Macroeconomic Dynamics, **17**, 2013, 621–645. Printed in the United States of America.
doi:10.1017/S136510051100040X

INTANGIBLE CAPITAL AND INTERNATIONAL INCOME DIFFERENCES

AAMIR RAFIQUE HASHMI
National University of Singapore

I add intangible capital to a variant of the neoclassical growth model that already features physical and human capital, and study the implications for international income differences. I calibrate the parameters associated with intangible capital by using new estimates of investment in intangibles by Corrado et al. [*Review of Income and Wealth* 55, 661–685 (2009)] and depreciation rates by Corrado and Hulten [*American Economic Review* 100, 99–104 (2010)]. I find that for a given efficiency difference between rich and poor countries, the model with intangible capital can explain more than double the income differences of the model without. Put another way, in the benchmark case, differences in intangible capital account for 14.3% of the observed income differences. I also examine the role played by intangible capital in versions of the model with barriers to accumulation. In all the variants that I consider, differences in intangible capital account for 10% to 22% of the observed income differences.

Keywords: International Income Differences, Barriers to Accumulation, Intangible Capital, Intermediate Goods

1. INTRODUCTION

Some intermediate goods continue to be useful in production after the first period of their use. Examples include computer software, output of research and development (R&D) activity, advertisement, management's time spent on promoting a business, and expenditure on training of workers and managers.[1] Because most of the intermediate goods in question are intangible and their benefits extend beyond the first periods of their use, I shall call the expenditure on these goods *investment in intangibles* and the accumulated value of this investment, after appropriate depreciation, *intangible capital*.[2] Expenditure on these intermediate goods ought

This paper is a revised version of the third chapter of my Ph.D. dissertation, written at the University of Toronto. I thank Diego Restuccia for his guidance and Michelle Alexopoulus, Rolando Avendano, Matthias Doepke, Igor Livshits, Robert Lucas, Fidel Perez-Sebastian, Guillaume Rocheteau, Richard Rogerson, Johannes Van Biesebroeck, Xiaodong Zhu, two anonymous referees, and an associate editor for very helpful comments. I also thank participants at the 2007 ACE International Conference (Hong Kong, China), the 2008 AEA Annual Meetings (New Orleans, LA, USA), the 2008 Southern Workshop in Macroeconomics (Auckland, New Zealand), the 2008 Canadian Economic Association Annual Meetings (Vancouver, BC, Canada), and the 2008 European Economic Association Annual Meetings (Milan, Italy) for comments. Address correspondence to: Aamir Rafique Hashmi, Department of Economics, National University of Singapore, 1 Arts Link, AS2/05-08, Singapore 117570; e-mail: aamir@nus.edu.sg.

to be treated as investment because "*any* use of resources that reduces current consumption in order to increase it in the future, qualifies as an investment" [Corrado et al. (2009, p. 66)]. However, in the National Income and Product Accounts (NIPA) of the United States, expenditure on these goods is treated as expenditure on intermediate goods and hence not included in gross domestic product (GDP).[3] This is a measurement error.

In this paper, I study the implications of correcting this measurement error for international income differences. I correct the error by treating the expenditure on the intermediate goods discussed as investment in intangible capital. I then write a one-sector neoclassical growth model that is very similar to the models in Mankiw et al. (1992) and Chari et al. (1996), except that it also includes intangible capital. In this respect the model is similar to the one in Parente and Prescott (1994). I then ask: how much more of the international income variation can the model explain when it is augmented with intangible capital?

The answer to this question depends on values of the parameters associated with intangible capital. To calibrate these parameters one needs, among other targets, an estimate of the size of intangible investment relative to GDP. Until recently, no credible estimate of this investment was available. Earlier studies by Parente and Prescott (1994) and Prescott (1998) implied that the size of this investment was around 40% and 32% of the GDP. In a couple of recent papers, Corrado et al. (2009) (from here on CHS) and Corrado and Hulten (2010) provide estimates of intangible investment in the U.S. economy for the postwar period. To my knowledge, they are the first to provide careful estimates of intangible investment and depreciation of intangible assets at the macro level.

The main contribution of the present study is to use the new estimates of intangible investment and show that when intangible capital is added to a standard one-sector neoclassical growth model, the multiplier effect of efficiency differences between rich and the poor countries is much higher. This conclusion is robust to the inclusion of barriers to accumulation of various types of capital in the model.

The paper contributes to the vast and still expanding literature on international income differences. McGrattan and Schmitz (1999) and Caselli (2005) are two excellent surveys of this literature. The novelty of the present paper is in considering intangible capital in the production function. Although this has been done before by Parente and Prescott (1994) and Prescott (1998), when those papers were written, credible estimates of investment in intangibles and depreciation of intangible assets were not available. The present paper uses new estimates of the size and depreciation of these assets and hence offers a more accurate and updated account of the role of intangibles in explaining international income differences.

Within the literature on international income differences, this paper is related to the subliterature that tries to answer the question of how much of international income variation is due to factor accumulation and how much to differences in efficiency (or total factor productivity, TFP). Important papers in this strand of literature include Mankiw et al. (1992), Parente and Prescott (1994), Chari et al. (1996), Klenow and Rodriguez-Clare (1997), Prescott (1998), Hall and Jones (1999), Kumar and Russell (2002), and Henderson and Russell (2005). The literature is still growing: recent contributions include Hulten and Isaksson (2007), Jerzmanowski (2007), Manuelli and Seshadri (2007), Erosa et al. (2010), Beaudry and Francois (2010), Boss et al. (2010), Hsieh and Klewnow (2010), and Jones (2011). This paper differs from earlier studies in that it takes into account all major sources of factor accumulation. The previous studies focused either on physical and human capital [e.g., Mankiw et al. (1992), Chari et al. (1996), Klenow and Rodriguez-Clare (1997), Manuelli and Seshadri (2007), and Erosa et al. (2010)] or on physical and intangible capital [e.g., Parente and Prescott (1994) and Prescott (1998)]. By taking all three types of capital into account, the present paper shows that a simple one-sector neoclassical growth model can explain a lot more variation in international incomes than previously thought. It also shows that differences in intangible capital contribute significantly in explaining the observed income differences.

The paper is also related to the literature on barriers to accumulation and technology adoption. Notable papers in this literature include Parente and Prescott (1994, 1999), Hall and Jones (1999), Caselli and Coleman (2001), Eaton and Kortum (2001), Restuccia and Urrutia (2001), Comin and Hobijn (2004), Ngai (2004), Aghion et al. (2005), Klenow and Rodriguez-Clare (2005), and Acemoglu et al. (2006). I add to this literature by quantifying the contribution of various barriers and showing that although barriers have the potential to explain a large fraction of the observed income differences, taking intangible capital into account still makes a difference.

