ASTEROID MASSES AND DENSITIES

JOACHIM SCHUBART
Astronomisches Rechen-Institut
Heidelberg

Before 1966, when Hertz (1966) published his first direct determination of the mass of Vesta, all our knowledge on asteroid masses was based on estimates. The masses of the first four minor planets resulted from the measured diameters by Barnard (1900) (see the paper by Dollfus in this volume[1]) and from estimated mean densities. The diameters of the smaller objects were derived from their brightness and an estimate of their reflectivity (usually the reflectivity of the Moon was adopted). In 1901, Bauschinger and Neugebauer (1901) derived a value for the total mass of the first 458 asteroids. All the diameters were computed from the brightness with an assumed value for the reflectivity. The diameter of Ceres found in this way is very close to Barnard's (1900) value. The mean density of the 458 asteroids was put equal to that of Earth, and their total mass resulted as 3×10^{-9} solar mass. Stracke (1942) used the same method with an increased material, but the addition of more than 1000 faint asteroids did not bring a significant change in the estimate of the total mass. The report on the McDonald asteroid survey (Kuiper et al., 1958) does not contain another estimate of the total mass of the asteroid ring, but it points to the possibility of a very rapid increase in the number of asteroids with decreasing absolute brightness. If this increase is strong enough, each interval of 1 mag in absolute magnitude can contribute the same amount to the total mass. In the range of magnitudes covered by the Palomar-Leiden survey (PLS) (van Houten et al., 1970), there are no indications for such a strong increase.

The attempts to find gravitational evidence on asteroid masses started with the total mass, but von Brunn (1910) demonstrated that at his time it was not possible to detect gravitational effects caused by the total mass of the asteroids. In the paper mentioned above, Stracke (1942) expressed the hope that accurate orbital theories of the first four minor planets and of Eros can answer the question of the gravitational effects of the total mass, if these theories are compared with the observations of a sufficiently long interval of time.

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[1]See p. 25.

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Actually, it was an orbital theory of 197 Arete that permitted the first gravitational determination for the mass of a single minor planet. Hertz discovered that Arete approaches Vesta within 0.04 AU once every 18 yr. Five such approaches have occurred since the discovery of Arete in 1879. A close commensurability corresponding to the ratio 4:5 of the mean motions of Arete and Vesta allows the repetition of the approaches, which cause the perturbations by Vesta in the mean longitude of Arete to accumulate. Although the total effect of the attraction of Vesta is small and the observations have various qualities, Hertz (1968) succeeded in determining the mass of Vesta from an orbital theory of Arete, which included numerical integrations and differential corrections. Using 72 observations from 28 oppositions of Arete, the mass of Vesta resulted as 1.20×10^{-10} solar mass, the formal mean error being 10 percent of the result. An earlier value based on only 59 observations of Arete was close to this result (Hertz, 1966).

I came in touch with the problems of asteroid masses when I studied the effects caused in the orbits of the first four minor planets by possible errors in the system of planetary masses (Schubart, 1970*b*). With the aid of numerical tests, I found that the members of the pairs Ceres-Pallas and Ceres-Vesta cause observable gravitational effects in the mean longitude of the respective other member if the whole span covered with observations is considered. Especially the mass of the largest body, Ceres, may not be neglected in an accurate theory of all the observations of Pallas or Vesta. The mass of Ceres can result from such a theory if it is introduced as an additional unknown in a differential correction. The tests showed that a theory of Pallas gives the best chance to determine the mass of Ceres. The reason for the observable interaction between Ceres and Pallas is given by the ratio of their mean motions, which is close to 1:1. As in the case of Vesta and Arete, the repetition of equal configurations causes an accumulation of the perturbations. Gauss realized this when he had obtained the first reliable orbital elements of Ceres and Pallas in 1802, and he thought of the possibility to determine the masses of the two planets from the accumulated effects after a sufficiently long interval of time (von Zach, 1874).

