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# CONTRACTIONARY TECHNOLOGY SHOCKS

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This paper adds to the large body of literature on the effects of technology shocks empirically and theoretically. Using a structural vector error correction model, we first provide evidence that not only hours but also investment decline temporarily following a technology improvement. This result is robust to important data and identification issues addressed in the literature. We then show that the negative response of inputs is consistent with an estimated monetary model in which the presence of strategic complementarity in price setting, in addition to nominal rigidities, lowers the sensitivity of prices to marginal costs, and monetary policy does not fully accommodate the shock.

Keywords: Technology Shocks, Inputs Dynamics, Structural Vector Error Correction Model, New Keynesian DSGE Model, Bayesian Inference

# 1. INTRODUCTION

A large body of literature has challenged the empirical relevance of the concept of technology-driven business cycles. The shift of interest from the analysis of sample correlations among macroeconomic time series to the analysis of their conditional counterparts has identified a countercyclical behavior of factor inputs following a technology shock. This result is apparently at odds with the predictions of a broad class of business cycle models that envisage technology shocks as one of the main determinants of the observed procyclical dynamics of factor inputs.

The idea that technology improvements can have contractionary effects is supported by several empirical studies that mainly focus on the emergence of a negative conditional correlation between productivity and hours worked in structural vector autoregressions (SVARs) identified with long-run restrictions [Galì (1999); Francis and Ramey (2005); Pesavento and Rossi (2005)].

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However, with the exception of Basu et al. (2006), who use a purified measure of technology in two-variable VARs, and Giuli and Tancioni (2012), who estimate a monetary model characterized by a particularly flat New Keynesian Phillips curve (NKPC), the evidence of an additional negative short-term response in investment has not generally been established in the empirical and theoretical literature on business cycles.

This paper develops this literature from both an empirical and a theoretical perspective. First, using a medium-scale structural vector error correction (SVEC) model, we show that both hours and investment respond negatively to a positive technology shock and that this result is robust with respect to the main control dimensions addressed in the literature. Second, we demonstrate that the negative response of inputs is consistent with an estimated monetary model in which (i) the presence of strategic complementarities in price setting, in addition to nominal rigidities, lowers the sensitivity of prices to marginal costs and (ii) monetary policy does not fully accommodate the technology shock.

Related to the main empirical findings, our results are not new. Our novel contribution is that the puzzling evidence in Basu et al. (2006) can be robustly replicated from a more standard empirical perspective and can be theoretically explained by a reasonably parameterized monetary model encompassing a number of alternative theoretical explanations in the literature.

The evidence on the contractionary effects of productivity improvements is highly debated among macroeconomists. The empirical controversy relates to the identifiability of technology shocks within the long-run SVAR approach when low-frequency movements in productivity [Fernald (2007)] and hours [Canova et al. (2010)] are present and to the role of monetary policy in the accommodation of shocks affecting the unobserved output potential [Galí et al. (2003)].

The empirical evaluation of structural macro-models also reaches conflicting conclusions. Basing the calibration of a monetary model on an impulse response matching strategy, Altig et al. (2011) obtain a positive short-term response of both factor inputs. Del Negro et al. (2005) and Smets and Wouters (2007) estimate monetary models with nominal and real frictions and find that whereas the response of hours is negative, that of investment is positive.<sup>1</sup>

From a theoretical point of view, different explanations of the contractionary effects of technology shocks have been proposed. Galì (1999) suggests a sticky price explanation based on a model in which monetary authorities adopt a partially exogenous money supply rule so that, following a productivity improvement, the weak response of real balances constrains the demand expansion, leading to a reduction in the use of labor.

Francis and Ramey (2005) show that a negative response in hours (but not in investment) is obtained in a flexible price model with real demand rigidities modeled in the form of consumption habits and capital adjustment costs. This is also Smets and Wouters's (2007) preferred interpretation for the negative response in hours.

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Lindé (2009) shows that the negative correlation between output, hours, and investment can emerge in a baseline RBC model in which the permanent technology shock is autocorrelated in growth rates. Under this hypothesis, the temporary contraction in inputs is due to the interaction of wealth and intertemporal substitution effects stemming from the expected increase in productivity.

Schmitt-Grohé and Uribe (2011) show that a flexible price model in which the common stochastic trend is driven by both neutral and investment-specific productivity shocks is also consistent with a temporary contraction of inputs following an investment-specific shock.<sup>2</sup>

Basu et al. (2006) take into account these different explanations and conclude that standard sticky price models, in which monetary policy follows a non-fully-accommodative rule, can account for the negative response in both hours and investment better than the alternative explanations. However, they do not support their argument with an analytical monetary model.<sup>3</sup>

Overall, there is no clear consensus on the robustness of the empirical results, and the competing theoretical explanations lack substantial empirical testing from a comparative perspective.

The analysis is organized in two stages. We first estimate a SVEC model using unprocessed data for per capita real variables and considering hours as stationary. The SVEC is specified to approximate a fairly general monetary model subject to permanent technology shocks. In particular, the explicit consideration of the stationary relations among nonstationary variables favors the separation between permanent and transitory components, thus improving the identifiability of the technology shock. The resulting cointegrating vectors are the theory-based stationary ratios among real and monetary variables, and the long-run effects matrix is restricted by imposing the hypothesis that only technology shocks can have permanent effects on per capita output.

The SVEC analysis confirms the existence of the short-term contractionary effects of productivity improvements. This finding provides new evidence that calls into question the ability of technology shocks to explain the unconditional procyclicality of investment and hours. Moreover, the responses of inflation and of the nominal interest rate signal that the monetary authority does not fully accommodate the shock.

From the robustness checks we find that, contrary to Fernald (2007) and Canova et al. (2010), under the SVEC specification, the consideration of breaks in productivity and hours is not crucial for the results in either a precrisis sample that is fully consistent with the information used in previous analyses or an extended sample including the recent economic contraction. Our evidence holds even considering exact balanced growth, a feature that, although standard in general equilibrium models, is not supported by the data. We also show that the use of alternative long-run identification strategies does not alter the main conclusions of our investigation.

The second stage of our analysis provides a theoretical interpretation of the SVEC evidence to be confronted with the data and with alternative explanations.

We set up and estimate a monetary model that can replicate the different empirical results and the different theoretical interpretations provided by the literature. In addition to the real and nominal rigidities that characterize standard monetary models, we assume that capital is firm-specific [Sveen and Weinke (2005); Woodford (2005); Altig et al. (2011)] and that the demand elasticity among differentiated goods is endogenous [Eichenbaum and Fisher (2007); Smets and Wouters (2007)].

Under these additional hypotheses, the slope of the NKPC depends not only on the frequency with which firms are allowed to reset their prices but also on the degree of strategic complementarities in price setting [Woodford (2005)]. This additional element lowers the slope of the NKPC, weakening the sensitivity of price inflation to variations in the marginal cost. The economic rationale for this result is that because capital must be accumulated by the firm, marginal costs are firm-specific and the incentive to cut prices following a productivity improvement is partially counterbalanced by the expected increase in marginal costs due to the expected increase in demand.

The importance of the aggregate demand response to productivity improvements highlights the role played by monetary policy. It is well known that for a wide class of monetary models, a policy rule that responds to the theory-consistent output gap can approximate the optimal policy, namely, the one that would minimize the volatility of the target variables around their natural levels. However, the implementation of this rule in real-life operations requires knowledge of the natural rate of interest or of the level of potential output, which is not within a monetary authority's information set. For this reason, we consider two alternative contemporaneous rules—one targeting output deviations from its trend (i.e., an "empirical" rule) and the other targeting the theory-based output gap—and let the data indicate which is the more empirically relevant.

The theoretical model is estimated with Bayesian techniques, counterfactually elicitating a prior parameterization for which the model does not replicate the SVEC-based evidence on the negative conditional correlation between per capita output and investment. We show that the dynamics of the estimated model is qualitatively similar to that produced by the SVEC.

As a further check of the validity of the SVEC analysis, we generate samples of artificial data by stochastically simulating the models at the posterior estimates and repeat the SVEC analysis. The IRFs show that our specification and identification strategies are able to replicate the dynamic properties of the true data-generating processes.

Our estimates also show that neither real rigidities in consumption and investment nor intertemporal substitution effects originating in expected productivity improvements are sufficient to explain the empirical evidence. Although an impulse response matching experiment shows that a flexible price version of the model is able to replicate the negative conditional correlation of labor and investment, direct estimates of the restricted models provide strong evidence in support of a New Keynesian interpretation. On one hand, the evidence in favor of the empirical rule indicates that the policy response to productivity improvements is not fully accommodative. On the other hand, a flat slope of the NKPC signals the presence of relevant rigidities in price setting that cannot be attributed exclusively to nominal rigidities, because this would imply a degree of price stickiness that is at odds with the evidence on the frequency of price optimization at the firm level. Firm-specific capital and endogenous demand elasticity are crucial to obtain a plausible estimate of the degree of price stickiness.

The paper is organized as follows. Section 2 presents the hypotheses and the results of the SVEC analysis. Section 3 presents a monetary DSGE model with firm-specific capital and endogenous demand elasticity. Section 4 provides details of the Bayesian estimates of the model. Section 5 discusses the results in light of the different theoretical explanations advanced in the literature. Section 6 concludes.

## 2. SVEC-BASED EVIDENCE

The empirical literature on the productivity–employment puzzle reaches conflicting conclusions on whether hours worked rise or fall after a productivity improvement. A controversial issue is whether hours should be assumed to be stationary or nonstationary in empirical trials [Christiano et al. (2004); Pesavento and Rossi (2005); Fernald (2007); Gil-Alana and Moreno (2009); Canova et al. (2010); Lovcha and Perez-Laborda (2015)]. The empirical controversy on the role of technology in macroeconomic dynamics shows that the imposition of long-run restrictions on highly persistent series (such as hours), or the presence of regime breaks, may lead to a problematic identification of the technology shock.

Closely related to the debate on the identifiability of technology shocks within the SVAR approach is the methodology detailed in Basu et al. (2006), which avoids these identification difficulties by using VARs in which a "direct" measure of technology is considered in the place of average productivity. Employing this alternative strategy, the authors find that the short-term responses of both hours and investment are negative conditional on productivity improvements. However, even though this approach has the advantage of eliminating the estimation biases induced by aggregation problems and the presence of low-frequency components in hours and productivity, this comes at the cost of adopting a complex methodology for the derivation of the technology measure.<sup>4</sup>

In this section we show that by adopting a SVEC representation, we can provide empirical evidence that robustly confirms Basu et al.'s (2006) conclusions even without considering processed data for productivity and regime-shift control dummies.

The choice of an SVEC specification has two major advantages. First, with respect to SVARs, it provides a more adequate approximation to a large class of structural models that predict long-run balanced growth and differential dynamic adjustments in real and monetary variables. Second, the explicit consideration of the stationary relations described by the cointegrating vectors (CVs) in the SVEC improves the identifiability of the technology shock, because the presence of linear relations that satisfy the stationarity requirements enhances the separation

between permanent and transitory components [Harvey and Stock (1988); King et al. (1991)].

In the following section we propose a benchmark SVEC and then evaluate the robustness of the results with respect to alternative samples, specifications of the long-run relations and identification schemes.

