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# Astronomical Observations of the 1869 Powell Expedition Through the Grand Canyon

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During his 1869 expedition down the Green River and through the Grand Canyon, Major John Powell made astronomical observations using a sextant and artificial horizon to fix the locations of key points along the rivers that were only poorly known at the time. Latitude was obtained from the altitude of Polaris or meridian transits of stars or Saturn. Local mean time was determined from equal altitude observations of the Sun. The swamping of one of the expedition's small boats ruined the chronometers, meaning that they could not be used to keep Greenwich mean time and hence find longitude. As a substitute a series of lunar distance observations were undertaken. In this paper observations recorded in Powell's journal are reduced and analysed.

KEYWORDS

1. Sextant. 2. Clock Corrections. 3. Log. 4. Navigation.

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1. INTRODUCTION. In 1869 Major John Wesley Powell led the first expedition through the Grand Canyon. The expedition's 10 members embarked in four small boats from Green River Station in Wyoming Territory on 24 May. Some 900 miles and 98 days later on 30 August they reached St. Thomas near the confluence of the Colorado and Virgin Rivers. Along the way they faced many hardships on dwindling rations but successfully navigated territory that was very little used and rarely visited even by Indians. One member of the team abandoned the expedition on 6 July. Three others who chose to do the same on 28 August were believed to have been killed by Indians.

The day-to-day events are recorded in journals kept by George Bradley, John Sumner and Powell himself. These have been fully transcribed by Ghiglieri (2015), which is the source used here for quotes made by expedition members. The astronomical observations from Powell's journals have also been transcribed by Quartaroli (2002), who has done much to encourage their analysis and interpretation. The published transcript does contain some errors, however, and for a detailed study the original manuscript, housed at the National Anthropological Archive of the Smithsonian Institution (Powell, 1869), should be consulted.

Despite their historical significance, no complete or systematic study had been done of the astronomical observations that Powell made to fix his position. Ghiglieri (2015) describes the efforts of an anonymous expert in historical methods of celestial navigation, but this is more or less anecdotal in nature. In this paper all recorded astronomical observations have been examined in detail and reduced. These span the period 13 July to 4 September 1869. A few of the journal pages from the early part of the expedition containing observations or calculations are faint and illegible. The locations where Powell took his most extensive rounds of astronomical sights were near well-defined landmarks such as river junctions and this information can be used to assess the quality of his sights.

The expedition carried a selection of scientific instruments including 'two sextants, four chronometers, a number of barometers' (Quartaroli, 2002, p. 132) that would allow latitude, longitude and elevations to be determined. On 11 July, while running the rapids at the southern end of Desolation Canyon, Powell's boat, named for his wife and second cousin *Emma Dean*, was swamped. Sumner records that this 'ruined $800 of watches' and caused the loss of three of their 10 rifles and all of the crew's bedding.

It appears that this represented a very serious blow to Powell's planned observing programme. Instead of being able to rely on chronometers to provide Greenwich mean time (GMT) and hence longitude, he was forced to fall back on lunar distance sights and a possible solar eclipse timing which he knew, in principle, could be used for that purpose but which he probably had little or no experience of actually performing. The calculations required to reduce these types of observations are more complex and specialised than those required when a working chronometer is available and there is no evidence in the journal of them being carried out. Powell presumably intended to record a sufficient number of observations that would allow their position to be accurately determined upon their return. Lacking working chronometers, an Elgin pocket watch was pressed into service as a partial replacement but it too shows evidence of behaving erratically.

Powell has his admirers and detractors, but that debate will not be entered into here. He seems to have been a diligent and committed observer who made every effort to perform the sights necessary to accurately track their position. This appears sometimes to have been at the frustration of expedition members, some of whom, faced with diminishing supplies, were keen to keep moving forward. Bradley wrote, on 2 August:

> In the same camp, doomed to be here another day, perhaps more than that, for Major has been taking observations ever since we came here and seems no nearer done now than when he began. He ought to get Latitude & Longitude of every mouth of a river not before known and we are willing to face starvation if necessary to do it but further than that he should not ask us to waite [sic] and he must go on soon or the consequences will be different from what he anticipates.

2. LOCATIONS. As noted previously, all the camp sites from which the expedition carried out extensive rounds of astronomical observations are described in sufficient detail in the members' journals (Ghiglieri, 2015) that their locations can be pinpointed with a fair degree of reliability:

> Bradley, 18 July: 'Our camp is pitched in the angle formed by the two rivers on a sand bank washed down by both of them. To our right, when looking south, comes the Green . . . The Grand flows in from the left . . . '
> Sumner, 4 August: 'Camped on the north side on a sandbar.'

Table 1. Locations of camp sites mentioned in expedition journals where rounds of astronomical observations were taken.

| | Dates | Latitude | | Longitude | | Elevation |
|---|---|---|---|---|---|---|
| Location | 1869 | ° | ′ | ° | ′ | m |
| Junction of Grand and Green Rivers | 16–20 Jul | 38 | 11·5 N | 109 | 53·2 W | 1,200 |
| Mouth of San Juan River | 31 Jul | 37 | 10·8 | 110 | 54·0 | 990 |
| Music Temple | 1–2 Aug | 37 | 09·8 | 110 | 55·4 | 990 |
| Pah Reah, Ute Creek | 4 Aug | 36 | 51·8 | 111 | 35·8 | 950 |
| Mouth of Flax River | 11–12 Aug | 36 | 11·5 | 111 | 47·9 | 830 |
| Silver Creek | 15–16 Aug | 36 | 06·0 | 112 | 05·5 | 750 |
| Separation Camp | 27 Aug | 35 | 49·4 | 113 | 34·2 | 370 |
| St George | 4 Sep | 37 | 06·1 | 113 | 34·7 | 820 |

> Bradley, 11 August: ‘Our camp is under the shelving edge of a cliff on the south side of the Chiquito [Little Colorado river] . . . ’

On his return to Washington D.C. Powell changed what he had named Silver Creek to Bright Angel Creek, the name it still bears today. Separation Camp is marked by a cenotaph plaque on the opposite bank, placed there in 1939 for the three men who left the expedition at that point.

The positions of the campsites as deduced from the journal descriptions and information above are listed in Table 1. A few locations from which a single meridian altitude sight for latitude was taken and only described by the camp number are not included. Elevations for each site are given and this information can be used in computing corrections for refraction. Two of the sites, at the mouth of the San Juan River and Music Temple, now lie under the waters of Lake Powell. The names in the table are the ones Powell and his men used in their journals at the time and may differ from the names by which these places are known today.

Powell recorded his observations in degrees, minutes and seconds of arc, and would have done so for position as well. In this work the modern convention of giving results up to tenths of a minute is followed.

It is assumed that Powell would have taken his sights near to the camp and these locations are used later when quoting differences between his true and observed position.

## 3. CELESTIAL NAVIGATION.

A navigator’s position on the surface of the Earth can be specified by the key quantities: latitude ($L$), the local mean time (LMT) and GMT. The difference, LMT − GMT, yields the longitude directly. Each of these three quantities is extracted from a distinct type of astronomical observation.

Observations for latitude are the simplest to perform. The sight requires following the body approaching the meridian with a sextant as its altitude increases, slows and then reverses direction. The required altitude is the observed maximum. The observer’s latitude is obtained after some basic sight corrections are applied and the body’s declination is factored in. In the northern hemisphere the altitude of the pole star, Polaris, approximates the observer’s latitude. After applying the appropriate corrections to the measured altitude, latitude can be accurately determined. In 1869 Polaris lay around 1·4° from the north celestial pole and without these corrections the observer’s position could be off by as much as

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±84 nautical miles in the north/south direction. In 2020 Polaris is 0·7° from the pole with the change being mainly due to precession over the intervening 150 years.

