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The Spectral Radius Formula for Fourier–Stieltjes Algebras

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Abstract. In this short note we first extend the validity of the spectral radius formula, obtained by M. Anoussis and G. Gatzouras, for Fourier–Stieltjes algebras. The second part is devoted to showing that, for the measure algebra on any locally compact non-discrete Abelian group, there are no non-trivial constraints among three quantities: the norm, the spectral radius, and the supremum of the Fourier–Stieltjes transform, even if we restrict our attention to measures with all convolution powers singular with respect to the Haar measure.

1 Introduction

We first collect some basic facts from Banach algebra theory and harmonic analysis in order to fix the notation (see [Ż] for Banach algebra theory and [R] for harmonic analysis). For a commutative unital Banach algebra A, the Gelfand space of A (the set of all multiplicative-linear functionals endowed with weak* topology) will be abbreviated $\triangle(A)$ and the Gelfand transform of an element $x \in A$ is a surjection $\widehat{x}\colon \triangle(A) \to \sigma(x)$ defined by the formula $\widehat{x}(\varphi) = \varphi(x)$, for $\varphi \in \triangle(A)$, where

$$\sigma(x) := \{\lambda \in \mathbb{C} : \mu - \lambda 1 \text{ is not invertible}\}$$

is the spectrum of an element x. Let G be a locally compact Abelian group with its unitary dual $\widehat{G}$, and let $M(G)$ denote the Banach algebra of all complex-valued Borel regular measures equipped with the convolution product and the total variation norm. The Fourier–Stieltjes transform will be treated as a restriction of the Gelfand transform to $\widehat{G}$, and $M(G)$ is also equipped with involution $\mu \mapsto \widetilde{\mu}$, where $\widetilde{\mu}(E) := \overline{\mu(-E)}$ for every Borel set $E \subset G$. A measure μ is Hermitian if $\mu = \widetilde{\mu}$ or, equivalently, if its Fourier–Stieltjes transform is real-valued. The ideal of measures with Fourier–Stieltjes transforms vanishing at infinity is denoted by $M_0(G)$.

Note that we have a direct sum decomposition $M(G) = L^1(G) \oplus M_s(G)$, where $L^1(G)$ is the group algebra identified via the Radon–Nikodym theorem with the ideal of all absolutely continuous measures and $M_s(G)$ is a subspace consisting of singular (supported on a set of Haar measure zero) measures. For $\mu \in M(G)$, we will write $\mu = \mu_a + \mu_s$ with $\mu_a \in L^1(G)$ and $\mu_s \in M_s(G)$.

The main result of the first section is presented in the framework of Fourier–Stieltjes algebras, so we recall some basic information [Ey, KL]. Let G be a locally compact group and let $B(G)$ be the Fourier–Stieltjes algebra on the group G, *i.e.*, the linear span

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of positive-definite continuous functions on G equipped with the norm given by the duality $B(G) = (C^*(G))^*$, where $C^*(G)$ is the full group C^*-algebra of G. It is well known that $B(G)$ with pointwise product is a commutative semisimple unital Banach algebra with G embedded in $\triangle(B(G))$ via point-evaluation functionals. As for the measure algebra, we have an orthogonal decomposition of $B(G) = A(G) \oplus B_s(G)$, where $A(G)$ is the Fourier algebra of the group G and $B_s(G)$ is a subspace consisting of singular elements (it was proved first in [A], see [OW] for a modern presentation). For $f \in B(G)$, we will write $f = f_a + f_s$ with $f_a \in A(G)$ and $f_s \in B_s(G)$.

In [AG] the following spectral radius formula for measure algebra on compact (not necessarily Abelian) group was shown to be true.