In 1970, I started with a first attempt to derive the mass of Ceres from observations of Pallas (Schubart, 1970*a*). I used an *N*-body program (Schubart and Stumpff, 1966) for the numerical integration of the orbit of Pallas. The computations started with Duncombe's (1969) elements of Ceres and Pallas. G. Struve (1911) published a list of 63 normal positions of Pallas, obtained from the same number of oppositions in the interval from 1803 to 1910. I selected 47 of these positions for my work, but I have to mention that this material is very inhomogeneous because Struve took a part of the places from much older sources without change. Combining the 47 positions with 27 positions of Pallas from 13 oppositions, 1927-68, I obtained the value 6.7×10^{-10} solar mass for the mass of Ceres in a differential correction (Schubart, 1970*a, b*). The formal mean error was less than 10 percent of the result, but this does not account for possible systematic effects. Tests showed

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that the uncertainties in the masses of Mars and Jupiter will not affect the result, and I do not believe that impacts have caused an observable effect in the motion of the planets under consideration. A real source of error is given by the systematic errors in the right ascensions of the reference stars used during various periods. Fortunately, the effects caused by Ceres in the observations of Pallas are much larger than these errors, but corrections are necessary in a more accurate theory.

To find an indication for the sign of a possible correction to my first value for the mass of Ceres, I started to explore the independent way of determining the mass from the observations of Vesta. Leveau (1896, 1910) derived a reliable and homogeneous set of 252 normal positions from 68 oppositions of Vesta for the interval from 1807 to 1904. He applied systematic corrections to the observations as far as he knew them. The positions appeared in two parts together with his theory of Vesta. From this material, I took a comparatively small selection of 70 places from 17 oppositions, 1807-1903/04. Combining this with 48 places of Vesta from 13 oppositions, 1923-68, I used the same method as before and obtained the smaller value of 5.1×10^{-10} solar mass for the mass of Ceres. A small mean error resulted again, but this is due to the large number of observations and to their small scatter. Because Vesta is less sensitive to changes in the mass of Ceres (Schubart, 1970*b*), the result derived from it should have a lower weight in comparison with that from Pallas. Systematic errors can affect the result from Vesta in a stronger way.

Quite recently, I examined the first observations of Pallas in Struve's (1911) list more closely. They are taken from a paper by Gauss that was unpublished at the time of his death, but a more original and accurate form of these positions is given elsewhere in his publications. Making use of these original positions and dropping one with a large residual, I obtained a decrease in my former result for the mass of Ceres by about 4 percent. Because the result from Vesta indicates that even this new value may be too large, I propose to adopt

$$(6.0 \pm 0.7) \times 10^{-10} \text{ solar mass}$$

as the result for the mass of Ceres, until a more reliable value becomes available. The mean error proposed here is an estimate.

The next important problem is the direct determination of a value for the mass of Pallas. This mass is probably much smaller than that of Ceres because the volumes of Pallas and Ceres are approximately in a ratio of 1:4 according to the measured diameters (Barnard, 1900). On principle, a treatment of the observations of Ceres should give a result for the mass of Pallas, but this will be subject to a large uncertainty and also will depend on the adoption of a value for the mass of Vesta (Schubart, 1970*b*). At the moment, I adopt 1/4 of the mass of Ceres as an estimate for the mass of Pallas.

In the case of Juno and all the asteroids discovered after Vesta, the method of estimating reflectivity and mean density is still the best one for a mass

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determination. According to their absolute magnitude g these bodies are all comparatively small (Kuiper et al., 1958). An estimation of the total mass of these bodies is especially interesting. To demonstrate a simple method for this, I assume that all these minor planets have the same reflectivity and mean density as Ceres. It is easy to correct the result for other mean values of reflectivity and density. In table I, N is the number of objects in half-magnitude intervals of g. Each interval is characterized by the mean value of g. The values of N are taken from Kuiper et al. (1958, table 15), but additions were made to account for the members of the Hilda and Trojan groups. N_1 is the number of objects with absolute magnitude g that would have a total mass equal to that of Ceres. N_1 results from

$$\log N_1 = 0.6(g - 4)$$

Therefore, N/N_1 is the mass contribution of each half-magnitude interval in units of the mass of Ceres. In this unit, the total mass of the asteroids with $6.0 < g < 10.5$ results as 0.74 from table I. The PLS (van Houten et al., 1970, fig. 6) allows an extension of table I to fainter asteroids, but their mass contribution is small. The intervals $10.5 < g < 13.5$ and $13.5 < g < 16.5$ contribute only 0.06 and 0.01, respectively, in the above unit. This gives the estimate of 0.8 mass of Ceres for the total mass of the objects considered. A lower value results if the average reflectivity is higher or if the mean density is lower than that of Ceres. If the sum of the masses of Pallas and Vesta is put equal to 0.45 mass of Ceres, the mass of Ceres results as nearly equal, or possibly even equal to the mass of the remaining minor planets with $g < 16.5$. According to my result, two masses of Ceres correspond to 1.2×10^{-9} solar mass, or to 2.4×10^{24} g. This value is lower than the early estimates of the total mass mentioned above. It is not very far from some of the more recent estimates (Allen, 1963; Anders, 1964, pp. 693-694; Dohnanyi, 1968, 1969).