Another important strand of literature argues that poor countries adopt inferior technologies because, conditional on their factor endowments, it is optimal to do so. Influetial papers in this literature include Zeira (1998), Acemoglu and Zilibotti (2001), and Kosempel (2004). More recently, Jerzmanowski (2007) and Sadik (2008) found empirical support for both type of theories, those based on barriers to accumulation and those based on the efficient technology hypothesis. In this paper, I do not explore the efficient technology hypothesis, but leave it for future research. I comment more on this in the Concluding Remarks.

The paper also makes contact with a recent literature that finds intangible capital to be important for our understanding of diverse phenomena such as investment dynamics [McGrattan and Prescott (2010)], effects of openness on development [McGrattan and Prescott (2009)], productivity [Bontempi and Mairesse (2008)], and growth [Corrado et al. (2009), Corrado and Hulten (2010), and Hulten (2010)]. The common theme is that intangible capital is important and has become more so over time.

However, this paper is inspired by and most closely related to Parente and Prescott (1994) (PP from here on). They were pioneers in introducing the idea of unmeasured investments that lead to intangible capital. They were also pioneers in showing that barriers to the accumulation of intangible capital can lead to sizable income differences across countries. The present paper, however, differs from PP in two important respects. First, the model in this paper includes human capital. This has important implications, as we shall see later. Second, this paper uses a different calibration strategy to pin down the parameters related to intangible capital. Whereas PP use the postwar experience of Japan to pin down the share parameter on intangible capital, the present paper uses direct estimates of investment in intangibles, as reported by Corrado and Hulten (2010). In PP the depreciation of intangible capital is determined by growth in world knowledge. In the present paper, I use estimates of depreciation of intangible capital as reported by Corrado and Hulten (2010). The inclusion of human capital and a different calibration strategy lead to different conclusions. The main conclusion in PP is that barriers to technology adoption (i.e., to accumulation of intangible capital) are important and small differences in these barriers can explain most of the cross-country income variation. The same conclusion does not hold in the present paper because the calibrated share of intangible capital is much smaller (13% compared to 50–55% in PP). However, the present paper confirms the general finding in PP that the one-sector neoclassical growth model can explain much greater income differences when it is augmented to include intangible capital.

This paper is also in the spirit of Jones (2011). He formalizes the idea that linkages between firms through intermediate goods generate a substantial multiplier effect that magnifies distortions in allocation of resources. In this paper, I also look at intermediate goods, but focus on those that can be classified as intangible capital. Hence my focus is on the capital accumulation multiplier when capital includes intangible assets.

The goal of this paper is to extend and recalibrate the models in PP and Prescott (1998). Hence I confine myself to a one-sector neoclassical growth model and explore different variations of it to examine the role played by intangible capital in explaining international income differences. I also confine myself to a steady-state analysis and leave the transitional dynamics implied by this model to future work. I do comment on transitional dynamics in Section 2.3.

The rest of the paper is organized as follows. In Section 2, I present the basic model, calibrate its parameters, and study its predictions about international income differences. In Section 3, I add barriers to accumulation of various types of capital to the baseline model and study the implications for income differences. In Section 4, I examine the sensitivity of my conclusions to changes in various parameters and targets. I conclude in Section 5.

2. THE MODEL

Consider a one-sector neoclassical growth model with three types of capital: physical (K), intangible (Z), and human (H).[4] Time is discrete. The aggregate production function is given by

$$Y_t = A_t K_t^{\theta_k} Z_t^{\theta_z} [(1 - u_t) H_t]^{\theta_h} L_t^{1-\theta_k-\theta_z-\theta_h}, \qquad (1)$$

https://doi.org/10.1017/S136510051100040X Published online by Cambridge University Press

where Y_t is output, A_t is total factor productivity (TFP), $1 - u_t$ is the fraction of human capital used in production, and L_t is raw labor. I assume that TFP grows exogenously at rate γ and all per capita variables grow in the steady state at rate g, which is defined as

$$g = (1+\gamma)^{\frac{1}{1-\theta_k-\theta_z-\theta_h}} - 1. \tag{2}$$

From this point on, I shall focus on quantities that are stationary in the steady state. Let $y_t \equiv Y_t/[(1+g)(1+n)]^t$, where n is the population growth rate. Let k_t, z_t, and h_t be defined in the same manner. Let $a \equiv A_t/(1+\gamma)^t$. With these new variables, the production function becomes

$$y_t = a k_t^{\theta_k} z_t^{\theta_z} [(1-u_t)h_t]^{\theta_h}. \tag{3}$$

I next specify the laws of motion for the three state variables: k, z, and h. The law of motion for physical capital is standard and given by

$$(1+g)(1+n)k_{t+1} = (1-\delta_k)k_t + x_{kt}, \tag{4}$$

where δ_k is the depreciation rate and x_k is investment in physical capital.

There are two popular approaches to modeling the accumulation of human capital. According to the first approach, human capital accumulation requires investment in the form of final goods [see, for example, Mankiw et al. (1992) and Chari et al. (1996)]. According to the second approach, human capital accumulation is time-intensive and hence a fraction of human capital has to be taken out of production and devoted to the accumulation of human capital. Examples of this approach include Lucas (1988) and Prescott (1998). I combine the two approaches and assume that the accumulation of human capital requires final goods as well as time.[5] The motivation for this human capital technology is that the quality of education, as measured by x_h (goods invested in the production of human capital), is important vis-à-vis the quantity of education, as measured by u (years of schooling). With this technology, the law of motion for human capital is

$$(1+g)(1+n)h_{t+1} = (1-\delta_h)h_t + (u_t h_t)^{\psi} x_{ht}^{\phi}, \tag{5}$$

where δ_h is the depreciation rate, u_t is the fraction of human capital devoted to the production of human capital, and x_{ht} is investment in the form of final goods in accumulation of human capital.

The law of motion for intangible capital is similar to the one for physical capital and is given by

$$(1+g)(1+n)z_{t+1} = (1-\delta_z)z_t + x_{zt}, \tag{6}$$

where δ_z is the depreciation rate and x_z is the investment in intangible capital. To ensure overall decreasing returns to accumulable factors, I impose the following restriction on parameters of the model:

$$\frac{1-\theta_k-\theta_z}{\theta_h} > \frac{\phi}{1-\psi}, \tag{7}$$

which simplifies to $\theta_k + \theta_h + \theta_z < 1$, if $\phi + \psi = 1$.[6]

https://doi.org/10.1017/S136510051100040X Published online by Cambridge University Press

It is important to note that with the addition of intangible capital, y is no longer the measured output, as in the National Income and Product Accounts (NIPA). Instead, it also includes investment in intangible capital. In symbols,

$$y = y_m + x_z, \tag{8}$$

where y_m is the measured output (as in NIPA) and x_z is the investment in intangible capital. I call y the *total output* and y_m the *measured output*.