Theorem 1.1 *Let G be a compact group and let $\mu \in M(G)$. Then the following formula holds: $r(\mu) = \max\{\sup_{\sigma\in\widehat{G}} r(\widehat{\mu}(\sigma)), \inf_{n\in\mathbb{N}} \|(\mu^{*n})_s\|^{\frac{1}{n}}\}$.*

Here $\widehat{G}$ is the set of all equivalence classes of irreducible unitary representations of the group G, and $\widehat{\mu}(\sigma)$ is the matrix-valued Fourier–Stieltjes transform.

In the first section we will give a short proof of the counterpart of this formula for Fourier–Stieltjes algebras. The aim of the second part is to construct examples of measures exhibiting that, other than the obvious ones, there are no connections among the norm, the spectral radius, and the supremum of the Fourier–Stieltjes transform.

2 The Spectral Radius Formula

Theorem 2.1 *Let G be a locally compact group and let $f \in B(G)$. Then the following formula holds true:*

$$r(f) = \max\{\|f\|_\infty, \inf_{n\in\mathbb{N}} \|(f^n)_s\|^{\frac{1}{n}}\}. \tag{2.1}$$

Proof Clearly, $r(f) \geq \|f\|_\infty$. Let us take $n \in \mathbb{N}$. Then $f^n = (f^n)_a + (f^n)_s$. This implies $\|f^n\| = \|(f^n)_a\| + \|(f^n)_s\| \geq \|(f^n)_s\|$. Taking the infimum on both sides and applying the spectral radius formula, we obtain the first inequality.

As $A(G)$ is an ideal in $B(G)$ we have the splitting of $\triangle(B(G))$:

$$\triangle(B(G)) = \triangle(A(G)) \cup h(A(G)) = G \cup h(A(G)), \tag{2.2}$$

where $h(A(G)) = \{\varphi \in \triangle(B(G)) : \varphi|_{A(G)} = 0\}$.

For $f \in B(G)$ and any $\varphi \in \triangle(B(G))$, we have $\varphi(f) = \varphi(f_a) + \varphi(f_s)$. If $\varphi \in \triangle(A(G))$, then, of course, $|\varphi(f)| = |f(x)|$ for some $x \in G$, and so $|\varphi(f)| \leq \|f\|_\infty$. In case $\varphi \in h(A(G))$, we have $|\varphi(f)| = |\varphi(f_s)| \leq r(f_s) \leq \|f_s\|$. The above considerations are summarized as the following inequality:

$$r(f) \leq \max\{\|f\|_\infty, \|f_s\|\}.$$

Let us fix $n \in \mathbb{N}$ and apply this inequality to f^n. As $r(f^n) = r(f)^n$, we get

$$r(f)^n \leq \max\{\|f\|_\infty^n, \|(f^n)_s\|\}.$$

To finish the proof of (2.1) we need only take the $\frac{1}{n}$ power on both sides and the infimum over n. ∎

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Corollary 2.2 *Let $f \in B(G)$ satisfy $r(f) > \|f\|_\infty$. Then at least one of the following situations occurs.*

(i) *For every $n \in \mathbb{N}$, we have $f^n \in B_s(G)$.*
(ii) $\|f\| > r(f)$.

Proof Suppose that $f^{n_0} \notin B_s(G)$ for some $n_0 \in \mathbb{N}$. Then by Theorem 1.1 we obtain

$$r(f) = \inf_{n\in\mathbb{N}} \|(f^n)_s\|^{\frac{1}{n}} \le \|(f^{n_0})_s\|^{\frac{1}{n_0}} < \|f^{n_0}\|^{\frac{1}{n_0}} \le \|f\|,$$

which finishes the proof. ∎

Our last objective for this section is to show that neither of the two items in Corollary 2.2 implies the other and, also, that both of them may hold at the same time, providing explicit examples of measures on the circle group. The last one will be also used in the next section.

