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Application of solitons to the study of laser propagation into a thermonuclear plasma in inertial confinement fusion

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Abstract

In this work, we study the laser propagation in a thermonuclear plasma corresponding to implosion of deuterium-tritium pellets in inertial confinement fusion, by injecting energy provided by high-power laser devices into a quiescent plasma and generating solitons. Having in mind that the electric field inside of plasma can be studied by means of a particular non-linear Schrödinger equation, we solve this equation as an inverse problem, using the Inverse Scattering Transform method, that is a 2×2 eigenvalue problem, known as the AKNS scheme, developed by Ablovitz, Kamp, Newell, and Shabat. We obtain the pseudopotentials q and r if we suppose that the eigenvalue is invariant in time, and is representative of a wave eigenvector, obtaining a solution that has a structure of the soliton type. In the process, one change of variable for space and another for time are applied, and the relation between the pseudopotentials is given by $r = -q^*$. Discretization of the non-linear Schrödinger equation, solved by inverse scattering transform are given by Ablovitz *et al.* (1999). These solitons are generated near the critical layer where $w_0 \cong w_p$, w_0 being the laser frequency and w_p the plasma frequency, exhibit a change in electronic density profile and are caused by the ponderomotive force of laser radiation. The electronic density is a function of the mean square of the electric field. The dispersion relation is representative of an inhomogeneous plasma. Finally, the electric field is obtained as a function of space and time, showing a structure of soliton type.

Keywords: Solitons; Laser; Inertial confinement fusion

1. INTRODUCTION: THE PARTICULAR NONLINEAR SCHRÖDINGER EQUATION

In this work, we study the laser propagation in a thermonuclear plasma corresponding to implosion of deuterium-tritium pellets in the inertial confinement fusion (Chen, 1990; Lindl, 1998). Solitons have been observed in thermonuclear devices and they have been generated by injecting energy of a high power into a quiescent ICF plasma. The data obtained by laser fusion exhibits a change in electronic density profile caused by the ponderomotive force of laser radiation near a critical layer where $w_p \cong w_0$, w_p being the plasma frequency and w_0 the laser frequency. The electronic density is a function of the mean square of the electric field, that is given by means of a particular nonlinear Schrödinger equation (Motz, 1979; Dodd *et al.*, 1982):

$$\frac{\partial^2 E}{\partial^2 y^2} + \alpha^2 E - \frac{w_p^2}{c^2}\left[1 - \left(\frac{w_p^2}{w_0^2}\right)\frac{\epsilon_0 |E|^2}{4KTn_0}\right]E - 2i\,\frac{w_0}{c^2}\,\frac{\partial E}{\partial t} = 0, \quad (1)$$

where E is the electric field that is a function of the position x and time t, α is given by

$$\alpha^2 = \frac{w_0^2}{c^2} - k^2, \quad (2)$$

where c is the light velocity in a vacuum and $k = \xi + i\eta$, the wave vector. The square of the wave vector k^2 is given by

$$k^2 = \frac{w_0^2}{c^2}\left[\left(1 - \frac{w_p^2}{w_0^2 + \nu_{ei}^2}\right) - \left(\frac{w_p^2}{w_0^2 + \nu_{ei}^2}\right)\cdot\frac{\nu_{ei}}{w}\,i\right], \quad (3)$$

where ν_{ei} is the electron–ion collision frequency. Having in mind the expressions (2) and (3), the differential equation (1) is transformed into

$$E_{xx} + aE + bE|E|^2 + ieE_t = 0, \quad (4)$$

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where a, b, and c are given by

$$a = \alpha^2 - \frac{1}{4KTn_0}\frac{w_p^2}{c^2} \tag{5}$$

$$b = \frac{w_p^2}{4KTn_0}\left(\frac{w_p^2}{w_0^2}\right)\epsilon_0 \tag{6}$$

$$e = -2\frac{w}{c^2}. \tag{7}$$

2. SOLUTION OF THE NONLINEAR SCHRÖDINGER EQUATION FOR LASER PROPAGATION IN A THERMONUCLEAR PLASMA BY MEANS OF THE INVERSE SCATTERING TRANSFORM METHOD