Local apparent time (LAT) is the time that is shown on a sundial and is equivalent to the local hour angle (LHA) that the true Sun makes with the observer's meridian. The true Sun does not move at a uniform rate along the ecliptic and hence LAT is not a uniform time scale. LMT is constructed as the average of LAT and is the time that an ideal chronometer would keep. The difference between LAT and LMT is called the equation of time (EqT), EqT = LAT–LMT.

With knowledge of the observer's latitude and the Sun's declination, a measurement of the Sun's altitude yields LAT. This is called a time sight and is best performed when the Sun is well off the meridian as it reduces the sensitivity to possible errors in the latitude. On the prime vertical, exactly east or west, the LAT obtained is independent of latitude, however, the Sun is comparatively low in the sky at such times and probably would not have been visible above the canyon walls. The same would be true for other solar system bodies. This may be the reason Powell took only one sight of this type, 'Al. Saturn time Aug 29', after exiting the canyon. However without a reliable measurement of GMT it provides no useful information.

Powell determined LMT using the method of equal altitudes (EA). A series of sights of the Sun's altitude over a 10–15 min period is made in the morning and afternoon. A set of altitudes is specified in advance and the chronometer time noted when the Sun attains each one. Averaging the times of morning and afternoon sights separately serves to suppress random errors. When reduced, the EA observations yield the chronometer time at which the Sun crosses the meridian. This is local apparent noon (LAN) and using the EqT, LMT is straightforwardly determined.

GMT can be read directly from a chronometer, provided it has been carefully rated and managed. In the absence of a chronometer, GMT may be determined by observation. The Moon moves sufficiently rapidly against background stars that careful measurements of its angular distance from the Sun, planets or certain stars when suitably located on or near the ecliptic can act as a clock from which GMT can be extracted. Along with the lunar distance, measurements of the altitudes of the Moon and other body all taken at nearly the same time are also required. The observations require a skilled observer, and the internal inconsistencies and inadequacies that can be seen in some of Powell's lunar distance observations, or 'lunars', suggest that this was not a method that he had practised. He may have only undertaken it upon loss or damage to the chronometers.

An attempt was made to observe the total solar eclipse on 7 August 1869, which was partial from the expedition's location. The time of the first or last contact of the Moon's disc on the face of the Sun could be used, along with latitude and LMT, to determine GMT and hence longitude.

> Bradley, 7 August: 'Major & brother have climbed the mountain to observe the eclipse . . . Cannot tell whether he saw it. If he did we shall have our Longitude.'
> Powell, 7 August: 'Failed clouds. Slept on mt. side, to [sic] dark to get all the way down.'

To use either of these methods to find longitude accurately, a clock running at a uniform rate is required to bring the observations of LMT and GMT together to a common instant in time.

With properly working chronometers, finding longitude from EA observations would have been relatively quick and easy. Without them, a considerable amount of extra calculation is needed. Tables of lunar distances were listed in the Nautical Almanac (American Ephemeris and Nautical Almanac, 1869) and made the process of reducing lunars far less burdensome than popularly believed but still required some effort. The reduction of eclipse observations is quite complex. It is therefore unlikely that Powell intended to perform the required calculations in the field but rather planned to accumulate observations that would be analysed upon his return.

3.1. *Making astronomical observations.* For the purposes of celestial navigation only altitudes and distances between bodies can be measured with sufficient accuracy to fix position. Angles such as azimuths cannot be used.

With no natural horizon being visible, altitude measurements would have to employ an artificial horizon. The specific type of artificial horizon used is not known but it likely consisted of a dish or pan of mercury, possibly covered by an angled glass roof to prevent the wind from rippling the surface. A degree of care is required to ensure that the surface remains free from contamination. In Powell's rounds of lunar sights there is a noticeable delay between altitude sights, made using the artificial horizon, and lunar distance sights which require the sextant only. These delays may reflect the time needed to safeguard and set up the mercury. On 31 July Powell successfully measured the altitude of the star $\beta$ Ceti (Diphda), which is visual magnitude 2, indicating that the surface was highly reflective and could not have been water, oil or alternative liquids that are sometimes used. As described in an account by Powell published in the *Chicago Tribune* on 19 July 1869 (Ghiglieri, 2015, p. 90), mercury was also carried on the expedition for the servicing and repair of the barometers.

The reflection off the surface of the artificial horizon means that the observed altitude angle is doubled. Powell indicates this in several places with notations like 'Al double' or '2 Alt'. Since a sextant can measure angles nominally up to 120° this angle doubling means that sights are restricted to objects below about 60° in altitude. Consequently Powell could not take noon sights for latitude using the Sun.

An altitude sight by sextant entailed bringing the direct and reflected images into coincidence and reading the angular distance off the sextant arc. Atmospheric refraction makes the observed body appear higher than its true altitude and this must be corrected for. In the case of the Sun and the Moon, the upper or lower limbs are brought together and the observation is adjusted to account for the finite radius or semi-diameter (SD).

For lunars the sextant is not used vertically, as it is for altitudes, but is tilted and swung to minimise the measured distance between the Moon and other body. To obtain the distance between the centres, the Moon's SD must be added to or subtracted from this, depending on whether the near or far limb was used. When the other body is the Sun, its SD must also be added.

In practice the index and horizon mirrors will seldom be perfectly adjusted. The degree of misalignment can be quantified by sighting a star and making the direct and reflected images coincide. Ideally the sextant should show zero but any index error (I.E.) can then be read from the scale. Readings of greater (less) than zero are referred to being 'on(off)-the-arc'. In daylight, as described in section 4.7, measurements of the disc of the Sun can be made to determine the I.E.

Taking sights with a sextant requires 'swinging the arc' with the right hand while adjusting the tangent screw to track and measure angles that are constantly changing with the left.

Major Powell had his right arm amputated below the elbow during the American Civil War and would have found this more difficult than an able-bodied observer. Many of his series of sights show sextant readings at regular intervals, suggesting that he was presetting the sextant and then waiting for the body to reach the specified altitude or distance. In meridian transit and Polaris sights for latitude, things change much more slowly and probably did not present the same level of difficulty.

Any sight with a sextant is pushing the limits of what is possible with a handheld instrument and the results obtained will be subject to random errors. Variations of 10″ or 20″ in the I.E. that Powell reported at the beginning and end of rounds of sights do not necessarily represent a change in the sextant itself.

## 4. MAJOR POWELL'S ASTRONOMICAL OBSERVATIONS.

Navigational books are an essential part of the navigator's kit. These include standard texts giving formulas and tables for sight reduction and the Nautical Almanac (American Astronomical Ephemeris and Nautical Almanac, 1869). The latter tabulates the coordinates of the Sun, Moon, planets and stars that Powell used in determining his position. Notably the Nautical Almanac of the time also contained tables of lunar distances for a selection of suitably placed objects that could be used for finding longitude.

In the analyses described here, the positions of astronomical objects were computed by the program MICA (USNO, 2005) or software package Skyfield (Rhodes, 2016) using JPL DE430 Lunar and Planetary Ephemerides (Folkner et al., 2014).

4.1. *Disambiguation and interpretation.* The astronomical observations recorded in Powell's journals were his personal notes and naturally employ notation and conventions that may not be universally understood. All times are recorded in 12 h watch time without an 'a.m.' or 'p.m.' designation and are approximately LMT. In practice, however, the nature and magnitude of the observations eliminate any possible confusion. Morning and afternoon EA observations are identifiable from the altitudes that increase and decrease with time, respectively. The recorded magnitude of observed lunar distances immediately pins down the Greenwich date and approximate time at which they were made.

