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SUSTAINABILITY OF PUBLIC DEBT AND INEQUALITY IN A GENERAL EQUILIBRIUM MODEL

Noritaka Maebayashi
The University of Kitakyushu

Kunihiko Konishi
Matsuyama University

This study investigates the relationship between the sustainability of public debt and inequality in an endogenous growth model with heterogeneous agents. We show that the threshold for the sustainability of public debt is related to not only the relative size of public debt but also inequality. In addition, this study examines the effects of budget deficit and redistributive policies on the sustainability of public debt and inequality. We show that an increase in the deficit ratio or the redistributive tax makes public debt less sustainable. If the economy falls into the unsustainable region as a result of the policy change, both public debt and inequality continue to increase.

Keywords: Fiscal Sustainability, Public Debt, Inequality, Redistributive Policy

1. INTRODUCTION

Since the recent default risk on Greek government debt exposed by the 2008–2009 global financial crisis, the concern over whether a government's deficit and debt are sustainable has been growing among countries whose public debt is large. In the euro area, sizable fiscal consolidation was implemented from 2011 to 2013. However, the Report on Public Finances in the EMU (2016) shows that sustainability needs assessed by the distance to the budgetary medium-term objective in the euro area remain high. In addition to increased public debt, wealth and income inequality have expanded in many developed countries.

Because the accumulation of public debt and an economy's inequality are both endogenous outcomes of the economic system, these factors can influence each

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other. Azzimonti et al. (2014) and Arawatari and Ono (2017) provide empirical evidence for inequality and public debt in OECD countries for the past three decades. They present a positive and highly significant correlation between inequality and public debt.[1,2] While Azzimonti et al. (2014) and Arawatari and Ono (2017) indicate that inequality increases public debt, Mankiw (2000) shows that a higher level of public debt affects inequality, which is based on the fact that the government bonds are interest-bearing assets and their interest payments benefit the rich (the savers). Therefore, it is important to investigate the relationship between public debt and inequality under a general equilibrium framework.

Recent studies examine fiscal sustainability under a public deficit rule by using the overlapping-generations (OLG) models (life-cycle models) developed by Diamond (1965). In OLG models, fiscal sustainability means that the ratio of public debt to GDP (or capital) converges to a stable level in the long run. Chalk (2000) examines a constant primary deficit rule and shows that if the initial public debt is large, it does not converge to a stable level but rather explodes, making it unsustainable. More recent studies (e.g., Bräuninger, 2005; Yakita, 2008; Arai, 2011; Teles and Mussolini, 2014; Agénor and Yilmaz, 2017) extend the analysis to an OLG model with an endogenous growth structure and a constant deficit/GDP rule following the criterion of the Maastricht Treaty and obtain a similar result to that of Chalk (2000).[3,4] Nevertheless, these studies ignore the relationship between the sustainability of public debt and inequality.

To our knowledge, some studies investigate how public debt affects inequality or wealth distribution in general equilibrium models (e.g., Mankiw, 2000; Michel and Pestieau, 2005; Pestieau and Thibaut, 2012). Although they reach no consensus on how public debt affects inequality or wealth distribution, they commonly indicate that these two factors are related to each other.[5] This is because public debt is absorbed by wealth in the economy through its asset market. However, these studies do not pay attention to the sustainability of public debt, instead focusing on the steady state at which public debt converges to a stable level.

Accordingly, no base model has thus far described endogenous mechanisms regarding inequality and the sustainability of public debt; that is, the relationship between these two factors is still unclear from both theoretical and empirical perspectives. Therefore, as a first step, we construct a simple and tractable OLG model in which the transition path of public debt and that of inequality are jointly determined. In this study, we focus on the extent to which households' savings and bequests (intergenerational transfers) affect inequality. Inequality caused by savings and bequests can influence the sustainability of public debt through the asset market in OLG models. By investigating the model, we address the following open questions:

(i) How are the sustainability of public debt and inequality related to each other?
(ii) How does public deficit policy affect the sustainability of public debt and inequality?
(iii) How does redistributive policy aiming to reduce inequality affect the sustainability of public debt?

The present model is based on Bräuninger (2005), who explores the sustainability of public deficit policy under a constant deficit/GDP rule in an AK model. We incorporate a mechanism generating an endogenous transmission of inequality into Bräuninger's (2005) model. The key mechanism generating inequality in this model is composed of (i) the heterogeneity of agents' subjective discount factors based on Becker (1980) and (ii) joy-of-giving bequest motives (e.g., Abel and Warshawsky, 1988; Andreoni, 1989, 1990). Our model splits the population into two classes, the rich and poor, and assumes that the rich have higher exogenous subjective discount factors than the poor (i.e., the rich are more patient than are the poor).[6] In addition, we assume that agents have joy-of-giving bequest motives independently of whether they are rich or poor. Under these assumptions, the rich save more and bequeath more wealth to their children, which becomes the source of inequality. Some empirical studies consider that such intergenerational linkages in saving behavior and wealth accumulation generate inequality. For example, Dynan et al. (2004), Bozio et al. (2013), Alan et al. (2014), and Néstor (2017) show that the rich have a higher savings rate than do the poor. Other empirical evidence supports that bequests are one of the major causes of inequality (e.g., Kotlikoff and Summers, 1981; Gale and Scholz, 1994).

By using this simple model, we illustrate that the transition paths of both public debt and inequality are determined in a two-dimensional phase diagram and we obtain the following results.

(i) There is a threshold of public debt for each level of inequality in order for the government to sustain fiscal policy, and the threshold of public debt is increasing in inequality. When the initial public debt is small, the economy can reach a stable equilibrium in which both public debt and inequality converge to the stable level. When the initial public debt is large, the economy with higher inequality can converge to a stable equilibrium, whereas the economy with low inequality cannot. If the economy is in the unsustainable region, both public debt and inequality continue to increase and the economy goes bankrupt in the long run.

(ii) An increase in the public deficit ratio makes public debt less sustainable. Therefore, if the economy with large public debt falls into the unsustainable region as a result of expanding the public deficit ratio, inequality increases as public debt grows during the bankruptcy path.

(iii) A redistributive policy that attempts to reduce inequality affects the sustainability condition. If the government taxes the bequests of the rich and redistributes the revenue to the poor, the economy is more likely to fall into the region in which public debt is not sustainable and inequality continues to increase. Thus, in the economy with large public debt, introducing such a redistributive policy might be risky.

(iv) The policy effects in the stable steady state are as follows. A rise in the public deficit ratio enlarges inequality and decreases the growth rate. An increase in the redistributive tax reduces both inequality and the growth rate, and hence leads to the trade-off between equality and growth.

The remainder of this paper is organized as follows. Section 2 establishes the base model whose objective is to derive the relationship between the sustainability of public debt and inequality. To achieve this as simply as possible, this base model excludes redistributive policy. Sections 3–5 explore this base model as follows. Section 3 derives the equilibrium condition and dynamic system of the economy. Section 4 derives the transition dynamics and relationship between inequality and the sustainability of public debt. Section 5 analyzes the effects of changes in the public debt finance ratio on the sustainability of public debt and inequality. Section 6 introduces redistributive policy into the base model and examines its effect on the sustainability of public debt and inequality. Section 7 notes several limitations of our specifications and discusses directions for future research. Section 8 concludes.

2. MODEL

2.1. Individuals

We consider a two-period OLG model following Diamond (1965). An individual lives for two periods and the cohort born in period t is called generation t. Therefore, two generations exist in period t: generation t (the young generation) and generation $t-1$ (the old generation). In each period, the size of the newly born cohort is given by N. There are two groups of families, "rich" and "poor," denoted by R and P, respectively. We assume that a constant fraction $\delta \in (0, 1)$ of individuals are the rich and a constant fraction $1-\delta$ of individuals are the poor. Each individual supplies one unit of labor inelastically and earns labor income in their young period, making total labor supply $L_t = N$. In the old period, they are retired, consume their savings, and leave bequests to their children. Individuals have perfect foresight.