Example A measure satisfying $\|\mu\| > r(\mu) > \|\widehat{\mu}\|_\infty$, but with non-singular convolution powers

Let us consider $\mu = \frac{1}{2}R - \frac{1}{2}\mathrm{m}$, where $R = \prod_{k=1}^{\infty}(1 + \cos(3^k t))$ is the classical Riesz product measure (understood as a weak* limit of finite products) and m is the normalized Lebesgue measure on $\mathbb{T}$. Let us recall that R is a continuous probability measure with independent powers, *i.e.*, $R^{*n} \perp R^{*m}$ for $n \neq m$ [BM1], and satisfying $\sigma(R) = \overline{\mathbb{D}} = \{z \in \mathbb{C} : |z| \le 1\}$ (see the chapter on Riesz products [GM]). Moreover,

$$\widehat{R}(\mathbb{Z}) = \{0\} \cup \left\{\frac{1}{2^n}\right\}_{n\in\mathbb{N}} \cup \{1\}.$$

We clearly have $\|\mu\| = 1$. It is also immediate that $\|\widehat{\mu}\|_\infty = \frac{1}{4}$. Taking into account (2.2), we get

$$\begin{aligned}\sigma(\mu) &= \widehat{\mu}(\mathbb{Z}) \cup \widehat{\mu}(\triangle(M(\mathbb{T})) \setminus \mathbb{Z}) \\ &= \{0\} \cup \left\{\frac{1}{2^{n+1}}\right\}_{n=1}^{\infty} \cup \frac{1}{2}\widehat{R}(\triangle(M(\mathbb{T})) \setminus \mathbb{Z}) \\ &= \frac{1}{2}\overline{\mathbb{D}} \text{ as } \sigma(R) = \overline{\mathbb{D}}.\end{aligned}$$

Example A measure with all convolution powers singular with the properties $\|\mu\| = r(\mu) > \|\widehat{\mu}\|_\infty$.

Put $\mu = \frac{1}{2}\left(R - R^{*2}\right)$. Then $\|\mu\| = 1$ and all convolution powers of μ are singular. By the spectral properties of Riesz products there exists $\varphi \in \triangle(M(\mathbb{T}))$ such that $\varphi(R) = -1$. This gives $\varphi(\mu) = -1$ and thus $r(\mu) = 1$. Also, $\|\widehat{\mu}\|_\infty = \frac{1}{8}$.

Example A measure with all convolution powers singular satisfying additionally $\|\mu\| > r(\mu) > \|\widehat{\mu}\|_\infty$.

Let $q_0(z) := \frac{1}{4}\left(z^5 - z^4 + z^2 - z\right) = \frac{1}{4}z(z^3 + 1)(z - 1)$. Then $q_0(1) = 0$, and by the maximum modulus principle we get

$$\sup_{z\in\{\frac{1}{2^n}\}_{n\in\mathbb{N}}\cup\{1\}} |q_0(z)| = \sup_{z\in\{\frac{1}{2^n}\}_{n\in\mathbb{N}}} |q_0(z)| < \sup_{z\in\overline{\mathbb{D}}} |q_0(z)|. \tag{2.3}$$

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Also, using the maximum modulus principle again,

$$\sup_{z\in\overline{\mathbb{D}}}|q_0(z)| = \sup_{z\in\mathbb{T}}|q_0(z)| \le \frac{1}{4}\sup_{z\in\mathbb{T}}|z^3+1|\cdot\sup_{z\in\mathbb{T}}|z-1| = 1.$$

However, as the supremum of $|z^3+1|$ on $\mathbb{T}$ is attained only for the roots of unity of order three and the supremum of $|z-1|$ as attained only for $z=-1$, we obtain

$$\sup_{z\in\overline{\mathbb{D}}}|q_0(z)| < 1. \tag{2.4}$$

It is convenient to use the following convention. For an algebraic polynomial $f(z) = a_n z^n + a_{n-1}z + \cdots + a_1 z + a_0$, we put $|f|_1 := \sum_{k=0}^{n}|a_k|$. Let $\mu := q_0(R)$, where R is the classical Riesz product. Then $\|\mu\| = |q_0|_1 = 1$ as R has independent powers. By the spectral mapping theorem

$$r(\mu) = \sup_{z\in\overline{\mathbb{D}}}|q_0(z)| \text{ as } \sigma(R) = \overline{\mathbb{D}}.$$

By (2.4) we thus have $r(\mu) < 1$. Using the properties of the functional calculus again, we get

$$\|\widehat{\mu}\|_\infty = \sup_{z\in\{\frac{1}{2^n}\}_{n\in\mathbb{N}}\cup\{1\}}|q_0(z)|.$$

The application of (2.3) finishes the argument.

