7)$$

But for a redundant robot, the elements of τ_{art} are no more independent (they verify the relation (3)). Therefore, relation (7) doesn't hold. In this case, the *Principle of Virtual Work* and Lagrange's multipliers are used to solve the problem. The *Principle of Virtual Work* is written $(V^T . G^T_{art\_s} - \dot{q}^T_{art}) . \tau_{art} = 0$. This relation states that the scalar product between the vector $B^T_{art\_s} = (V^T . G^T_{art\_s} - \dot{q}^T_{art})$ and the vector τ_{art} is equal to zero while relation (3) states that the r scalar products between the vector τ_{art} and the rows of matrix E_{art_s} are equal to zero. The vector B_{art_s} and the r rows of matrix E_{art_s} are thus orthogonal to the same vector of the joint space. They are thus linearly dependent via r Lagrange's multipliers that will be grouped in a vector V_{int}. This dependency can be written $B_{art\_s} + E^T_{art\_s} . V_{\text{int}} = 0$. With this expression of B_{art_s}, the inverse kinematic model is obtained as follows:

$$\dot{q}_{art} = G_{art\_s} . V + E^T_{art\_s} . V_{\text{int}} \quad (8)$$

When the operational speed is null, joint speeds depend only upon V_{int}. Thus joint displacements appear when V_{int} varies even if the end effector remains fixed. Vector V_{int} is therefore called a vector of internal speeds. Note that internal speeds V_{int} in relation (8) are arbitrary. To solve the redundancy, we must compute these parameters. If this is done by considering $V_{\text{int}} = 0$, relation (8) becomes $\dot{q}_{art} = G_{art\_s} . V$. This solution minimises the norm of joint speeds because G_{art_s} is the pseudo inverse of J_{art_s}.

4.3. *Noticeable relations*

Taking into account relations (1) and (4) and considering the independence of operational efforts F_i, we obtain ($I_{m \times m}$ *is the* $m \times m$ identity matrix):

$$J_{art\_s} . G_{art\_s} = I_{m \times m} \quad (9)$$

This relation remains always true, whatever the form of G_{art_s}. It is, in particular, not necessary that $G_{art\_s} = J^T_{art\_s} . (J_{art\_s} . J^T_{art\_s})^{-1}$.

Conversely, taking into account relations (1) and (3) and considering the independence of operational efforts F_i, we obtain:

$$J_{art\_s} . E^T_{art\_s} = 0 \quad (10)$$

The columns of matrix $E^T_{art\_s}$ are thus in the kernel of J_{art_s}. They constitute a base of this kernel. If needed, this base can be replaced by one whose vectors are orthogonal and eventually normalized to unity. In this last case, we obtain $E_{art\_s} . E^T_{art\_s} = 1_{r \times r}$ ($1_{r \times r}$ is the r $\times$ r identity matrix).

Relation (10) allows us to introduce an alternative method to get the inverse kinematic model. Taking into account that $E_{art\_s} . G_{art\_s} = 0$ when G_{art_s} is the pseudo inverse of J_{art_s} and multiplying relation (8) by E_{art_s}, we obtain:

$$V_{\text{int}} = (E_{art\_s} . E^T_{art\_s})^{-1} . E_{art\_s} . \dot{q}_{art} \quad (11)$$

Building, as usually found in the literature,[11,12] an augmented operational speed vector composed of operational speeds V and internal speeds V_{int}, we obtain:

$$V_a = \begin{bmatrix} V \\ V_{\text{int}} \end{bmatrix} = \begin{bmatrix} J_{art\_s} \\ (E_{art\_s} . E^T_{art\_s})^{-1} . E_{art\_s} \end{bmatrix} . \dot{q}_{art} = J_{a\_art\_s} . \dot{q}_{art} \quad (12)$$

$$\dot{q}_{art} = J^{-1}_{a\_art\_s} . V_a \quad (13)$$

The simplicity of relation (11) is obtained thanks to the fact that G_{art_s} is the pseudo inverse of J_{art_s}. The relation $E_{art\_s} . G_{art\_s} = 0$ does not hold for another choice of G_{art_s}. In this case, it would have been necessary that the internal speed depends on V (that is internal speeds depend on end effector's operational speeds) to find a relation of the same form as relation (8). Hence the interest of the choice of the pseudo inverse for G_{art_s}. Beware, however, that this solution, that minimises the norm of joint speeds, is only adapted when all joints are of the same type. If not, it is meaningless.