In the partial differential Eq. (4) we do the followings changes of variable:

$$t = \left(\frac{e}{b}\right)t' \tag{8}$$

$$x = \left(\frac{1}{\sqrt{b}}\right)x' \tag{9}$$

$$E = q, \tag{10}$$

obtaining

$$q_{t'} + iq_{x'x'} + iq|q|^2 + idq = 0, \tag{11}$$

where

$$d = a/b. \tag{12}$$

This particular nonlinear Schrödinger equation can be solved as an inverse problem, using the inverse scattering transform method, that is a 2×2 eigenvalue problem, known as the AKNS scheme, developed by Ablovitz, Kamp, Newell, and Shabat (Ablovitz & Segur, 1981), that is given by

$$\psi_{1x} = -i\varsigma\psi_1 + q\psi_2 \tag{13}$$

$$\psi_{2x} = i\varsigma\psi_2 + r\psi_1, \tag{14}$$

the linear time dependence being

$$\psi_{1t} = A\psi_1 + B\psi_2 \tag{15}$$

$$\psi_{2t} = C\psi_1 + D\psi_2, \tag{16}$$

where A, B, C, and D are scalar functions independent of ψ_i. If we substitute (13) in (14), we obtain

$$\psi_{2xx} - \frac{r_x}{r}\psi_{2x} - \left(qr - i\varsigma\frac{r_x}{r} - \varsigma^2\right)\psi_2 = 0. \tag{17}$$

We consider that

$$r = -q^* \tag{18}$$

and that

$$\frac{q_x^*}{q^*}$$

is $\ll$ and $\cong 0$, and (17) can be written as

$$\psi_{2xx} + (\varsigma^2 + |q|^2)\psi_2 = 0, \tag{19}$$

and $|q|^2$ can be considered as an eigenfunction of (19). With the condition

$$\lim_{x\to\infty} q = 0, \tag{20}$$

Eq. (19) is transformed into

$$\psi_{2xx} + \varsigma^2\psi_2 = 0. \tag{21}$$

If $r = -1$, Eq. (17) may be reduced to the Schrödinger scattering problem

$$\psi_{2xx} + (\varsigma^2 + q)\psi_2 = 0 \tag{22}$$

and ς^2 is the eigenvalue λ of the operator L,

$$L\psi = \lambda\psi \tag{23}$$

$$\psi_t = M\psi \tag{24}$$

$$L = \partial_{xx}^2 + q, \tag{25}$$

where L is the operator of spectral problem and M is the operator related with a time evolution equation. The conditions for compatibility of Eqs. (23) and (24) are the following:

$$\text{Cross differentiation: } (\psi_{ix})_t = (\psi_{it})_x \tag{26}$$

$$\text{Isospectrality: } \frac{\partial\varsigma}{\partial t} = 0. \tag{27}$$

We finally obtain the following system of coupled differential equations, doing $D = -A$:

$$A_x = qe - rB \tag{28}$$

$$B_x + 2i\varsigma B = q_t - 2aq \tag{29}$$

$$C_x - 2i\varsigma C = r_t + 2Ar. \tag{30}$$

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We suppose that

$$A = \sum_{i=0}^{3} a_i \varsigma^i \tag{31}$$

$$B = \sum_{i=0}^{3} b_i \varsigma^i \tag{32}$$

$$C = \sum_{i=0}^{3} c_i \varsigma^i. \tag{33}$$

Finally we obtain

$$A = a_3\varsigma^3 + a_2\varsigma^2 + \frac{1}{2}(a_3 qr + a_1)\varsigma + \frac{1}{2}a_2 qr - i\frac{a_3}{4}(qr_x - q_x r) + a_0, \tag{34}$$

and other similar equations for B and C, obtaining the evolution equation for q

$$q_t = \frac{i}{4}a_3(q_{xxx} - 6qrq_x) + \frac{1}{2}a_2(q_{xx} - 2q^2 r) - ia_1 q_x - 2a_0 q, \tag{35}$$

and another similar evolution equation for r.