Each individual $i \in \{R, P\}$ born in period t maximizes utility,

$$U_t^i = (1-\alpha_i)\log c_t^{1i} + \alpha_i\left[(1-\beta)\log c_{t+1}^{2i} + \beta \log b_{t+1}^i\right], \quad (1)$$

where c_t^{1i} is consumption when young, c_{t+1}^{2i} is consumption when old, and b_{t+1}^i is the bequest passed onto the child. Note that utility depends on the amount b_{t+1}^i. This reflects a joy-of-giving savings motive. $\alpha_i \in (0, 1)$ is the intertemporal preference parameter and $\beta \in (0, 1)$ is the relative importance parameter of consumption when old and the bequest. We assume that β does not differ between the rich and poor following Bossmann et al. (2007). In addition, we assume that $\alpha_R > \alpha_P$ based on Becker (1980) and other empirical evidence (e.g., Lawrance, 1991; Harrison et al., 2002). This assumption generates the mechanism by which the rich save a larger proportion of their income than do the poor, which is empirically supported by Dynan et al. (2004) for the United States, Bozio et al. (2013) for the United Kingdom, Alan et al. (2014) for Canada, and Néstor (2017) for Latin America. In addition, we assume that the wealth endowment of the rich old generation in the initial period ($t = 0$) is larger than that of the poor old generation. As a result, the rich bequeath larger wealth than the poor do. Let s_t^i be savings in youth. The budget constraint of generation t can be written as follows:

$$c_t^{1i} = (1-\tau_t)w_t - s_t^i + b_t^i, \tag{2}$$

$$c_{t+1}^{2i} = \left[1 + (1-\tau_{t+1})r_{t+1}\right] s_t^i - b_{t+1}^i, \tag{3}$$

where w_t, r_{t+1}, and τ_t represent the wage rate, interest rate, and tax on wage and interest income. By solving the intertemporal utility maximization, we obtain the following optimal conditions:

$$c_t^{1i} = (1-\alpha_i)\left[(1-\tau_t)w_t + b_t^i\right], \tag{4a}$$

$$s_t^i = \alpha_i\left[(1-\tau_t)w_t + b_t^i\right], \tag{4b}$$

$$c_{t+1}^{2i} = (1-\beta)\left[1 + (1-\tau_{t+1})r_{t+1}\right] s_t^i, \tag{4c}$$

$$b_{t+1}^i = \beta\left[1 + (1-\tau_{t+1})r_{t+1}\right] s_t^i. \tag{4d}$$

From (4b) and (4d), savings are determined as follows:

$$s_t^i = \alpha_i\left[(1-\tau_t)w_t + \underbrace{\beta\{1 + (1-\tau_t)r_t\}s_{t-1}^i}_{\text{the bequest from parents}}\right]. \tag{5}$$

Equation (5) indicates the following. First, the savings of the current generation s_t^i are linked to the savings of parents s_{t-1}^i. This is because the bequest from parents depends on their wealth income $[1 + (1-\tau_t)r_t]s_{t-1}^i$ from (4d). Second, from assumption $\alpha_R > \alpha_P$, the rich save more than the poor do and leave more wealth to their offspring, who in turn tend to do the same. This means that the rich tend to accumulate more wealth than the poor do.

The total assets (savings) held by young agents in period t, $A_t \equiv \delta s_t^R N + (1-\delta)s_t^P N$, are composed of public bonds, D_{t+1}, and private capital, K_{t+1}. Hence, the asset market clears as follows:

$$K_{t+1} + D_{t+1} = A_t. \tag{6}$$

2.2. Production

There is a large number of identical firms denoted by j. Firm j produces a single final good by using the production technology given by $Y_{j,t} = \Gamma K_{j,t}^{\gamma}\left(a_t L_{j,t}\right)^{1-\gamma}$ $(0 < \gamma < 1)$, where $Y_{j,t}$, $K_{j,t}$, and $L_{j,t}$ represent the output level, private capital, and labor input of firm j, respectively. a_t is the labor efficiency at time t. From profit maximization in competitive markets, factor prices become equal to the marginal products: $r_t = \partial Y_{j,t}/\partial K_{j,t} = \gamma\Gamma(K_{j,t}/L_{j,t})^{\gamma-1}a_t^{1-\gamma}$ and $w_t = \partial Y_{j,t}/\partial L_{j,t} = (1-\gamma)\Gamma(K_{j,t}/L_{j,t})^{\gamma}a_t^{1-\gamma}$.

Following Romer (1986), we assume that the average capital per worker has positive external effects on labor productivity and specify $a_t = K_t/L_t$, where K_t is the average stock of private capital and L_t is the average labor input in the economy. In the equilibrium, $K_{j,t} = K_t$ and $L_{j,t} = L_t$ hold for all j, and thus the factor prices and aggregate output, Y_t, in period t can be written as follows:

$$w_t = \Gamma(1-\gamma)\frac{K_t}{L_t}, \tag{7a}$$

$$r_t = \Gamma\gamma, \tag{7b}$$

$$Y_t = \Gamma K_t. \tag{7c}$$

2.3. Government

The government in period t imposes a tax on income, $w_t L_t + r_t A_{t-1}$, and issues bonds, $D_{t+1} - D_t$, to finance public spending, G_t and interest payments for public debt, $r_t D_t$. Tax revenue in period t, $\tau_t(w_t L_t + r_t A_{t-1})$, is rewritten as $\tau_t(Y_t + r_t D_t)$ by using (6), (7a), (7b), and (7c). Thus, the budget constraint of the government is

$$D_{t+1} - D_t + \tau_t(Y_t + r_t D_t) = G_t + r_t D_t. \tag{8}$$

As mentioned in the Introduction, we are interested in the sustainability of public deficit policy such as the criterion of the Maastricht Treaty. Thus, following Bräuninger (2005), we assume that a constant proportion, $g \in (0, 1)$, of national income, Y_t, is used for public expenditure: $G_t = gY_t$. In addition, the government borrows a constant proportion, $\lambda \in (0, 1)$, of GDP, that is, the government fixes the deficit ratio as follows:

$$D_{t+1} - D_t = \lambda Y_t. \tag{9}$$

As in Bräuninger (2005), Yakita (2008), Arai (2011), and Teles and Mussolini (2014), when g and λ are kept constant, the government must adjust the income tax rate, τ_t, to satisfy the budget constraint (8). By using (7b), (7c), (8), and (9), we obtain

$$\tau_t = 1 - \frac{1+\lambda-g}{1+\gamma x_t}, \tag{10}$$

where $x_t \equiv D_t/K_t$. A higher level of public debt means that a higher level of income taxation must be used to pay for the interest payments on the debt. Therefore, an increase in the ratio of public debt to private capital x raises the income tax rate: $d\tau_t/dx_t > 0$ as in Bräuninger (2005), Yakita (2008), Arai (2011), and Teles and Mussolini (2014). We call this the *tax burden effect*.

3. EQUILIBRIUM CONDITION AND DYNAMIC SYSTEM

From individuals' budget constraints (2) and (3), the distribution of output, $Y_t = r_t K_t + w_t L_t$, the government's budget constraint (8), and the asset market-clearing condition (6), the market equilibrium satisfies the following resource constraint: $Y_t = \delta c_t^{1R} N + (1-\delta) c_t^{1P} N + \delta c_t^{2R} N + (1-\delta) c_t^{2P} N + K_{t+1} - K_t + G_t$.

We then characterize the equilibrium paths in this economy. By substituting (5) into $A_t = \delta s_t^R N + (1-\delta)s_t^P N$, we obtain

$$A_t = \bar{\alpha}(1-\tau_t)w_t N + \beta[1+(1-\tau_t)r_t]\left[\alpha_R s_{t-1}^R \delta N + \alpha_P s_{t-1}^P (1-\delta)N\right], \quad \textbf{(11)}$$

where $\bar{\alpha} \equiv \delta\alpha_R + (1-\delta)\alpha_P$. By using (6) and the definition of x_t, we obtain

$$K_t = (1+x_t)^{-1}A_{t-1}. \quad \textbf{(12)}$$

Equation (12) indicates that an increase in x_t reduces investment in private capital because public bonds account for a larger proportion of aggregate assets, A_{t-1}. We call this the *crowding-out effect*.

By dividing (11) by A_{t-1} and substituting (7a), (7b), (10), and (12) into (11), we obtain the growth in aggregate savings A as follows:

$$\frac{A_t}{A_{t-1}} = \frac{\bar{\alpha}(1-\gamma)\mu_1}{(1+x_t)(1+\gamma x_t)} + \beta\left(1+\frac{\gamma\mu_1}{1+\gamma x_t}\right)\left[(\alpha_R-\alpha_P)\varphi_{t-1}+\alpha_P\right]$$
$$\equiv G^A(x_t, \varphi_{t-1}), \quad \textbf{(13)}$$

where $\varphi_t \equiv \frac{\delta s_t^R N}{A_t}$ and $\mu_1 \equiv \Gamma(1+\lambda-g)$. Note that $1-\varphi_t = \frac{(1-\delta)s_t^P N}{A_t}$ holds from the definition of φ_t and A_t. In this study, because φ_t represents the ratio of total savings of the rich to aggregate savings, φ_t serves as a convenient measure of inequality. If φ_t is close to 0.5, the economy expresses weak inequality (i.e., equality). Conversely, if φ_t is close to 0 or 1, the economy expresses strong inequality. Because the wealth of the rich is larger than that of the poor in the real economy, we assume that $\varphi_t > 0.5$ for all t.