The total output can be used either for consumption or for investment in physical, intangible, or human capital. Hence the aggregate resource constraint is

$$c_t = y_t - x_{kt} - x_{ht} - x_{zt}. \tag{9}$$

The social planner chooses the sequence $\{c_t, k_{t+1}, z_{t+1}, h_{t+1}, u_t\}_{t=0}^{\infty}$, given k_0, z_0, and h_0, to maximize the present discounted value of utility, $u(c_t)$. More specifically, the planner's problem is

$$\max_{\{(c_t,k_{t+1},z_{t+1},h_{t+1},u_t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{10}$$

subject to (3), (4), (5), (6), and (9). I assume CRRA preferences and define the period utility function as

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \tag{11}$$

where σ is the inverse of the intertemporal elasticity of substitution.[7]

In the rest of the paper I focus on the steady state equilibrium of the model. The steady state equilibrium is a set of allocations $\{c, k, z, h, u\}$ such that, given the constraints, utility is maximized and the steady state variants of (3), (4), (5), (6), and (9) are satisfied.

When the model is solved for its steady state, the steady state level of output is

$$y = ba^{\xi}, \tag{12}$$

where b is a constant whose value depends on parameters (other than a) of the model and ξ (the TFP elasticity of output) is given by

$$\xi = \frac{1-\psi}{(1-\psi)(1-\theta_k-\theta_z)-\phi\theta_h}. \tag{13}$$

As is well known, the steady state solution implies that if we add another type of capital to the model, the elasticity of output with respect to capital will increase. The contribution of the present paper is to quantify this effect and decompose it to highlight the individual contributions of various types of capital in explaining international income differences. For international income comparisons I assume that all the countries are in the steady state. I further assume that technology and preferences are the same across countries and the only thing that differs is the TFP.

Hence b is the same across countries and output of country i relative to that of country j is

$$\frac{y_i}{y_j} = \left(\frac{a_i}{a_j}\right)^{\xi}. \tag{14}$$

In international income comparisons, ξ is the key parameter. For a given difference in the TFPs, a higher value of ξ would generate greater income differences. It is clear from (13) that if θ_z is positive, i.e., intangible capital is considered a factor of production together with physical and human capital, the value of ξ will be larger than when $\theta_z = 0$. What is not clear is the magnitude of change in ξ due to the addition of intangible capital to the model. To quantify this effect we need to calibrate the parameters of the model. This is what I do in the next section.

Before proceeding to calibration, it is instructive to introduce a simple decomposition of the right-hand side of (14). In each country, in the steady state, the values of k, h, and z are constant and depend on a and other parameters of the model. Using this fact and (3), we can rewrite (14) as

$$\frac{y_i}{y_j} = \left(\frac{a_i}{a_j}\right)\left[\frac{k_i(a_i)}{k_j(a_j)}\right]^{\theta_k}\left[\frac{z_i(a_i)}{z_j(a_j)}\right]^{\theta_z}\left[\frac{h_i(a_i)}{h_j(a_j)}\right]^{\theta_h}. \tag{15}$$

Note that u does not appear in (15). This is because it depends on other parameters of the model than a and those parameters are the same across countries. If we substitute the steady-state values of k, h, and z into (15), we get

$$\frac{y_i}{y_j} = \left(\frac{a_i}{a_j}\right)\left(\frac{a_i}{a_j}\right)^{\theta_k\xi}\left(\frac{a_i}{a_j}\right)^{\theta_z\xi}\left(\frac{a_i}{a_j}\right)^{\theta_h\xi\phi/(1-\psi)}.$$

Taking logs and dividing by $\log\left(y_i/y_j\right)/100$ gives

$$100 = \underbrace{B}_{\text{due to } TFP} + \underbrace{\theta_k\xi B}_{\text{due to } K} + \underbrace{\theta_z\xi B}_{\text{due to } Z} + \underbrace{\theta_h\xi\phi B/(1-\psi)}_{\text{due to } H}, \tag{16}$$

where $B = 100 \times \log(a_i/a_j)/\log(y_i/y_j)$. This decomposition is useful for at least two reasons. First, it breaks down the income differences into various components, and one can clearly see the percentage share of each component in explaining income differences. Second, it makes the results in this paper directly comparable to those of the other papers in the literature that use similar decompositions.

2.1. Calibration

I calibrate the parameters of the model such that the steady state of the model is consistent with certain long-run features (targets) of the U.S. economy. I report the targets and the calibrated parameters in Table 1 and provide details of the calibration strategy in the Appendix. Of the 12 parameters reported in Table 1, 8 remain unchanged whether we include intangible capital in the model (i.e., assume that $\theta_z > 0 \Longrightarrow x_z > 0$) or exclude it (i.e., assume that $\theta_z = 0 \Longrightarrow x_z = 0$).

TABLE 1. Calibrated parameter values and targets

Parameter	Parameter value: Model w/o Z	Parameter value: Full model	Target	Target value
g	0.02	0.02	Growth in p.c. consumption	2.34%
n	0.01	0.01	Growth in population	1.17%
β	0.94	0.94	Implied discount rate	5%
σ	2.00	2.00	Empirical literature	—
δ_k	0.05	0.05	Capital–output ratio	2.46
δ_h	0.05	0.05	See text	—
θ_k	0.36	0.32	x_k/y_m	0.20
θ_h	0.29	0.25	x_h/y_m	0.127
ψ	0.37	0.37	u	0.21
ϕ	0.63	0.63	$1-\psi$	—
δ_z	—	0.22	Estimates in Corrado and Hulten (2010)	—
θ_z	—	0.14	x_z/y_m	0.128
ξ	2.82	3.51		

These parameters are g, n, β, σ, δ_k, δ_h, ϕ, and ψ. Two parameters take different values depending on whether we include intangible capital in the model or not: θ_k and θ_h. When I discuss calibration of these parameters, I also explain why they are different in the two versions of the model. The remaining two parameters (δ_z and θ_z) are specific to the model with intangible capital.

According to Heston et al. (2006), from 1950 to 2004, the average population growth rate in the United States has been 1.17% and the per capita consumption growth rate has been 2.34%. Hence I set $n = 0.0117$ and $g = 0.0234$. I choose β so that the implicit subjective discount rate is 5%. I choose σ to be equal to 2. This is at the lower end of the range of values used in the literature.[8]

I choose θ_k such that the steady state ratio of investment in physical capital (x_k) to measured output (y_m) is 0.2. I denote this ratio by ι_k (see (A.1) in the Appendix). It is important to note that θ_k is the share of capital in total output (y). If we exclude intangible capital from the model (i.e., assume that $\theta_z = 0 \Longrightarrow x_z = 0$), then $y = y_m$ and θ_k is equal to 0.36. This is a familiar number in the literature for the share of capital in output. However, when intangible capital is included (i.e., $\theta_z > 0 \Longrightarrow x_z > 0$), $y = x_z + y_m$ and $\theta_k = 0.36/(1 + \iota_z) = 0.32$, where $\iota_z = x_z/y_m$. Hence, when we compare the share of capital in total output in the models with and without intangible capital, the share is smaller in the former because the total output is greater. And the total output is greater because it includes investment in intangible capital.