5. PARALLEL ROBOTS

5.1. *Kinematics*

Parallel robots are composed of a set of open kinematic chains arranged in parallel and connecting a base to an end-effector. In the literature, each chain has mostly only one actuated joint that is mainly prismatic, and is mostly connected to the base and to the end effector by a universal joint and a spherical joint, respectively. The most famous example is the *Gough Stewart Platform*. In such conditions, joint speeds $\dot{q}_{art}$ can always be computed from operational speeds V. Thus, kinematics are considered first and the inverse kinematic model is written under the following form:

$$\dot{q}_{art} = G_{art\_p} . V \qquad \text{with } G_{art\_p} \text{ an } n \times m \text{ matrix} \quad (14)$$

In this paper, we will use a more extensive definition of parallel robots. In fact, each chain can have any number of actuated joints, providing that it is not kinematically redundant. With this definition, relation (14) always holds. This definition allows, in particular, to handle parallel robot with serial linkages as a leg, as for example the platform device designed by professor Iwata.[13]

For a non-redundant robot (when $n = m$), system (14) has the same number of equations as unknowns. This system can thus be inverted out of the singularities to obtain the direct kinematic model:

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$$V = J_{art\_p} \cdot \dot{q}_{art} \qquad \text{with } J_{art\_p} = G_{art\_p}^{-1} \text{ an } n \times n \text{ matrix} \tag{15}$$

But for a redundant robot, system (14) has more equations n than unknowns m which are the operational speeds V_i. Therefore, when the rank of matrix G_{art_p} is full (equal to m), there exist necessarily $r = n - m$ linear relations of compatibility between the joint speeds $\dot{q}_i$. These relations can be written:

$$C_{art\_p} \cdot \dot{q}_{art} = 0 \qquad \text{with } C_{art\_p} \text{ an } r \times n \text{ matrix} \tag{16}$$

The kinematic constraint matrix C_{art_p}, as the static constraint matrix E_{art_s}, can be computed by writing that the r characteristic determinants of system (14) are equal to zero.

Relation (17), that gives a solution for the over determined system (14), constitutes a direct kinematic model. The choice of $J_{art\_p} = (G^T_{art\_p} \cdot G_{art\_p})^{-1} \cdot G^T_{art\_p}$ will be explained further.

$$V = J_{art\_p} \cdot \dot{q}_{art}$$

$$\text{with } J_{art\_p} = (G^T_{art\_p} \cdot G_{art\_p})^{-1} \cdot G^T_{art\_p} \text{ an } m \times n \text{ matrix} \tag{17}$$

5.2. *Statics*

Static models are written based on the results obtained for kinematic models. Under the assumption that the joints are frictionless, the *Principle of Virtual Work* states that:

$$V^T \cdot F = \dot{q}^T_{art} \cdot \tau_{art} \tag{18}$$

Taking into account relations (14) and (18) and considering the independence of operational speeds V_i, the direct static model is obtained as follows:

$$F = G^T_{art\_p} \cdot \tau_{art} \tag{19}$$

For a non-redundant robot, the elements of $\dot{q}_{art}$ are independent. Thus, taking into account relations (15) and (18), the inverse static model is obtained as follows:

$$\tau_{art} = J^T_{art\_p} \cdot F \qquad \text{with } J^T_{art\_p} = G^{T\,-1}_{art\_p} \text{ an } n \times n \text{ matrix} \tag{20}$$