3 The Construction

In this section we will show that there are no non-trivial constraints among the quantities $\|\mu\|$, $r(\mu)$, and $\|\widehat{\mu}\|_\infty$ even if we restrict our attention to measures with all convolution powers singular.

Theorem 3.1 *Let a, b be two fixed numbers satisfying $0 < b \le a \le 1$, and let G be a locally compact Abelian non-discrete group. Then there exists a measure $\mu \in M_0(G)$ with all convolution powers singular such that $\|\widehat{\mu}\|_\infty = b$, $r(\mu) = a$, and $\|\mu\| = 1$.*

The proof of this theorem depends on the existence of measures with special properties and we combine Theorem 6.1.1 and Corollary 7.3.2 from [GM] to obtain the formulation adequate for our needs.

Theorem 3.2 *Let G be a locally compact Abelian non-discrete group. Then there exists a Hermitian independent power probability measure in $M_0(G)$. Moreover, the spectrum of this measure is the whole unit disc.*

An application of the functional calculus to a measure described in Theorem 3.2 shows that, in order to prove Theorem 3.1, it is enough to construct a polynomial with the properties specified below.

Theorem 3.3 *Let a, b be two fixed numbers satisfying $0 < b \le a \le 1$. Then there exists an algebraic polynomial p with the following properties.*

(i) $|p|_1 = 1$.
(ii) $\sup_{z\in\overline{\mathbb{D}}}|p(z)| = a$.
(iii) $\sup_{x\in[-1,1]}|p(x)| = b$, $p(1) = b$.

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Proof We divide the argument into three steps.

Step 1: $\frac{-5+4\sqrt{2}}{7} =: b_0 < b < a \le 1$. The polynomial

$$q_0(z) = \frac{1}{4}z(z-1)(z+1)(z^2-z+1) = \frac{1}{4}z(z-1)(z^3+1) = \frac{1}{4}(z^5-z^4+z^2-z)$$

introduced in the previous section has the following properties:

- $|q_0|_1 = 1$,
- $q_0(1) = q_0(0) = q_0(-1) = 0$,
- $\|q_0\|_{C(\overline{\mathbb{D}})} < 1$.

Let us consider an inductive procedure $q_{n+1}(z) = q_n(z) \cdot q_0(z^{\deg q_n + 1})$ with q_0 defined as before. It is elementary to verify that, for every $n \in \mathbb{N}$, the polynomial q_n satisfies

- $|q_n|_1 = 1$,
- $q_n(1) = q_n(0) = q_0(-1) = 0$,
- $\|q_n\|_{C(\overline{\mathbb{D}})} \le \|q_0\|_{C(\overline{\mathbb{D}})}^{n+1}$.

The first property follows from the fact that at each step we multiply q_n by a polynomial with gaps between powers longer than $\deg q_n$. Moreover, observing first that $|q_n(x)| \le |q_1(x)| \cdot \|q_0\|_{C(\overline{\mathbb{D}})}^{n-1}$, we get an elementary estimate of q_n on the real axis

$$|q_n(x)| \le c(1-x)^2(1+x)^2 \cdot \|q_0\|_{C(\overline{\mathbb{D}})}^{n-1} \quad \text{for } x \in [-1,1] \text{ and } n \ge 1, \tag{3.1}$$

where c is a numerical constant.