I pick δ_k such that the steady state capital–output ratio is 2.46. This target capital–output ratio is based on the estimates of per capita U.S. capital stock and real output as reported in Caselli and Coleman (2006, Table A.1, p. 519).

I set δ_h (the depreciation rate of human capital) equal to δ_k. This is a standard practice in the literature [for example, see Mankiw et al. (1992) and Chari et al. (1996)]. I shall say more about this parameter in Section 4.

The value of ψ depends on the steady state value of u, i.e., the fraction of productive time spent accumulating human capital (see (A.2) in the Appendix). I assume this fraction to be equal to the ratio of average years of schooling to the sum of average years of schooling and the time spent in the labor force. The average years of schooling in the United States in 2000 was 12.25 [Barro and Lee (2000)]. I assume that the time spent in the labor force is 47 years (i.e., $65 - 18$). This gives $u = 0.21$.

I assume $\phi = 1 - \psi$. This restriction is required to ensure that the steady state growth rate of human capital is the same as that of output, physical capital, and intangible capital. This restriction is desirable, as it ensures, among other things, that the ratio of human capital to physical capital is constant in the steady state. In the absence of the restriction, this ratio would approach either infinity or zero, both of which are unreasonable outcomes. I choose θ_h to match investment in human capital as a fraction of GDP. According to Haveman and Wolfe (1995), this fraction is 12.7%.[9] Hence I set $\iota_h = 0.127$.

When we compare parameter θ_h for the two versions of the model, we see that θ_h is smaller in the model with intangible capital. The reason is the same as for θ_k. The share of human capital in total output is smaller in the model with intangible capital because the total output is greater. And the total output is greater because it includes investment in intangible capital.

There are two parameters related to intangible capital: δ_z and θ_z. The first, δ_z, is the depreciation rate of intangible capital. Earlier studies, including CHS, assumed a very high (around 30%) depreciation rate for intangible capital.[10] However, Corrado and Hulten (2010) provide an updated estimate of 21.5%. I use this updated estimate and later check the sensitivity of my results to changes in this parameter.[11]

The other parameter, θ_z, is identified by choosing a target for investment in intangible capital as a fraction of the measured output (see (A.4) in the Appendix). I denote this fraction by ι_z. Here I closely follow CHS. Their definition of investment is based on the idea that "*any* use of resources that reduces current consumption in order to increase it in the future qualifies as an investment." They distinguish between tangible and intangible investments. In the tangible category they include the usual investments in structures, tools, and machinery. For intangibles, they identify three main categories of investment. The first category is *computerized investment* and consists mainly of computer software. The second category is *innovative property*, which is divided into two subcategories. The first subcategory is *scientific R&D* and consists of the National Science Foundation's industrial R&D series. The second subcategory is *nonscientific R&D*,

which includes revenues of the nonscientific commercial R&D industry, spending for new product development by financial services and insurance firms, and the cost of development of new products by the entertainment industry. The third category is *economic competencies*. This is also divided into two subcategories. The first subcategory is *brand equity* and consists of a fraction of advertisement expenditure. The second subcategory is *firm-specific resources* and includes a fraction of the cost of employer-provided worker training and management time devoted to enhancing the productivity of the firm.[12] According to the estimates in CHS, average investment in intangibles was 15.7% of the measured output during the period from 2000 to 2003. Corrado and Hulten (2010) update this number to 12.8% for the period 1995–2007.[13] I use this latter number as my target for intangible investment. This gives θ_z equal to 0.14.

2.2. International Income Differences

The implications of the model for international income differences depend on the value of ξ. I first consider the special case of the model in which $\theta_z = 0$, which implies that $x_z = 0$. When the model is calibrated for this special case, the value of ξ is 2.82. This implies that in order to explain a 40-fold difference in output between the rich and the poor countries, TFP in the former must be 3.70 times higher ($3.70^{2.82} = 40$).[14] In other words, in the absence of intangible capital, the model can magnify a TFP ratio of 3.70 to an output ratio of 40. Here it is instructive to compare these results with those of some earlier studies. The parameter estimates in Mankiw et al. (1992) imply a value of ξ equal to 2.44. The calibration in Erosa et al. (2010) implies a value of ξ equal to 2.77. Hence my results are very close to those of Erosa et al. (2010). However, it is not the absolute value of ξ that is important for the question of interest. What is important is the increase in the value of ξ once intangible capital is added to the model.

Next I calibrate the full model using the target values of x_z/y_m and δ_z as reported in Table 1. For the full model the value of ξ is 3.51.[15] With this value of ξ we need a TFP ratio of 2.86 to explain a 40-fold difference in output ($2.86^{3.51} = 40$). Recall that this ratio was 3.70 in the model without intangible capital. One objection to this comparison could be that the TFP is just a residual. It is not clear how big the improvement is if this residual drops from 3.70 to 2.86. To respond to this objection, I compute the income difference generated by the full model using the TFP ratio of the model without intangible capital. I denote this number by y^R to signify that it is the income of a rich country relative to that of a poor country in the full model, given that the relative income under the model without intangible capital was 40. A simple calculation shows that $y^R = 99$ ($3.70^{3.51} = 99$). In words, the same TFP ratio that could explain a 40-fold income difference in the model without intangible capital can explain a 99-fold income difference in the model with intangible capital. This is one of the main results of the paper, that the addition of intangible capital to a standard neoclassical growth model more than doubles the model's ability to explain cross-country income variation.[16]

TABLE 2. Decomposition of income differences (baseline model)

Percentage income difference explained by difference in	Model without Z	Model with Z	Bils and Klenow[a] (2000)
TFP	35.4	28.5	36
K	35.9	31.8	34
H	28.7	25.4	30
Z	—	**14.3**	—
Required TFP ratio	3.70	2.86	—
y^R	99		—

[a]This is decomposition (g) in Henderson and Russell (2005, Table 5, p. 1187).

Another way to compare the two versions (with and without intangible capital) of the model is to use the decomposition defined in (16). The results are in Table 2. First consider the model without intangible capital (the second column in the table). The decomposition suggests that 35.4% of the observed income differences are the direct result of TFP differences. Of the rest, 35.9% are due to differences in physical capital and 28.7% to differences in human capital. These numbers are similar to those reported by others in the literature. For example, Henderson and Russell (2005, Table 5, p. 1188) summarize a few decompositions from the literature. These decompositions are based on models featuring physical and human capital. I report one such decomposition in the last column of Table 2. The results of this decomposition are very similar to mine.