It is found that all sights involving stars or the planet Saturn were made in the evening hours, with the exception of one round of sights of Polaris for latitude, which was made in the morning hours of 12 August. On that occasion, Powell wrote in the journal, '11th Aug Astr. Date, 12th Common date'. In 1869 (up to and including 1924) the American Nautical Almanac was tabulated using astronomical time and this needed to be remembered when entries were being looked up. A given astronomical date began at 0 h at noon, twelve hours after the beginning of the corresponding civil or common date. Thus Powell correctly indicates that his round of sights began at '4 h–46′-30″ [a.m.]' on 12 August 1869 civil time but that it was still 11 August according to local astronomical time.

Powell kept careful records of the sextant I.E. throughout the journal. Modern terminology makes a distinction between index correction (I.C.) and I.E. which are equal in magnitude but opposite in sign. The observation of the meridian altitude of Altair made on 4 September in Saint George, Utah leaves no doubt as to the convention Powell was using. The I.E. on that occasion is listed as the relatively large +24′30″ and an incorrect interpretation of this sight by artificial horizon would displace the position obtained by 24·5 nm. It is found that Powell's I.E. must be added to the raw sextant reading before further reductions are performed and hence would be referred to as the I.C. in modern usage. With this interpretation Powell's sight places him squarely in present day Saint George. His usage is adopted here.

4.2. *Reduction formulas.* The altitude, $h$, of a celestial body is related to its declination, $\delta$, LHA, $t$, and the observer's latitude, $L$, by

$$\sin h = \sin L \sin \delta + \cos L \cos \delta \cos t \tag{1}$$

which follows directly from the cosine rule of spherical trigonometry. Several of the reduction formulas that Powell needed can be derived from it.

4.3. *Refraction.* All celestial bodies have their apparent altitudes elevated from the effect of atmospheric refraction which must be subtracted to obtain the true altitude. In the present work the semi-empirical formula given in the Nautical Almanac (2020) is used. For a body observed at altitude $h$, in degrees, the correction for refraction in degrees to be subtracted is

$$R = \frac{4{\cdot}676 \times 10^{-3} P}{(T + 273)} \cot \left( h + \frac{7{\cdot}32}{(h + 4{\cdot}32)} \right) \tag{2}$$

where $P$ is the atmospheric pressure in millibars (mb) and $T$ is the temperature in °C. Bradley wrote, 'The thermometer indicates above 100° most of the time' (23 July) and 'The thermometer seldom gets lower than 100°F except just before sunrise when it falls a little' (31 July). In the present work a temperature of $T = 37{\cdot}8$°C (100°F) is used throughout.

The expedition began at an elevation of 1,850 m and recorded sights at a lowest elevation of 370 m from Separation Camp. To allow for this the atmospheric pressure used in Equation (2) is computed based on the U.S. Standard Atmosphere Model (NOAA, 1976) from

$$P = 1010 \left( 1 - \frac{H}{44398} \right)^{5{\cdot}256} \tag{3}$$

in which $H$ is the observer's altitude in metres above sea level.

4.4. *Latitude by meridian transit.* If the altitude, $h$, of the Sun, Moon, planet or star is measured as it crosses the meridian, then the observer's latitude, $L$, is

$$L = 90^\circ - h + \delta \tag{4}$$

where $\delta$ is the body's declination.

Table 2 lists Powell's meridian transit observations along with the latitude that is obtained from them. When the site of the expedition's camp appears in Table 1, the difference from the observed location is given in nautical miles that correspond to one arc minute subtended on the Earth's surface. These are generally found to be within the expected range for sights with a handheld sextant. The comparatively large deviation on 4 August has its I.E. in the journal annotated with something resembling a reflected '?'. The observation of 15 August is noted as 'Meridian Alt of Altair same night by Sumner'. Perhaps coincidentally this latitude corresponds to within one to two miles of a dry canyon known as Silver Grotto.

Of some interest is the 'Obs for Lat on Star in leg of Ophuci [Ophiuchus]' on 29 July. This appears to be a tongue-in-cheek reference to Saturn and its location at that time. A faint sketch is drawn (Figure 1) showing four stars in the constellation of Ophiuchus with a line connecting the star Sabik ($\eta$ Ophiuci) to Saturn. In standard representations of Ophiuchus,

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Table 2. Meridian altitude sights recorded in Powell's journal.

| Date 1869 | | Location | Object | $H_{A.H.}$ ° | ′ | ″ | I.E. | ′ | ″ | Refr. ′ | Observed latitude ° | ′ | Diff. nm |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Jul | 13 | Camp No 7 | Saturn | 61 | 06 | 40 | + | 3 | 55 | 1·3 | 38 | 55·0 N | |
| | 14 | Camp No 8 | Saturn | 61 | 47 | 50 | + | 0 | | 1·3 | 38 | 36·6 | |
| | 19 | Junc. Grand & Green | Saturn | 62 | 40 | 20 | − | 1 | | 1·3 | 38 | 11·7 | 0·2 |
| | 29 | Camp 19[1] | Saturn | 64 | 05 | 50 | + | 11 | 20 | 1·3 | 37 | 23·4 | |
| | 31 | Mouth San Juan[2] | $\beta$ Ceti | 68 | 06 | 20 | + | 9 | 20 | 1·2 | 37 | 11·2 | 0·4 |
| Aug | 4 | Pah Reah, Ute Creek[3] | Saturn | 65 | 00 | 30 | + | 10 | 30 | 1·3 | 36 | 56·2 | 4·5 |
| | 15 | Silver Creek[4] | Altair | 123 | 45 | 20 | + | 11 | 50 | 0·4 | 36 | 33·6 | 27·6 |
| | 27 | Separation Camp | Altair | 125 | 13 | 00 | + | 12 | 20 | 0·4 | 35 | 49·5 | 0·2 |
| | 29 | | Altair | 124 | 45 | 50 | + | 10 | 10 | 0·5 | 36 | 4·2 | |
| Sep | 4 | St George | Altair | 122 | 29 | 00 | + | 24 | 30 | 0·4 | 37 | 5·4 | −0·6 |

Note: The double altitude as measured using the artificial horizon is given in the column '$H_{A.H.}$' and index error as recorded in the journal in column 'I.E.'. Refr. gives the correction for atmospheric refraction applied to obtain the 'Observed latitude'. (1) 'Star in leg of Ophuci'; (2) 'Lat 37°-11′-47″ aprox'; (3) I.E. marked with '(?)'; (4) 'same night by Sumner'.

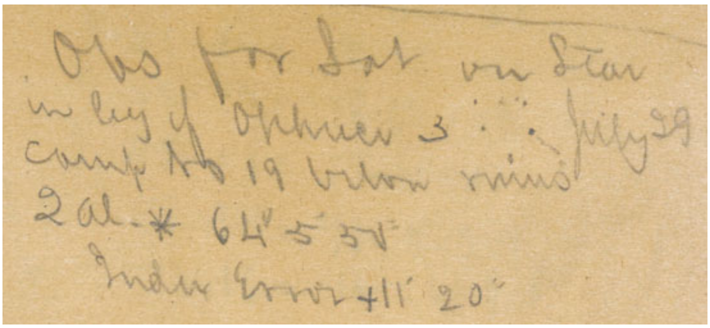


Figure 1. Apparent reference to Saturn as 'Star in leg of Ophuci [Ophiuchus]'. A sketch to left of the words 'July 29' shows four stars in the constellation of Ophuchus and a line drawn from the star Sabik at the lower right of the trapezoid to Saturn just above the 's' in the word 'ruins'. This closely matches the relative positions of the stars and Saturn as Powell would have seen them. (Smithsonian Institution, NAA MS 1975a-v2-145b).

Sabik lies at the serpent bearer's hip and Saturn therefore temporarily formed a leg. There is no bright star in Ophiuchus that would have transited near the recorded altitude and therefore the observation can only refer to Saturn itself.

