IAU North Poles and Rotation Parameters for Natural Satellites

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Abstract. In 1970 the IAU defined any object's *north* pole to be that axis of rotation which lies north of the solar system's invariable plane. A competing definition in widespread use at some institutions followed the "right hand rule" whereby the "north" axis of rotation was generally said to be that that of the rotational angular momentum

A Working Group has periodically updated the recommended values of planet and satellite poles and rotation rates in accordance with the IAU definition of north and the IAU definition of prime meridian.

In this paper we review the IAU definitions of *north* and of the location of *prime meridian* and we present the algorithm which has been employed in determining the rotational parameters of the natural satellites.

1. Introduction

For newly discovered objects, the IAU in 1970 recommended that the *north* pole on a planet or satellite be defined as that rotation axis which is above the invariable plane of the solar system (de Jager and Jappel 1971). For objects on which a body-fixed coordinate system has not yet been established, the IAU further defined the location of the prime meridian "...by the sub-planetary intersection of the satellite's equator and the plane containing the center of the satellite, the planet and the Sun at the first superior heliocentric conjunction of the satellite after 1950.0" (Contopoulos and Jappel 1974) .

A Working Group sponsored by the IAU, COSPAR and IUGG has periodically updated the recommended values of planet and satellite poles and rotation rates (Davies *et al.* 1980, 1983, 1986, 1989, 1992). The IAU guidelines have been somewhat controversial ever since their adoption. The various trade-offs are discussed in Davies *et al.* (1980) as well as in Kerr (1989).

Lieske (1993) outlines the IAU algorithm which is employed for defining north poles and prime meridians on natural satellites and other bodies on which physical features have not been observed heretofore. This paper presents a summary of the algorithm.

2. IAU and Angular Momentum Poles

In this paper we will mean planeto*centric* longitudes when we refer to *longitude* without qualification. The poles and rotation rates which follow the IAU guidelines are designated by α_0, δ_0 for the right ascension and declination of the pole and by W for the prime meridian measured along the satellite's equator from the point Q,

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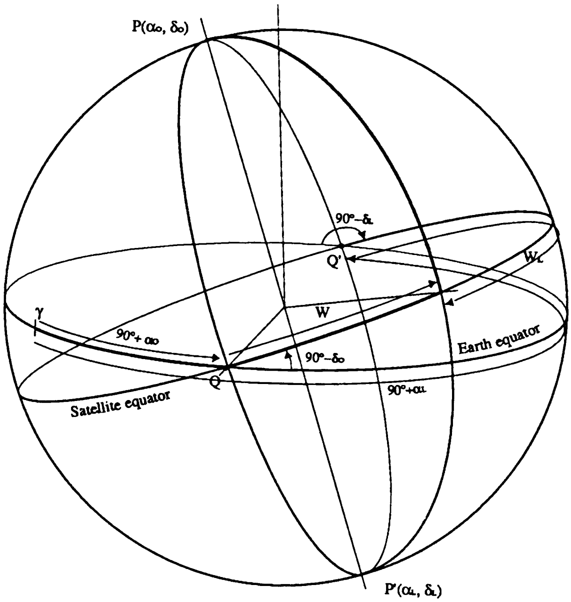

Fig. 1. Geometry relating IAU and angular momentum poles when $\mathbf{p}_L \cdot \mathbf{k}_I < 0$. The IAU parameters are denoted by α_0, δ_0, and W while the right-hand-rule parameters are denoted by α_L, δ_L and W_L. In cases where $\mathbf{p}_L \cdot \mathbf{k}_I \geq 0$, the IAU parameters are the same as those of the L-pole.

as depicted in Fig. 1 . The satellite pole has coordinates $\mathbf{p}_{IAU} = \mathbf{p}(\alpha_0, \delta_0)$ where the direction cosines are given by

$$\mathbf{p}(\alpha, \delta) = \begin{bmatrix} \cos\alpha\cos\delta \\ \sin\alpha\cos\delta \\ \sin\delta \end{bmatrix}. \tag{1}$$

We will denote the pole and prime meridian which follow the right-hand-rule (when the z-axis points along the rotational angular momentum $\mathbf{L}$) by the symbols α_L, δ_L and W_L. In systems which follow the right-hand-rule, the value of W_L always increases with time since the z-axis points along the rotational angular momentum vector. The IAU pole $\mathbf{p}_{IAU}$ and the angular momentum pole $\mathbf{p}_L$ parameters are identical if $\mathbf{p}_L \cdot \mathbf{k}_I \geq 0$:

$$\begin{aligned} \alpha_0 &= \alpha_L \\ \delta_0 &= \delta_L \\ W &= W_L, \end{aligned} \tag{2}$$