Next consider the full model. The corresponding decomposition is in the third column of Table 2. It turns out that differences in intangible capital contribute a nontrivial 14.3% to the observed income differences. In the absence of intangible capital, this 14.3% is wrongly attributed to TFP and physical and human capital. Another implication of these decompositions is that the share of TFP in explaining income differences drops from 35.4% in the model without intangible capital to 28.5% in the model with intangible capital. In other words, factor accumulation can explain 71.5% of the observed income differences when intangible capital is included in the model. In the absence of intangible capital, this share is 64.6%.

2.3. Some Other Implications of the Model

In this section I discuss two implications of the model that might be of interest to some readers. The first is the implication for rates of return to various types of capital. Here the model of this paper behaves like a standard neoclassical growth model. The steady state rates of return (net of depreciation) for all three types of capital are given by $(1+g)^\sigma(1+\rho)-1$, where ρ is the subjective discount rate. The calibration of the baseline model implies that this rate of return is 9.97% $\left[1.0234^2 \times 1.05 - 1\right]$. It should be no surprize that this is higher than the riskless rate of return of, say, 5%. As Gomme and Rupert (2007) discuss in some detail, this is a normal feature of the models calibrated in a similar way.

The second is the implication for transitional dynamics. With the combined share of the three types of capital in output of 0.71 $(\theta_k + \theta_h + \theta_z = 0.71)$, the rate of transition to steady state in this model is slower than the model that features only physical capital with a share of one-third in output. This implication is an important departure from PP. Their calibration strategy for θ_z is to match the transitional dynamics of the model to the Japanese postwar development experience. The calibration in the present paper suggests a slower transition to the steady state, which is more in line with 2% per annum, as suggested by Barro and Sala-i-Martin (1992), than what is suggested by the experience of Japan and other Asian miracle economies.

3. THE MODEL WITH BARRIERS

In this section I extend the model to include barriers to accumulation. Economists generally agree that various types of barriers hamper the economic performance of poor countries and if the barriers were lowered or removed, the economies of these countries would perform much better with the existing resources. Some examples of barriers include higher relative prices of capital goods [Restuccia and Urrutia (2001) and Hsieh and Klenow (2007)], vested interests that obstruct the adoption of new technologies [Parente and Prescott (1994, 1999, 2000)], and the poor quality of institutions [Hall and Jones (1999) and Acemoglu et al. (2005)].[17]

I incorporate barriers into the model in the same way as Parente and Prescott (1994) did. The only difference is that they introduced barriers to the accumulation of intangible capital alone, whereas, in line with the recent literature and for some other reasons that I explain later, I add barriers to the accumulation of all three types of capital in the model. The main advantage of using barriers in PP's way is that one can use a one-sector model to do what would otherwise require a multisector model. I comment more on this subsequently.

The first barrier that I consider is a barrier to the accumulation of physical capital in the spirit of Restuccia and Urrutia (2001). There is sufficient evidence in the literature that price of investment goods relative (to that of consumption goods) is higher in poor countries [Restuccia and Urrutia (2001) and Hsieh and Klenow (2007)]. I model this by rewriting the law of motion for physical capital as

$$(1+g)(1+n)k_{t+1} = (1-\delta_k)k_t + \frac{x_{kt}}{\pi_{x_k}}, \qquad \textbf{(17)}$$

where π_{x_k} is the price of investment goods relative to that of consumption goods. The interpretation of π_{x_k} is as follows. Suppose $\pi_{x_k} = 2$. This means that we need two units of consumption goods to produce one unit of capital good. Here we see why this methodology is attractive: we are able to incorporate different prices for consumption and investment goods within the framework of a one-sector growth model.

The second barrier that I consider is a barrier to the accumulation of human capital in the form of fewer years of schooling. The data on years of schooling compiled by Barro and Lee (2000) show that the average years of schooling in poor countries are very low compared to those in the rich countries. I model this barrier by rewriting the law of motion of human capital as

$$(1+g)(1+n)h_{t+1} = (1-\delta_h)h_t + \left(\frac{u_t h_t}{\pi_u}\right)^{\psi} x_{ht}^{\phi}, \tag{18}$$

where π_u is the barrier to the accumulation of human capital. The interpretation of π_u is slightly different from that of π_{x_k}. Suppose $\pi_u = 2$. Because u represents the fraction of total working time spent in accumulating human capital, a reasonable interpretation of $\pi_u = 2$ would be that people wasted half of the time that they were supposed to spend on learning. There could be many reasons for this. It could be that schools were too far from where the students lived and hence they spent a lot of time commuting to school. Schools might remain closed very often because of political unrest or strikes. Even when the students were in school, they might not spend all the time on learning because of high rates of teacher absenteeism. They might also drop out early for various personal or social reasons. These barriers to accumulation of human capital are assumed to be exogenous; i.e., the decision makers have no control over them, and even the social planner takes them as given while solving the optimization problem.

The third barrier that I consider is a barrier to the accumulation of intangible capital. There is some evidence that the rate of investment in intangibles varies directly with the standard of living [Van Ark et al. (2009)]. Hence the poor countries are likely to make a smaller investment in intangibles than the rich countries do. I model this barrier in exactly the same way as PP did. The law of motion for intangible capital is then written as

$$(1+g)(1+n)z_{t+1} = (1-\delta_z)z_t + \frac{x_{zt}}{\pi_{x_z}}, \tag{19}$$

where π_{x_z} is the barrier to the accumulation of intangible capital.

The introduction of the barriers does not affect the other equations of the model. When the model with barriers is solved for the steady state, the output is given by

$$y = ba^{\xi} \pi_{x_k}^{-\xi\theta_k} \pi_u^{-\frac{\xi\psi\theta_h}{(1-\psi)}} \pi_{x_z}^{-\xi\theta_z}. \tag{20}$$

Note that the exponents on π's are all negative: the higher the barrier, the lower the steady state output. The effect of each barrier on output depends on the size of the barrier itself, the value of ξ, the share of the relevant type of capital in output, and the relevant elasticity parameters. Consider, for example, π_{x_k}, the barrier on accumulation of physical capital. Its effect on output depends on its own size, the value of ξ, and the share of physical capital in total output. The effects of barriers associated with human capital accumulation are slightly more complicated because human capital is used in production as well as in the accumulation of human capital.

3.1. International Income Differences

Assuming that there are no barriers in the benchmark rich country, i.e., all π's are equal to one in the rich country, the relative output is given by

$$\frac{y_i}{y_j} = \left(\frac{a_i}{a_j}\right)^{\xi} \pi_{x_k}^{\xi\theta_k} \pi_u^{\frac{\xi\psi\theta_h}{(1-\psi)}} \pi_{x_z}^{\xi\theta_z}, \tag{21}$$

where all π's refer to barriers in the poor country (i.e., country j). It is obvious that if one or more barriers in the poor country have numerical values greater than one, the TFP ratio required to explain the observed income differences will be smaller in this model than in the model without barriers. In other words, the barriers are additional sources of income differences. The question of interest, then, is how big a role does intangible capital play in this extended model to explain international income differences. Clearly, in the presence of the barriers, the quantitative effect of intangible capital (and for that matter, of any type of capital) will be smaller. However, as I show later, it is still nontrivial. Specifically, when reasonable numerical values are assigned to the barriers, the differences in intangible capital (and in the barriers to the accumulation of intangible capital) explain from 10% to 22% of the observed income differences in various versions of the model.