Apart from the observations of 4 and 15 August which are flagged as questionable in the journal the differences listed in the right-hand column of Table 2 are typical of the accuracy that can be attained by a skilled land-based observer with a handheld sextant. The difference recorded for the observation from Saint George on 4 September should be taken as indicative only as Powell's exact location within the town is not known.

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4.5. *Latitude by altitude of Polaris.* Let $L$ be the required observer's latitude and $h$ be the true altitude of Polaris. Writing $L = h + x$, Equation (1) becomes

$$\sin h = \sin(h + x)\cos p + \cos(h + x)\sin p \cos t \tag{5}$$

where $p$ is the polar distance, defined as the complement of the declination, $p = 90^\circ - \delta$, and $t$ is the LHA. For Polaris $x$ and $p$ are small quantities. Expanding the right-hand side as a series to second order in $x$ gives a quadratic equation which can be solved in the usual way. Expanding the solution to second order in $p$ yields

$$L = h - p\cos t + \frac{1}{2}(p \sin t)^2 \tan h \tag{6a}$$

in radian measure. When $p$ is expressed in units of seconds of arc this is sometimes written as

$$L = h - p\cos t + \frac{1}{2}\sin 1''\,(p \sin t)^2 \tan h \tag{6b}$$

since $\sin 1'' \approx 1''$ expressed in radians (Chauvenet, 1863, p. 255).

Table 3 lists the observations for latitude by Polaris that Powell recorded during the expedition. The times and altitudes given are the averages of those that appear in the journal. Once again the difference from the latitude of known camp sites is listed when known. The results are dependent on the LHA, $t$, or equivalently LMT. For the purposes of the table, LMT has been calculated from the known longitude and chronometer time at local mean noon (LMN) on the date in question in Table 5. The values in the differences (Diff.) column therefore represent a combination of observational and chronometer errors. The sight of 29 August cannot be used with any degree of reliability since no EA sights had been taken since 16 August and the chronometer error is therefore highly uncertain.

Again within these uncertainties the differences are typical of the accuracy that can be attained by a skilled land-based observer with a handheld sextant.

4.6. *Management of chronometers.* A chronometer provides a relatively convenient and straightforward way to find longitude. It was set to show GMT and the difference from LMT, typically obtained by observing the Sun, when converted to an angle immediately yields the longitude.

In practice, however, chronometers were expensive and delicate instruments that needed to be treated gently and required careful attention to be paid to their management (Shadwell, 1861). Several would be carried so that a departure of one from the consensus could be detected. The expedition set out with four (Quartaroli, 2002). The chronometers would essentially never show GMT exactly and were very seldom reset. Instead careful records were kept of the current chronometer error (CE) and chronometer rate (CR). The former is the amount by which chronometer time is fast or slow of GMT at a certain time and the latter is the amount it is losing or gaining per day. These would be applied as corrections to the time shown by the chronometer itself. The requirement for a good working chronometer was only that it marked time at a predictable rate.

Starting from a known location it could be expected that Powell would want to make frequent observations to monitor the chronometers' performance. EA observations make it relatively easy both to find longitude and track the CE and CR of each chronometer. Effectively, longitude is read directly from the chronometer (after correcting for CE and CR) at LMN and the exact 24-hour period that elapses between the observations of LMN

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Table 3. Observations of latitude by Polaris. For all dates but 29 August, the chronometer time (CT), artificial horizon altitude ($H_{A.H.}$) and index error (I.E.) are the average over multiple sights.

| Date 1869 | | Location | CT h | CT m | CT s | | $H_{A.H.}$ ° | $H_{A.H.}$ ′ | $H_{A.H.}$ ″ | I.E. | I.E. ′ | I.E. ″ | Refr. ′ | Observed latitude ° | Observed latitude ′ | | Diff. nm |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| July | 18 | Junc. Grand & Green | 9 | 53 | 54 | PM | 75 | 20 | 17 | – | 1 | 55 | 1·0 | 38 | 11·2 | N | −0·3 |
| August | 1 | Music Temple | 8 | 45 | 16 | PM | 72 | 54 | 50 | + | 10 | 40 | 1·1 | 37 | 8·5 | | −1·2 |
| | 12 | Mouth of Flax River | 4 | 52 | 12 | AM | 74 | 59 | 58 | + | 10 | 50 | 1·1 | 36 | 11·0 | | −0·6 |
| | 29 | | 8 | 58 | 18 | PM | 71 | 41 | 40 | + | 10 | 0 | 1·2 | 35 | 52·2 | | |

Note: Refr. is the correction for atmospheric refraction applied to obtain ‘Observed latitude’.

taken from the same location on consecutive days simplifies the task of monitoring CE and CR.

4.7. *LMT by EA of the Sun.* Recording the times when the Sun is at the same altitude in the morning and afternoon allows the time of LAN to be determined. Define a timescale $t = \text{CT}–\text{CT}_{\text{LAN}}$ where $\text{CT}_{\text{LAN}}$ is the time the chronometer would show when the Sun is on the meridian at LAN. This is the Sun's LHA expressed in time. Let $t_{\text{AM}}$ and $t_{\text{PM}}$ be times in the morning and afternoon when the altitude of the Sun is equal. Their average will be denoted by $\bar{t} = \frac{1}{2}(t_{\text{PM}} + t_{\text{AM}})$ and their difference by $\Delta t = t_{\text{PM}} - t_{\text{AM}}$ and both are expressed in hours. Hence $t_{\text{AM}} = \bar{t} - (\Delta t/2)$ and $t_{\text{PM}} = \bar{t} + (\Delta t/2)$.

Let $\bar{\delta}$ be the Sun's declination at time $t = \bar{t}$ and $\delta'$ be the rate of change of declination in °/hour.

If $h$ is the Sun's observed altitude at $t_{\text{AM}}$ and $t_{\text{PM}}$ then from Equation (1)

$$\begin{aligned}\sin h &= \sin L \sin\left(\bar{\delta} - \delta' \frac{\Delta t}{2}\right) + \cos L \cos\left(\bar{\delta} - \delta' \frac{\Delta t}{2} \cdot\right) \cos\left(\bar{t} - \frac{\Delta t}{2}\right) \\ &= \sin L \sin\left(\bar{\delta} + \delta' \frac{\Delta t}{2}\right) + \cos L \cos\left(\bar{\delta} + \delta' \frac{\Delta t}{2}\right) \cos\left(\bar{t} + \frac{\Delta t}{2}\right)\end{aligned} \tag{7}$$

where the arguments of the trigonometric functions are converted to angular measure as necessary.

Performing a series expansion to first order in the small quantities $\bar{t}$ and $\delta' \Delta t$ gives

$$\bar{t} = \frac{\delta' \Delta t}{30} \left\{ \frac{\tan L}{\sin(\Delta t/2)} - \frac{\tan \bar{\delta}}{\tan(\Delta t/2)} \right\} \tag{8}$$

From the definition of $t$ given at the start of this section it follows that at LAN the chronometer reads $\text{CT}_{\text{LAN}} = \overline{\text{CT}} - \bar{t}$ where $\overline{\text{CT}}$ is the average of the morning and afternoon chronometer times. Up to a sign the quantity $\bar{t}$ is traditionally known as the equation of equal altitudes (Admiralty Manual of Navigation, 1922, Chapter XIII, p. 204) or noon correction as it is added to the average of the chronometer times to obtain the chronometer time of LAN.

Note that there is no need to know what the value of the altitude, $h$, actually is and hence if conditions are the same at the a.m. and p.m. sights there is no need to apply the usual corrections for I.E., refraction, SD and parallax. The only requirement is that the a.m. and p.m. sights be treated in the same manner. If significant changes have occurred or very high accuracy is required the true time of EA may be computed using the known rate of change of the Sun's altitude.