# 2.1. Data and the Baseline SVEC Model

The VEC model is estimated using U.S. quarterly time series for real per capita output  $(y_t)$ , consumption of nondurable goods and services  $(c_t)$  and fixed investment  $(i_t$ —which includes consumption of durable goods), hourly labor compensation  $(wr_t)$ , inflation  $(\pi_t)$ , per capita hours worked  $(h_t)$ , and the short-term nominal interest rate  $(r_t)$ . The civilian noninstitutional population aged 16 and older is used as a normalizing variable. The statistical sources and data transformations are fully consistent with the information used in other empirical trials in the literature [e.g, Smets and Wouters (2007); Canova et al. (2010); Altig et al. (2011)]. The reference sample period considers a large time span ranging from 1948:1 to 2008:4, extended to 2014:3 for a robustness check. A detailed description of the data and their manipulations is provided in Table A.1 in the Appendix.

From the Phillips–Perron (PP) and KPSS tests, we find that real variables are all I(1) in levels irrespective of how the deterministic components are specified. Price inflation and per capita hours are shown to be I(0) according to both the PP unit root test and the KPSS test for stationarity. The tests are inconclusive with respect to the nominal interest rate, resulting in I(1) according to the PP test and I(0) according to the KPSS test. The results are summarized in Table B.1 in the Appendix.

On the basis of the very weak decay of the autocorrelation function for inflation and the interest rate, we assume that both monetary variables,  $r_t$  and  $\pi_t$ , are I(1). The dependence of results on this latter hypothesis is evaluated in the robustness checks.

A convenient structural formulation of the *m*-dimensional VEC representation for the endogenous variables  $\mathbf{x}'_t = [y_t \ \pi_t \ h_t \ r_t \ c_t \ i_t \ w_t]$  can be specified by assuming no contemporaneous correlations among variables in the SVEC,

$$\Gamma(L) \Delta \mathbf{x}_t = \Pi \mathbf{x}_{t-1} + \mathbf{B}\varepsilon_t, \tag{1}$$

where  $\Gamma(L) = \Gamma_0 - \Gamma_1 L - ... - \Gamma_{p-1} L^{p-1}$  are structural coefficient matrices and  $\Gamma_0 = \mathbf{I}_m$ . Under this hypothesis, **B** contains the contemporaneous structure of the system, which is thus modeled in the stochastic component. The long-run relations matrix  $\Pi$ , in the presence of cointegration, is a reduced-rank matrix and can be decomposed as  $\Pi = \alpha \beta'$ , where  $\alpha$  and  $\beta$  are  $m \times r$  full-column-rank matrices containing, respectively, the loading coefficients and the *r* cointegrating vectors. The vector of disturbances  $\varepsilon_t \sim (\mathbf{0}, \mathbf{I}_m)$  contains the orthonormal structural

innovations. The system of linear equations relating the estimated reducedform errors  $\mathbf{u}_t$  to the structural shocks is thus  $\mathbf{u}_t = \mathbf{B}\varepsilon_t$ , which implies that  $\Omega = \mathbf{u}\mathbf{u}' = \mathbf{B}\mathbf{B}'$ .

#### 2.2. Long-Run Components and CI Space

We impose a third-order memory for the starting VAR, a lag order that ensures serially uncorrelated errors. Considering a VEC structure with unrestricted constants, the LR trace test indicates the presence of five stationary components at the 90% significance level. The results are basically unaffected for lower and higher lag order specifications of the VAR. According to this evidence, the system is driven by two permanent components and five transitory shocks. The rank test results are summarized in Table B.2 in the Appendix.

We assume that the permanent component observed in the four real variables  $y_t$ ,  $c_t$ ,  $i_t$ , and  $w_t$  is due to the stochastic trend in technology [King et al. (1991); Pesaran and Smith (1995); Garratt et al. (2003)] and that the permanent component observed in the monetary variables  $r_t$  and  $\pi_t$  is due to the way the central bank adjusts its policy target [Vlaar (2004)].<sup>5</sup> Among other checks, in the robustness analysis we will evaluate the effects of considering inflation and the nominal interest rates stationary, as predicted by standard monetary models.

In terms of the variables' ordering, the first CI relation in  $\beta'$  defines stationary hours; that is, we impose a CI relation in which only  $h_t$  enters the corresponding CV. The second CI relation defines the Fisher interest parity, namely, the stationary real interest rate  $(r_t - \beta_{22}\pi_t)$ . The last three CVs define the stationary "great ratios" of the economy,  $c_t - \beta_{31}y_t$ ,  $i_t - \beta_{41}y_t$ , and  $w_t - \beta_{51}y_t$ .

The estimated coefficients of the great ratios are all significant but only marginally consistent with the hypothesis of exact balanced growth, i.e., with the restriction  $\beta_{31} = \beta_{41} = \beta_{51} = -1$ :

$$\beta' = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -3.7 & 0 & 1 & 0 & 0 \\ -1.2 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1.1 & 0 & 0 & 0 & 0 & 1 & 0 \\ -0.9 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Consistent with sample evidence, the estimated long-run relations indicate that per capita consumption and investment grew more than per capita output, whereas the real wage grew less. By construction, the CI space defines the structure of the long-run effects matrix. Given the estimated  $\beta'$ , a permanent technology shock leading to a 1% increase in long-run output will increase consumption, investment, and the real wage by 1.2%, 1.1%, and 0.9%, respectively.

# 2.3. Identification

The permanent components are identified by imposing exclusion restrictions on the long-run effects matrix  $\mathbf{C}(1)\mathbf{B}$  in the structural vector moving average (SVMA) representation  $\mathbf{x}_t = \mathbf{C}(1)\mathbf{B}\sum_{i=1}^t \varepsilon_i + \mathbf{C}^0(L)\mathbf{B}\varepsilon_t + \widetilde{\mathbf{x}}_0$ . We espouse the standard hypothesis that only technology shocks can have permanent effects on real variables [Blanchard and Quah (1989); Shapiro and Watson (1989); Galì (1999); Francis and Ramey (2005)]. This provides one exclusion restriction on the long-run effects of inflation shocks on per capita output (the element  $c_{12}$  of the  $\mathbf{C}(1)\mathbf{B}$  matrix is zero).

The assumption of a lower triangular structure for the  $(m - r) \times (m - r)$  upper left block of **C**(1)**B** separates the real from the nominal permanent component in the system. This hypothesis is consistent with the predictions of a broad class of business cycle models.<sup>6</sup> The orthogonality among permanent and transitory components ensures that the dynamic effects of a technology shock on  $\mathbf{x}_t$  do not depend on the identification of the transitory components [King et al. (1991)]. A more detailed description of the identification strategy is provided in Appendix C.

The estimated long-run effect of a productivity improvement, i.e., the elements of the first column in C(1)B, show that the long-run responses of real variables are all positive, with those of consumption, investment, and the real wage defined by their estimated long-run relations with output:

 $\mathbf{C}(1) \mathbf{B}'_{i=1} = \begin{bmatrix} 0.56 & -0.02 & 0.00 & -0.07 & 0.66 & 0.63 & 0.51 \end{bmatrix}.$ 

## 2.4. Impulse Responses and Variance Decomposition

Figure 1 depicts the impulse responses to a one-standard-deviation productivity shock. Solid lines denote the IRF point estimates of the benchmark specification, in which long-run balanced growth is not imposed, whereas dotted lines define the corresponding 90% confidence intervals. The bulleted lines denote the IRFs obtained when the extended sample 1948:1–2014:3 is considered. In this case, a long-run dummy accounting for the liquidity trap period is considered in the CI vectors.

The results of the IRF analysis can be summarized as follows.

- The medium-run (20 periods) responses of real variables (output, consumption, and investment) are positive, confirming the standard theoretical prediction that positive technology shocks are expansionary. Following a technology improvement, the output response reaches its long-run value after 14 quarters, while at the same time horizon consumption and investment reach 95% and 84% of their long-run effects.
- 2. Although the short-term responses of output and consumption are basically consistent with the predictions of standard business cycle models, those of hours and investment are not. Hours decline immediately after a supply shock (-0.3) and remain negative for eight quarters. The 90% error bands indicate that the contraction is significant over approximately five quarters. Similarly, the impact response of investment is negative (-0.9) and persistent, with the point estimate crossing the zero line only



**FIGURE 1.** Impulse responses to a technology improvement. Solid lines: benchmark specification. Bulleted lines: extended sample. Dotted lines: 90% confidence interval.

after seven quarters. According to the 90% confidence intervals, the negative response is significant over approximately four quarters. The contraction of investment on impact is aligned with that obtained by Basu et al. (2006), whereas that of hours is lower (-1.1 and -0.6, respectively). However, the latter contraction is consistent with the findings from the extended SVAR estimates in Galì (1999) and Francis and Ramey (2005) and the results in Canova et al. (2010), indicating an impact reduction in hours by between 0.2 and 0.3. Because these analyses adopt the BLS total hours index or the hours-to-population ratio as labor input measure, whereas Basu et al. (2006) use hours per employee, we conjecture that the differences obtained in the impact response are mainly related to the different scaling used for this variable.<sup>7</sup>

- 3. The IRFs of inflation and the interest rate are consistent with the theoretical predictions of monetary models. Inflation and interest rate responses are significantly negative, and the response of the nominal rate denotes monetary policy inertia and gradual accommodation. The reduction in inflation indicates that in the short run, the monetary policy does not fully accommodate the increase in productivity.
- 4. The consideration of a sample including the postcrisis period does not alter the main findings of our analysis. The IRFs of all the variables in the SVEC are shown to be aligned to those obtained with the restricted sample. The short-term response of hours and investment is confirmed to be negative in the short term.

The FEVDs, reported in Table 1, confirm that technology shocks are important but are not the main driver of economic fluctuations. The percentage of variance explained by the technology shock on impact is 0.3% for output, 21% for

	Variable									
Period	y <sub>t</sub>	$\pi_t$	$h_t$	$r_t$	C <sub>t</sub>	<i>i</i> <sub>t</sub>	wr <sub>t</sub>			
1	0.3	26.0	29.6	20.0	21.0	4.0	0.2			
4	2.6	28.4	15.0	17.9	27.1	1.3	2.5			
8	15.3	31.6	9.4	13.6	45.6	5.9	7.1			
12	31.3	29.1	11.2	11.1	63.6	11.3	14.3			
16	42.0	27.5	15.6	9.9	73.6	14.1	22.2			
20	47.7	27.2	17.7	9.6	79.0	14.9	29.6			
40	61.4	27.6	17.5	11.3	88.5	18.7	51.7			
$\infty$	100	25.5	0.00	14.0	100	100	100			

**TABLE 1.** Forecast error variance decomposition (%)

Notes: Fraction of FEV attributed to a technology shock.

consumption, 29% for hours, 4.0% for investment, and 0.2% for the real wage. By construction, the technology shock asymptotically tends to explain all the variability of real variables. These results, which are basically aligned to those obtained in previous analyses [e.g., King et al. (1991); Basu et al. (2006)], show that technology shocks, even if expansionary in the medium term, explain only a limited fraction of the total variability of real series at business cycle frequencies. In particular, the technology shock is unable to explain the business-cycle variations in investment.

# 2.5. Robustness

The robustness of our results can be evaluated in several ways. Here, we focus on three major aspects of the analysis: (i) the relevance of not imposing the balanced growth hypothesis in the long-run identification strategy; (ii) the importance of considering price inflation and the nominal interest rate as cointegrated I(1) processes; (iii) the importance of the VAR specification and of the long-run identification strategy.