To study the role of intangible capital in this model, I first consider one barrier at a time, quantify the size of the barrier by using information from the available literature, and study the role played by intangible capital in explaining income differences in the presense of the barrier. I then consider a version of the model in which all three barriers are present.

Barriers to the accumulation of physical capital (π_{x_k}). Given my interpretation of these barriers as the relative price of investment, the literature has assigned various values to this barrier ranging from 2 to 3 [Hsieh and Klenow (2007)]. Here I follow Manuelli and Seshadri (2007) and set $\pi_{x_k} = 2.73$, which is well within the range in the literature. I set $\pi_{x_z} = \pi_u = 1$ for this exercise. The results are in columns (2) and (3) of Table 3.

The first observation is that this barrier is important and can account for 27.5% to 30.4% of the observed income differences. The second observation is that although in the presence of the barrier, factor accumulation is relatively less important, intangible capital can still explain 9.9% of the observed income differences. The third observation is that the addition of intangible capital affects the relative contribution of various factors in explaining income differences in nontrivial and interesting ways. The contribution of TFP is reduced by 5.8% (from 25.7% to 19.9%). The combined contribution of physical and human capital is reduced by 7% (from 46.8% to 39.8%). The contribution of the barrier increases by 2.9% (from 27.5% to 30.4%). The reason for the decline in the share of TFP is obvious. However, it is not so obvious why the combined share of physical and human capital declines and that of barriers increases. The decline in the contribution of

TABLE 3. Decomposition of income differences (model with barriers)

	Assumptions about π_{x_k}, π_u, and π_{x_z}							
	$\pi_{x_k} = 2.73$ $\pi_u = 1.00$ $\pi_{x_z} = 1.00$		$\pi_{x_k} = 1.00$ $\pi_u = 4.46$ $\pi_{x_z} = 1.00$		$\pi_u = 1.00$ $\pi_{x_k} = 1.00$ $\pi_{x_z} = 2.00$		$\pi_{x_k} = 2.73$ $\pi_u = 4.46$ $\pi_{x_z} = 2.00$	
Percentage income	Model		Model		Model		Model	
difference explained	w/o Z	w/ Z	w/o Z	w/ Z	w/o Z	w/ Z	w/o Z	w/ Z
by difference in (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
TFP	25.7	19.9	28.6	22.4	35.4	25.8	18.8	11.1
K	26.0	22.1	28.9	25.0	35.9	28.8	19.1	12.4
H	20.8	17.7	23.2	20.0	28.7	23.0	15.2	9.9
Z	—	**9.9**	—	**11.2**	—	**12.9**	—	**5.6**
π_{x_k}	27.5	30.4	—	—	—	—	27.5	30.4
π_u	—	—	19.3	21.3	—	—	19.3	21.3
π_z	—	—	—	—	—	**9.4**	—	**9.4**
Required TFP ratio	2.58	2.08	2.87	2.29	3.70	2.59	2.00	1.51
y^R	85		89		139		109	

physical and human capital is due to lower values of θ_k and θ_h. I have explained the reason for this in Section 2.1. The increase in the contribution of barriers is due to the bigger value of the multiplier reflected in a higher value of ξ.

Another way to compare the difference that intangible capital makes in the ability of this model to explain income differences is to compute y^R as defined in Section 2. In this case $y^R = 85$. In words, the same TFP difference that could generate a 40-fold income difference in the model without intangible capital can now generate an 85-fold income difference. This is not as high as 99 (which was the value of y^R in the baseline model) because now, even in the absence of intangible capital, the barrier accounts for a sizable fraction of the income differences. But still, $y^R = 85$ suggests that when intangible capital in added, the ability of the model to explain income differences increases by more than a factor of two.

Barriers to the accumulation of human capital (π_u). According to Manuelli and Seshadri (2007), the average years of schooling in the richest decile of countries is 4.46 times higher than that in the poorest decile of countries. According to my interpretation of the barrier to the accumulation of human capital, this implies $\pi_u = 4.46$. For this exercise, I set $\pi_{x_k} = \pi_{x_z} = 1$. The results are in columns (4) and (5) of Table 3. This barrier accounts for close to 20% of the observed differences in incomes. Intangible capital, when added to this model, can explain 11.2% of the income differences. In this case $y^R = 89$, which suggests that intangible capital increases the ability of the model to explain income differences by more than a factor of two.

Barriers to the accumulation of intangible capital (π_{x_z}). To the best of my knowledge, no estimates of investment in intangibles by the poor countries are available. Even if they were, it would be difficult to map the differences in investment rates into π_{x_z}. So I confine myself to a thought experiment. Suppose that $\pi_{x_z} = 2$. This implies that the barriers to the accumulation of intangible capital are such that a two-unit investment in intangible capital leads to a one-unit gross increase in the stock of intangible capital. The results of this experiment are reported in columns (6) and (7) of Table 3. Here intangible capital contributes to income differences through two channels: first, the differences in Z, and second, the differences in π_{x_z}. The total contribution of these two channels explains 22.3% of the observed income differences. In this case $y^R = 139$, which suggests that when intangible capital in added, the ability of the model to explain income differences increases by more than a factor of three.

Barriers to the accumulation of all three types of capital. For this exercise, I consider all three barriers simultaneously. The motivation for doing so is twofold. First, if all three barriers are important, it will be interesting to see how they interact in this model. Second, one might suspect that when all three barriers are present, they will explain so much more of the variation in income that adding intangible capital would not make a significant difference. The results of this exercise are in columns (8) and (9) of Table 3. In this version of the model, the total contribution of intangible capital to explaining income differences is 15% (5.6% + 9.4%). y^R in this case is 109. Both these numbers are higher than their counterparts in the baseline model.

To sum up, the barriers to accumulation are an important source of variation in income across countries. When barriers are present, the role of capital accumulation in explaining income differences is somewhat diminished. Nonetheless, the intangible capital remains important, despite the barriers. In the various versions of the model that I have discussed, intangible capital accounts for 10% to 22% of the observed income differences.

4. SENSITIVITY ANALYSIS

I have shown that the ability of the neoclassical growth model to explain international income differences is significantly improved when intangible capital is added to the model together with physical and human capital. In this section I study the sensitivity of this conclusion to changes in some of the parameters and targets about which, in my opinion, we have less reliable information than others. My strategy for analysis in this section is the following. I pick a parameter (or target), one at a time, and try two different values for it other than the one used in the main analysis. I then study what happens to the comparison between the model without intangible capital and the full model at different values of the parameter or the target.