From (13), $G^A(x_t, \varphi_{t-1})$ satisfies (a) $\frac{\partial G^A(x_t,\varphi_{t-1})}{\partial x_t} < 0$ and (b) $\frac{\partial G^A(x_t,\varphi_{t-1})}{\partial \varphi_{t-1}} > 0$. The former, (a), comes from the following two channels. First, an increase in x_t reduces the wage rate through the *crowding-out effect* (see (12)). Second, an increase in x_t decreases disposable wage and bequest incomes through the *tax burden effect* (see (10)). Both have negative effects on the growth in aggregate savings. The latter, (b), indicates the following. The rich accumulate more wealth than the poor do and hold a larger proportion of aggregate wealth (see (5)). Then, inequality φ driven by the rich contributes to the growth in aggregate savings $G^A(\cdot)$.

By using (5), (7a), (7b), (10), (12), and the definition of φ_t, we obtain the growth in the rich's savings as

$$\frac{\delta s_t^R N}{\delta s_{t-1}^R N} = \frac{\alpha_R}{\varphi_{t-1}}\left[\frac{\delta(1-\gamma)\mu_1}{(1+x_t)(1+\gamma x_t)} + \beta\left(1+\frac{\gamma\mu_1}{1+\gamma x_t}\right)\varphi_{t-1}\right]$$
$$\equiv G^R(x_t, \varphi_{t-1}). \quad \textbf{(14)}$$

Substituting (7c) into (9) and using $D_t/K_t \equiv x_t$ yields the growth in public debt as

$$\frac{D_{t+1}}{D_t} = 1 + \frac{\lambda\Gamma}{x_t} \equiv G^D(x_t). \quad \textbf{(15)}$$

From (6) and (9), we obtain $K_{t+1} + \lambda Y_t + D_t = A_t$. Dividing both sides of this by K_t and using (7c) yield $K_{t+1}/K_t = (A_t/A_{t-1})(A_{t-1}/K_t) - (x_t + \lambda\Gamma)$. By substituting (12) and (13) into it, we obtain

$$\frac{K_{t+1}}{K_t} = (1 + x_t)G^A(x_t, \varphi_{t-1}) - (x_t + \lambda\Gamma) \equiv G^K(x_t, \varphi_{t-1}). \tag{16}$$

The growth in private capital $G^K(\cdot)$ is linked positively with $G^A(\cdot)$ through the asset market-clearing condition (see (12)). Because $\frac{\partial G^A(x_t,\varphi_{t-1})}{\partial \varphi_{t-1}} > 0$ holds, a higher level of inequality generates a higher growth rate of K_t. That is, $\frac{\partial G^K(x_t,\varphi_{t-1})}{\partial \varphi_{t-1}} > 0$ holds.

From (13), (14), and the definition of φ_t, we obtain

$$\frac{\varphi_t}{\varphi_{t-1}} = \frac{\frac{\delta s_t^R N}{\delta s_{t-1}^R N}}{\frac{A_t}{A_{t-1}}} = \frac{G^R(x_t, \varphi_{t-1})}{G^A(x_t, \varphi_{t-1})}. \tag{17}$$

The growth in inequality decreases (increases) when $G^A(\cdot)$ is larger (smaller) than $G^R(\cdot)$. This fact is attributed to the definition of φ. From (15) and (16), we obtain

$$\frac{x_{t+1}}{x_t} = \frac{G^D(x_t)}{G^K(x_t, \varphi_{t-1})} = \frac{1 + \lambda\Gamma/x_t}{(1 + x_t)G^A(x_t, \varphi_{t-1}) - (x_t + \lambda\Gamma)}. \tag{18}$$

The above two difference equations (17) and (18) together with the initial values φ_{-1} and x_0 characterize the dynamics of the economy. Note that both x_t and φ_{t-1} in period t are predetermined variables.

4. TRANSITION DYNAMICS OF INEQUALITY AND THE PUBLIC DEBT/PRIVATE CAPITAL RATIO

In this section, we derive the global transition dynamics of the economy and investigate how the accumulation of public debt and inequality relate with each other.

We begin with the derivation of the $\varphi_t = \varphi_{t-1}$ locus on the (x_t, φ_{t-1}) plane. Setting $\varphi_t = \varphi_{t-1}$ in (17), that is, $1 = \frac{G^R(x_t,\varphi_{t-1})}{G^A(x_t,\varphi_{t-1})}$, yields

$$\beta(1 + x_t)\left[\mu_1^{-1}(1 + \gamma x_t) + \gamma\right] = \frac{1 - \gamma}{(\alpha_R - \alpha_P)(1 - \varphi_{t-1})}\left(\bar{\alpha} - \frac{\delta\alpha_R}{\varphi_{t-1}}\right). \tag{19}$$

Let us define the left- and right-hand sides of (19) as $\varepsilon(x_t)$ and $\eta(\varphi_{t-1})$, respectively. By examining (19), we arrive at the following.

LEMMA 1.

(i) The $\varphi_t = \varphi_{t-1}$ locus is an upward-sloping curve on the (x_t, φ_{t-1}) plane.
(ii) The $\varphi_t = \varphi_{t-1}$ locus has an asymptote $\varphi_{t-1} = 1$ when $x_t \to \infty$ and takes a lower limit $(0, \tilde{\varphi})$ on the (x_t, φ_{t-1}) plane. $\tilde{\varphi}$ is defined in Appendix A.

Proof. See Appendix A in the supplementary material.

Next, we derive the $x_{t+1} = x_t$ locus on the (x_t, φ_{t-1}) plane. Setting $x_{t+1} = x_t$ in (18), that is, $1 = \frac{1+\lambda\Gamma/x_t}{(1+x_t)G^A(x_t,\varphi_{t-1})-(x_t+\lambda\Gamma)}$, leads to

$$\varphi_{t-1} = \frac{\zeta(x_t)}{\alpha_R - \alpha_P}, \tag{20}$$

where

$$\zeta(x_t) \equiv \frac{(1+x_t)(1+\gamma x_t)\left(1+\frac{\lambda\Gamma}{x_t}\right) - \bar{\alpha}(1-\gamma)\mu_1}{\beta(1+x_t)\left[1+\gamma(x_t+\mu_1)\right]} - \alpha_P.$$

By examining (20), we arrive at the following.

LEMMA 2. *Suppose that* $\gamma(1-g) > \lambda$.[7]

(i) The $x_{t+1} = x_t$ *locus is a U-shaped curve on the* (x_t, φ_{t-1}) *plane.*
(ii) The $x_{t+1} = x_t$ *locus has the asymptotes* $\lim_{x_t\to\infty} \varphi_{t-1} = \frac{\beta^{-1}-\alpha_P}{\alpha_R-\alpha_P} > 1$ *and* $\lim_{x_t\to 0} \varphi_{t-1} = +\infty$ *on the* (x_t, φ_{t-1}) *plane.*

Proof. See Appendix B in the supplementary material.

Figure 1 depicts the $\varphi_t = \varphi_{t-1}$ and $x_{t+1} = x_t$ loci on the (x_t, φ_{t-1}) plane.

Finally, we examine the regions in which $K_{t+1}/K_t \geq 0$, that is, $(1 + x_t)G^A(x_t, \varphi_{t-1}) - (x_t + \lambda\Gamma) \geq 0$ from (16) so far.[8] This condition can be rewritten as

$$\varphi_{t-1} \geq \frac{1}{\alpha_R - \alpha_P}\left[\zeta(x_t) - \frac{(1+\gamma x_t)\left(1+\frac{\lambda\Gamma}{x_t}\right)}{\beta(1+x_t)[1+\gamma(x_t+\mu_1)]}\right] \equiv \frac{\Lambda(x_t)}{\alpha_R - \alpha_P}. \tag{21}$$

Thus, we can recognize that $K_{t+1}/K_t > 0$ is satisfied above the $K_{t+1}/K_t = 0$ locus. By examining (21), we arrive at the following. From (20) and (21), the $K_{t+1}/K_t = 0$ locus is always below the $x_{t+1} = x_t$ locus. In addition, Appendix C in the supplementary material shows that the $K_{t+1}/K_t = 0$ locus is an upward-sloping curve if $\bar{\alpha}(1-\gamma)(1+\lambda-g) > \lambda$ and has an asymptote $\lim_{x_t\to\infty} \varphi_{t-1} = \frac{\beta^{-1}-\alpha_P}{\alpha_R-\alpha_P} > 1$ on the (x_t, φ_{t-1}) plane.[9] Figure 1 depicts the $K_{t+1}/K_t = 0$ locus as the broken curve.