Imposing exact balanced growth. Most monetary models display exact balanced growth in real variables such that the resulting stationary great ratios satisfy the restrictions  $\beta_{31} = \beta_{41} = \beta_{51} = -1$ . By construction, the estimated long-run proportionality in the  $\beta$  matrix is reflected in the elements of the long-run effects matrix C(1)B, i.e.,  $c_{51}/c_{11} = -\beta_{31}$ ,  $c_{61}/c_{11} = -\beta_{41}$ , and  $c_{71}/c_{11} = -\beta_{51}$ . The exact balanced growth restriction thus ensures homogeneity in the long-run effects of the technology shock on real variables.

We evaluate whether our results, obtained with estimated  $\beta$ s, are robust to the imposition of this theoretical constraint. Compared to the baseline specification, the IRFs are only marginally affected (Check 1 in Figure 2), showing that the



**FIGURE 2.** Impulse responses to a productivity improvement: robustness checks. Bulleted lines: benchmark specification. Solid lines: robustness checks. Dotted lines: 90% confidence intervals. Check 1: SVEC with exact balanced growth. Check 2: interest rate and inflation as stationary processes. Check 3: neglecting CI: BQ long-run identification (SVAR). Check 4: alternative specification of the CVs.

choice of relaxing the exact balanced growth assumption is not crucial for the main findings of our analysis.

*Price inflation and the interest rate as stationary processes.* The baseline SVEC was specified assuming two permanent and five stationary components.

Given the strong evidence on stationary hours, the remaining four stationary components were interpreted in terms of the three great ratios and the Fisher interest parity. To evaluate the dependence of the results on the inclusion of the last relation, we reestimate and simulate the SVEC, assuming that both inflation and the interest rate are stationary, as predicted by most monetary models. In this case, the long-run behavior of the system is driven by the technology (real) component only, whose identification is provided by the zero restrictions on the long-run effects matrix implied by the six transitory components (the last six columns of C(1)B are zero vectors) and by the orthogonality between the permanent and the transitory components.<sup>8</sup>

Even in this case, the IRFs are only marginally affected by the hypothesis of stationary inflation and nominal interest rate (Check 2 in Figure 2). The short-term responses of hours and investment remain negative and significant, although for investment this evidence is observed over a slightly shorter period. Unsurprisingly, the major differences are observed for the IRFs of inflation and the interest rate, which display negative but slightly less persistent responses.

Alternative specifications and long-run identification strategies. Two alternative long-run identification strategies and SVAR specifications, both consistent with the stochastic properties of the data, are evaluated. We first consider a secondorder stationary SVAR with the real variables entering in first differences, in which the technology shock is identified with a standard recursive scheme for the long-run effects matrix [Blanchard and Quah (1989); Galì (1999)]. Then, following Christiano et al. (2005) and Altig et al. (2011), we consider a thirdorder stationary SVAR including differenced output and the consumption, investment, and wage-to-output ratios in place of the corresponding level variables; i.e.,  $\pi_t$   $h_t$   $r_t$   $c_t - y_t$   $i_t - y_t$   $wr_t - y_t$ ], in which the permanent  $x'_t = [\Delta y_t]$ technology shock is identified with the instrumental variables method detailed in Shapiro and Watson (1989) and Francis and Ramey (2005).<sup>9</sup> Under this peculiar SVAR representation, the hypothesis of stationary inflation and nominal interest rate is adopted along with that of exact balanced growth, so that the three control dimensions of the robustness analysis are jointly considered.

The IRFs clearly show that with the former SVAR (Check 3), the results are only marginally aligned to those obtained with the SVEC specifications. In particular, the negative responses of hours and investment are shortened, with the latter returning positive results after one quarter only. In contrast, with the second SVAR specification, the results confirm those obtained with the baseline and the alternative SVECs (Check 4).

Because the two long-run identification strategies are basically equivalent, the difference in results should be attributed to the fact that the former SVAR omits the consideration of the stationary ratios. In the presence of CI, this omission can bias the results through the misspecification and weak instrumentation issues. In fact, the exclusion of the stationary ratios implies the omission of the error-correcting component from the model and does not allow the separation between

permanent and transitory components [Harvey and Stock (1988); King et al. (1991)], weakening the identifiability of the technology shock.

#### 3. THE MODEL

In this section, we describe the linearized version of a cash-in-advance monetary model that is able to encompass alternative theoretical explanations for the contractionary effects of technology improvements and to reproduce the contrasting empirical results in the literature.

To obtain these model features, we consider the main factors that can constrain the aggregate demand response to a productivity improvement and allow for intertemporal substitution effects by assuming that technology shocks are permanent and autocorrelated in growth rates [Lindé (2009)].

In addition to standard nominal and real rigidities, the model is characterized by the presence of strategic complementarity in price setting, emerging from the hypotheses of firm-specific capital [Sveen and Weinke (2005); Woodford (2005); Altig et al.(2011)] and endogenous demand elasticity (Eichenbaum and Fisher (2007); Smets and Wouters (2007)].

The model economy is populated by maximizing households and firms, whereas monetary and fiscal authorities follow exogenous policy rules. Final sector firms operate in a perfectly competitive environment as simple aggregators of the differentiated goods produced by intermediate sector firms. These combine labor and capital services, employing a Cobb–Douglas production technology that is subject to permanent productivity shocks, giving rise to a common stochastic trend and to long-run stationary ratios among real variables. This feature makes the model consistent with the nonstationary and co-trending behavior of the data addressed by the SVEC analysis.

Each intermediate firm rents differentiated labor services from the households and makes an investment decision to adjust its capital stock to the desired level, taking into account a capital adjustment cost. Intermediate sector firms can reoptimize their prices only infrequently, according to a random duration Calvo lottery. Households maximize a separable utility function defined over consumption and leisure. Their preferences exhibit persistence in external consumption habits and are assumed to be log-linear in consumption and CRRA in leisure to guarantee balanced growth. The presence of differentiated labor services implies some monopoly power in labor supply, and wages are set in staggered contracts according to a Calvo scheme.

The linearized model is expressed in stationary form, which is necessary because we deal with the hypothesis of nonstationary technology shocks, which induce a common stochastic trend in the real variables [Juillard et al. (2008)]. To obtain model stationarity, we first scale the real variables with respect to the stochastic technology level  $Z_t$  by imposing the transformation  $X_t = \hat{X}_t Z_t$ , where the circumflex indicates that level variables are expressed in terms of stationary ratios. The model is then log-linearized around the steady state of the scaled variables. Lowercase letters with a circumflex denote log deviations in the corresponding detrended variables.

#### 3.1. Production

The linearized aggregate production function is

$$\hat{y}_t = \alpha \hat{k}_{t-1} + (1 - \alpha) h_t - \alpha \log g_t^z,$$
 (2)

where we assume that firms produce their output  $\hat{y}_t$  by combining, in a Cobb– Douglas production function, their accumulated (thus firm-specific) capital endowment  $\hat{k}_{t-1}$  with hired labor services  $h_t$ . The parameter  $\alpha$   $(1 - \alpha)$  denotes the capital (labor) share in production. The term  $\log g_t^z$  is the growth rate of the labor-augmenting technology, which is assumed to follow a first-order autoregressive process  $\log g_t^z = (1 - \rho_z) \log \gamma_z + \rho_z \log g_{t-1}^z + \varepsilon_t^z$ , where  $\gamma_z$ is the deterministic long-run growth rate. Under this specification, the evolution of the technology level has a nonstationary second-order autoregressive representation, because  $\log Z_t = \log Z_{t-1} + \log g_t^z$  can be rewritten as  $\log Z_t = (1 - \rho_z) \log \gamma_z + (1 + \rho_z) \log Z_{t-1} - \rho_z \log Z_{t-2} + \varepsilon_t^z$ .

This choice for the technology process is motivated by the need to separate the model-specific dynamics from that potentially emerging from a fairly general specification of the stochastic components. In fact, when technology is autocorrelated in growth rates, even flexible-prices models can be made consistent with the contractionary effects of positive technology shocks because of the operation of wealth and intertemporal substitution effects [Lindé (2009)].

We assume that firms face convex adjustment costs of changing their fixed asset holdings, which become productive with a one-period lag. By log-linearizing the capital adjustment cost function, we obtain the following law of motion for capital:

$$\hat{k}_t = \frac{(\delta + \gamma_z - 1)}{\gamma_z} \hat{\iota}_t + \frac{(1 - \delta)}{\gamma_z} \hat{k}_{t-1} + \frac{(\delta - 1)}{\gamma_z} \log g_t^z,$$
(3)

where  $\hat{i}_t$  is the stationary log deviation of gross investment, and the parameter  $\delta$  denotes capital depreciation.

#### 3.2. Pricing Behavior of Firms

The aggregate price dynamics  $\pi_t$  is described by the following specification of the NKPC:

$$\pi_t = \iota_p \pi_{t-1} + \beta E_t \left( \pi_{t+1} - \iota_p \pi_t \right) + \kappa \widehat{\mathrm{mc}}_t + \log u_t^{\pi}, \qquad (4)$$

where  $\beta$  is the discount factor,  $\widehat{\mathrm{mc}}_t = \widehat{w}_t^r - \widehat{y}_t + h_t$  is the log-linearized real marginal cost ( $\widehat{w}_t^r$  is the real wage),  $\kappa$  is the reduced-form NKPC slope coefficient, and the stochastic term log  $u_t^{\pi}$  denotes a cost-push disturbance that is assumed to follow the stationary first-order autoregressive process log  $u_t^{\pi} = \rho_{\pi} \log u_{t-1}^{\pi} + \varepsilon_t^{\pi}$ .

The backward-looking component in (4) emerges from the hypothesis of partial indexation (of degree  $l_p$ ).

Equation (4) is compatible with a large class of monetary models. The adoption of the firm-specific or the rental capital specification (FSK or RK, respectively) and of constant or endogenous demand elasticity (CDE or EDE, respectively) only affects the convolution of parameters defining the reduced-form slope coefficient  $\kappa$  [Eichenbaum and Fisher (2007)]. Under RK and CDE, the slope coefficient is given by  $\kappa_{\rm RK} = \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}$ , where  $\theta_p$  defines the random fraction of firms that are not allowed to reset their price. Under FSK, the NKPC slope coefficient can be written as  $\kappa_{\rm FSK} = \kappa_{\rm RK} \Lambda$ , where  $\Lambda$  is a function of the model's parameters.

The computation of  $\Lambda$  is not straightforward and can only be obtained using the undetermined coefficients method. Sveen and Weinke (2005) and Woodford (2005) provide the useful approximation  $\Lambda \simeq \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$  in terms of the capital share in production and the elasticity of substitution  $\epsilon$ , where equality holds exactly in the case of constant capital [Woodford (2005)]. With respect to the standard rental capital specification, the multiplicative term  $\Lambda$  reduces the slope of the NKPC for any model parameterization.

The economic rationale for the reduced price sensitivity to changes in the marginal cost under FSK is that because firms operate with a predetermined (firm-specific) stock of capital, their marginal cost increases with the level of output. Compared with a situation in which capital services can be chosen period by period in a RK market, this implies that reoptimizing firms facing a positive productivity shock are induced to cut prices by a smaller amount because they anticipate that price reductions eventually lead to higher marginal costs as a result of increased demand and output at the firm level.