In the case where the IAU pole and the rotational angular momentum pole are in opposite hemispheres, $\mathbf{p}_L \cdot \mathbf{k}_I < 0$, then the IAU and right-hand-rule parameters are related in the following manner as depicted in Fig. 1:

$$\alpha_0 = 180 + \alpha_L$$
$$\delta_0 = -\delta_L \tag{3}$$
$$W = 180 - W_L.$$

3. Location of the Prime Meridian

The prime meridian for new objects is related to the sub-planetary point at what might be called "the first full moon after 1950." There are several ambiguities in the IAU guideline: (1) what plane is referenced in in the phrase "superior heliocentric conjunction" and (2) what is meant by "1950.0" as an epoch in the phrase "...at the first superior heliocentric conjunction of the satellite after 1950.0."

In actual practice, the "conjunction" referred to is interpreted to be heliocentric conjunction in *ecliptic* orbital longitude between the host planet and the satellite. And the basic epoch after which the first superior conjunction defines the location of the prime meridian is actually taken to be J1950 rather than Besselian epoch 1950 which might be inferred from the resolution.

We assume that the satellite rotation axis is normal to the mean orbit plane and that the satellite is in synchronous rotation. In order to obtain an approximate value for the first time t_1 after J1950 at which the satellite is in heliocentric superior conjunction, it is convenient to compute the configuration in a coordinate frame in which the true anomaly of the satellite increases with time, *viz.* the L-system in which the z-axis points along the satellite's orbital momentum vector.

If $t_0 = t_{J1950}$ is the initial estimate for the time of superior heliocentric conjunction, then the next approximation is obtained from $\Delta\psi = \psi_p(t_0) - \psi_s(t_0)$ where ψ is the orbital longitude in the L-system and where the subscripts p and s represent the planet and satellite, respectively. If one can ignore the eccentricity of the satellite and if one ignores the motion of the planet in the interval $t_1 - t_0$, then the approximate date of first superior heliocentric conjunction is given by

$$t_1 \approx t_0 + \frac{[\psi_p(t_0) - \psi_s(t_0)]}{n} \tag{4}$$

where n is the satellite mean motion. A more general approach is to regard $\Delta\psi$ as the true anomaly of the satellite at t_1 minus the true anomaly of the satellite at t_0 and to solve the Lambert problem for the difference $t_1 - t_0$ (Lieske 1993).

If t_2 represents the converged final value for the first superior conjunction after $J1950$, there is a simple geometric interpretation to the value $W_L(t_2)$ and the true orbital longitude $\psi_s(t_2)$ measured relative to Q :

$$W_L(t_2) = \psi_s(t_2) + 180°. \tag{5}$$

They are related to the satellite mean longitude λ at $t_0 =$ J1950 by

$$W_L(t_0) = \lambda(t_0) + 180° + [f_s(t_2) - M(t_2)]$$

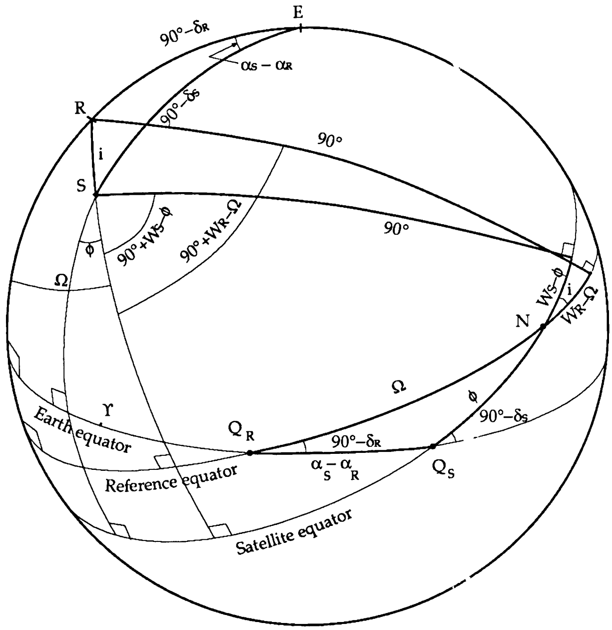

Fig. 2. Geometry in the Earth equatorial J2000 system relating the satellite parameters α_S, δ_S, and W_S to the reference parameters α_R, δ_R, and W_R The distance $W_S - \phi$ measured along the satellite equator is equal to the distance $W_R - \Omega$ measured along the reference plane from the point N. The pole of the Earth equator is E, the pole of the reference equator is R, and the pole of the satellite equator is S.

where λ is the mean longitude measured from Q and where f and M are the true and mean anomalies, respectively, at t_2.