Consider the family of polynomials p_α indexed by the parameter $\alpha \in [b,1]$,

$$p_\alpha(z) = \frac{1}{2}(\alpha+b)z^4 - \frac{1}{2}(\alpha-b)z^2 + (1-\alpha)q_n(z), \tag{3.2}$$

where $n > 1$ will be chosen later, but let us take into account that the form of q_n implies $|p_\alpha|_1 = 1$ for $\alpha \in [b,1]$. Clearly, we also get $p_\alpha(1) = p_\alpha(-1) = b$. Applying elementary calculus to the function $f(x) = \frac{1}{2}(\alpha+b)x^4 - \frac{1}{2}(\alpha-b)x^2$ on the interval $[0,1]$ (this is sufficient, as f is even and the estimate of $|p_\alpha|$ is also even-type), we find the local extremum at the point $x_0 = \sqrt{\frac{\alpha-b}{2(\alpha+b)}}$ equal to

$$f(x_0) = -\frac{1}{8}\frac{(\alpha-b)^2}{\alpha+b}.$$

Taking the supremum over $\alpha \in [b,1]$, we obtain $|f(x_0)| \le \frac{1}{8}\frac{(1-b)^2}{1+b}$. Solving the inequality $\frac{1}{8}\frac{(1-b)^2}{1+b} < b$, gives $b > \frac{-5+4\sqrt{2}}{7} = b_0$ implying $|f(x_0)| < b$ for $b > b_0$.

Let us fix ε satisfying

$$\varepsilon < \min\left\{ a-b, b - \frac{1}{8}\frac{(1-b)^2}{1+b} \right\}, \tag{3.3}$$

and take n (depending only on ε) such that $\|q_n\|_{C(\overline{\mathbb{D}})} < \varepsilon$. As $|p_\alpha(x_0)| \le |f(x_0)| + \|q_n\|_{C(\overline{\mathbb{D}})}$, we obtain $|p_\alpha(x_0)| < b$ for $\alpha \in [b,1]$.

Further analysis of f shows that the function $x \mapsto |f(x)|$ has the following properties:

- $f(0) = 0$,
- increases on the interval $[0, x_0]$ up to value $|f(x_0)|$,
- decreases on the interval $[x_0, x_1]$ up to value 0, where $x_1 = \sqrt{\frac{\alpha-b}{\alpha+b}}$,
- increases on the interval $[x_1, 1]$ up to value b and $f(x) > 0$ in this region.

It follows from the above discussion that for $x \in [0, x_1]$ we are allowed to use the same estimates as for the point x_0 to obtain the conclusion $|p_\alpha(x)| \le b$. For the interval $[x_1, 1]$ we proceed as follows (here we use (3.1)):

$$\begin{aligned}|p_\alpha(x)| &\le |f(x)| + (1-\alpha)c\|q_0\|^{n-1}_{C(\overline{\mathbb{D}})}(1-x)^2(1+x)^2\\ &= \frac{1}{2}x^2\big((\alpha+b)x^2 - (\alpha-b)\big) + (1-\alpha)c\|q_0\|^{n-1}_{C(\overline{\mathbb{D}})}(1-x)^2(1+x)^2\\ &\le bx^2 + \widetilde{c}\|q_0\|^{n-1}_{C(\overline{\mathbb{D}})}(1-x)^2,\end{aligned}$$

where $\widetilde{c}$ is another numerical constant. The last expression can be made smaller than b by taking sufficiently big n (depending on b only) as it is a non-negative quadratic function with positive leading coefficient, so one must take care of the endpoints of the interval $[0,1]$ for which we have values $\widetilde{c}\|q_0\|^{n-1}_{C(\overline{\mathbb{D}})}$ and b.