Another theoretical hypothesis that can induce strategic complementarity in price setting is the EDE assumption [Eichenbaum and Fisher (2007); Smets and Wouters (2007)]. In such a case, the coefficient relating inflation to the marginal cost in the EDE model  $(k^{\text{EDE}})$ , irrespective of the RK or FSK specification, is reduced by the factor  $(\frac{1}{(\epsilon-1)}\phi_k + 1)^{-1}$ , where  $\phi_k$  is the percentage of change in the demand elasticity evaluated in the steady state due to a 1% change in the relative price of the good [Kimball (1995)]. The relation between the CDE and the EDE specifications of the reduced-form NKPC slope coefficient is thus approximated by the following equation:

$$\kappa^{\text{EDE}} = \kappa^{\text{CDE}} \frac{1}{\frac{1}{\epsilon - 1}\phi_k + 1}.$$

The fact that both the FSK and the EDE models differ from the baseline rental capital model only for the size of the NKPC slope coefficient makes the different models observationally equivalent when a specification is considered in which the reduced-form coefficient  $\kappa$  enters the NKPC. This allows us to estimate the model without specifying whether capital is firm-specific or rental and whether the elasticity of substitution among differentiated goods is constant or endogenous

[Altig et al. (2011)]. From the estimated value of  $\kappa$ , given assumptions about  $\epsilon$  and  $\phi_k$ , we can infer the price duration in each model.

#### 3.3. Pricing Behavior of Wage Setters

Concerning the pricing behavior of monopolistically competitive wage setters, the following real wage equation holds:

$$\hat{\pi}_{t}^{w} = \beta E_{t} \left[ \hat{\pi}_{t+1}^{w} + (\pi_{t+1} - \iota_{w}\pi_{t}) + \log g_{t+1}^{z} \right] - \log g_{t}^{z} - (\pi_{t} - \iota_{w}\pi_{t-1}) + \kappa^{w} \left( \widehat{\mathrm{mrs}}_{t} - \hat{w}_{t}^{r} - \log \chi_{t} \right) + \log u_{t}^{\pi^{w}},$$
(5)

where  $\hat{\pi}_t^w = \hat{w}_t^r - \hat{w}_{t-1}^r$  is real wage growth,  $\widehat{\operatorname{mrs}}_t = \frac{\gamma_z}{\gamma_z - \lambda} [\widehat{c}_t - \frac{\lambda_c}{\gamma_z} (\widehat{c}_{t-1} - \log g_t^z)] + \eta h_t$  denotes the (real) marginal rate of substitution between consumption  $\widehat{c}_t$  and labor, and  $\lambda_c$  is the degree of habit persistence in consumption. The parameter  $\kappa^w = \frac{(1 - \beta \theta_w)(1 - \theta_w)}{\theta_w} \frac{1}{1 + \epsilon_H \eta}$  is the reduced-form slope coefficient, where the parameter  $\theta_w$  denotes the degree of nominal wage stickiness, and the parameters  $\eta$  and  $\epsilon_H$  are the inverse Frisch labor elasticity and the elasticity of substitution among differentiated labor services, respectively. The stochastic wage-push disturbance  $\log u_t^{\pi^w}$  is assumed to follow the first-order autoregressive process  $\log u_t^{\pi^w} = \rho_{\pi^w} \log u_t^{\pi^w} + \varepsilon_t^{\pi^w}$ . Even in this case, the backward-looking component in (5) is due to partial indexation to past inflation (of degree  $\iota_w$ ).

#### 3.4. Demand Side

Concerning the demand side of the economy, the dynamics of consumption  $\hat{c}_t$  resulting from the corresponding Euler equation is described by

$$\hat{c}_{t} = \frac{\lambda_{c}/\gamma_{z}}{1 + \lambda_{c}/\gamma_{z}} \left( \hat{c}_{t-1} - \log g_{t}^{z} \right) + \left( 1 - \frac{\lambda_{c}/\gamma_{z}}{1 + \lambda_{c}/\gamma_{z}} \right) E_{t} \left( \hat{c}_{t+1} + \log g_{t+1}^{z} \right) \\ - \frac{1 - \lambda_{c}/\gamma_{z}}{1 + \lambda_{c}/\gamma_{z}} \left[ (r_{t} - E_{t}\pi_{t+1} - \rho) - E_{t} \left( \Delta \log \chi_{t+1} \right) \right],$$
(6)

where  $r_t$  and  $\rho$  are the current and the steady state nominal interest rates. Current consumption thus depends on the expected real interest rate and on a weighted average of past and future consumption, with weights depending on the degree of external habit persistence  $\lambda_c$ . The term log  $\chi_t$  denotes a consumption preference shock and is assumed to follow the stationary first-order autoregressive process log  $\chi_t = \rho_{\chi} \log \chi_{t-1} + \varepsilon_t^{\chi}$ .

The investment dynamics depends on the firm's choices in relation to capital accumulation, defined by the log-linear capital Euler equation

$$\hat{k}_{t} = \frac{1}{(1+\beta)}\hat{k}_{t-1} + \frac{\beta}{(1+\beta)}E_{t}\hat{k}_{t+1} + \frac{1-\beta\gamma_{z}^{-1}(1-\delta)}{\epsilon_{\psi}\gamma_{z}(1+\beta)}E_{t}\widehat{\mathrm{ms}}_{t+1}$$

$$-\frac{1}{\epsilon_{\psi}\gamma_{z}(1+\beta)}(r_{t}-E_{t}\pi_{t+1}-\rho)-\frac{1}{(1+\beta)}\log g_{t}^{z}$$
  
+
$$\frac{\beta}{(1+\beta)}\log g_{t+1}^{z}+\frac{1}{\epsilon_{\psi}\gamma_{z}(1+\beta)}\left[\beta\gamma_{z}^{-1}(1-\delta)E_{t}\log\zeta_{t+1}-\log\zeta_{t}\right],$$
 (7)

where  $\beta = (1 + \rho)^{-1}$  and  $\widehat{\mathrm{ms}}_{t+1} = \widehat{w}_{t+1}^r + h_{t+1} - \widehat{k}_t + \log g_{t+1}^z$  is the expected stationary log deviation of the return on capital, which, under firm-specific capital, is expressed in terms of the firm's marginal savings on labor costs. Current installed capital thus depends on its past and expected future values, expected marginal savings, and expected real interest rates. The dynamics of investment/capital is affected by a stationary first-order autoregressive disturbance to the convex capital adjustment cost function,  $\log \zeta_t = \rho_{\zeta} \log \zeta_{t-1} + \varepsilon_t^{\zeta}$ , whose steady-state elasticity is  $\epsilon_{\psi} > 0$ .

## 3.5. Model Closure

The model is closed by the aggregate resource constraint and the policy reaction function. The log-linear constraint is given by

$$\hat{y}_{t} = \left(1 - \psi - g^{y}\right)\hat{c}_{t} + \psi\hat{i}_{t} + g^{y}\hat{g}_{t},$$
(8)

where  $\psi = \frac{\alpha(\delta+\gamma_c-1)}{[\epsilon/(\epsilon-1)]\gamma_c(\rho+\delta)}$  is the steady-state investment-to-output ratio. The term  $\hat{g}_t$  is an AR(1) measurement error capturing public expenditure and other exogenous components affecting the aggregate resource constraint. The coefficient  $g^y$  denotes the steady state public expenditure-to-GDP ratio.

Two alternative monetary policy reaction rules are considered. The first targets inflation deviations from a nonzero policy target  $\pi^*$  and output growth deviations from the deterministic long-run rate of growth  $\Delta y_t - \log \gamma_z = \Delta \hat{y}_t + \log g_t^z - \log \gamma_z$ . The second targets inflation deviations and the theory-based output gap  $\hat{y}_t - \hat{y}_t^p$ , where  $\hat{y}_t^p$  is the level of output that would prevail in the absence of nominal rigidities. The policy instrument is adjusted gradually, giving rise to interest rate smoothing, whose degree is defined by  $\rho_r$ :

$$r_{t} = \rho + \rho_{r} r_{t-1} + (1 - \rho_{r}) \left[ \phi_{\pi} \left( \pi_{t} - \pi^{*} \right) \right] + \phi_{y} \left( \Delta \hat{y}_{t} + \log g_{t}^{z} - \log \gamma_{z} \right) + \log u_{t}^{r}, \qquad (9)$$

$$r_{t} = \rho + \rho_{r} r_{t-1} + (1 - \rho_{r}) \left[ \phi_{\pi} \left( \pi_{t} - \pi^{*} \right) \right] + \phi_{y} \left( \hat{y}_{t} - \hat{y}_{t}^{p} \right) + \log u_{t}^{r}.$$
(10)

The parameters  $\phi_{\pi}$  and  $\phi_{y}$  define the strength of the policy reaction to inflation and output deviations from the respective targets. The stochastic term log  $u_{t}^{r} = \varepsilon_{t}^{r}$ denotes an i.i.d. monetary policy error.

Considering a productivity improvement, a policy rule targeting potential output would be more accommodative than a policy rule targeting the long-term growth rate. In fact, under the empirical rule (9), the authorities underestimate the actual growth rate of natural output, resulting in a not fully accommodative interest rate response.

There are at least two reasons that justify the use of an empirical rule. First, targeting the theory-based output gap requires knowledge of the natural level of output, which by definition is unobservable. Second, given that real-time data on potential output are subject to relevant imperfections, under model uncertainty and when technology evolves according to a random walk with drift process, the estimated long-term deterministic growth component  $\gamma_z$  might represent the best prediction for output growth.

In the estimation process, we will evaluate the relevance of both the empirical and the theory-based monetary policy reaction functions and let the data decide which model—and thus which rule—is to be preferred.

The linearized system is composed of four behavioral equations, (6), (7), (4), and (5), the production function (2), the permanent inventory equation (3), the aggregate resource constraint (8), and a Taylor rule (9 or 10). Four definition equations for  $\hat{w}_t^r$ ,  $\widehat{\mathrm{ms}}_t$ ,  $\widehat{\mathrm{mrs}}_t$ , and  $\widehat{\mathrm{mc}}_t$  complete the economic system.

#### 4. BAYESIAN ESTIMATION AND SIMULATION

The strategy adopted for the parameterization of theoretical models is key in the face of the conflicting SVAR-based evidence discussed in Section 2. Some influential contributions supporting the procyclicality of investment and hours worked [Christiano et al. (2005); Altig et al. (2011)] use monetary models parameterized through estimators that minimize the weighted distance between the theoretical and the SVAR-based impulse responses. In our view, results obtained using a matching estimator applied to a model with particularly flexible dynamic properties cannot be considered conclusive because they do not add much to the evidence implied by the SVAR-based impulse responses.

This consideration leads us to use a calibration strategy that does not rely upon our SVEC evidence but is based on a direct estimate of model's parameters. This section provides details of the estimation methodology and the evaluation of the empirical relevance of the two alternative specifications of the monetary policy rule. Note that by estimating a reduced form slope coefficient of the NKPC, we do not impose any prior assumptions about the CDE/EDE or the RK/FSK specifications of the model.

#### 4.1. The Posterior Distribution and Model Comparison

We derive the posterior distribution for the *j*th model's parameters  $P(\theta_j | \mathbf{Y}_T, M_j)$  by nesting prior beliefs for models  $M_j$  (j = 1, 2, ...) and structural parameters  $\theta_j$ —i.e., the prior distribution  $P(\theta_j, M_j)$ —with sample information—i.e., the conditional distribution  $P(\mathbf{Y}_T | \theta_j, M_j)$ —where  $\mathbf{Y}_T = {\mathbf{y}_t}_{t=1}^T$  contains sample information.

