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Secular dynamics of a coplanar, non-resonant planetary system, consisting of a star and two planets

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Abstract. We consider an exoplanetary system consisting of a star and two planets. The masses of the planets are significantly less than the mass of the star. The evolution of the orbital motion of exoplanets is studied within the framework of a double averaged unrestricted three-body problem. The main attention is paid to coplanar configurations, when the star and planets move in a certain plane that preserves a constant position. The possibility of reversing the orbital motion of the inner planet is noted.

Keywords. Three-body problem, secular evolution, averaging technique

1. Introduction

In the general case, the motion of three gravitating points is rather complicated. The situation is simplified when we can apply the averaging technique to study their dynamics. Several cases are known when the averaged problem is integrable. The first such case was described by Harrington (1968): a point moves around a pair of points at a distance significantly greater than the distance between the points in the pair. The next case was found by Ziglin (1976): two points move in close orbits around the third, whose mass is much greater than their masses. From considerations of symmetry, we can conclude that if the motion of these points occurs in some constant plane, then the double averaged problem will be integrable for any ratio between the major semi-axes of their orbits. The study of this case was started by Pauwels (1983). Further investigations of the planar double averaged unrestricted problem were stimulated by the discovery of exoplanetary systems (e. g., Michtchenko and Malhotra (2004); Migaszewski and Gozdziewski (2008)). However, there is still no classification of the dynamical behavior of the system even by the number of stationary solutions.

2. Double Averaged Equations of Motion

The evolution of the coplanar motion of two planets can be described by the equations

$$\frac{d\varpi_\Delta}{dt} = \mu\left[\frac{\chi\sqrt{1-e_1^2}}{\bar{\mu}_1 e_1\sqrt{a}}\frac{\partial W}{\partial e_1} - \frac{\sqrt{1-e_2^2}}{\bar{\mu}_2 e_2}\frac{\partial W}{\partial e_2}\right] \tag{1}$$

$$\frac{de_1}{dt} = -\mu\chi\frac{\sqrt{1-e_1^2}}{\bar{\mu}_1 e_1\sqrt{a}}\frac{\partial W}{\partial\varpi_\Delta}, \qquad \frac{de_2}{dt} = -\mu\frac{\sqrt{1-e_2^2}}{\bar{\mu}_2 e_2}\frac{\partial W}{\partial\varpi_\Delta}.$$

In system (1), indices 1 and 2 correspond to the inner planet and the outer planet, respectively. The variable $\varpi_\Delta = \varpi_1 - \varpi_2$ is the difference in the longitudes of the pericenters

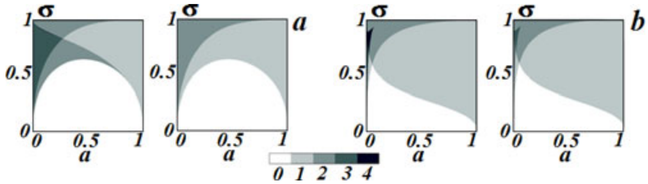

Figure 1. The total number of stationary solutions and the number of stable stationary solutions: a - $\bar{\mu}_1 = 0.25$, b - $\bar{\mu}_1 = 0.75$

of the planets, the variables e_1 and e_2 are the averaged values of the eccentricities of the osculating orbits of the planets. The units of measurement are chosen in such a way that the total mass of the star and planets is equal to 1, the average value of the major semi-axis of the outer planet is equal to 1, and, finally, the value of the gravitational constant is also equal to 1. The parameter μ characterizes the part of the mass of planets in the total mass of the system, parameters $\bar{\mu}_1$ and $\bar{\mu}_2$ set the ratio of the masses of the planets ($\bar{\mu}_1 + \bar{\mu}_2 = 1$). If the direction of rotation of the inner planet coincides with the direction of rotation of the outer planet, the factor χ in system (1) is equal to 1; in the case of retrograde motion of the inner planet $\chi = -1$.

The function W in (1) is a double averaged disturbing function. When constructing W numerically, it is convenient to use the fact that after averaging over the orbital motion of the first planet, the second averaging is equivalent to averaging over the motion in the field of the Gaussian ring specified by the force function W_{Gauss} (for details see Kondratyev (2012)):

$$W = \frac{\bar{\mu}_1 \bar{\mu}_2}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \frac{d\lambda_1 d\lambda_2}{|\mathbf{r}_1 - \mathbf{r}_2|} = \frac{\bar{\mu}_1 \bar{\mu}_2}{2\pi} \int_0^{2\pi} W_{Gauss} d\lambda_2.$$

The parameter μ determines only the rate of secular evolution. The qualitative properties of motion depend on the ratio of the masses of the planets, on the value of the major semiaxis a of the inner planet, and on the value of the normalized angular momentum of the system

$$\sigma = \left[\chi \bar{\mu}_1 \sqrt{a(1 - e_1^2)} + (1 - \bar{\mu}_1)\sqrt{1 - e_2^2}\right] / \left[1 - \bar{\mu}_1(\sqrt{a} - 1)\right] .$$

3. Results of Investigation

We used equations (1) to study the secular evolution of the orbital motion of the planets. Similar to Michtchenko and Malhotra (2004), special attention was paid to apsidal resonances - stationary solutions of equations (1). The number of apsidal resonances for different values of the system parameters has been established, and their stability has been studied. Some examples are provided by Figure 1.

As it turned out, under certain conditions the direction of the orbital motion of the inner planet changes. Although this is a known effect (e.g., see Kaplan *et al.* (2008)), the scenarios of evolution with alternating forward and retrograde motion of the inner planet in the planar problem have not been considered before.

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