We obtain the pseudopotentials q and r if we suppose that the eigenvalue is invariant in time and is representative of wave eigenvector $k = \xi + i\eta$ and applying the Marchenko's equation (Ghost Roy, 1991).

$$K_1(x,y) + F^*(x,y) \pm \int_x^\infty \int_x^\infty K_1(x,z)F(z+s)F^*(s+y)\,ds\,dz = 0. \tag{36}$$

For $N = 1$, that is to say one eigenvalue,

$$F(x) = -ic\exp(-i\varsigma x). \tag{37}$$

We multiply by $\exp(i\varsigma y)$ both sides of (36) and we operate with

$$\int_x^\infty \exp(i\varsigma x)\,dy.$$

We obtain for $q(x)$ and $K_1(x,y)$ the expressions

$$K_1(x,y) = ic^*\exp[i\varsigma^*(x+y)]\left\{1 - \frac{|c|^2}{(c-c^*)}\exp[2i(\varsigma - \varsigma^*)]\right\}^{-1} \tag{38}$$

$$\begin{aligned} q(x) &= -2K_1(x,x) \\ &= -2i(c/c^*)\eta\exp(-2i\varsigma x)\mathrm{Sech}[2(\eta x - \phi)], \end{aligned} \tag{39}$$

where

$$\frac{|c|^2}{4\eta^2} = \exp(4\phi) \tag{40}$$

$$c = c_0\exp[-2A\_(\varsigma)t]. \tag{41}$$

In accordance with the inverse scattering transform method, the evolution equation for Eq. (11) having in mind Eq. (35) is given by

$$q_t + \frac{1}{2}a_2 q_{x'x'} - iq^2 r + 2a_0 q = 0, \tag{42}$$

where $a_3 = 0$, $a_2 = 2i$, $a_1 = 0$, $a_0 = -id/2$.

Discretizations methods to solve the nonlinear Schrödinger equation that is given by Eq. (11) by the inverse scattering transform method are given by Ablovitz *et al.* (1999).

We consider that the pseudopotentials q and r are related by $r = -q^*/2$, where $(^*)$ is the symbol of conjugate complex and we build the operator that results when $x \to \infty$ and the value of $|q|^2 \to 0$. We obtain

$$A\_(\zeta) = \lim_{x\to\infty} A(\zeta) = 2i\zeta^2 + id \tag{43}$$

$$d = C - Di. \tag{44}$$

The real and imaginary parts of $A\_(\zeta)$ are given by

$$\mathrm{Re}\,A\_(\zeta) = D + 4\xi\eta \tag{45}$$

$$\mathrm{Im}\,A\_(\zeta) = i[2(\zeta^2 - \eta^2) + C] \tag{46}$$

and the solution of partial differential equation (42) is given by

$$q(x',t') = 2\eta\cdot\exp[-2i\xi x]\cdot\exp[2i\,\mathrm{Im}\,A\_(\zeta)t'] \times \exp[-i(\phi_0 + \pi/2)] \tag{47}$$

$$\mathrm{Sech}[2\eta x' + 2\,\mathrm{Re}\,A\_(\zeta)t' - x_0]. \tag{48}$$

We do the variable changes

$$x' = x''\lambda_D \tag{49}$$

$$t' = t''w_p \tag{50}$$

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where λ_D is the Debye length and w_p the plasma frequency of the ICF plasma. Doing $x_0 = 0$ and $\phi_0 = 0$, we obtain

$$|q|^2 = 4\eta^2 \,\mathrm{Sech}^2[2\eta x' + 2\,\mathrm{Re}\,A\_(\zeta)t']. \tag{51}$$

3. RELATION BETWEEN THE ELECTRIC FIELD SQUARE MODULUS AND THE CHANGE IN THE DENSITY PROFILE OF ELECTRONS

The high-frequency motion of the electrons is governed by

$$\frac{\partial u}{\partial t} = -\frac{e}{m_e}E - \frac{3KT_e}{m_e n_0}\frac{\partial n}{\partial x} \tag{52}$$

where u is the electron velocity and n the electronic density. If we do not consider the thermal correction and