But for a redundant robot, the relation (20) doesn't hold because the elements of $\dot{q}_{art}$ are no more independent (they verify the relation (16)). In this case, equation (18) is written $(F^T \cdot J_{art\_p} - \tau^T_{art}) \cdot \dot{q}_{art} = 0$. This relation states that the vector $B^T_{art\_p} = (F^T \cdot J_{art\_p} - \tau^T_{art})$ is orthogonal to the vector $\dot{q}_{art}$ which is also orthogonal to the r rows of matrix C_{art_p} (see equation (16)). The vector B_{art_p} and the r rows of matrix C_{art_p} are thus orthogonal to the same vector of the joint space. They belong to the same hyperplan of R^n and are thus linearly dependent via r Lagrange's multipliers that will be grouped in a vector F_{int}. This dependency can be written $B_{art\_p} + C^T_{art\_p} \cdot F_{\text{int}} = 0$. With this expression of B_{art_p}, we obtain the following expression of the inverse static model:

$$\tau_{art} = J^T_{art\_p} \cdot F + C^T_{art\_p} \cdot F_{\text{int}} \tag{21}$$

When operational forces are null, joint torques depend only upon F_{int}. Consequently, joint torques $\tau_{art} = C^T_{art\_p} \cdot F_{\text{int}}$ represent a system of auto equilibrated joint torques, as noted by Wen and Wilfinger.[14] Vector F_{int} is therefore called a vector of internal forces. Note that the elements of F_{int} in relation (21) are arbitrary. To solve the redundancy, we must compute these parameters. If $F_{\text{int}} = 0.$ is chosen, we obtain $\tau_{art} = J^T_{art\_p} \cdot F$. This solution minimises the norm of joint torques because $J^T_{art\_p}$ is the pseudo inverse of $G^T_{art\_p}$.

5.3. *Noticeable relations*

Taking into account relations (14) and (17) and considering the independence of operational speeds V_i, we obtain relation (22), in which $I_{m \times m}$ is the $m \times m$ identity matrix, and which holds whatever the form of J_{art_p}. It is, in particular, not necessary that $J_{art\_p} = (G^T_{art\_p} \cdot G_{art\_p})^{-1} \cdot G^T_{art\_p}$.

$$J_{art\_p} \cdot G_{art\_p} = I_{m \times m} \tag{22}$$

Conversely, taking into account relations (14) and (16) and considering the independence of the elements of the operational speed vector V, we obtain:

$$G^T_{art\_p} \cdot C^T_{art\_p} = 0 \tag{23}$$

The columns of matrix $C^T_{art\_p}$ are thus in the kernel of $G^T_{art\_p}$. They constitute a base of this kernel. If needed, this base can be replaced by one whose vectors are orthogonal and eventually normalized to unity. In this last case, we obtain $C_{art\_p} \cdot C^T_{art\_p} = 1_{r \times r}$ (where $1_{r \times r}$ is the $r \times r$ identity matrix).

Relation (23) allows us to introduce an alternative method to obtain the inverse static model. Taking into account that $C_{art\_p} \cdot J^T_{art\_p} = 0$ when $J^T_{art\_p}$ is the pseudo inverse of $G^T_{art\_p}$ and multiplying relation (21) by C_{art_p}, we get:

$$F_{\text{int}} = (C_{art\_p}, C^T_{art\_p})^{-1} \cdot C_{art\_p}, \tau_{art} \tag{24}$$

Building, as for the speeds of a redundant serial robots, an augmented operational force vector composed of operational forces F and internal forces F_{int}, we obtain:

$$F_a = \begin{bmatrix} F \\ F_{\text{int}} \end{bmatrix} = \begin{bmatrix} G^T_{art\_p} \\ (C_{art\_p}, C^T_{art\_p})^{-1}, C_{art\_p} \end{bmatrix} \cdot \tau_{art} = G^T_{a\_art\_p} \cdot \tau_{art} \tag{25}$$

$$\tau_{art} = G^{T\,-1}_{a\_art\_p} \cdot F_a \tag{26}$$

The interest of the pseudo inverse form of J_{art_p} appears here. This particular form leads to the relation C_{art_p}, $J^T_{art\_p} = 0$, which would not have been true for another form of J_{art_p}. This would not be necessarily annoying but to find a relation of the same form as relation (21), it would have been necessary that the internal forces depend on F, i.e. the internal forces are functions of the forces and torques applied on the end effector. Beware, however, that this solution, that minimises the norm of joint torques, is only adopted when all joints are of the same type.