Now, we investigate the steady states of the economy wherein both x_t and φ_{t-1} are constant. In this study, we use an asterisk to represent variables in the steady state (i.e., $x_t = x_{t+1} = x^*$ and $\varphi_{t-1} = \varphi_t = \varphi^*$). As shown in Figure 1, the steady-state values of (x^*, φ^*) are determined by the intersections of the curves, $\varphi_t = \varphi_{t-1}$ and $x_{t+1} = x_t$ loci, on the (x_t, φ_{t-1}) plane. From Lemmas 1 and 2, there are either two long-run equilibria or none. Figure 1 shows the case of two steady states, which is obtained if **C1** is satisfied.[10]

C1: $\varepsilon(\bar{x}) > \eta(\bar{\varphi})$, where $\bar{\varphi} \equiv \zeta(\bar{x})$.

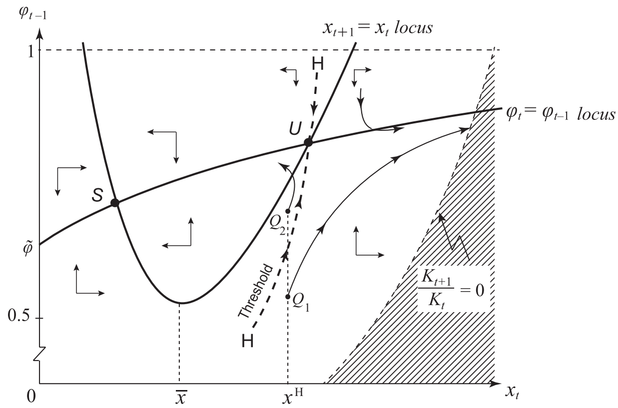

FIGURE 1. Phase diagram on the (x_t, φ_{t-1}) plane.

In addition, **C2** allows us to focus on the economy in which the wealth of the rich is larger than that of the poor (i.e., $\varphi_{t-1} > 0.5$ for all t).[11]

C2: $\tilde{\varphi} \equiv \eta^{-1}(\varepsilon(0)) > 0.5.$

Let us refer to (x_k^*, φ_k^*) as the steady state k ($k \in \{S, U\}$). We obtain $x_S^* < x_U^*$ and $\varphi_S^* < \varphi_U^*$ because the $\varphi_t = \varphi_{t-1}$ locus is increasing in an upward-sloping curve. From (7c), (15), (16), and constant x_k^*, the steady-state growth rate at each state is as follows:

$$\hat{Y}_k^* \equiv \left(\frac{Y_{t+1}}{Y_t}\right)^* = \left(\frac{K_{t+1}}{K_t}\right)^* = \left(\frac{A_t}{A_{t-1}}\right)^* = \left(\frac{D_{t+1}}{D_t}\right)^* = 1 + \frac{\lambda\Gamma}{x_k^*} for k \in \{S, U\}. \quad (22)$$

Therefore, $x_S^* < x_U^*$ implies that the growth rate of the steady state S is higher than that of the steady state U. As shown in Appendix D in the supplementary material, Figure 1 illustrates a phase diagram of this economy, highlighting that the steady state S is stable and the steady state U is saddle-point stable.[12] The dotted line HH in Figure 1 represents the stable arm converging to the steady state U. Because x_t and φ_{t-1} are predetermined variables at time t, as mentioned before, we must note the following two points. First, the initial state of the economy is given by a point (x_0, φ_{-1}) on the (x_t, φ_{t-1}) plane. Second, the saddle arm, HH, is a knife-edge.

These facts lead to the following two cases. When an economy starts at the initial state (x_0, φ_{-1}) in the upper-left of the saddle arm, HH, it converges to the steady state S. At the steady state S, both x_t and φ_{t-1} are constant, and the government can run the constant budget deficit policy permanently because private capital grows at the constant rate $\hat{Y}_S^*$. By contrast, when (x_0, φ_{-1}) is in the lower-right of the saddle arm, HH, an economy will not converge to any steady states. In this case, public debt grows more than private capital does:

$G^D(\cdot) > G^K(\cdot)$; finally, the economy falls into the $K_{t+1}/K_t < 0$ region in which the public debt/private capital ratio, x_t, becomes too large to sustain investment in private capital. Therefore, the dotted line HH in Figure 1 represents the *threshold of public debt* for each level of inequality in order for the government to sustain fiscal policy. This threshold level of public debt has a positive relationship with inequality. This is a noticeable result that departs from previous studies that do not include inequality (e.g., Bräuninger, 2005; Yakita, 2008; Arai, 2011; Teles and Mussolini, 2014).

In summary, we can state the following proposition.

PROPOSITION 1. *There is a threshold of public debt for each level of inequality in order for the government to sustain its policy. The threshold of public debt is increasing in inequality.*

To consider the intuition behind Proposition 1, let us begin with the case in which the initial public debt/private capital ratio, x, is sufficiently large to be near the saddle arm, HH, for example, $x_0 = x^H$ in Figure 1. In this situation, the growth rates of both aggregate savings and private capital at time $t = 0$ (i.e., $G^A(x_0, \varphi_{-1})$ and $G^K(x_0, \varphi_{-1})$) tend to be small. As in Section 3, we show that $\frac{\partial G^A(x_t,\varphi_{t-1})}{\partial \varphi_{t-1}} > 0$ and $\frac{\partial G^K(x_t,\varphi_{t-1})}{\partial \varphi_{t-1}} > 0$ hold. Therefore, if the initial wealth of the rich is sufficiently larger than that of the poor (i.e., φ_{-1} is sufficiently high), the wealth accumulation of the rich can be sufficiently strong to reinforce the growth in aggregate savings and private capital at time $t = 0$. In the next period, the public debt/private capital ratio, x, decreases, which implies that both the *tax burden effect* and the *crowding-out effect* become small from (10) and (12), respectively, resulting in relatively high wage and bequest incomes. In such an environment, aggregate savings and private capital can continue to grow strongly. As a result, both public debt/private capital ratio, x, and inequality, φ, converge to the steady state S. That is, the economy with high inequality, as represented by Q_2, can sustain its public debt.

By contrast, in the economy with low inequality, as represented by Q_1, the initial growth in aggregate savings, $G^A(x_0, \varphi_{-1})$, tends to remain small. In this case, public debt grows more than private capital does, and then the economy cannot sustain its public debt. It is of great interest that inequality increases as public debt grows during the bankruptcy path. An intuitive reason for this is as follows. As public debt grows, the growth in the wage rate keeps decreasing through both the *tax burden effect* and the *crowding-out effect*. In this situation, bequest income plays a more important role in wealth accumulation than does wage income. Because the rich leave more wealth to their offspring than do the poor, inequality increases and the absorption of larger public debt tends to rely more on wealth accumulation by the rich. However, this situation does not last long and the economy goes bankrupt in the long run.

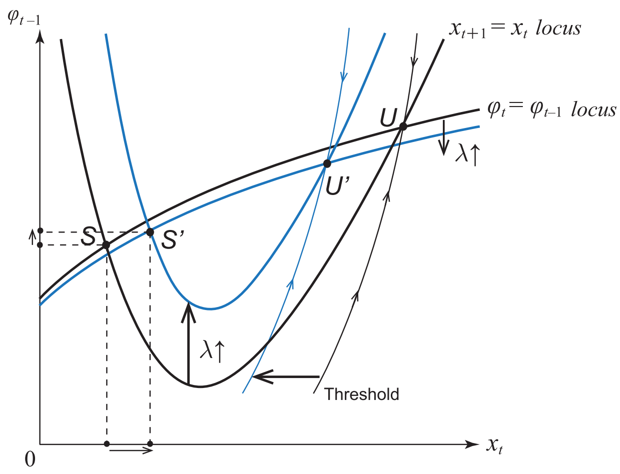

FIGURE 2. Effects of an increase in λ.