TABLE 4. Sensitivity analysis

Sensitivity parameter or target	Model w/o Z		Full model			
	ξ_0[a]	TFPR_0[b]	ξ_F[c]	TFPR_F[d]	y^{R}[e]	% explained by Z
$\sigma =$						
2.0	**2.82**	**3.70**	**3.51**	**2.86**	**99**	**14.3**
2.5	3.56	2.82	4.68	2.20	128	14.8
3.0	4.97	2.10	7.28	1.66	222	15.4
$\delta_h =$						
0.013	5.30	2.00	7.25	1.66	152	14.3
0.046	**2.82**	**3.70**	**3.51**	**2.86**	**99**	**14.3**
0.070	2.51	4.34	3.09	3.30	93	14.3
$\delta_z =$						
0.31	2.82	3.70	3.41	2.95	87	13.5
0.215	**2.82**	**3.70**	**3.51**	**2.86**	**99**	**14.3**
0.05	2.82	3.70	4.37	2.33	304	19.9
$\iota_h =$						
0.127	**2.82**	**3.70**	**3.51**	**2.86**	**99**	**14.3**
0.086	2.24	5.20	2.72	3.88	89	14.3
0.054	1.93	6.79	2.32	4.91	85	14.3
ι_z[f] $=$						
0.15	2.82	3.70	3.64	2.75	117	16.4
0.128	**2.82**	**3.70**	**3.51**	**2.86**	**99**	**14.3**
0.100	2.82	3.70	3.35	3.01	80	11.4
$u =$						
0.15	2.65	4.01	3.28	3.08	95	14.3
0.21	**2.82**	**3.70**	**3.51**	**2.86**	**99**	**14.3**
0.25	3.01	3.40	3.77	2.66	101	14.3
COR $=$						
2.00	2.38	4.72	2.91	3.55	91	14.3
2.46	**2.82**	**3.70**	**3.51**	**2.86**	**99**	**14.3**
3.00	3.66	2.74	4.69	2.20	113	14.3
$\rho =$						
0.03	2.14	5.59	2.54	4.28	79	13.3
0.05	**2.82**	**3.70**	**3.51**	**2.86**	**99**	**14.3**
0.07	4.33	2.35	6.01	1.85	170	15.2

[a] $\xi_0 = \xi$ in the model without intangible capital.
[b] TFPR_0 = TFP ratio in the model without intangible capital.
[c] $\xi_F = \xi$ in the full model.
[d] TFPR_F = TFP ratio in the full model.
[e] $y^R = (\text{TFP}R_0)^{\xi_F}$.
[f] In the model without intangible capital, $\iota_z = 0$.

All the relevant numbers are reported in Table 4. The rows in **bold** show the parameter (target) values and corresponding values of ξ and TFP ratios used in the analysis in Section 2. These are my preferred values.Although I have reported

https://doi.org/10.1017/S136510051100040X Published online by Cambridge University Press

the values of ξ and the TFP ratio in Table 4, in the sensitivity analysis I shall focus on two numbers.

The first number is y^R. As defined in Section 2.2, it is the income of a rich country relative to that of a poor country under the full model, given that the relative income under the model without intangible capital is 40. y^R is reported in the second to last column of Table 4. The main result of this paper is that y^R is 99; i.e., if the model without intangible capital can generate 40-fold income differences with a certain TFP ratio, the full model can generate 99-fold income differences with the same TFP ratio. When I change a parameter (or a target) and y^R remains close to 99 or increases above 99, I shall conclude that my main result is robust to the change in the parameter (or the target). However, if as a result of the change, y^R falls well below 99 (I use $y^R = 90$ as the threshold), I shall conclude that my main result is sensitive to the change in the parameter (or the target).

The second number is the contribution of differences in intangible capital to explaining the observed income differences. This number is reported in the last column of Table 4. In the benchmark case, this number is 14.3%. Here too, as in the case of y^R, if a change in a parameter (or a target) results in a value of this number that is higher than or close to 14.3%, I shall conclude that my main result about the importance of intangible capital is robust to the change in the parameter (or the target). However, if this number drops well below 14.3%, I shall conclude that my main conclusion is sensitive to the change in the parameter (or the target).

A quick look at the last two columns of Table 4 shows that in case of a change in the following four parameters (or targets), y^R either increases or does not change much:[18] σ, δ_h, u, and COR. The percentage of income differences explained by differences in intangible capital (see the last column in Table 4) is also not much affected by the given changes in these parameters (or targets). I conclude that the main result of the paper is not sensitive to the choice of these parameters (targets). However, I would still like to comment on δ_h, i.e., the depreciation rate of human capital.

There is no reliable estimate of δ_h available. Earlier studies have assumed different deprecation rates for human capital. Lucas (1988), for example, assumes zero depreciation. Mankiw et al. (1992) and Chari et al. (1996) assume that the depreciation rate of human capital is equal to that of physical capital. There is a large literature that tries to measure the value of human capital in an economy.[19] There are especially quite a few studies on the U.S. economy. However, the results are all over the place. At one extreme, some studies conclude that the value of the stock of human capital is the same as or even less than the value of the stock of physical capital. At the other extreme, some studies find the stock of human capital to be 20 times as valuable as the stock of physical capital. With such inconclusive evidence, it is hard to make a precise statement about the relative size of human capital, which could help us pin down the depreciation parameter. In the main analysis I have followed Mankiw et al. (1992) and Chari et al. (1996) and assumed that the depreciation rate of human capital is equal to that of physical capital. Another possibility is to look at the percentage of people retiring from the labor force every year and take this to be an estimate of the depreciation of human capital. One such calculation gives $\delta_h = 0.013$.[20] This is one of the alternative values of δ_h that I use for sensitivity checks. Not surprisingly, a lower depreciation rate leads to a much higher value of ξ. However, the contribution of intangible capital is almost the same whether the depreciation rate of human capital is 0.013 or 0.046.

I would like to comment on the other four targets in Table 4, which, when changed in a particular direction, result in $y^R < 90$. The first such target is δ_z, the depreciation rate of intangible capital. If I use the earlier estimate of 31% from CHS, $y^R = 87$ and intangible capital explains 13.5% of the income difference. These numbers are smaller than the benchmark but are still high enough so that our conclusion about the importance of intangible capital still holds. Moreover, the latter estimates of the depreciation of intangible capital are more reliable, as they are based on a more refined definition of intangible capital.

The second such target is ι_h. This is the target level of investment in human capital and was used to calibrate θ_h, the share parameter for human capital. Although small changes in this target do not affect the results much, when I reduce this target from 0.127 to 0.054 the value of y^R drops from 99 to 85. This suggests that a very low investment in human capital adversely affects the capital accumulation multipliers and hence the overall ability of the model to explain income differences. Note, however, that a lower investment target for human capital does not affect the percentage of income differences that can be explained by differences in intangible capital. Hence our main conclusion is not sensitive to changes in this target.