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Table I. Duality between serial and parallel robots (1)

Kinematics of serial robots	Statics of parallel robots
$V=J_{art\_s}.\dot{q}_{art}$	$F=G^T_{art\_p}.\tau_{art}$
$\dot{q}_{art}=J^T_{art\_s}.(J_{art\_s}.J^T_{art\_s})^{-1}.V+E^T_{art\_s}.V_{\text{int}}$	$\tau_{art}=G_{art\_p}.(G^T_{art\_p}.G_{art\_p})^{-1}.F+C^T_{art\_p}.F_{\text{int}}$
$E_{art\_s}.\dot{q}_{art}=V_{\text{int}}$	$C_{art\_p}.\tau_{art}=F_{\text{int}}$
Redundancy resolution with minimum Euclidean norm:	Redundancy resolution with minimum Euclidean norm:
$V_{\text{int}}=0$ and $E_{art\_s}.\dot{q}_{art}=0$	$F_{\text{int}}=0$ and $C_{art\_p}.\tau_{art}=0$
Statics of serial robots	**Kinematics of parallel robots**
$\tau_{art}=J^T_{art\_s}.F$	$\dot{q}_{art}=G_{art\_p}.V$
$E_{art\_s}.\tau_{art}=0$	$C_{art\_p}.\dot{q}_{art}=0$
$F=(J_{art\_s}.J^T_{art\_s})^{-1}.J_{art\_s}.\tau_{art}$	$V=(G^T_{art\_p}.G_{art\_p})^{-1}.G^T_{art\_p}.\dot{q}_{art}$

6. DUALITY BETWEEN SERIAL AND PARALLEL MANIPULATORS

Relations (1) to (13) for serial robots are similar to relations (14) to (26) for parallel robots. These similarities are summarized in Table I.

Relations between speeds in serial robots and forces in parallel robots are of the same form and *vice versa*. This duality is summarized in Table II.

While some of these relations are classical, the duality between internal speeds in redundant serial robots and internal forces in redundant parallel robots, that is very general and does not depend on the method used to solve the redundancy, does not appear in the literature and is demonstrated here for the first time.

7. CONCLUSION

In this paper, kinematic and static models of kinematically redundant or not serial robots and of statically redundant or not parallel robots are written in great detail. It allows one to show a strong duality between operational speeds of serial robots and operational forces of parallel robots, as well as between internal speeds in redundant serial robots and internal forces in redundant parallel robots. This last property is clearly demonstrated for the first time.

We should point out that the models are given in the most general form, taking into account internal speeds in redundant serial robots and internal forces in redundant parallel robots. It allows one to handle any type of robots, even if they have different types of joints. The redundancy can then be solved in any way. The solution proposed here uses the notion of pseudo inverse that leads to the minimum Euclidean norm of joint speeds or joint torques. This solution is valuable only when all joints are of the same type. If not, redundancy could be solved either by controlling the internal movements or forces in the robot,[12] or the stiffness of the system,[10] or by using the notion of weighted pseudo inverse. However, this problem is beyond the scope of this paper.

The most important and newest feature of this article is the proof of duality between redundant serial and parallel manipulators. They can thus be handled in the same manner and solutions adopted to solve the duality in serial robots can be adopted to solve the duality in parallel robots and *vice versa*. This property is of particular value for the control of such structures. It should be used as a guide to apply the same types of solutions found for one type of redundant robots to the dual type of manipulators, leading to improved performance or functionalities.

Table II: Duality between serial and parallel robots (2)

Serial robot		Parallel robot
V, F	⇔	F, V
$\dot{q}_{art}$, τ_{art}	⇔	τ_{art}, $\dot{q}_{art}$
J_{art_s}	⇔	$G^T_{art\_p}$
E_{art_s}	⇔	C_{art_p}
V_{int}	⇔	F_{int}

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