5. CHANGES IN THE PUBLIC DEBT FINANCE RATIO λ

Some studies investigate the relationship between public debt and inequality (e.g., Mankiw, 2000; Michel and Pestieau, 2005) and show that increases in public debt finance raise wealth (or income) inequality. However, these studies focus only on the steady state, which corresponds to the stable steady state S in our study. Therefore, they neglect the possibility of unstable (unsustainable) paths. The main objective here is to study how increases in public debt finance affect inequality and the sustainability of public debt.

We begin with the effects of an increase in the public debt finance ratio λ on the $\varphi_t = \varphi_{t-1}$ locus. Taking the total differentials of (19) yields

$$\left.\frac{d\varphi_{t-1}}{d\lambda}\right|_{\varphi_t=\varphi_{t-1}} = -\frac{\beta\Gamma\mu_1^{-2}(\alpha_R-\alpha_P)(1+x_t)(1+\gamma x_t)(1-\varphi_{t-1})}{\eta(\varphi_{t-1})(\alpha_R-\alpha_P)+(1-\gamma)\delta\alpha_R\varphi_{t-1}^{-2}} < 0. \qquad \textbf{(23)}$$

Note that $0 < \varphi_{t-1} < 1$ holds from the definition of φ_{t-1}. We next investigate the effect of changes in λ on the $x_{t+1} = x_t$ locus. By differentiating (20) with respect to λ, we obtain

$$\left.\frac{d\varphi_{t-1}}{d\lambda}\right|_{x_{t+1}=x_t} = \frac{\Gamma(1+\gamma x_t)\left[\left(1+x_t^{-1}\right)(1+\gamma\mu_1-\gamma\lambda\Gamma)-(1-\gamma)\bar{\alpha}\right]}{(\alpha_R-\alpha_P)\beta(1+x_t)\left[1+\gamma(x_t+\mu_1)\right]^2}. \qquad \textbf{(24)}$$

Because of $1+\gamma\mu_1-\gamma\lambda\Gamma = 1+\gamma\Gamma(1-g) > 1$ (from the definition of μ_1) and $(1-\gamma)\bar{\alpha} \in (0,1)$, $\left.\frac{d\varphi_{t-1}}{d\lambda}\right|_{x_{t+1}=x_t} > 0$ holds. Thus, we obtain the following.

LEMMA 3. *When the government increases the public debt finance ratio λ, (i) the $\varphi_t = \varphi_{t-1}$ locus shifts downward from (23), and (ii) the $x_{t+1} = x_t$ locus shifts upward from (24). These are represented in Figure 2.*

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TABLE 1. Deficit ratio and steady-state values

λ	x_S^*	φ_S^*	$\hat{Y}_S^*$	x_U^*	φ_U^*	$\hat{Y}_U^*$
0.01	0.0646	0.6585	2.8582	1.6121	0.6835	1.0744
0.02	0.1431	0.6596	2.6770	1.4470	0.6806	1.1659
0.03	0.2454	0.6611	2.4669	1.2588	0.6773	1.2860
0.04	0.4033	0.6635	2.1901	1.0156	0.6732	1.4726

From Lemma 3, we can observe that the steady state U shifts left and downward and therefore the saddle arm, HH, also shifts left and downward, as depicted in Figure 2. That is, the threshold of public debt for each level of inequality in order for the government to sustain fiscal policy becomes lower. This result implies that the range in which the deficit policy is sustainable is shrunk by an increase in λ. Thus, we obtain the following proposition.

PROPOSITION 2. *An increase in the deficit ratio reduces the range of the sustainable initial public debt.*

A larger budget deficit reinforces both the *tax burden effect* and the *crowding-out effect*, which leads to a decline in aggregate savings. Therefore, even if the initial level of inequality is somewhat high, the economy is more likely to fall into an unsustainable path. This result leads to the following policy implication. In the economy with large public debt, an increase in the public debt finance ratio not only makes public debt less sustainable but can also induce inequality to increase persistently.

In the rest of this section, we shed light on the steady state S and investigate the effects of changes in λ on the public debt/private capital ratio, inequality, and the long-run growth rate. Lemma 3 shows that at the steady state S, an increase in λ raises the public debt/private capital ratio, x_S^*, but the effect on inequality, φ_S^*, is ambiguous. Moreover, the effect of λ on the long-run growth rate at the steady state S is ambiguous. Then, we conduct a numerical analysis. We adopt the following benchmark parameters: $\gamma = 0.2$, $\Gamma = 12$, $g = 0.2$, $\delta = 0.5$, $\beta = 0.3$, $\alpha_R = 0.45$, and $\alpha_P = 0.25$.[13] Table 1 shows the steady-state values for each deficit ratio and leads to the following result.

RESULT 1. *At the steady state S, an increase in λ (i) raises inequality and (ii) reduces the long-run growth rate.*

Result (i) is similar to that of Mankiw (2000) and Michel and Pestieau (2005). As mentioned in the paragraph below Proposition 2, a higher budget deficit decreases aggregate savings, which reduces the growth in the wage rate because a decline in aggregate savings reduces investment in private capital (see (7a) and (12)). In this study, from (5), the income of the young consists of wage and bequest incomes. Lower wage income indicates that bequest income becomes more important for the accumulation of wealth. The rich tend to hold a larger proportion of total wealth, and then inequality increases. Furthermore, result (ii)

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is similar to that of Bräuninger (2005). A higher budget deficit implies that an increase in public debt raises the interest payment of the government. To satisfy the government's budget constraint, the income tax rate increases. This reduces total savings. In addition, a higher level of public debt crowds out investment in private capital. As a result, the long-run growth rate declines.

6. REDISTRIBUTIVE POLICY

We have thus far considered the relationship between the sustainability of public debt and inequality. In many developed countries, policies aiming for a reduction in inequality are implemented.[14] Thus, we wonder how redistributive policy affects the sustainability of public debt, inequality, and economic growth. To tackle this problem, we introduce a redistributive policy in the following simple way.

The government taxes the bequests of the rich at rate τ^b and redistributes its revenue to the poor in youth. This is the reduced form of the redistributive policy considered in Bossmann et al. (2007).[15] The budget constraint for the rich is given by $c_t^{1R} = (1-\tau_t)w_t - s_t^R + b_t^R$ and $c_{t+1}^{2R} = \left[1 + (1-\tau_{t+1})r_{t+1}\right] s_t^R - (1+\tau^b)b_{t+1}^R$ and that for the poor is given by $c_t^{1P} = (1-\tau_t)w_t - s_t^P + b_t^P + T_t$ and $c_{t+1}^{2P} = \left[1 + (1-\tau_{t+1})r_{t+1}\right] s_t^P - b_{t+1}^P$, where T_t is the uniform lump-sum transfer under the redistributive policy. The government's redistributive policy is represented as $\delta N \tau^b b_t^R = (1-\delta)NT_t$.

Appendix F in the supplementary material shows that the growth in aggregate savings and that in private capital are given by

$$G^A(x_t, \varphi_{t-1}; \tau^b) = \frac{\bar{\alpha}(1-\gamma)\mu_1}{(1+\gamma x_t)(1+x_t)} + \beta\left(1 + \frac{\gamma\mu_1}{1+\gamma x_t}\right)\left[\frac{\alpha_R - \alpha_P}{1+\tau^b}\varphi_{t-1} + \alpha_P\right], \tag{25}$$

and

$$G^K(x_t, \varphi_{t-1}; \tau^b) = (1+x_t)G^A(x_t, \varphi_{t-1}; \tau^b) - (x_t + \lambda\Gamma). \tag{26}$$

Furthermore, Appendix F in the supplementary material shows that the $\varphi_t = \varphi_{t-1}$ and $x_{t+1} = x_t$ loci are rewritten as

$$\varepsilon(x_t) = \frac{1+\tau^b}{1 - \frac{\tau^b \alpha_P}{(\alpha_R - \alpha_P)(1-\varphi_{t-1})}}\eta(\varphi_{t-1}) \equiv \tilde{\eta}(\varphi_{t-1}), \tag{27}$$

and

$$\varphi_{t-1} = \frac{1+\tau^b}{\alpha_R - \alpha_P}\zeta(x_t), \tag{28}$$

respectively. As shown in Appendix G in the supplementary material, the introduction of redistributive policy does not affect the main properties of the $\varphi_t = \varphi_{t-1}$ locus that we observe in Section 4 if we assume $\alpha_R > (1+\tau^b)\alpha_P$. Then, we easily recognize that (i) the $\varphi_t = \varphi_{t-1}$ locus (27) is an upward-sloping