The consideration of the alternative policy rules corresponds to the evaluation of two different model structures (j = A, B), one adopting the empirical policy rule (9), the other the theory-based rule (10). The empirical relevance of the alternative feedback rules is evaluated by estimating the two competing models  $M_A$  and  $M_B$  with parameter vectors  $\theta_A$  and  $\theta_B$  and deriving the Bayes factor from the log-marginal likelihood.

#### 4.2. Measurement Equations, Priors, and Posterior Distributions

*Measurement equations.* We use the same U.S. data as the SVEC analysis briefly described in Section 2.3. The reference sample is thus composed of quarterly series for the period 1948:1–2008:4. Seven variables are considered: the log differences of real per capita GDP  $\Delta y_t$ , consumption  $\Delta c_t$ , and investment  $\Delta i_t$ , the log differences of the real hourly wage  $\Delta w_t$ , and the log levels of per capita hours  $h_t$ , GDP price inflation  $\pi_t$ , and the federal funds rate  $r_t$ . The vector of observables is thus

$$\mathbf{x}_t' = \begin{bmatrix} \Delta y_t \ \Delta c_t \ \Delta i_t \ \Delta wr_t \ h_t \ \pi_t \ r_t \end{bmatrix}.$$

Because we express the models in log deviations around the stochastic growth path (log  $Z_t$ ), the measurement equations linking the model variables to observables are the following:

$$\begin{aligned} \Delta y_t &= \hat{y}_t - \hat{y}_{t-1} + \log g_t^z, \\ \Delta c_t &= \hat{c}_t - \hat{c}_{t-1} + \log g_t^z, \\ \Delta i_t &= \hat{i}_t - \hat{i}_{t-1} + \log g_t^z, \\ \Delta wr_t &= \hat{w}_t - \hat{w}_{t-1} + \log g_t^z, \\ h_t &= h_t, \\ \pi_t &= \pi_t + \log g^p, \\ r_t &= r_t - \log (\beta) + \log \gamma_z + \log g^p, \end{aligned}$$
(11)

where  $\log g_t^z = (1 - \rho_z) \log \gamma_z + \rho_z \log g_{t-1}^z + \varepsilon_t^z$ .

*Priors.* We initialize the estimates over a parameter space for which both  $M_A$  and  $M_B$  do not replicate the SVEC-based evidence on the persistent contractionary effects of productivity improvements on investment. Outside this choice, reflected mainly in the prior mean for the autoregressive component in the monetary authority's reaction rule, we adopt a common prior parameterization (i.e.,  $\theta_A = \theta_B$ ) defined according to sample information or considering the results obtained in previous analyses.

We impose six dogmatic priors by fixing the discount factor  $\beta$  to 0.995, the steady state values for the elasticity of substitution among differentiated goods  $\epsilon$  and labor services  $\epsilon_H$  to the customary value of 11, consistent with a price/wage mark-up  $\epsilon(\epsilon - 1)^{-1}$  of 10%, the capital depreciation rate  $\delta$  to 0.025, and the parameters defining the degree of price and wage indexation to past inflation,  $\iota_p = \iota_w = 0.^{10}$ 

All the remaining parameters are estimated. Prior distributions are summarized in Tables 1 and 2 together with the posterior mode and mean estimates. Some choices in the elicitation of priors deserve discussion. The reduced-form NKPC slope coefficient  $\kappa$  is described by a weak beta-distributed prior with mean 0.05 and s.d. 0.025. Under a RK specification, this prior implies a Calvo parameter value  $\theta_p = 0.8$ , which is in line with the available macroeconometric evidence [Galì and Gertler (1999); Smets and Wouters (2003); Del Negro et al. (2005); Eichenbaum and Fisher (2007)]. Under the FSK and EDE hypotheses, given the demand elasticity parameter and a curvature parameter of the Kimball aggregator  $\phi_{\kappa} = 10$ , the prior implies a Calvo parameter close to 0.5, in line with the microdata-based evidence produced by Bils and Klenow (2004), suggesting an average price duration of nearly two quarters.

The coefficients of the monetary policy reaction rule are assumed to follow a normal distribution with prior means  $\phi_{\pi} = 1.5$  and  $\phi_y = 0.25$ , both with s.d. 0.1. The interest rate smoothing coefficient  $\rho_r$  is beta-distributed with prior mean 0.5 and s.d. 0.1. This value is lower than that adopted and estimated in other applications but ensures that the estimates are initialized over a parameterization for which monetary policy is sufficiently accommodative to rule out the emergence of contractionary effects on investment.

Even if we assume that all shocks but the monetary policy shock are serially correlated, we adopt differentiated priors for the autoregressive coefficients. A low degree of autocorrelation is assumed for the stationary disturbances to favor the separation between stationary and nonstationary components [Smets and Wouters (2003)].<sup>11</sup>

*Posterior distributions.* Table 2 reports the posterior mode and mean estimates of the parameters for models A and B.<sup>12</sup> Table 2(a) presents the estimates of the 11 parameters defining the model structure, and Table 2(b) presents those of the parameters defining the persistence and size of the 7 stochastic components. According to the estimated posterior standard deviations and the implied pseudo-t values, all parameter estimates appear significant at the standard level in both model specifications, except for the autoregressive coefficient of the technology growth process. This result indicates an absence of autocorrelation in technology growth.

Outside the measurement error in the aggregate resource constraint and the capital adjustment cost shock, a moderate degree of autocorrelation is obtained for all the remaining shock processes. This is more evident for  $M_A$ , signaling that the empirical specification of the policy rule tends to tone down the relevance of the stochastic sources of persistence.

The posterior mean estimates for the behavioral and policy parameters are close to our priors and to the results obtained in the literature. However, two exceptions deserve some discussion.

First, the estimated reduced-form NKPC slope coefficient is lower than the prior mean value under both model specifications, indicating a weak transmission

	Prior distribution		Posterior distribution								
	Models A and B			Mod	lel A		Model B				
	Distr	Mean St. Dev.	Mode St. Dev.	Mean	5%	95%	Mode St. Dev.	Mean	5%	95%	
				(a) Stri	ictural pa	ramete	rs				
$\gamma_z$	$\mathcal{N}$	1.004	1.005	1.005	1.004	1.006	1.004	1.004	1.003	1.005	
$\gamma^{p}$	$\mathcal{N}$	1.009	1.010	1.010	1.009	1.011	1.010	1.010	1.009	1.011	
α	${\mathcal B}$	0.360	0.237	0.241	0.219	0.262	0.152	0.152	0.132	0.172	
η	$\mathcal{N}$	1.000	1.297	1.303	0.942	1.660	1.644	1.662	1.319	1.996	
$\lambda_c$	${\mathcal B}$	0.700	0.631	0.658	0.565	0.748	0.664	0.694	0.626	0.757	
$\epsilon_{\psi}$	$\mathcal{N}$	5.000	5.516	5.504	5.145	5.852	6.067	6.038	5.838	6.272	
κ	${\mathcal B}$	0.050	0.014	0.015	0.009	0.022	0.013	0.014	0.010	0.019	
$\theta_w$	${\mathcal B}$	0.500	0.871	0.870	0.830	0.910	0.800	0.794	0.755	0.833	
$\rho_r$	${\mathcal B}$	0.500	0.602	0.595	0.526	0.665	0.575	0.570	0.515	0.627	
$\phi_{\pi}$	$\mathcal{N}$	1.500	1.375	1.406	1.292	1.527	1.226	1.256	1.175	1.339	
$\phi_y$	$\mathcal{N}$	0.250	0.401	0.415	0.316	0.512	0.133	0.139	0.084	0.194	
		0.100	0.000	(b) S	shock pro	cesses	0.055				
$\rho_z$	$\mathcal{N}$	0.000	-0.001	0.006	-0.078	0.087	-0.091	-0.078	-0.155	0.001	
$ ho_{\zeta}$	${\mathcal B}$	0.750	0.943	0.939	0.921	0.958	0.990	0.987	0.978	0.996	
$ ho_g$	${\mathcal B}$	0.750	0.992	0.990	0.986	0.995	0.993	0.992	0.987	0.996	
$ ho_{\chi}$	${\mathcal B}$	0.750	0.767	0.729	0.622	0.840	0.858	0.815	0.732	0.900	
$\rho_{\pi^w}$	${\mathcal B}$	0.250	0.412	0.413	0.344	0.482	0.425	0.433	0.365	0.503	
$ ho_{\pi}$	${\mathcal B}$	0.250	0.499	0.501	0.435	0.566	0.489	0.498	0.435	0.558	
$\sigma_z$	$\mathcal{IG}$	0.010	0.011	0.011	0.011	0.012	0.010	0.010	0.010	0.011	
$\sigma_{\zeta}$	$\mathcal{IG}$	0.010	0.012	0.012	0.010	0.014	0.011	0.011	0.010	0.012	
$\sigma_{g}$	$\mathcal{IG}$	0.010	0.014	0.014	0.013	0.015	0.015	0.015	0.014	0.016	
$\sigma_{\chi}$	$\mathcal{IG}$	$0.010_{2}^{-10}$	0.014	0.016	0.011	0.020	0.016	0.017	0.014	0.021	
$\sigma_{\pi^w}$	$\mathcal{IG}$	$0.010_{2}^{-10}$	0.004	0.004	0.003	0.004	0.003	0.003	0.002	0.003	
$\sigma_{\pi}$	$\mathcal{IG}$	0.010	0.002	0.002	0.002	0.003	0.003	0.003	0.003	0.004	
$\sigma_{u^r}$	$\mathcal{IG}$	0.010	$0.005 \\ 0.001$	0.005	0.004	0.006	0.003	0.003	0.003	0.004	

TABLE 2. Priors and posterior distribution of structural parameters and of shock processes

*Notes*:  $\mathcal{N}$ ,  $\mathcal{B}$  and  $\mathcal{IG}$  are normal, beta, and inverted gamma distributions, respectively. Posterior mean estimates obtained with 500, 000 Metropolis–Hastings replications.

mechanism from marginal costs to price inflation. The estimated slope is basically the same as that obtained by Altig et al. (2011).<sup>13</sup>

Second, the estimated parameters defining the interest rate response to real activity in the two alternative monetary policy reaction functions depart from the common prior mean, but in opposite directions:  $\hat{\phi}_y$  is 0.40 under  $M_A$  and 0.13 under  $M_B$ . The degree of interest rate smoothing is only slightly higher than the prior under both  $M_A$  ( $\hat{\rho}_r = 0.60$ ) and  $M_B$  ( $\hat{\rho}_r = 0.58$ ).

These differences signal that under  $M_B$ , the estimates tend to highlight the sources of persistence in the model. This result can be attributed to the fact that the higher stabilizing effects implied by the theory-based monetary policy reaction rule are counteracted by higher estimates of the economic and stochastic sources of persistence.

The log-marginal likelihood is 5, 939.1 for Model *A* and 5, 884.4 for Model *B*, so that the Bayes factor is  $B_{A,B} = e^{[\log P(Y_T/M_A) - \log P(Y_T/M_B)]} = e^{54.7}$ , a value that, according to Jeffrey 's scale of equivalence, indicates that the evidence in favor of Model *A* is decisive.<sup>14</sup>

#### 5. MODEL DYNAMICS

In this section, we provide an evaluation of the dynamic properties of the models using stochastic simulations based on posterior mean estimates, summarized in Figure 3. In discussing our results, we focus on the economic mechanisms determining the sign and the persistence of the response of factor inputs to a positive technology shock.