$$u = u_0 \exp(-iw_0 t), \tag{53}$$

we obtain

$$|E|^2 \cong \frac{w_0^2 m^2}{e^2}|u|^2. \tag{54}$$

The change in electronic profile can be written as

$$\partial n_e = n_0 \partial n_e' \tag{55}$$

$$\partial n_e' = -\frac{1}{4}|u'|^2 \tag{56}$$

$$u' = \frac{u}{\sqrt{\dfrac{KT_e}{m_e}}}. \tag{57}$$

With Eqs. (53), (54), and (55) we have

$$\partial n_e = -\frac{1}{4}\frac{e^2}{KT_e w^2 m_e}|E|^2 \tag{58}$$

4. RESULTS AND CONCLUSIONS

In the present work, we consider a high-frequency ICF electronic plasma and the input data are: electronic density $n_0 = 10^{30}\,\mathrm{m}^{-3}$, laser frequency $w_0 = 1.884\cdot 10^{15}$ Hz, $KT_e = 10$ keV. With this data input, we obtain $\xi = 6.01738\cdot 10^6$ and $\eta = -5.94521\cdot 10^8$. The square of the absolute value of the electric field can be written as a function of variables $x'' = x'/\lambda_D$ and $t'' = t' w_p$, λ_D being the Debye length of the ICF plasma. We obtain

$$|E|^2 = 1.413829\cdot 10^{18}\cdot \mathrm{Sech}^2[-5.59075x'' + 0.160439t'']. \tag{59}$$

The electronic density scales as $n \propto |E|^2$, causing the generation of solitons as a consequence of ponderomotive force. We can see that $|E|^2$ is a function that has a structure of a

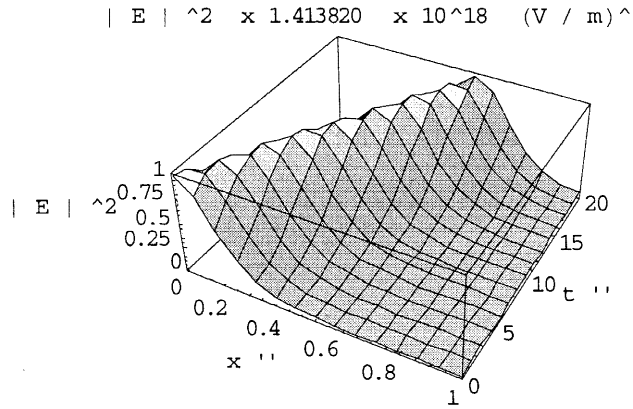

Fig. 1. $|E|^2$ as a function of x'' and t''.

soliton type. Figure 1 shows in tridimensional space a detailed picture of $|E|^2$ as a function of x'' and t''. The selected ranges vary in natural units from 0 to 1 for x'' and from 0 to 1 for t''. These structures have been calculated at $n_0 \cong 10^{30}\,\mathrm{m}^{-3}$ corresponding to implosion states.

The negative change in electronic density profile δn_e has been calculated in the Eq. (57), and we can see that it depends on the square of the modulus of the electric field and has been observed experimentally (Miramar, 1997; Miramar & Alos, 1999).

The representation of $\mathrm{Re}[q[x'', t'']] = \mathrm{Re}[E]$ can be written as

$$\mathrm{Re}[E] = 2\eta\cos[2\xi x' + (2(\xi^2 - \eta^2) - C)t' + \pi/2] \times \mathrm{Sech}[2\eta x' - (8\xi\eta + 2D)t'], \tag{60}$$

and substituting ξ and η by their numerical values we have

$$\mathrm{Re}[E] = 2\eta\cos[0.056586\cdot x'' + 7.92493\cdot t'' + \pi/2] \times \mathrm{Sech}[-5.59075\cdot x'' + 0.160439\cdot t'']. \tag{61}$$

We can see that $\mathrm{Re}[E]$ is a function that has a structure of a modulated soliton type. Figure 2 shows in tridimensional

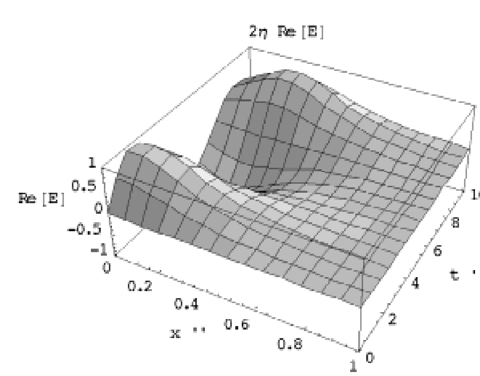

Fig. 2. $\mathrm{Re}[E]$ as a function of x'' and t''.

space a detailed picture of Re$[E]$ as a function of x'' and t''. The selected ranges vary in natural units from 0 to 1 for x'' and from 0 to 10 for t''.

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