4. A General Formulation for the Pole Parameters

We will assume that the reference pole $\mathbf{p}_R$ is in the same hemisphere as the satellite orbital angular momentum vector so that $\mathbf{p}_R \cdot \mathbf{p}_L > 0$. Lieske (1993) provides formulae also for the case when $\mathbf{p}_R \cdot \mathbf{p}_L < 0$. Let the satellite pole be represented by right ascension α_S and by declination δ_S, so that $\hat{\mathbf{p}}_S = \mathbf{p}(\alpha_S, \delta_S)$. We use the generally adopted body-fixed reference frame as opposed to instantaneous axis of angular momentum, which can lead to ambiguities of the sort described by Standish (1981) in the definition of the dynamical equinox.

Then from the spherical triangles depicted in Fig. 2, we have the following relations:

$$\begin{aligned} \cos\delta_S \cos(\alpha_S - \alpha_R) &= \cos i \cos\delta_R + \sin i \sin\delta_R \cos\Omega \\ \cos\delta_S \sin(\alpha_S - \alpha_R) &= \sin i \sin\Omega \\ \sin\delta_S &= \sin\delta_R \cos i - \cos\delta_R \sin i \cos\Omega. \end{aligned} \tag{6}$$

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The preceding equations enable one to relate the satellite values of α_S and δ_S to the reference values α_R and δ_R and to the orbital parameters.

In order to calculate the location of the prime meridian W_S, it is convenient to draw a small circle of radius $R = W_S - \phi$, which is the distance of the satellite prime meridian W_S from the point N. The pole of the small circle is at the intersection N of the satellite equator and the reference equator. We then define the auxiliary quantity W_R measured along the reference equator in relation to the quantity W_S measured along the satellite equator so that

$$W_S - \phi = W_R - \Omega = R, \tag{7}$$

as indicated in Fig. 2. Hence, in order to relate W_S to W_R we need to derive the angles ϕ and Ω depicted in the figure from the relationships given in the spherical triangle $Q_R Q_S N$ using the auxiliary quantity ζ where

$$\zeta = \phi - \Omega = W_S - W_R$$

and

$$\begin{aligned} \sin(\Omega + \zeta) &= \sin\Omega \cos\delta_R / \cos\delta_S \\ \cos(\Omega + \zeta) &= \cos\Omega \cos(\alpha_S - \alpha_R) + \sin\Omega \sin(\alpha_S - \alpha_R) \sin\delta_R. \end{aligned} \tag{8}$$

We then obtain

$$\begin{aligned} \cos\delta_S \cos(W_S - W_R) &= \cos\delta_R \left[1 - 2\sin^2\frac{i}{2}\cos^2\Omega\right] + \sin i \cos\Omega \sin\delta_R \\ \cos\delta_S \sin(W_S - W_R) &= 2\sin^2\frac{i}{2}\sin\Omega\cos\Omega\cos\delta_R - \sin i \sin\Omega \sin\delta_R, \end{aligned} \tag{9}$$

which allows us to compute the location of the prime meridian W_S.

5. Series Expansions for the Pole Parameters

From Eq. (6) one can derive a two-term Fourier expansion for $\alpha_S - \alpha_R$ in the form

$$\begin{aligned} \alpha_S - \alpha_R &= A_1 \sin\Omega + A_2 \sin 2\Omega + \cdots \\ A_1 &= \frac{\tan i}{\cos\delta_R} \\ A_2 &= \frac{-\tan^2 i \tan\delta_R}{2\cos\delta_R}. \end{aligned} \tag{10}$$

One can obtain $\delta_S - \delta_R$ by expanding the expression for $\tan(\delta_S - \delta_R)$ of Eq. (6) in the form

$$\begin{aligned} \delta_S - \delta_R &= B_0 + B_1 \cos\Omega + B_2 \cos 2\Omega + \cdots \\ B_0 &= -\frac{1}{4}\tan^2 i \tan\delta_R \\ B_1 &= -\tan i \\ B_2 &= \frac{1}{4}\tan^2 i \tan\delta_R. \end{aligned} \tag{11}$$

Finally, from Eq. (9) one can derive the expansion

$$\begin{aligned} W_S - W_R &= C_1 \sin\Omega + C_2 \sin 2\Omega + \cdots \\ C_1 &= -\sin i \tan\delta_R \\ C_2 &= \sin^2\frac{i}{2} + \frac{1}{2}\sin^2 i \tan^2\delta_R. \end{aligned} \tag{12}$$

The short Fourier expansions given in Equations (10)–(12) are adequate for most satellites. In exceptional cases (*e.g.* Triton, Nereid) where the short series expansions are inadequate one can directly expand Equations (6) and (9) using Fast Fourier Transform (FFT) techniques to numerically obtain the expansions as a function of Ω.

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