To verify the validity of the SVEC analysis, we simulate models *A* and *B* parameterized at the posterior estimates to generate samples of artificial data (250 quarterly observations) and check whether our empirical identification strategy is able to replicate the dynamic properties of the true data-generating process. The artificial data SVEC IRFs are reported in the same graphs in Figure 3, together with the IRFs at the prior parameterization.

#### 5.1. Posterior Impulse Responses

At least four indications are worth highlighting. First, the estimates for Model A, but not those for Model B, confirm the results obtained with the SVEC analysis presented in Section 2. With  $M_A$ , the investment and hours responses are qualitatively in line with those obtained with the SVEC-based impulse response analysis. The investment response is negative in the short run and becomes positive only after some periods, when the demand constraint becomes less binding and the standard expansionary mechanisms display their effects. The posterior hours response is also negative in the short and in the medium run. With  $M_B$ , both hours and investment respond positively to the productivity improvement even on impact.



**FIGURE 3.** Prior and posterior impulse responses to a productivity improvement. Solid lines: IRFs computed at the model's posterior mean estimates. Bulleted solid lines: artificial data SVEC-based IRFs. Bulleted lines: model's prior IRFs. Dotted lines: 90% confidence of artificial data SVEC-based IRFs. Model A: empirical policy rule. Model B: theory-based policy rule.

Second, the short-run interest rate reduction, which obtains under both model specifications, signals that the estimated monetary policy rules are fairly accommodating, but not enough to prevent a short-run decrease in inflation. The impulse responses of inflation and the interest rate, however, are quite different from those obtained with the SVEC, in which a more persistent contraction of both variables is observed. Differences are greater for the posterior IRFs obtained under  $M_B$ .

Third, consumption and output responses are standard under both model specifications: consumption rises smoothly in response to the expected permanent increase in productivity and output, driving the expansionary aggregate demand response. Unsurprisingly, the speed of convergence toward the new steady state is higher under  $M_B$  than under  $M_A$ .

Fourth, the SVECs estimated over artificial data for  $M_A$  and  $M_B$  are able to replicate the dynamic properties of the true data-generating processes, validating the empirical evidence obtained with real data. It is worth highlighting that such a result is not trivial, because models in which slowly changing variables such as the capital stock are present—such as our models—generally do not display a finite-order VARMA representation in a subset of the model variables [Fry and Pagan (2005)]. In these cases, finite-order VAR approximations are often affected by serious truncation biases. These results show that this is not the case for our structural model estimates.

## 5.2. Theoretical Insights and Model Comparison

The literature suggests alternative theoretical explanations for the contractionary effects of productivity improvements. Our model specification allows us to consider some of them from a comparative perspective, basically focusing on the real, nominal, and monetary factors.

*Real factors.* Our estimates rule out explanations based on intertemporal substitution effects due to expected increases in productivity. As shown by Lindè (2009) and Hamilton and Francis (2014), this result can arise in flexible price models when the permanent technology shock is autocorrelated in growth rates. The size of the autoregressive coefficient  $\rho_z$  is, in fact, estimated to be not statistically different from zero. We have verified that, in the flexible-price version of our model and absent real frictions, the contractionary effects on inputs emerge only for values of  $\rho_z$  well above 0.8, which is the value used by Lindè (2009) in his RBC model simulation.

The estimates also do not support explanations based on real rigidities directly affecting consumption and investment decisions [Francis and Ramey (2005); Smets and Wouters (2007)]. In the absence of nominal frictions, i.e., considering the flexible price/real rigidities version of the model, and given a standard calibration for the capital adjustment cost parameter ( $\epsilon_{\psi} = 5$ ), a negative hours—but not investment—response to a positive technology shock can be observed on impact only for high degrees of habit persistence ( $\lambda_c > 0.9$ ). By assuming  $\epsilon_{\psi} = 10$ , the



**FIGURE 4.** Impulse responses to a technology improvement. Solid lines: benchmark SVECbased IRFs. Dashed lines: flex-price-matched IRFs. Dotted lines: 90% confidence interval of the SVEC-based IRFs.

threshold  $\lambda_c$  value for observing a negative impact response of hours is reduced to 0.85.

For a further evaluation of the empirical relevance of explanations based on real factors, we adopt the following procedure: First, we calibrate the flexible price/real frictions model with an impulse response matching procedure targeting the 20-period IRFs of the four real variables and hours obtained with the baseline SVEC, i.e.,  $x_h = [y_h, c_h, i_h, wr_h, h_h]$ . Formally,

$$\underset{\theta \in \Theta}{\operatorname{arg\,min}} \left[ \mathbf{x}_{h}^{\text{SVEC}} - \mathbf{x}_{h}^{\text{Model}} \right]' \Psi^{-1} \left[ \mathbf{x}_{h}^{\text{SVEC}} - \mathbf{x}_{h}^{\text{Model}} \right],$$

where  $\theta$  is the vector of matching parameters  $\theta' = [\alpha, \eta, \lambda_c, \epsilon_{\psi}, \rho_z], x_h^{\text{SVEC}}$  and  $x_h^{\text{Model}}$  are the (20 × 5) vectors of SVEC and model-based IRFs, respectively, and  $\Psi$  is a diagonal matrix containing the standard deviations of the SVEC IRFs. Next, we adopt the resulting calibrated values as priors for a direct estimate of the flexible price model, for which we use the same sample information employed for the SVEC and  $M_A/M_B$  estimates. Two flexible price versions of our model are estimated, one considering the real rigidities in consumption and investment, i.e., habits and capital adjustment costs, and the other assuming a standard RBC specification ( $M_C$  and  $M_D$ , respectively).

As shown in Figure 4, outside the wage, the flexible price model is able to qualitatively match the SVEC-based IRFs [Lindé (2009)]. However, this result

			Posterior mode estimates for selected parameters							
Model	Log marg. lik.	α	η	$\lambda_c$	$\epsilon_\psi$	$ ho_z$	κ	$ ho_r$	$\phi_{\pi}$	$\phi_y$
$M_A$	5939.1	0.24	1.30	0.63	5.52	-0.00	0.014	0.60	1.37	0.40
$M_B$	5884.4	0.15	1.64	0.66	6.07	-0.09	0.013	0.57	1.23	0.13
$M_C^{\rm IRM}$	_	0.28	0.93	0.58	0.94	0.95	_	_	_	_
$M_C$	4482.7	0.31	1.06	0.79	5.57	-0.07	_	_	_	_
$M_D$	4482.3	0.29	1.34	-	—	-0.01	—	—	—	_

TABLE 3. Alternative model specifications: Summary of results

Notes: Models  $M_C$  and  $M_D$  denote the flexible price versions with and without real demand frictions, respectively.

is obtained at the cost of a parameterization that is hardly supported by existing evidence. Even if the estimates for the capital share, labor disutility, and habits parameters are reasonable ( $\alpha = 0.28$ ;  $\eta = 0.93$ ;  $\lambda_c = 0.58$ ), the capital adjustment cost parameter is estimated to be very low ( $\epsilon_{\psi} = 0.94$ ) and the autoregressive coefficient for technology growth very high ( $\epsilon_{\psi} = 0.95$ ).

These values are not supported by the empirical literature and the data, as shown in Table 3, which provides a summary of results from the Bayesian estimation of the alternative model specifications. The table reports the marginal likelihood and the posterior mode estimates of the key parameters for the conditional dynamics of hours and investment. The results for  $M_A$  and  $M_B$  and for the impulse response matching parameterization ( $M_C^{IRM}$ ) are reported for comparison.

Nominal and monetary factors. In the presence of an accommodative policy reaction function (Model B), the estimated nominal and real frictions cannot generate the negative response of inputs. A temporary contraction can be obtained for hours, but not for investment, by increasing the level of real demand frictions to values that are higher than those needed in the flexible price model and by lowering the interest rate reaction to inflation to values close to one (or by assuming a backward-looking policy rule). The reason for this result lies in the strongly procyclical investment response that characterize the monetary models. These models imply that, because (i) nominal frictions themselves lead to the opening of positive gaps in Tobin's q and (ii) the central bank's reactions operate in the same direction as increased expected capital returns, the incentive to invest is higher than in the flexible price economy. This is the basic reason why Galì and Gertler (2007) argue that, other things being equal, monetary models generally predict that firms invest more than they would under flexible prices.

Our results thus allow us to specify Basu et al. (2006)'s conclusion that monetary models can account for the contractionary effects of productivity improvements. First, the negative hours and investment responses to productivity improvements emerge only by considering monetary policy informational lags or empirical rules such as those adopted in Model A. Second, under a RK-CDE specification, the estimated slope of the NKPC would imply an excessively high level of nominal frictions compared with survey evidence. This is not the case if strategic complementarities in price-setting emerging under the FSK-EDE model specification, along with nominal rigidities, are considered.

A low degree of accommodation of monetary policy and the flatness of the NKPC are thus the most empirically relevant factors explaining the negative response of hours and investment. When a technology improvement hits the economy, the degree to which real activity follows its natural level depends on the resulting price cut. A small NKPC slope coefficient implies that, following a productivity improvement, the contraction of the marginal cost is followed by a weak reduction in prices. For low degrees of monetary policy accommodation, the aggregate demand response is insufficient to meet the increase in productivity, leading to a reduction in the use of inputs.

The role of these two key factors is addressed in more detail in the next two subsections.

Nominal and real rigidities and the slope of the NKPC. As stressed in Section 3.2, a weak relation between marginal costs and price inflation is the result of both nominal and real rigidities, the latter defined in terms of strategic complementarity in price-setting. From the estimated reduced-form coefficient  $\kappa$ , given assumptions about the demand elasticity coefficient  $\epsilon$  and the coefficient defining its degree of endogeneity  $\phi_{\kappa}$ , we can obtain the degree of nominal rigidity that emerges under alternative model specifications.

Considering a RK-CDE specification, the estimated slope coefficient implies a price Calvo parameter  $\theta_p$  close to 0.87 (0.88 in Model *B*), consistent with a frequency of price optimization of 8 quarters (12 in Model *B*). These values are distant from those implied by the available firm-level evidence, indicating an average frequency of roughly two quarters [Bils and Klenow (2004)]. This micromacro puzzle persists, but to a reduced extent, when the FSK-EDE specification is considered: In this case, given that  $\epsilon = 11$  and  $\phi_{\kappa} = 10$ , our estimates point to a sticky price parameter value of 0.71 (0.72 under Model *B*). Table 4 shows the sensitivity of the NKPC slope coefficient to different values of  $\epsilon$  and  $\theta_p$ , given  $\phi_{\kappa} = 10$ .

Unfortunately, the parameters  $\epsilon$  and  $\phi_{\kappa}$  are unobservable.<sup>15</sup> Bowman's (2003) recent estimates of the price mark-up, which point to a value close to 4% for the U.S. economy, would suggest an implied elasticity parameter of nearly 26. On this basis, the resulting price stickiness parameter is reduced to values slightly above 0.6, consistent with a frequency of price optimization close to 2.5 periods, in accordance with Bils and Klenow's (2004) firm-level evidence and macro estimetes [Altig et al., 2011; Smets and Wouters (2007);<sup>16</sup> Riggi and Tancioni (2010); Altig et al. (2011)].

The importance of the real rigidities entailed by the FSK and EDE hypotheses is that because of their effects on the relation between marginal costs and prices, low estimates of the NKPC slope are consistent with a degree of nominal stickiness that is close to the available firm-level evidence on the frequency of price adjustments.