The directly determined masses of Ceres and Vesta in combination with the measured diameters allow an attempt to derive the mean densities. Barnard's

TABLE I.—*Mass Contribution of Half-Magnitude Intervals in Absolute Magnitude* g

g	N	$\log N_1$	Mass (Ceres = 1)
6.25	3	1.35	0.134
6.75	3	1.65	.067
7.25	11	1.95	.124
7.75	14	2.25	.079
8.25	32	2.55	.090
8.75	61	2.85	.086
9.25	90	3.15	.063
9.75	150	3.45	.053
10.25	240	3.75	.043

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(1900) diameter of Ceres is 768 km, so that a mass of 1.2×10^{24} g leads to a mean density of 5 g-cm^{-3} for Ceres. Dollfus (1970) estimates the error of the diameter as about ±6 percent, so that the value of the density is uncertain by about 20 percent according to this. The uncertainty coming from the mass is only about 10 percent. The mean density of Vesta came out much larger at first (Hertz, 1968), but the value was based on Barnard's diameter, which is probably too small. A recent measurement published by Dollfus (1970) makes the volume of Vesta equal to 1/5 the volume of Ceres. Because the masses are in the same ratio, this measurement points to the same densities for Vesta and Ceres. However, the relative uncertainty in the measured diameter of Vesta is comparatively large.

T. Gehrels directed my attention to the way of getting a lower limit for the mean density of a rotating asteroid (Gehrels et al., 1970; Gehrels, 1970). If the period of rotation is less than a critical value depending on the density, the planet will be rotationally unstable and tend to break up. This was, for instance, mentioned by Kuiper (1950). Asteroid 1566 Icarus has the shortest known period of rotation (Gehrels, 1970), and this might require a density greater than 3 g-cm^{-3}, but further considerations are necessary because cohesive forces can probably not be neglected.

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DISCUSSION

VEVERKA: You referred to the mean density of Icarus as being 3.0 g/cm^3. I believe this number is taken from a paper by Gehrels, Roemer, Taylor, and Zellner (1970) and is only a plausible guess to which undue physical importance should not be attached.

BRATENAHL: Is there a search for other close encounters besides 197 Arete and Vesta?

SCHUBART: I do not know. It is possible that Hertz made such a search when he discovered the case of Arete and Vesta.

HERGET: Yes. One must find a close approach to one of the more massive minor planets, and there just is none such, less than several million miles, amongst the known objects.

RABE: How large (approximately) are the longitude perturbations produced by the mutual actions of Ceres and Pallas?

SCHUBART: I found residuals of 40 arcsec between some of the early observations of Pallas and a computation based on modern orbital elements when I neglected the mass of Ceres. (See my earlier paper, Schubart, 1970.) I expect the effects in the longitude of Ceres to be comparatively small; but Dollfus mentioned in his paper given here[2] that the diameter of Pallas measured by Barnard may be too small. Therefore, my estimate of the mass of Pallas can be too low. If this is so, it will not be so difficult to determine the mass of Pallas from the observations of Ceres.

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[2]See p. 25.

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SCHUBART (in reply to a question by Roosen): My estimate of the total mass of the asteroids refers to the observable objects. The mass contribution of the unobservable small asteroids with a diameter of less than 1 km is unknown.

KIANG: I may point out, that many decades ago attempts were made by Harzer to determine the total mass of the asteroid ring from gravitational effects using the perturbations on the orbit of Mars. A rather large, but extremely uncertain value of about one-tenth the mass of Earth was obtained.

SCHUBART: Harzer made his determination before the effects of relativity became known. It is, therefore, not based on real effects due to the asteroids; compare also von Brunn's (1910) work. We should not use gravitational determinations of the total mass, unless they are confirmed with modern computing techniques.

DISCUSSION REFERENCES

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