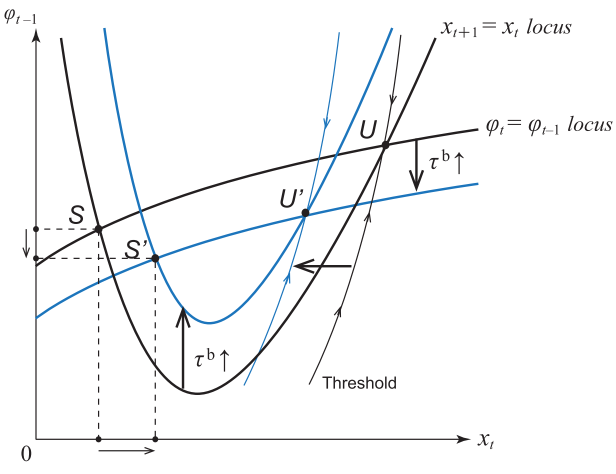

FIGURE 3. Effects of an increase in τ^b.

curve and the $x_{t+1} = x_t$ locus (28) is a U-shaped curve on the (x_t, φ_{t-1}) plane and (ii) two steady states corresponding to S and U exist as depicted in Figure 3. We next investigate the effects of a rise in the bequest tax rate. Taking the total differentials of (27) yields

$$\left.\frac{d\varphi_{t-1}}{d\tau^b}\right|_{\varphi_t=\varphi_{t-1}} = -\frac{\tilde{\eta}(\varphi_{t-1})\left[\alpha_R(1-\varphi_{t-1})+\alpha_P\varphi_{t-1}\right]}{(1+\tau^b)\left[\tilde{\eta}(\varphi_{t-1})(\alpha_R-\alpha_P)+(1+\tau^b)(1-\gamma)\delta\alpha_R\varphi_{t-1}^{-2}\right]} < 0.$$

Thus, when the government increases τ^b, the $\varphi_t = \varphi_{t-1}$ locus shifts downward. We then examine the effect of changes in the τ^b on $x_{t+1} = x_t$ locus. By differentiating (28) with respect to τ^b, we obtain

$$\left.\frac{d\varphi_{t-1}}{d\tau^b}\right|_{x_{t+1}=x_t} = \frac{\zeta(x_t)}{\alpha_R-\alpha_P} > 0.$$

Therefore, when the government increases τ^b, the $x_{t+1} = x_t$ locus shifts upward. These results imply that the effects of an increase in the bequest tax rate are qualitatively similar to those of a rise in the public debt finance ratio. We can state the following proposition in summary.

PROPOSITION 3. *Taxing the bequests of the rich and redistributing the revenue to the poor (or a rise in the bequest tax) reduces the range of the sustainable initial public debt and makes fiscal policy less sustainable.*

An intuitive explanation is as follows. The redistributive policy increases the income and savings of the poor, whereas it decreases the income and savings of the rich. We find that the latter dominates the former and then the growth in aggregate savings declines because of an increase in τ^b as follows:

$$\frac{dG^A(x_t,\varphi_{t-1};\tau^b)}{d\tau^b} = -\beta\left(1+\frac{\gamma\mu_1}{1+\gamma x_t}\right)\frac{\alpha_R-\alpha_P}{(1+\tau^b)^2}\varphi_{t-1} < 0. \quad \textbf{(29)}$$

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TABLE 2. Effects of an increase in the bequest tax rate

λ	Δx_S^*	$\Delta \varphi_S^*$	$\Delta \hat{Y}_S^*$	Δx_U^*	$\Delta \varphi_U^*$	$\Delta \hat{Y}_U^*$
0.01	1.76%	−2.46%	−1.12%	−4.28%	−6.17%	0.31%
0.02	2.27%	−2.64%	−1.39%	−4.75%	−5.75%	0.71%
0.03	3.25%	−2.87%	−1.87%	−5.65%	−5.27%	1.33%
0.04	6.61%	−3.23%	−3.37%	−8.62%	−4.67%	3.03%

Note: The changes in the steady-state values are expressed in percentage points.

This is because the rich are more patient than the poor are. If the initial public debt/private capital ratio, x, is sufficiently large, the growth in private capital declines sufficiently to be below that of public debt. Thus, the redistributive policy makes the government's budget deficit policy less sustainable. If the economy falls into a region where public debt is unsustainable, then inequality increases during the bankruptcy path. Hence, it is noticeable that the redistributive policy can result in widening inequality.

We investigate the effects of changes in the redistributive tax at the steady state S. When the government raises τ^b, the steady state S shifts rightward. Hence, the public debt/private capital ratio at the steady state S increases (i.e., $dx_S^*/d\tau^b > 0$). From (22), it is obvious that an increase in τ^b reduces the growth rate at the steady state S. The result of this declining growth rate is brought about by the reduction in the growth in aggregate savings, as in (29). On the contrary, the effect of τ^b on inequality at the steady state S is ambiguous because whether the steady state S shifts upward or downward it uncertain. To clarify this ambiguity, we conduct a numerical analysis with the following parameters: $\gamma = 0.2$, $\Gamma = 12$, $g = 0.2$, $\delta = 0.5$, $\beta = 0.3$, $\alpha_R = 0.45$, and $\alpha_P = 0.25$. Let us denote the steady-state values after the policy change as x_k^{**}, φ_k^{**}, and $\hat{Y}_k^{**}$. The effects of the policy change are measured by $\Delta x_k^* \equiv \left(x_k^{**} - x_k^*\right)/x_k^*$, $\Delta \varphi_k^* \equiv \left(\varphi_k^{**} - \varphi_k^*\right)/\varphi_k^*$, and $\Delta \hat{Y}_k^* \equiv \left(\hat{Y}_k^{**} - \hat{Y}_k^*\right)/\hat{Y}_k^*$ $(k = \{S, U\})$.

Table 2 represents the percentage changes in the steady-state values when the government increases the bequest tax rate, τ^b, from 0 to 0.3. This numerical analysis shows the following. An increase in τ^b reduces inequality. However, the effect of decreasing inequality is relatively small (only about 2% or 3% changes in φ_S^* even by a 30% increase in τ_b). This is attributed to the adverse effect of τ^b on inequality. As mentioned in the paragraph below Proposition 3, a rise in τ^b not only redistributes income from the rich to the poor but also decreases the long-run growth rate and investment in private capital. The latter leads to a decrease in wage income (from (7a)), and bequest income plays a more important role in savings. Because the rich receive more bequests, inequality increases. This mitigates the effect of τ^b on inequality. We summarize the redistributive policy effect at the steady state S.

RESULT 2. *An increase in τ^b reduces inequality and the growth rate at the steady state S. The effect of decreasing inequality is relatively small.*

Our investigation throughout this section leads to the following policy implication. The redistributive policy aimed at reducing inequality faces a trade-off between equality and growth at the steady state S. Moreover, in the economy with a large public debt/private capital ratio, implementing such a redistributive policy might be risky because it can take the economy on an unstable path from which inequality and public debt continue to increase.

7. DISCUSSION

To clarify our main arguments, we employ a simple model with some restrictive specifications such as AK production function and constant marginal propensity to save. Furthermore, we ignore other sources of inequality (e.g., human capital accumulation, trade/financial globalization) and productive public spending that is linked to economic growth (e.g., public investment in infrastructure). In this section, we note several limitations of our specifications and discuss directions for future research.

First, we assume the AK production technology in order to make the analysis tractable. Under this assumption, the real interest rate is constant and exogenous in equilibrium (see (7b)). Therefore, some readers might wonder whether public debt can really become unsustainable under endogenous real interest rate. The results of Chalk (2000) are helpful for this point; Chalk (2000) assumes that the productivity of private capital is marginally decreasing; that is, the interest rate is decreasing in private capital. Under certain conditions, Chalk (2000) presents the following possibility: If the deficit-to-GDP ratio is sufficiently low, there are stable and unstable steady states. In this case, when the initial public debt/private capital ratio is sufficiently high, the crowding-out effect of public debt on private capital is so large that the economy goes bankrupt in the long run. This finding is consistent with our result (see discussion in Section 4). Hence, we predict that our main results would not change under endogenous real interest rate.