The evaluation of Equation (8) for the EA observations of 18 July is recorded in the journal (Figure 2). The calculations have been replicated with the steps labelled in Table 4 using the same values given on that journal page. In order to avoid subtractions, 10 is added to all logarithms that would otherwise be negative. Differences in the last digit of logarithms are likely rounding error in the look-up tables that were used. Possibly due to interpolation errors, there is a significant discrepancy when antilogarithms are taken but the final result is only off by about half a second.

The SD of the Sun changes by at most 0·28″ per day. The influence of atmospheric refraction depends on the temperature and the altitude of the body observed. It decreases as the altitude increases. For the lowest sight that Powell took, the Sun was at around 42°

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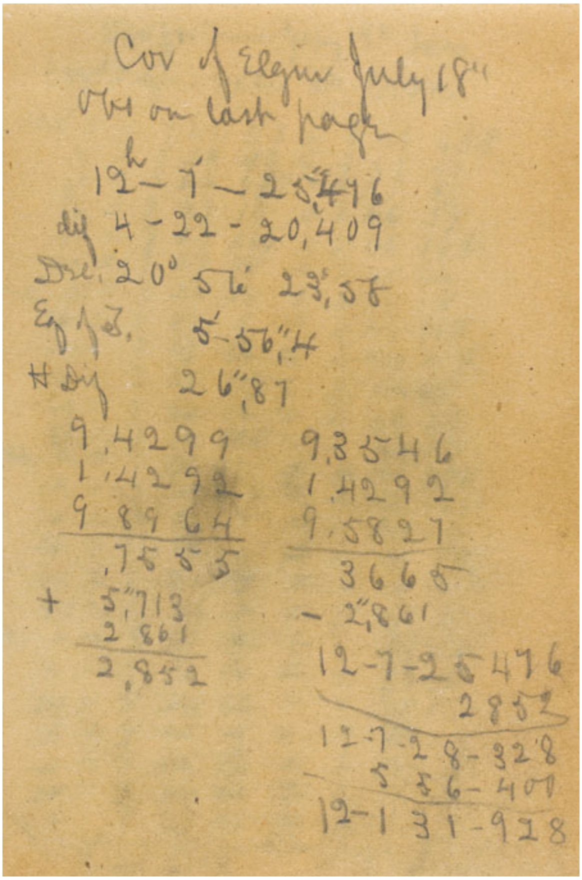


Figure 2. Reduction of the observations of the equal altitude observations of 18 July 1869 at the junction of the Grand and Green Rivers. The final number is the time that the Elgin watch was reading at local mean noon. (Smithsonian Institution, NAA MS 1975a-v2-039v).

altitude and the temperature between morning and afternoon sights would need to change by at least 30°C to have an observable effect.

It is possible the sextant's I.E. might change significantly and the EA observations of 11, 12 and 16 August show that Powell has measured it using the disc of the Sun during a period when the Elgin watch appeared to behaving erratically. For example on 12 August the journal records

| | | | |
|---|---|---|---|
| AM | +41′00″ | PM | +41′30″ |
| | −20′40″ | | −20′00″ |

These are limb to limb measurements of Sun's disc made both on and off the arc. Halving their algebraic sum gives the I.E., which in Powell's convention would give I.E. −10′10″ and −10′45″ respectively. Subtracting the two readings and dividing by four provides a measurement of the Sun's apparent SD which at the time was 15·8′ and is useful as a check. Powell's results give 15·4′ and they are also low on both 11 and 16 August which suggests the presence of sextant side error.

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Table 4. Replication of the equal altitude observations of 18 July 1869 using the values that appear in the journal and carrying the same number of decimals.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Date | 18 July, 1869 | | | | | | | | |
| | h | m | s | | | | | | |
| Average, $\bar{t}$ | 12 | 7 | 25·476 | | | | | | |
| Difference, $\Delta t$ | 4 | 22 | 20·409 | | | | | | |
| | ° | ′ | ″ | | | | | | |
| Declination, $\delta$ | 20 | 56 | 23·58 | N | | | | | |
| Hourly variation, $\delta'$ | | – | 26·87 | ″ | | | | | |
| | | m | s | | | | | | |
| Equation of time | – | 5 | 56·4 | | | | | | |
| $\log_{10}$ $\Delta t/(30 \sin \Delta t/2)$ | | | 9·4299 | | $\log_{10}$ $\Delta t/(30 \tan \Delta t/2)$ | | | 9·3545 | |
| $\log_{10}\delta'$ | | | 1·4293 | | $\log_{10}$ $\delta'$ | | | 1·4293 | |
| $\log_{10}$ tan $L$ | | | 9·8964 | | $\log_{10}$ tan $\delta$ | | | 9·5828 | |
| Sum | | | .7556 | | Sum | | | .3666 | |
| Antilogarithm | | | 5·696 | s | Antilogarithm | | | 2·326 | s |
| | + | | 5·696 | s | | | | | |
| | – | | 2·326 | s | | h | m | s | |
| | | | 3·370 | s | CT at $\bar{t}$ | 12 | 7 | 25·476 | |
| | | | | | Correction | | | 3·370 | |
| | | | | | CT at LAN | 12 | 7 | 28·846 | |
| | | | | | Eq. of Time | – | 5 | 56·400 | |
| | | | | | CT at LMN | 12 | 1 | 32·446 | |

Note: Differences in the last digit of logarithms may be rounding errors in the tables Powell was using. The differences in antilogarithms may be due to interpolation errors.

To make the EA observation, a sequence of 10–25 regularly spaced altitudes was specified and the watch time in the morning and afternoon when each was attained was noted. Their averages are given in Table 5 along the chronometer time (CT) of LAN and LMN derived from them. Their difference, (CT at LMN)–(CT at LAN), is the EqT.

For observations made at a common location, precisely 24 h elapses between LMN on consecutive days. The observations on 18, 19 and 20 July were all recorded at the junction of the Grand and Green Rivers but do not show a constant change from one day to the next and indicate that the Elgin watch was not running at a constant rate. Moreover the observations of 1 and 2 August that were also made at a common site show a large jump. Subsequent observations appear to indicate that the watch was behaving erratically and Powell was operating without a reliable chronometer. The advance seen in CT at LMN is far too large to be accounted for by the party's westward progress.

4.8. *Longitude by lunar distance.* To determine GMT by the method of lunar distances or lunars, the observer uses the sextant to measure the angular distance from the illuminated limb of the Moon to the Sun, star or planet. The altitudes of the Moon and the other body are ideally measured at the same instant that the lunar distance (LD) measurement is made. Depending on whether the near/far or lower/upper limb was measured, the Moon's SD is added to or subtracted from LD and the Moon's altitude to obtain the LD to the Moon's centre, $\mathrm{LD}_S$, and the altitude of the Moon's centre, $h_M$. Similar adjustments

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Table 5. Equal altitude observations made with the Elgin watch.

| Date | CT of $t_{AM}$ | | | CT of $t_{PM}$ | | | LAT of $\bar{t}$ 12$^h$ | | CT at LAN 12$^h$ | | | CT at LMN 12$^h$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1869 | h | m | s | h | m | s | | s | | m | s | | m | s |
| 18 Jul | 9 | 56 | 20·7 | 2 | 18 | 44·0 | – | 3·4 | + | 7 | 35·7 | + | 1 | 39·2 |
| 19 Jul | 9 | 31 | 12·2 | 2 | 42 | 38·5 | – | 3·8 | + | 6 | 59·1 | + | | 58·5 |
| 20 Jul | 9 | 16 | 1·9 | 2 | 56 | 46·6 | – | 4·0 | + | 6 | 29·0 | + | | 25·0 |
| 1 Aug | 9 | 6 | 54·0 | 2 | 58 | 58·3 | – | 5·7 | + | 3 | 1·8 | – | | 20·1 |
| 2 Aug | 8 | 59 | 46·0 | 3 | 35 | 32·8 | – | 6·2 | + | 17 | 45·6 | + | 14 | 14·0 |
| 11 Aug | 10 | 39 | 12·4 | 2 | 48 | 20·1 | – | 6·0 | + | 43 | 52·3 | + | 38 | 54·1 |
| 12 Aug | 11 | 22 | 52·3 | 2 | 26 | 25·8 | – | 5·8 | + | 54 | 44·9 | + | 49 | 37·0 |
| 16 Aug | 10 | 26 | 14·9 | 2 | 7 | 33·8 | – | 6·5 | + | 17 | 0·9 | + | 11 | 14·9 |