We start with the estimates for the uniform norm of p_α on the unit disc. The upper bound is

$$\|p_\alpha\|_{C(\overline{\mathbb{D}})} \le \alpha + \|q_n\|_{C(\overline{\mathbb{D}})} < \alpha + \varepsilon. \tag{3.4}$$

In order to get the lower bound, we simply observe that

$$\|p_\alpha\|_{C(\overline{\mathbb{D}})} \ge |p_\alpha(i)| = |\alpha + (1-\alpha)q_n(i)|. \tag{3.5}$$

Consider now the function $F(\alpha) := \|p_\alpha\|_{C(\overline{\mathbb{D}})}$ for $\alpha \in [b, 1]$. Using (3.4) and (3.5) we get $F(b) \le b + \varepsilon < a$ and $F(1) = 1$. It is immediate that F is continuous (in fact Lipschitz continuous), so there exists $\alpha_0 \in [b, 1]$ such that $F(\alpha_0) = a$ and then the polynomial p_{α_0} satisfies the assertion of the theorem.

Step 2: $0 < b < a \le 1$. We find first $k \in \mathbb{N}$ (depending on b only) such that $\sqrt[k]{b} > b_0$. Let us fix ε restricted as in (3.3) with b, a replaced by $\sqrt[k]{b}$, $\sqrt[k]{a}$, respectively. Construct the polynomial q_n starting with the data $\sqrt[k]{b}$, $\sqrt[k]{a}$, and ε as in Step 1 and let us form the family p_α for $\alpha \in [\sqrt[k]{b}, 1]$ as in (3.2). Now we perform another inductive procedure to be executed k-times:

$$w_{l+1,\alpha}(z) = w_{l,\alpha}(z) \cdot w_{0,\alpha}(z^{4\deg w_{l,\alpha}+1}) \text{ with } w_{0,\alpha} = p_\alpha.$$

As $4\deg w_{l,\alpha} + 1 > \deg w_{l,\alpha}$, we get $|w_{l,\alpha}|_1 = 1$ for $l \in \mathbb{N}$ and $\alpha \in [\sqrt[k]{b}, 1]$.

Let us define $w_\alpha := w_{k-1,\alpha}$ for $\alpha \in [\sqrt[k]{b}, 1]$. Then $w_\alpha(1) = (p_\alpha(1))^k = b$, and also $\|w_\alpha\|_{C[-1,1]} \le \|p_\alpha\|^k_{C([-1,1])} \le b$. Moreover, by (3.4), we obtain

$$\|w_\alpha\|_{C(\overline{\mathbb{D}})} \le (\alpha + \varepsilon)^k \le \alpha^k + c\varepsilon,$$

where c is a numerical constant. Decreasing ε if necessary, we get

$$\|w_{\sqrt[k]{b}}\|_{C(\overline{\mathbb{D}})} \le b + c\varepsilon < a. \tag{3.6}$$

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For the lower estimate, we use (3.5)

$$\|w_1\|_{C(\overline{\mathbb{D}})} \geq |w_1(i)| = \prod_{l=0}^{k-1} |p_1(i^{4deg w_{l,1}+1})| = (p_1(i))^k = 1. \tag{3.7}$$

The same continuity argument as in Step 1, using (3.6) and (3.7), finishes the proof.

Step 3: $b = a = 1$ and $b = a < 1$. If $b = a = 1$, then the polynomial $p(z) = z$ does the job. In case $b = a < 1$, we take the polynomial p with the properties $|p|_1 = 1$, $\|p\|_{C(\overline{\mathbb{D}})} = a$, and $\|p\|_{C([-1,1])} = \widetilde{b}$ for some positive $\widetilde{b} < a$ constructed in Steps 1 and 2. Let $z_{\max}$ be the point on the circle for which $|p(z_{\max})| = \|p\|_{C(\overline{\mathbb{D}})}$ and consider the polynomial $p_{\max}(z) := p(z_{\max} \cdot z)$. Then $|p_{\max}|_1 = 1$, $\|p_{\max}\|_{C(\overline{\mathbb{D}})} = \|p\|_{C(\overline{\mathbb{D}})} = a$, and $\|p_{\max}\|_{C([-1,1])} = |p_{\max}(1)| = |p(z_{\max})| = a$. ∎

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