Second, in Proposition 1, we show that there is a threshold of public debt for each level of inequality in order for the government to sustain its policy. We explore this threshold (the dotted line HH in Figure 1) through a numerical analysis. We adopt the following parameters: $\gamma = 0.2$, $\Gamma = 12$, $g = 0.2$, $\delta = 0.5$, $\beta = 0.3$, $\alpha_R = 0.45$, and $\alpha_P = 0.25$. Figure 4 represents the threshold of public debt for each level of inequality under $\lambda = 0.03$.[16]

The numerical result is consistent with our theoretical result of Proposition 1. That is, the threshold of public debt is increasing in inequality. In this study, we assumed that higher inequality leads to higher aggregate savings because the rich have a higher marginal propensity to save. Therefore, higher inequality enlarges the maximum public debt/private capital ratio for fiscal sustainability. However, our numerical result suggests that the slope of the threshold is steep, indicating that the effect of inequality on aggregate savings is not large. This result may be because of the exogenous marginal propensity to save. Some recent studies (e.g., Auclert and Rognlie, 2017, 2018) show that the marginal propensity to

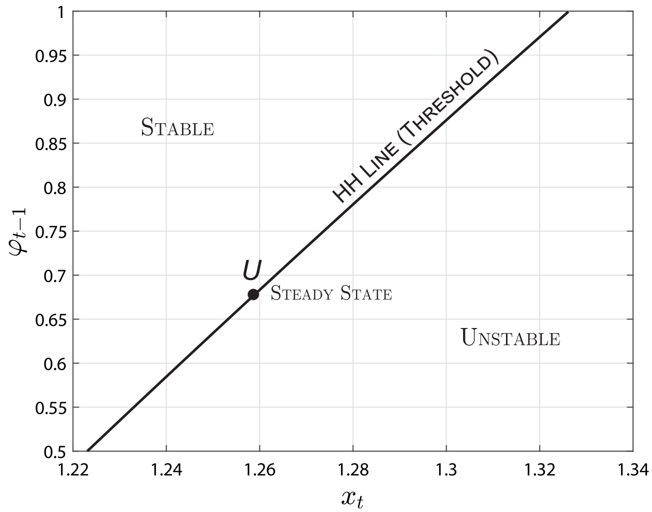

FIGURE 4. HH line under a numerical example.

save increases with wealth. Incorporating this factor could enhance the effect of inequality on aggregate savings and sustain higher public debt. Thus, under endogenous marginal propensity to save, the slope of the threshold could become milder.

Third, this study used an OLG model with the AK production structure and assumed that government expenditure is public consumption. Incorporating productive government spending or public capital that enhance economic growth (e.g., Barro, 1990; Futagami et al., 1993) is an important direction for the extension of this study. Some recent empirical studies indicate that public investment in infrastructure may reduce inequality (e.g., Calderón and Servén, 2004). However, the recent trend of enhanced power by the elite could result in a more limited provision of productive government spending, as pointed by Dabla-Norris et al. (2015). Furthermore, investigating the effects of deficit-financing public investment (e.g., Greiner and Semmler, 2000; Ghosh and Mourmouras, 2004; Tamai, 2014, 2016) could provide interesting insights into public finance.

Fourth, we did not consider the heterogeneity of wage income and mobility between generations. Incorporating human capital accumulation generates an endogenous disparity of wage income and mobility between generations (e.g., Galor and Moav, 2004). An increase in inequality may lower growth by keeping the poor away from accessing better education and health services that contribute to human capital accumulation. Incorporating these mechanisms into the investigation of the sustainability of public debt would, therefore, be an important direction for future research.

Fifth, there have been various discussions on the causes of inequality (e.g., education, technological change, trade globalization, financial globalization, changes in labor market institutions, and the stance of redistributive policies).[17] Future research should, threrfore, examine the relationship between the sustainability of public debt and inequality, resulting from these factors.

8. CONCLUSION

This study constructed an endogenous growth model with heterogeneous agents to examine the relationship between the sustainability of public debt and inequality. We showed that there is a threshold of public debt for each level of inequality in order for the government to sustain fiscal policy and that the threshold of public debt is increasing in inequality. In addition, we investigated the effects of budget deficit and redistributive policies on the sustainability of public debt. We showed that an increase in the deficit ratio or the redistributive tax reduces the range of sustainable public debt. That is, in the economy with large public debt, such a policy change makes the economy fall into the unsustainable region in which both public debt and inequality continue to increase.

SUPPLEMENTARY MATERIAL

To view supplementary material for this article, please visit https://doi.org/10.1017/S1365100519000336.

NOTES

1. The empirical analysis of Azzimonti et al. (2014) is included in their online appendix.

2. From a theoretical perspective, both Azzimonti et al. (2014) and Arawatari and Ono (2017) investigate the relationship between public debt and inequality in politico-economic models. However, they focus mainly on the stable equilibrium and do not address the relationship between the sustainability of public debt and inequality.

3. In these endogenous growth models, large outstanding public debt can induce higher growth in public debt than in private capital and output and can make fiscal policy unsustainable.

4. In representative infinitely-lived agent models, many studies investigate fiscal sustainability under various public deficit or debt policy rules by using endogenous growth models. For example, Minea and Villieu (2012) consider deficit/GDP rules following the criterion of the Maastricht Treaty. Greiner (2007, 2011, 2012, 2015) and Kamiguchi and Tamai (2012) consider the primary surplus rule, which positively depends on the debt-to-GDP ratio. Futagami et al. (2008), Minea and Villieu (2013), Maebayashi et al. (2017), and Morimoto et al. (2017) consider debt/GDP rules following the criterion of the Maastricht Treaty.

5. Mankiw (2000) and Michel and Pestieau (2005) show that public debt increases steady-state inequality, whereas Pestieau and Thibaut (2012) show that public debt redistributes wealth from the most wealthy to those less well off.

6. As for such the classification of agents' types, Borissov and Lambrecht (2009, p. 99) point out: "In the terminology of Mankiw (2000), this classes might be called savers and spenders." Some empirical studies show that the rich are more likely to be patient than are the poor (e.g., Lawrance, 1991; Harrison et al., 2002).

7. This is satisfied if λ is not so large and (γ, g) takes conventional parameters used in the literature, such as Bräuninger (2005) and Michel et al. (2010). For example, by taking a parameter set $(\gamma, g) = (0.2, 0.2)$, this condition is satisfied under $\lambda < 0.16$.

8. In the region where $K_{t+1}/K_t < 0$, the asset market-clearing condition (6) indicates that D_{t+1} becomes larger than A_t. In this situation, public debt cannot be absorbed by aggregate savings and then no capital is installed in the production sector.

9. Similar to the assumption of $\gamma(1-g) > \lambda$ in Lemma 2, $\bar{\alpha}(1-\gamma)(1+\lambda-g) > \lambda$ is satisfied if λ is not so large and $(\gamma, g, \bar{\alpha})$ takes conventional parameters used in the literature, such as Bräuninger (2005) and Michel et al. (2010). For example, by taking a parameter set $(\gamma, \bar{\alpha}, g) = (0.2, 0.35, 0.2)$, this condition is satisfied under $\lambda < 0.31$.

10. The case of no steady state is realized and fiscal policy is always unsustainable when the public debt finance ratio, λ, is large, as in Bräuninger (2005). We rule out this case because of the same discussion by Bräuninger (2005).

11. $\tilde{\varphi} \equiv \eta^{-1}(\varepsilon(0)) > 0.5$ holds if and only if

$$\varepsilon(0) > \eta(0.5) \Leftrightarrow \beta\left(\mu_1^{-1} + \gamma\right) > -\frac{2(1-\gamma)[\delta\alpha_R - (1-\delta)\alpha_P]}{\alpha_R - \alpha_P}.$$

12. In Appendix E, we consider the local stability at each steady state by using numerical examples.

13. The values of γ, Γ, and g follow the methodology in Bräuninger (2005). $\gamma = 0.2$ and $\Gamma = 12$ represent $r = \gamma\Gamma = 2.4$. If we assume that the time period is about 30 years, $r = 2.4$ implies that the annual interest rate is 4.2%. We employ $\alpha_R = 0.45$ and $\alpha_P = 0.25$ to satisfy the conventional value of $\bar{\alpha} = \delta\alpha_R + (1-\delta)\alpha_P = 0.35$. The value of δ follows the methodology in Galí et al. (2007). The results presented here are robust to other parameter values, δ. We conduct robustness checks in the technical appendix, which is available on request.

14. For instance, France, Germany, Japan, the United States, and the United Kingdom have inheritance or estate taxes.

15. In Bossmann et al. (2007), the government taxes the bequests of both the rich and the poor and redistributes the revenue through a lump-sum transfer to the young generation independently of whether the young belong to the rich or the poor.