Note: The AM and PM time columns are the averages of morning and afternoon sights. The right-hand column gives the chronometer time (CT) at local mean noon (LMN). When observations are made from a fixed location, the interval between LMN on consecutive days is precisely 24 h.

for SD are applied when the other body is the Sun. The remainder of the clearing process is identical for all bodies, which henceforth will be generically referred to as a star. The azimuth difference between the Moon and the star, $\Delta Z$, can be shown, by means of the cosine rule of spherical trigonometry, to satisfy the relation

$$\cos \Delta Z = \frac{\cos \mathrm{LD}_S - \sin h_M \sin h_S}{\cos h_M \cos h_S} \tag{9}$$

where $h_S$ is the star's observed altitude. In order to obtain the geocentric or cleared $\mathrm{LD}_{S0}$, corrections are applied to the observed altitudes to account for refraction and, if necessary, parallax. These corrections are entirely in the vertical direction[1] and can be incorporated by applying adjustments $\Delta h_M$ to $h_M$ and $\Delta h_S$ to $h_S$. In doing so the angle, $\Delta Z$, is unchanged and hence the cleared lunar distance satisfies

$$\begin{aligned} \cos \mathrm{LD}_{S0} &= \sin(h_M + \Delta h_M) \sin(h_S + \Delta h_S) \\ &\quad + \cos(h_M + \Delta h_M) \cos(h_S + \Delta h_S) \cos \Delta Z \end{aligned} \tag{10}$$

Historical texts (Mendoza y Rios, 1801; Chauvenet, 1864; Wilberforce Clarke, 1885) describe a variety of methods for clearing lunar distance observations. They amount to rearrangements or approximations to Equations (9) and (10) in order to streamline their evaluation by manual calculation. A navigator equipped with the appropriate tables could clear a lunar distance in a matter of minutes rather than hours, as is popularly believed.

Although high precision is not required in the altitudes, $h_M$ and $h_S$, ideally they should be measured as near as possible to the time that the lunar distance was taken. In order to achieve this it is recommended that two competent assistants be employed. Jackson (1847, p. 296) describes the procedure and cautions

> The lunar method of finding the longitude requires the utmost exactness on the part of the observers, otherwise a considerable error will be made.

[1] For very high accuracy parallax in azimuth arising from the ellipsoidal figure of the Earth needs to be taken into account and can be added as a correction to $\Delta Z$ in Equation (10). Its effect is small and is not included in the present analysis.

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Jackson continues

> When assistance cannot be obtained, one person may take a set of observations in the following order, noting the times by a watch:–1, the altitude of the sun or star; 2, the altitude of the moon; 3, any number of distances; 4, the altitude of the moon; 5, the altitude of the sun or star.

He gives a numerical example demonstrating how it is carried out in practice. This is the procedure that Powell is seen performing in the journal.

Wilberforce Clarke (1885) offers additional advice

> A clear head, a steady hand, a sharp eye, a calm mind, are necessary for success as an observer.
> Everything tending to agitate the nervous system should be avoided—running, carrying instruments to the place of observation, star-gazing.

He repeats the numerical example that appears in Jackson (1847) and further states

> All these observations, with their times should be completed within ten minutes of time, or the observations cannot (as is necessary) be reduced to one moment of time.

The observation times are averaged as are the lunar distances giving $LD_S$. The initial and final altitudes of the Moon and star are interpolated to find the altitudes $h_M$ and $h_S$ at the averaged time. The values obtained are inserted into Equation (9).

The clear message is that lunars must be carried out with great care and precision. Indeed an error of 1 minute of arc in the measured LD produces an error of approximately 1·8 minutes in GMT or 27′ of longitude.

Powell diligently performed some very long sequences of lunar distances, with the consequence that the initial and final altitude measurements were separated by a correspondingly long time interval, in some cases over an hour. The upshot is that simple linear interpolation of the altitudes is no longer reliable. This in itself is not a fatal flaw, however, and can be remediated by replacing simple linear interpolation with a more accurate calculation of the altitudes. The measured altitudes of the Moon and other body taken at the beginning and end of the sequences of lunar distance sights yield their respective LHAs at those times. Equation (1) can then be used to compute the altitudes at the required time average of the LD sights. In this calculation, in addition to adjusting the LHA, changes in right ascension (R.A.) and declination over the interval must also be included.

All of Powell's lunar distance observations were made from well-defined landmarks whose positions are given in Table 1. Using these known positions it is possible to examine Powell's sights for accuracy and internal consistency. There are several ways to go about this. A navigator schooled in modern techniques might calculate lines of position to confirm that they can be made to cross near a single point. Alternatively the final altitude sights can be computed from the initial altitude sight and the changes in LHA, R.A. and declination of the body being observed. This is then compared with the observed values. Unfortunately whatever method is used indicates that there are problems with all of the sights and, taken on face value, they cannot be used reliably to find longitude. Powell's lunar distance measurements all show the expected linear trend with time and the residual root-mean-square errors are too small for random fluctuations to be the source of the discrepancies.

The actual observations are summarised in Table A2 in the appendix. The 'Diff.' column in Table A2 gives the difference between the observed altitudes recorded in the journal and those calculated from the GMT inferred from the lunar distance measurements and the known location. The first lunar taken on 18 July is particularly problematic. Quite apart from its implausibly large initial I.E. of $-2°20'$ it has not been found to be possible to make the altitudes and LD fall in line even when the initial I.E. is interpreted as $-2'20''$. The observations of 20 July showing a 'Diff.' of around $-35'$ in the altitude of the Moon may have come about by inadvertently measuring the altitude of the lower limb rather than the upper as it is labelled in the journal and as would be appropriate given the Moon's orientation at the time. The Moon was three and a half days from full, or 94% illuminated, and this type of error might conceivably occur. A measurement to the lowest point on the Moon's terminator would reduce the effective SD by $20''$ for the first sight and $8''$ for the second. The 'Diff.'s then show a root-mean-square value of 3·4′ compared with 24·9′ for the upper limb. It is of interest to note that on a given date the Nautical Almanac tabulates lunar distances for bodies that are considered suitable or 'within distance'. On 20 July Saturn is not one of the listed objects (American Ephemeris and Nautical Almanac, 1869, p. 124).

The observations of 1 and 2 August exhibit some potentially telling features. The differences are approaching 1° in magnitude but are relatively consistent between initial and final altitude observations. The differences for the Sun and the Moon are of opposite signs. Such a pattern could be produced by a timing offset for bodies on opposite sides of the meridian and the altitude of one is increasing, and the other decreasing, with time. The observations cannot be brought into line while maintaining the observer's known longitude by introducing an additional CE. Consistency can only be achieved by altering the elapsed time between the altitude and LD sights which requires adjusting the recorded LD in some manner. For the observations of 1 August, increasing the I.E. by 1·8′ brings the largest altitude difference down to 3·0′ and makes the longitude error from the LD measurement 7·4′. However this requires adopting an I.E. that is inconsistent with the $+11'20''$ that Powell measured at the end of his round of sights.