					$\theta_p$				
$\epsilon$	0.5	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90
3	0.0653	0.0482	0.0350	0.0248	0.0170	0.0111	0.0067	0.0036	0.0016
6	0.0791	0.0584	0.0425	0.0302	0.0208	0.0137	0.0084	0.0046	0.0021
11	0.0617	0.0457	0.0333	0.0237	0.0164	0.0108	0.0067	0.0037	0.0017
21	0.0381	0.0282	0.0206	0.0147	0.0102	0.0068	0.0042	0.0024	_
26	0.0316	0.0234	0.0171	0.0122	0.0085	0.0056	0.0035	0.0020	_
41	0.0209	0.0155	0.0113	0.0081	0.0056	0.0037	0.0024	_	_
RK	0.5010	0.3691	0.2675	0.1892	0.1292	0.0838	0.0504	0.0268	0.0113

**TABLE 4.** The NKPC slope coefficient  $\kappa$ 

*Notes*:  $\theta_p$  is the Calvo parameter (nominal rigidity);  $\epsilon$  is the demand elasticity parameter;  $\kappa$  is obtained with the undetermined coefficient method. Kimball curvature parameter  $\phi_{\kappa} = 10$ .

In contrast, under a standard rental capital specification, a flat NKPC estimate implies unrealistically low probabilities of price re-optimization.

*Productivity improvements and monetary policy.* Our estimates show that the data support the hypothesis that monetary policy follows an empirical rule rather than a theory-based reaction rule. This, together with the flat slope of the NKPC, explains the negative short-term hours and investment responses.

The degree to which a rule targeting a measure of potential output does not fully accommodate the technology shock depends on the measure being used. The policy rule adopted in Model *A* considers the long-run deterministic trend in potential output, which by definition does not respond to stochastic variations in technology. Consider a technological improvement that increases the potential and actual levels of output. The general rise in economic activity leads to a positive *measured* output gap, counteracting the interest rate drop stimulated by the induced deflation. If demand is constrained by the presence of real and/or nominal rigidities, the *true* output gap is instead negative. Consequently, the policy response suggested by the measured gap is the opposite of what the actual gap would indicate.

From a normative point of view, neutral technology shocks do not pose a relevant policy trade-off. The monetary authority could fully stabilize the economy by employing an optimal rule [from a timeless perspective, Clarida et al. (1999)]. Nevertheless, this could not happen in the real world because of the difficulties in the identification of the specific source of the shocks and the impossibility of determining a reliable measure of the natural real interest rate through the real-time informational content of macroeconomic data.<sup>17</sup>

Our analysis, conducted from a positive point of view, aims to evaluate whether and under which parameterization a monetary model can replicate the empirical evidence on the contractionary effects of productivity improvements. What the data unambiguously tell us is that monetary policy does not fully accommodate the productivity improvement. This is evident in both the SVEC and model-based impulse responses. Clearly, this conclusion is conditional on the specific policy rules and models being tested and cannot be generalized to the vast set of options that are present in the literature.

From this perspective, our results relate to a large body of literature addressing the implications of a not fully accommodative rule for the propagation mechanics of the productivity shocks. Galì (1999) proposes an interpretation of the contractionary effects of technology improvements based on a partially exogenous money supply rule. Galì et al. (2003) evaluate a similar interpretation from the perspective of not fully accommodative contemporaneous Taylor rules. Basu et al. (2006) propose an interpretation based on sticky prices and monetary policy reaction rules targeting past measures of the gaps, consistent with the idea that "the central banks observe technology shocks only with a long lag." Moreover, a recent literature focusing on the implications of the liquidity trap environment for the effectiveness of monetary policy has shown that central banks could be unable to stabilize a deflationary shock, irrespective of the possibility of having full information about the specific source of variability [Eggertsson et al. (2014)].

The estimates, in which we do not impose any prior weight on the rule to be preferred, confirm this view by showing that Model *A* maximizes the posterior marginal likelihood and is able to qualitatively reproduce the impulse responses provided by the SVEC analysis, notwithstanding the unappealing normative implications of the resulting model.

#### 6. CONCLUSIONS

This paper addresses the contractionary effects of positive technology shocks. With a SVEC model, we show that the short-term response of both hours and investment to a positive technology shock is negative and that this result is robust to important data and identification issues addressed in the literature.

We then show that the SVEC-based results are consistent with an estimated monetary model in which firm-specific capital and endogenous demand elasticity lower the price sensitivity to marginal costs (i.e., the slope of the NKPC) and monetary policy follows a not fully accommodative interest rate rule. Conditional on productivity improvements, these factors lead to the emergence of relevant demand constraints and ensure that the negative hours and investment responses are also observed for reasonable degrees of real and nominal rigidity.

Our results are consistent with those obtained by Basu et al. (2006), who use a purified measure of the Solow residual in VAR estimates, but they contrast with some of the conclusions in the macro literature.

With respect to SVAR-based results, the reasons for these different outcomes are to be found in the use of the SVEC representation. Such a representation, which is consistent with the nonstationary and co-trending properties of the data, improves the identifiability of the permanent productivity shock underlying the common trend among real variables, because the explicit consideration of the stationary ratios enhances the separation between permanent and transitory components. With respect to model-based results, our analysis shows that the key assumption that prevents the emergence of the short-term contraction of investment in other contributions is the one of monetary authorities targeting flexible price output. The data indicate that an empirical rule is preferred to a theory-based rule.

Our results thus provide additional evidence challenging the empirical relevance of flexible price models addressing neutral technology shocks as the main driver of the observed procyclicality of productivity, investment, and hours. The analysis also allows a comparative evaluation of some of the theoretical explanations of the contractionary effects of productivity improvements suggested by the literature. In this respect, our main conclusion is that both real and nominal rigidities, along with a weakly accommodative policy rule, are needed to explain the apparent puzzle within a monetary model apparatus.

However, the key real rigidities are different from those that directly affect the dynamics of consumption and investment. Although habit persistence and capital adjustment costs may contribute to explaining the observed persistence in the real variables and, for some model calibrations, the negative response of hours, they are, in fact, unable to produce a negative investment response to technology improvements. The emergence of this phenomenon requires a weak relation between marginal costs and firms' pricing behavior, which can be brought about by the additional real rigidities implied by the strategic complementarities generated by capital firm-specificity and endogenous demand elasticity.

#### NOTES

1. Del Negro et al. (2005)'s baseline specification adopts a monetary policy rule that responds to technology shocks. Smets and Wouters (2007) assume instead that the monetary authority targets flexible price output. In both cases, the policy rule accommodates the technology shock.

2. The temporary contraction in inputs appears to be driven by the short-term negative response of both types of productivity to the investment-specific productivity shock, which is interpreted as indicating the operation of technological diffusion delays. A similar argument—based on the wealth and intertemporal substitution effects implied by the presence of technological diffusion delays or by expected increases in productivity—has been proposed by Rotemberg (2003) and Beaudry and Portier (2007).

3. Their conclusion refers to Basu's (1998) model, in which the policy rule responds to lagged inflation and the lagged output gap.

4. This measure is obtained using a bottom-up growth accounting method, first estimating a purified Solow residual by controlling for capacity utilization in Hall-style regressions and then obtaining the aggregate technology as a weighted sum of the industry residuals. This methodology requires the use of industry-level annual data, the estimation of theory-based proxies for unobserved utilization and the use of a bandpass filter to isolate the frequencies of interest in the hours series.

5. Theoretically, this assumption is justified by the fact that in the absence of inflation biases, the long-run output gap is zero and long-run inflation is determined by the nonstationarity of the monetary authority's policy target.

6. Theory may suggest including a further exclusion restriction for the long-run effects of productivity shocks on hours [Francis and Ramey (2005)]. We do not take such a restriction into account, on the ground that in the presence of permanent productivity shocks, its theoretical validity is limited to a specification of utility where income and substitution effects exactly cancel out in the long run.

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7. By regressing per capita hours growth on four lags of Basu et al.'s (2006) purified technology measure growth rate, we have verified that the size of the impact coefficient is smaller than that obtained in their analysis [Table 3 in Basu et al. (2006)]. The same difference in results is obtained when the estimated (and annualized) real permanent component obtained from the SVEC-based historical decomposition is considered.

8. The zero restrictions on the  $m \times r$  right block of **C**(1)**B** are implied by the six cointegrating vectors  $\pi_t$ ,  $h_t$ ,  $r_t$ ,  $c_t - \beta_{51}y_t$ ,  $i_t - \beta_{61}y_t$  and  $wr_t - \beta_{71}y_t$ .

9. The identification of the technology shock is obtained by estimating the first equation of the VAR with the contemporaneous values and p - 1 lags of  $\Delta \pi_t$ ,  $\Delta h_t$ , and  $\Delta r_t$  and with the contemporaneous values and p lags of  $\Delta(c_t - y_t)$ ,  $\Delta(i_t - y_t)$ , and  $\Delta(wr_t - y_t)$ . p and p + 1 lags of the level variables, respectively, are used as instruments. The shock of the first equation is then included as a regressor in the remaining five equations of the VAR to capture the contemporaneous correlation between the technology shock and the other variables, i.e., to ensure orthogonality between the permanent components of interest and the stationary components.

10. No dynamic indexation is assumed, to enhance empirical identification and to allow an interpretation of results on the estimated NKPC slope in terms of the frequency of price adjustments. In fact, because under dynamic indexation prices are changed each period according to past inflation, the Calvo parameter loses its direct link with the frequency at which firms reset their prices.

11. This choice enhances the identification of the economic (endogenous) as opposed to the stochastic (exogenous) sources of persistence.

12. The posterior mode is estimated with the Simsoptimizer, and numerical integration is performed by employing 500,000 Metropolis–Hastings (M-H) replications. The fraction of the drops in the initial parameter vector estimates is set at 30%. The scale parameter for the variance of the jump distribution is calibrated to obtain an acceptance rate of nearly 30%.

13. Both  $M_A$ - and  $M_B$ -based values lie well within the wide range of NKPC slope coefficient estimates reported in the literature. Schorfheide (2008) provides a survey of these findings.

14. By estimating models *A* and *B* with full dynamic indexation, which is not supported by the data, the results do not qualitatively change. The main effect is a reduction of the estimated autocorrelation coefficients for the price and the wage push shock processes.

15. This explains the wide array of values adopted in the literature for the demand elasticity parameter, ranging from a value of 3, as in Smets and Wouters (2007), to a value of 101, as in the benchmark specification of Altig et al. (2011).

16. However, given their assumptions on the demand elasticity coefficients ( $\epsilon = 3$ , and  $\phi_{\kappa} = 10$ ) and their EDE model specification, the implied NKPC slope coefficient is near 0.028, a value that is above our estimate.

17. Because potential output is inherently unobservable, the consideration of different statistical measures can, at best, reduce but not solve this problem, as evidenced by the fact that the most frequently used measures of the output gap do not match the theoretical measures [Orphanides and van Norden (2002)]. In contrast, model-based measures are theory-specific and thus subject to the problems implied by model uncertainty. For these reasons, policy effectiveness resulting from targeting a misperceived output gap can be inferior to that of a policy rule responding to output deviations from the trend [Orphanides (2003a, 2003b, 2007); Del Negro et al. (2005)].