16. The value of $\lambda = 0.03$ follows the criterion of the Maastricht Treaty.

17. Dabla-Norris et al. (2015) survey some of the causes of inequality and make a number of findings as follows. First, new technology leads to improvements in productivity but drives up the skill premium (the rate of return to education) and increases the earnings gap between high- and low-skilled workers. Second, trade globalization enabled by technological advances drives income inequality. Furthermore, financial globalization causes the concentration of foreign assets and liabilities in some higher skill- and technology-intensive sectors, leading to inequality. Third, greater flexibility in labor market institutions can pose challenges for workers, especially low-skilled labors, and may explain inequality developments. Finally, the progressivity of tax systems has declined in some advanced economies over recent decades, and the effect of mitigating inequality has weakened.

REFERENCES

Abel, A.B. and M. Warshawsky (1988) Specification of the joy of giving: Insights from altruism. *Review of Economics and Statistics* 70, 145–149.

Agénor, P.-R. and D. Yilmaz (2017) The simple dynamics of public debt with productive public goods. *Macroeconomic Dynamics* 21, 1059–1095.

Alan, S., K. Atalay, and T.F. Crossley (2014) Do the rich save more? Evidence from Canada. *Review of Income and Wealth* 61, 739–758.

Andreoni, J. (1989) Giving with impure altruism: Applications to charity and Ricardian equivalence. *Journal of Political Economy* 97, 1447–1458.

Andreoni, J. (1990) Impure altruism and donations to public goods: A theory of ‘warm glow’ giving. *Economic Journal* 100, 464–477.

Arai, R. (2011) Productive government expenditure and fiscal sustainability. *FinanzArchiv: Public Finance Analysis* 67, 327–351.

Arawatari, R. and T. Ono (2017) Inequality and public debt: A positive analysis. *Review of International Economics* 25, 1155–1173.

Auclert, A. and M. Rognlie (2017) Aggregate demand and the top 1 percent. *American Economic Review* 107, 588–592.

Auclert, A. and M. Rognlie (2018) Inequality and aggregate demand. *NBER Working Paper* No. w24280.

Azzimonti, M., E. de Francisco, and V. Quadrini (2014) Financial globalization, inequality, and the rising public debt. *American Economic Review* 104, 2267–2302.

Barro, R.J. (1990) Government spending in a simple model of endogenous growth. *Journal of Political Economy* 98, 103–125.

Becker, R.A. (1980) On the long-run steady state in a simple dynamic model of equilibrium with heterogeneous households. *Quarterly Journal of Economics* 95, 375–382.

Borissov, K. and S. Lambrecht (2009) Growth and distribution in an AK-model with endogenous impatience. *Economic Theory* 39, 93–112.

Bossmann, M., C. Kleiber, and K. Wälde (2007) Bequests, taxation and the distribution of wealth in a general equilibrium model. *Journal of Public Economics* 91, 1247–1271.

Bozio, A., C. Emmerson, C. O'Dea, and G. Tetlow (2013) Savings and wealth of the lifetime rich: Evidence from the UK and US. *IFS (Institute for Fiscal Studies) Working Paper* W13/30.

Bräuninger, M. (2005) The budget deficit, public debt, and endogenous growth. *Journal of Public Economic Theory* 7, 827–840.

Calderón, C. and L. Servén (2004) The effects of infrastructure development on growth and income distribution. *World Bank Policy Research Working Paper* 3400.

Chalk, N.A. (2000) The sustainability of bond-financed deficits: An overlapping generations approach. *Journal of Monetary Economics* 45, 293–328.

Dabla-Norris, E., K. Kochhar, N. Suphaphiphat, F. Ricka, and E. Tsounta (2015) Causes and consequences of inequality: A global perspective. Staff Discussion Note, SDN/15/13. Washington: International Monetary Fund.

Diamond, P.A. (1965) National debt in a neoclassical growth model. *American Economic Review* 55, 1126–1150.

Dynan, K.E., J. Skinner, and S.P. Zeldes (2004) Do the rich save more? *Journal of Political Economy* 112, 397–444.

European Commission (2016) Report on Public Finances in EMU 2016. INSTITUTIONAL PAPER 045 Luxembourg: Publications Office of the European Union, 2016.

Futagami, K., T. Iwaisako and R. Ohdoi (2008) Debt policy rule, productive government spending, and multiple growth paths. *Macroeconomic Dynamics* 12, 445–462.

Futagami, K., Y. Morita, and A. Shibata (1993) Dynamic analysis of an endogenous growth model with public capital. *Scandinavian Journal of Economics* 95, 607–625.

Gale, W.G. and J.K. Scholz (1994) Intergenerational transfers and the accumulation of wealth. *Journal of Economic Perspectives* 8, 145–160.

Galí, J., J.D. López-Salido, and J. Vallés (2007) Understanding the effects of government spending on consumption. *Journal of the European Economic Association* 5, 227–270.

Galor, O. and O. Moav (2004) From physical to human capital accumulation: Inequality and the process of development. *Review of Economic Studies* 71, 1001–1026.

Ghosh, S. and I.A. Mourmouras (2004) Endogenous growth,welfare, and budgetary regimes. *Journal of Macroeconomics* 26, 623–635.

Greiner, A. (2007) An endogenous growth model with public capital and sustainable government debt. *Japanese Economic Review* 58, 345–361.

Greiner, A. (2011) Economic growth, public debt, and welfare: Comparing three budgetary rules. *German Economic Review* 12, 205–222.

Greiner, A. (2012) Public capital, sustainable debt, and endogenous growth. *Research in Economics* 66, 230–238.

Greiner, A. (2015) Public debt, productive public spending and endogenous growth. *Japanese Economic Review* 66, 520–535.

Greiner, A. and W. Semmler (2000) Endogenous growth, government debt and budgetary regimes. *Journal of Macroeconomics* 22, 363–384.

Harrison, G.W., M.I. Lau, and M.B. Williams (2002) Estimating individual discount rates in Denmark: A field experiment. *American Economic Review* 92, 1606–1617.

Kamiguchi, A. and T. Tamai (2012) Are fiscal sustainability and stable balanced growth equilibrium simultaneously attainable? *Metroeconomica* 63, 443–457.

Kotlikoff, L.J. and L.H. Summers (1981) The role of intergenerational transfers in aggregate capital accumulation. *Journal of Political Economy* 98, 706–732.

Lawrance, E. C. (1991) Poverty and the rate of time preference: Evidence from panel data. *Journal of Political Economy* 99, 54–77.

Maebayashi, N., T. Hori, and K. Futagami (2017) Dynamic analysis of reductions in public debt in an endogenous growth model with public capital. *Macroeconomic Dynamics* 21, 1454–1483.

Mankiw, N.G. (2000) The savers-spenders theory of fiscal policy. *American Economic Review* 90, 120–125.

Michel, P. and P. Pestieau (2005) Fiscal policy with agents differing in altruism and ability. *Economica* 72, 121–135.

Michel, P., L. Von Thadden, and J.-P. Vial (2010) Debt stabilizing fiscal rules. *Journal of Public Economic Theory* 12, 923–941.

Minea, A. and P. Villieu (2012) Persistent deficit, growth, and indeterminacy. *Macroeconomic Dynamics* 16, 267–283.

Minea, A. and P. Villieu (2013) Debt policy rule, productive government spending, and multiple growth paths: A note. *Macroeconomic Dynamics* 17, 947–954.

Morimoto, K., T. Hori, N. Maebayashi, and K. Futagami (2017) Debt policy rules in an open economy. *Journal of Public Economic Theory* 19, 158–177.

Néstor, G. (2017) Do the rich save more in Latin America? *Journal of Economic Inequality* 15, 75–92.

Pestieau, P. and E. Thibaut (2012) Love thy children or money: Reflections on debt neutrality and estate taxation. *Economic Theory* 50, 31–57.

Romer, P.M. (1986) Increasing returns and long-run growth. *Journal of Political Economy* 94, 1002–1037.

Tamai, T. (2014) Public capital, deficit financing, and economic growth in a stochastic economy. *Journal of Macroeconomics* 42, 14–26.

Tamai, T. (2016) Public investment, the rate of return, and optimal fiscal policy in a stochastically growing economy. *Journal of Macroeconomics* 49, 1–17.

Teles, V.K. and C.C. Mussolini (2014) Public debt and the limits of fiscal policy to increase economic growth. *European Economic Review* 66, 1–15.

Yakita, A. (2008) Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting. *Journal of Public Economics* 92, 897–914.