As noted previously, the I.E. would be measured using a star with the sextant set near 0°. An offset of the pivot point of the index arm from the centre of the sextant arc can however produce additional errors for angles measured away from 0° and does not affect the I.E. An angle measured as $a$ is corrected by (Wrangel, 1897, p. 389)

$$a \rightarrow a + \frac{2E}{r}\left(\sin b - \sin\left(b - \frac{a}{2}\right)\right)$$

where $E$ is the pivot point offset, $r$ is the radius of the sextant arc and $b$ is the angle on the sextant's arc closest to the pivot point. Ironically finding longitude by EA and chronometer as Powell had intended to do is very insensitive to sextant centring errors and produces no measureable impact. If such a centring error were present, however, it would affect the Polaris and meridian sights for latitude which generally appear to give reliable results.

Ignoring the sights of 18 July and assuming those of 20 July were made using the lower limb, an adjustment of between $-25''$ and $1'50''$ applied to the I.E. recorded in the journal brings the root-mean-square of the altitude 'Diff.'s to between 0·3′ and 3·8′ and the corresponding error in longitude coming from the determination of GMT from lunars to between 0·4 and 5·9 nm. No single adjustment produces the best fit for the altitudes on all days.

Lunar distance yields GMT, but to find longitude LMT is also required. This could be determined from the EA observations carried forward by chronometer. Alternatively the LMT could be determined from the altitude sights of the Moon or other body. The chronometer is still required to carry LMT forward or backward to the time of the LD sight, but the time interval involved is shorter and therefore less susceptible to irregularities in its rate.

From Table 5 it appears that, for the lunars in July, the Elgin watch was operating fairly reliably and would allow LMT to be read to within about ±5 s, resulting in an additional $\pm 1\,{}^{1}/{}_{4}'$ or ±1 nm error in longitude. By August the Elgin's apparent erratic rate would have rendered it unusable for this purpose. Depending on their relative signs, the errors could add or partially cancel.

The error in LMT obtained from a time sight depends on the accuracy of the measured altitude, $h$, as well as the body's azimuth, $Z$, measured from north in an eastward direction, and the observer's latitude, $L$. It can be shown that $\mathrm{d}t/\mathrm{d}h = \csc Z \sec L$, where $t$ is the LHA expressed in the same units as $h$. Hence an error of one minute of arc in altitude produces an error of ($\csc Z \sec L$) minutes of arc in longitude. The results in the 'Diff.' column in Tables 2 and 3 indicate that Powell could measure altitudes to within $1'$ and his LMT by time sight should not produce an additional error in longitude of more than a few nautical miles.

## 5. CONCLUSIONS.

The astronomical observations made by Major John Powell in the course of the 1869 expedition through the Grand Canyon and recorded in his original journal (Powell, 1869) have been examined, reduced and analysed. Sights were taken using one of two sextants and an artificial horizon, likely comprised of a dish of mercury. Many of the observations were made from well-defined locations such as the confluences of tributaries to the Grand (Colorado) River, which allows the quality of Powell's sights to be assessed. Errors in latitude are consistent with the accuracy typically attainable by a skilled land-based observer.

The loss of functioning chronometers due to the swamping of Powell's boat greatly complicated the procedures needed to find longitude. For that purpose Powell attempted to fall back on lunar distance observations, or lunars, and a possible eclipse timing with which he probably had little or no experience. The rounds of lunars spanned an unusually long time period, which was not standard methodology, and complicates their reduction. Most exhibit internal inconsistencies between lunar distances and measured altitudes and therefore do not reliably fix longitude. The lunar distance observations were not reduced during the expedition and therefore offered no feedback on any shortcomings or how well the observations were being performed.

It is found, however, that on any given day the lunar distance observations, altitude measurements, and known longitude from which the sights were made can all be brought into line by a single adjustment applied to the sextant's I.E. This suggests the presence of a systematic error of unknown origin.

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## APPENDIX

This appendix records the rounds of lunar distance sights that Major John Powell made from 18 July to 15 August 1869. The locations from where the observations were made are given in Table 1. The dates given in the tables are the local dates as recorded in his journals.

In Tables A1 to A3, the chronometer times are labelled as AM (ante-meridian) and PM (post-meridian). For each date the first and last two rows represent altitude, with UL and LL indicating whether the observation was upper or lower limb. The figures in the left-hand columns, are the chronometer time (CT), the sextant measurement ($H_s$) and index error (I.E.) taken directly from Powell's journal. The third row is the average of the lunar distance observation times and measurements, again from Powell's journal, along with the I.E. that has been assumed in subsequent analysis. The abbreviations NL and FL are used to indicate whether the lunar distance was taken to the near or far limb. The column $H_o$ lists the observed lunar distance between the centres of the Moon and other body, $H_o = H_s + \text{I.E.} \pm \text{SD}$, where the semi-diameter (SD) is added or subtracted according to whether the observation is near or far limb. For the Sun, the semi-diameters of both the Sun and the Moon are added. The GMT column gives the time when this lunar distance (LD) would be observed from the known location. The listed GMT can be checked by inputting it and LD = $H_s$ + I.E. along with location, temperature of 100°F and pressure, calculated using Equation (3), into Frank Reed's online Lunars Calculator: http://reednavigation.com/lunars/lunars_v4.html.

Table A1. Altitude and lunar distance sights recorded in Powell's journal for 18 July 1869. The time and artificial horizon altitude in the lunar distance (LD) row are averages over the 11 observations.

| 18 July | | CT (PM) | | | $H_s$ | | | I.E. | |
|---|---|---|---|---|---|---|---|---|---|
| Spica | | $8^h$ | $19^m$ | $32^s$ | 55° | 10′ | 0″ | −2° | 20′ |
| Moon | UL | 8 | 23 | 23 | 70 | 18 | 50 | −2° | 20′ |
| LD | NL | 8 | 50 | 3 | 41 | 43 | 53 | −2′ | 10″ |
| Moon | UL | 9 | 10 | 59 | 64 | 50 | 0 | −2 | 0 |
| Spica | | 9 | 22 | 2 | 38 | 4 | 0 | −2 | 0 |

The GMT thus obtained is used to compute the applicable chronometer error and hence GMT for each of the altitude sights and is listed in their respective rows. The column $H_o$ gives the observed altitude of the body's centre, $H_o = (H_s + \text{I.E.})/2 \pm \text{SD}$, where SD is added or subtracted according to whether the observation is of the lower or upper limb. These are the $h_M$ and $h_S$ to be used in Equations (9) and (10) as described in the text. The Diff. column is the difference between the observed altitude of the centre, uncorrected for refraction or parallax, and what should be observed based on the known location and listed GMT. A positive value indicates that Powell's observed altitude is greater than the one calculated for that time and position.

Because of special problems that appear to be present in the LD sights of 18 July, no attempt has been made to compute GMT and the observations are recorded as extracted directly from the journal.