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# APPENDIX A: SOURCES OF DATA AND THEIR TRANSFORMATIONS

GDP ( $Y_t$ ), personal consumption expenditure for nondurable goods and services ( $C_t$ ), fixed investment including durable goods consumption ( $I_t$ ), and the GDP deflator ( $PY_t$ ) are taken from the U.S. Department of Commerce–Bureau of Economic Analysis (BEA) database. Civilian noninstitutional population aged 16 and older ( $P16_t$ ), employment level aged 16 and older ( $N16_t$ ), the average weekly hours worked in nonfarm business index ( $H_t$ ), and the hourly compensation in nonfarm business index are taken from the U.S. Department of Labor—Bureau of Labor Statistics (BLS) database. The nominal interest rate ( $R_t$ ) is the effective federal funds rate from 1954 and the three-month interest rate prior to 1954, both taken from the Federal Reserve Board economic database (FRED).

Real GDP is expressed in chained 2009 dollars. Nominal consumption, investment, and hourly compensation are deflated using the chained price GDP deflator. Real GDP, consumption, investment, and hours are scaled with respect to active population so that per capita figures are obtained. These choices make our dataset fully consistent with that employed in previous analyses in the literature.

The use of a common deflator eliminates the positive trend in the investment share resulting from the almost flat dynamics of the investment price index [Del Negro et al. (2005); Smets and Wouters (2007); Altig et al. (2011)]. The use of the hours-to-population ratio as the labor supply measure is standard in the literature [Christiano et al. (2004); Del Negro et al. (2005); Smets and Wouters (2007)]. All series are seasonally adjusted and entered in percent logs. The quarterly nominal interest rate is obtained by simply dividing the original series by four. Table A.1 summarizes the data sources and details their manipulations.

Variable	Source	Definition	Table/code	Transformation
Y <sub>t</sub>	BEA	Gross domestic product (GDP)	NIPA Table 1.1.5	$y_t = \log\left(\frac{Y_t}{\frac{PY_t}{100} \frac{P16_t}{P16_{2009}}}\right) 100$
$C_t$	BEA	Cons. of nondurables and services	NIPA Table 1.1.5	$c_t = \log\left(\frac{C_t}{\frac{PY_t - P16_t}{100 P16_{2009}}}\right) 100$
$I_t$	BEA	Fixed investment and durable cons	NIPA Table 1.1.5	$i_t = \log\left(\frac{I_t}{\frac{PY_t}{100} \frac{P16_t}{P16_{2009}}}\right) 100$
$PY_t$	BEA	Implicit price defl. for GDP 2009 = 100	NIPA Table 1.1.9	$\pi_t = \log\left(\frac{PY_t}{PY_{t-1}}\right) 100$
$P16_t$	BLS	Civilian non inst. population 16 years and older	LNU00000000	-
$N16_t$	BLS	Employment level 16 years and older	LNS12000000	-
W <sub>t</sub>	BLS	Hourly compensation idx. Nonfarm business	PRS85006103	$w_t = \log\left(rac{W_t}{rac{PY_t}{100}} ight) 100$
$H_t$	BLS	Avg. weekly hours worked idx. Nonfarm business	PRS85006023	$h_t = \log\left(\frac{\frac{N16_t}{N16_{2009}}H_t}{\frac{P16_t}{P16_{2009}}}\right) / h100$
$R_t$	FRB	Effective Federal Funds Rate	FF	$r_t = \frac{R}{4}$

# TABLE A.1. Sources of data and their transformations

# APPENDIX B: TESTS FOR (NON)STATIONARITY AND COINTEGRATION

Table B.1 provides summary information for the results of the Phillips–Perron (PP) unit root test and of the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test of stationarity for the variables used in the SVEC and in the structural model estimates, discussed in Section 2.1. The specification of the deterministic components considers the appropriate process under the alternative hypothesis of stationarity: a  $\tau_{\mu}$  specification is preferred for nonzero mean variables and a  $\tau_{\beta}$  specification for trending variables. The results summarized in the table are obtained by considering the 1948:1–2008:4 sample. We have verified that the consideration of the extended 1948:1–2014:3 sample does not change the statistical properties of the series considered in the analysis.

Table B.2 provides a summary of Johansen's trace tests results for the seven variables VEC discussed in Section 2.1. The results summarized in the table consider an unrestricted constant specification. The table reports the results for the different lag order specifications of the starting VAR suggested by the Bayesian information criteria. The Schwartz Bayesian criterion (SBC) and the Akaike information criterion (AIC) indicate one and three lags, respectively, in the 1948:1–2008:4 sample and one and five lags, respectively, in the extended 1948:1–2014:3 sample. The critical values are obtained from MacKinnon et al. (1999).

			Differences				Levels			
	Det comp	PP		KPSS		РР		KPSS		
Variable	(levels)	Test	5% cv	Test	5% cv	Test	5% cv	Test	5% cv	
<i>y</i> <sub>t</sub>	$ au_eta$	-11.0	-2.87	0.28	0.46	-2.37	-3.43	0.20	0.15	
$h_t$	$\tau = 0$	-13.4	-1.94	0.05	0.46	-2.91	-1.94	0.44	0.46	
$\pi_t$	$ au_{\mu}$	-24.2	-1.94	0.04	0.46	-5.92	-2.87	0.38	0.46	
$r_t$	$ au_{\mu}$	-12.7	-1.94	0.11	0.46	-2.37	-2.87	0.48	0.46	
$C_t$	$ au_eta$	-13.7	-2.87	0.15	0.46	-1.42	-3.43	0.37	0.15	
$i_t$	$ au_{eta}$	-11.8	-2.87	0.12	0.46	-2.72	-3.42	0.20	0.15	
$w_t$	$ au_eta$	-18.1	-2.87	0.11	0.46	-2.02	-3.43	0.39	0.15	

 TABLE B.1. PP and KPSS test results for model variables: Sample 1948:1–2008:4

*Notes*:  $\tau_{\mu,\beta}$  defines the specification of the deterministic component;  $\tau_{\mu}$ : constant;  $\tau_{\beta}$ : trend.

		LR tı					
Rank	1948:1-	-2008:4	1948:1-	-2014:3	Critical	Critical values*	
	$p = 1^a$	$p = 3^{b}$	$p = 1^a$	$p = 5^{b}$	90%	95%	
0	233.64	156.18	222.28	177.08	120.37	125.61	
1	144.09	112.02	140.28	124.45	91.11	95.75	
2	98.70	76.60	94.41	85.57	65.82	68.82	
3	56.92	50.55	56.73	52.88	44.49	47.86	
4	28.71	28.10	26.91	30.48	27.07	29.79	
5	7.03	9.86	6.51	13.27	13.43	15.49	
6	0.00	0.32	0.34	1.30	2.70	3.84	

TABLE B.2. Johansen's trace test results

Note: p is the lag order. <sup>a</sup>SBC. <sup>b</sup>AIC. \*MacKinnon et al. (1999).

# APPENDIX C: IDENTIFICATION OF THE TECHNOLOGY SHOCK

Consider the SVMA representation of the SVEC,

$$\mathbf{x}_{t} = \mathbf{C}(1) \sum_{i=1}^{t} \mathbf{B}\varepsilon_{i} + \mathbf{C}^{0}(L) \mathbf{B}\varepsilon_{t} + \widetilde{\mathbf{x}}_{0}, \qquad (\mathbf{C.1})$$

where the long-run effects matrix  $\mathbf{C}(1) = \beta \perp (\alpha' \perp \Gamma \beta \perp)^{-1} \alpha' \perp$ ,  $\Gamma = \mathbf{I}_m - \sum_{i=1}^{p-1} \Gamma_i$ , and  $\alpha \perp$ ,  $\beta \perp$  are the orthogonal complements of the loading coefficients and the long-run equilibrium matrices, respectively.  $\mathbf{C}^0(L) = \sum_{j=0}^{\infty} \mathbf{C}_j^0 L^j$  is a convergent infinite-order polynomial in the lag operator denoting the impact and interim multipliers of the shocks and  $\widetilde{\mathbf{x}}_0 = \mathbf{C}(1)\mathbf{x}_0$  depends on initial conditions [Johansen (1995)].

Leaving out the coefficients attached to the lagged variables, the reduced-form VEC provides m(m + 1)/2 = 28 nonredundant coefficients in the dispersion matrix  $\Omega$ , whereas the SVEC has  $m^2 = 49$  unknown structural coefficients in **B**. Once errors are orthonormalized, the order conditions for identification require the imposition of m(m - 1)/2 = 21 restrictions. Because the rank of the  $m \times m$  total impact matrix **C**(1) is given by the number of permanent components in the system, which is m - r = 2, the last five columns of **C**(1), which correspond to the *r* transitory components in the model (the CI vectors), are zero vectors. However, given the reduced rank of **C**(1), CI provides only (m - r)r = 10 constraints for the long-run response matrix **C**(1), leaving (m - r)(m - r - 1)/2 = 1 additional restrictions to exactly identify the permanent shocks and r(r - 1)/2 = 10 restrictions to exactly identify the transitory shocks.

Given the orthogonality between permanent and transitory components, the identification strategy of the latter does not affect the identification of the technology shock, so that to our scopes leaving the transitory shocks unidentified (i.e., the  $m \times r$  right block of **B**) unrestricted is equivalent to any other short-run identification strategy imposing restrictions on the  $m \times r$  right block of the impact effects matrix.

# APPENDIX D: BAYESIAN ESTIMATION AND MODEL SELECTION

The posterior density is obtained by employing the Bayes rule,

$$P\left(\theta_{j} \mid \mathbf{Y}_{T}, M_{j}\right) = \frac{P\left(\mathbf{Y}_{T} \mid \theta_{j}, M_{j}\right) P\left(\theta_{j}, M_{j}\right)}{P\left(\mathbf{Y}_{T}, M_{j}\right)},$$
(D.1)

where  $P(\mathbf{Y}_T, M_j)$  is the marginal data density, which can be normalized because it does not depend on  $\theta_j$ . Because the posterior density of interest is a complex nonlinear function of the deep parameters  $\theta_j$ , its analytical calculation is not generally feasible. For this reason, we calculate the posterior distribution via numerical integration. Operationally, the Bayesian MCMC posterior estimates are obtained in a two-step procedure, first employing the Kalman smoother to approximate the conditional distribution and then the Metropolis– Hastings (M-H) algorithm to perform Monte Carlo integration. Bayesian model selection is based on the Bayes factor. Considering Bayes' theorem, this posterior distribution can be expressed in terms of the posterior probabilities of the models, i.e.,

$$P(M_A, \mathbf{Y}_T) = \frac{P(\mathbf{Y}_T/M_A)P(M_A)}{P(\mathbf{Y}_T/M_A)P(M_A) + P(\mathbf{Y}_T/M_B)P(M_B)},$$
 (D.2)

where  $P(\mathbf{Y}_T/M_j) = \int P(\mathbf{Y}_T/\theta_j, M_j)P(\theta_j, M_j)d\theta_j$ , j = A, B, is the marginal distribution. The ratio between the posterior distributions of the two models gives the posterior odds ratio, which can be expressed as the priors ratio  $P(M_A)/P(M_B)$  times the Bayes factor  $P(\mathbf{Y}_T/M_A)/P(\mathbf{Y}_T/M_B)$ . Because we do not have any prior preference for one of the two models, we assume that  $P(M_A) = P(M_B)$ , so that the posterior odds is equivalent to the Bayes factor:

$$B_{A,B} = \mathrm{PO}_{A,B} = \frac{P(\mathbf{Y}_T/M_A)}{P(\mathbf{Y}_T/M_B)}.$$
 (D.3)