Table A2. Lunar distance and altitude sights from Powell's journal. The GMT column gives Greenwich mean time extracted from the lunar distance and known location at which the sight was made. The $H_o$ column lists altitudes and distances between the centres of the bodies concerned based on the observations and correcting for their semi-diameters.

| 19 July | | | CT (PM) | | | $H_s$ | | | I.E. | | | GMT | | $H_o$ | | Diff. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Altair | | $9^{h}$ | $6^{m}$ | $6^{s}$ | 84° | 29′ | 30″ | + | 0′ | 50″ | $4^{h}$ | $29^{m}$ | $41^{s}$ | 42° | 15·2′ | −45·9′ |
| Moon | UL | 9 | 2 | 49 | 64 | 56 | 30 | + | 0 | 50 | 4 | 26 | 24 | 32 | 13·0 | −1·6 |
| LD | FL | 9 | 31 | 6 | 48 | 15 | 27 | + | 0 | 55 | 4 | 54 | 42 | 48 | 0·7 | |
| Moon | LL | 9 | 55 | 12 | 62 | 21 | 30 | + | 1 | 0 | 5 | 18 | 47 | 31 | 26·9 | +11·2 |
| Altair | | 9 | 52 | 18 | 99 | 29 | 40 | + | 1 | 0 | 5 | 15 | 53 | 49 | 45·3 | −44·8 |
| **20 July** | | | **CT (PM)** | | | **$H_s$** | | | **I.E.** | | | **GMT** | | **$H_o$** | | **Diff.** |
| Saturn | | $8^{h}$ | $21^{m}$ | $27^{s}$ | 62° | 12′ | 50″ | + | | 0″ | $3^{h}$ | $41^{m}$ | $54^{s}$ | 31° | 6·4′ | −1·6′ |
| Moon | UL | 8 | 25 | 29 | 52 | 17 | 10 | + | | 0 | 3 | 45 | 56 | 25 | 53·0 | −36·3 |
| LD | NL | 8 | 52 | 16 | 19 | 2 | 27 | − | | 5 | 4 | 12 | 43 | 19 | 17·9 | |
| Moon | UL | 9 | 12 | 33 | 58 | 26 | 0 | − | | 10 | 4 | 33 | 0 | 28 | 57·4 | −34·0 |
| Saturn | | 9 | 16 | 30 | 61 | 36 | 20 | − | | 10 | 4 | 36 | 57 | 30 | 48·1 | +1·8 |
| **1 August** | | | **CT (AM)** | | | **$H_s$** | | | **I.E.** | | | **GMT** | | **$H_o$** | | **Diff.** |
| Sun | LL | $9^{h}$ | $13^{m}$ | $16^{s}$ | 96° | 20′ | 0″ | + | 11′ | 20″ | $16^{h}$ | $46^{m}$ | $46^{s}$ | 48° | 31·4′ | −59·1′ |
| Moon | UL | 9 | 24 | 28 | 88 | 22 | 20 | + | 11 | 20 | 16 | 57 | 58 | 44 | 1·5 | +54·8 |
| LD | NL | 9 | 38 | 12 | 78 | 33 | 48 | + | 11 | 20 | 17 | 11 | 42 | 79 | 16·2 | |
| Moon | UL | 9 | 49 | 44 | 79 | 5 | 20 | + | 11 | 20 | 17 | 23 | 14 | 39 | 23·1 | +56·4 |
| Sun | LL | 9 | 54 | 16 | 111 | 26 | 20 | + | 11 | 20 | 17 | 27 | 46 | 56 | 4·6 | −59·4 |
| **2 August** | | | **CT (AM)** | | | **$H_s$** | | | **I.E.** | | | **GMT** | | **$H_o$** | | **Diff.** |
| Sun | UL | $9^{h}$ | $6^{m}$ | $50^{s}$ | 89° | 0′ | 0″ | + | 11′ | 30″ | $16^{h}$ | $24^{m}$ | $37^{s}$ | 44° | 20·0′ | −46·6′ |
| Moon | UL | 9 | 16 | 35 | 117 | 4 | 30 | + | 11 | 30 | 16 | 34 | 22 | 58 | 22·5 | +39·3 |
| LD | NL | 9 | 49 | 58 | 66 | 49 | 33 | + | 11 | 25 | 17 | 7 | 45 | 67 | 32·2 | |
| Moon | UL | 10 | 11 | 0 | 98 | 40 | 0 | + | 11 | 20 | 17 | 28 | 47 | 49 | 10·1 | +51·1 |
| Sun | UL | 10 | 13 | 55 | 113 | 51 | 20 | + | 11 | 20 | 17 | 31 | 42 | 56 | 45·6 | −50·1 |

(*continued*)

Table A2. Continued

| 11 August | | | CT (PM) | | | $H_s$ | | | I.E. | | | GMT | | $H_o$ | | Diff. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sun | UL | 2^h | 52^m | 51^s | 109° | 30′ | 0″ | + | 9′ | 15″ | 21^h | 36^m | 28^s | 54° | 33·8′ | −37·0′ |
| Moon | UL | 3 | 12 | 48 | 97 | 54 | 0 | + | 9 | 15 | 21 | 56 | 25 | 48 | 45·0 | +16·3 |
| LD | NL | 3 | 21 | 40 | 55 | 17 | 50 | + | 9 | 15 | 22 | 5 | 17 | 55 | 59·4 | |
| **12 August** | | | **CT (PM)** | | | **$H_s$** | | | **I.E.** | | | **GMT** | | **$H_o$** | | **Diff.** |
| Altair | | 8^h | 35^m | 26^s | 89° | 57′ | 40″ | + | 10′ | 10″ | 3^h | 10^m | 12^s | 45° | 3·9′ | −10·7′ |
| Moon | LL | 8 | 30 | 52 | 57 | 23 | 30 | + | 10 | 10 | 3 | 5 | 38 | 29 | 3·1 | +7·3 |
| LD | FL | 8 | 47 | 19 | 86 | 11 | 25 | + | 10 | 15 | 3 | 22 | 5 | 86 | 5·4 | |
| Moon | LL | 9 | 23 | 11 | 39 | 56 | 10 | + | 10 | 20 | 3 | 57 | 57 | 20 | 19·5 | +13·5 |
| Altair | | 9 | 19 | 28 | 104 | 32 | 30 | + | 10 | 20 | 3 | 54 | 14 | 52 | 21·4 | −13·4 |
| **15 August** | | | **CT (PM)** | | | **$H_s$** | | | **I.E.** | | | **GMT** | | **$H_o$** | | **Diff.** |
| Altair | | 8^h | 27^m | 47^s | 104° | 24′ | 30″ | + | 11′ | 50″ | 3^h | 40^m | 9^s | 52° | 18·2′ | +11·9′ |
| Moon | LL | 8 | 31 | 39 | 64 | 55 | 50 | + | 11 | 50 | 3 | 44 | 1 | 32 | 49·6 | −5·7 |
| LD | FL | 8 | 52 | 37 | 50 | 54 | 10 | + | 11 | 50 | 4 | 5 | 0 | 50 | 50·3 | |
| Moon | LL | 9 | 5 | 57 | 59 | 41 | 50 | + | 11 | 50 | 4 | 18 | 19 | 30 | 12·6 | −5·8 |
| Altair | | 9 | 9 | 40 | 116 | 0 | 0 | + | 11 | 50 | 4 | 22 | 2 | 58 | 5·9 | +12·5 |

Table A3. Lunar distance (LD) and altitude sights for 20 July 1869 assuming that, notwithstanding their labelling in the journal, the altitudes represent the Moon's lower limb. The initial and final $H_o$ values incorporate a reduction of 20″ and 8″ respectively in the Moon's semi-diameter to account for the measurement being made to the lowest point on the terminator rather than the limb.

| 20 July | | CT (PM) | | | $H_s$ | | | I.E. | | GMT | | | $H_o$ | | Diff. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Saturn | | $8^h$ | $21^m$ | $27^s$ | 62° | 12′ | 50″ | + | 0″ | $3^h$ | $41^m$ | $54^s$ | 31° | 6·4′ | −1·6′ |
| Moon | LL | 8 | 25 | 29 | 52 | 17 | 10 | + | 0 | 3 | 45 | 56 | 26 | 23·8 | −5·6 |
| LD | NL | 8 | 52 | 16 | 19 | 2 | 27 | − | 5 | 4 | 12 | 43 | 19 | 17·9 | |
| Moon | LL | 9 | 12 | 33 | 58 | 26 | 0 | − | 10 | 4 | 33 | 0 | 29 | 28·3 | −3·0 |
| Saturn | | 9 | 16 | 30 | 61 | 36 | 20 | − | 10 | 4 | 36 | 57 | 30 | 48·1 | +1·8 |

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