The third such target is ι_z, the ratio of investment in intangibles. Following estimates in CHS, I chose $\iota_z = 0.128$. This is based on their estimates of investment in intangible capital in the United States during the period 1995–2007. However, according to CHS, this investment has been rising over time and if we compute the average for the post-WWII period, it is close to 0.10. It is instructive to see how the model fares when a lower target for ι_z is chosen. When ι_z is lowered from 0.128 to 0.10, y^R declines from 99 to 80 and the contribution of intangible capital drops from 14.3% to 11.4%. This is hardly surprising. In fact, the main point of this paper is that this investment is not too small, and hence, by excluding intangible capital from the analysis, we omit some of the variation in output that the neoclassical model is capable of explaining. Nonetheless, even with this lower target for investment in intangibles, the full model can explain twice as large income differences as the model without intangible capital. So even if the investment in intangible capital is as low as 10%, ignoring it in the studies of international income comparison can distort the relative incomes by a factor of two.

The fourth such target is ρ, the implicit discount rate. When the implicit discount rate is reduced from 5% to 3%, y^R drops from 99 to 79 and the share of intangible capital drops from 14.3% to 13.3%. Although the results are not very sensitive to this change in the value of the implicit discount rate, it is worth some thought that the addition of intangible capital has a smaller effect when people are more patient. The reason is that when people are more patient (ρ is smaller), it is optimal for the planner to allocate more resources to investment. However, we calibrate the model to fixed investment targets. Hence when ρ is smaller, the calibrated parameters change in such a way that the investment rates remain unchanged. Specifically, when the implicit discount rate is reduced from 5% to 3%, the share parameters $\{\theta_h, \theta_k, \theta_z\}$ drop from $\{0.25, 0.32, 0.14\}$ to $\{0.20, 0.27, 0.13\}$ and the overall multiplier ξ drops from 3.51 to 2.54.

To conclude, the main results of this paper are not very sensitive to the choice of various targets (and parameters) and they still hold for a reasonable range of these targets.

5. CONCLUDING REMARKS

In this paper I construct a variant of the neoclassical growth model to study its implications for international income differences. The model features intangible capital in addition to physical and human capital. The main finding is that the addition of intangible capital to an otherwise standard neoclassical growth model more than doubles the model's ability to account for cross-country variation in income. Specifically, the same efficiency difference (i.e., the TFP ratio) that generates a 40-fold income difference in the model without intangible capital can generate a 99-fold income difference with intangible capital in the model. I also find, using a simple decomposition, that intangible capital explains 14.3% of the observed variation in income. These results are robust to different parameterizations of the model.

I also examine a version of the model in which countries differ in the size of barriers to accumulation of various types of capital. The main conclusion from this exercise is that differences in intangible capital explain 10% to 22% of the observed income differences.

Given the increasing role of intangible capital in the economy, it is just a matter of time before we shall have estimates of this type of capital for a number of countries, including some poor countries. In fact, work in this direction is already under way. For example, Van Ark et al. (2009) provide estimates of investment in intangible capital for a set of European countries. Once these estimates are available for a larger set of countries, we shall be in a better position to examine the true contribution of differences in intangible capital to differences in international incomes. In the absence of these statistics, the present study provides some evidence that intangible capital is important for understanding international income differences. I hope this evidence will grow stronger with the availability of more data.

An interesting direction for future work will be to apply the efficient technology hypothesis to a model with intangible capital. The objective will be to understand why rates of investment in intangible capital differ accross countries. Is it because there are barriers to the accumulation of intangible capital or because it is optimal for poor countries not to invest in intangible capital.

NOTES

1. Some of these items are clearly *services*, but for simplicity of exposition I shall call them *goods*.

2. Human capital is also intangible. However, in this paper I distinguish between human capital and intangible capital. The reason is that many authors have studied the implications for international income differences of adding human capital to the models of growth. In this paper, I study the implications for income differences when intangible capital, as defined earlier, is added to a growth model that already includes human capital (also see note 4).

3. The only exception is expenditure on computer software, which has been treated as investment since 1999. However, the U.S. Bureau of Economic Analysis (BEA) is continuing with its efforts to include intangible investment in its core GDP accounts. An important development in this regard is that the BEA expects to incorporate R&D investment and investment in artistic originals (mainly motion pictures and sound recordings) into GDP accounts in 2013 [Aizcorbe et al. (2009)].

4. I distinguish between human and intangible capital. Human capital is the result of investments that are included in GDP but treated as consumption rather than investment. Examples include expenditures on education, health, and upbringing of children. Intangible capital, on the other hand, is the result of investments that have not been included in the GDP. Instead they were treated as expenditures on intermediate goods. For a thorough discussion of this concept of intangible capital see Corrado et al. (2009). The BEA is conducting research to construct measures of the size of human capital in the U.S. economy. This initiative is separate from the BEA's efforts to measure intangible capital as defined in this paper.

5. For further details on this human capital technology, see Erosa et al. (2010) and Manuelli and Seshadri (2007).

6. When I calibrate the parameters, I ignore this restriction and choose the parameters to match the targets. I then check whether the calibrated parameters satisfy the restriction. For the results that I report in this paper, the restriction is never violated.

7. In (10), β is the modified discount factor. Given CRRA preferences, β is equal to $\tilde{\beta}(1+n)(1+g)^{1-\sigma}$, where $\tilde{\beta}$ is the discount factor.

8. See Ljungqvist and Sargent (2004, p. 426) for a discussion of the value of σ.

9. This includes private as well as public expenditure on children aged 0–18. For details, see Table 1 in Haveman and Wolfe (1995).

10. Also see Table 3 in Aizcorbe et al. (2009) for depreciation rates used in studies that attempt to measure intangible capital in some other countries.

11. The calibration in PP implies a 3% depreciation in intangible capital. If I use this estimate, the main conclusion of the paper will be strengthened greatly (see, for example, the row corresponding to $\delta_z = 0.05$ in Table 4). However, I use the depreciation rate given in Corrado and Hulten (2010) because it appears to be more realistic given the type of investments included in intangible capital.

12. For further details see CHS.

13. This estimate is much lower than the 40% or 32% implied by Parente and Prescott (1994) and Prescott (1998). I also used PP's strategy to calibrate the share of intangible capital. This involved matching the transition dynamics of the model to Japan's post-WWII real output. With human capital in the model, their strategy implied an investment in intangible capital of around 20% of the measured income. This is higher than CHS's estimate but much lower than the 40% implied by the original calibration in PP. The reason for not using PP's strategy to calibrate the share of intangible capital is that if we choose a different country to match the transition dynamics of the model, the calibrated share of intangible capital will be different. However, given the direct estimates of investment in intangibles, we get a unique estimate of the share of intangible capital in output.

14. According to Heston et al. (2006), in the year 2000 the ratio of real GDP per capita of the richest 10% countries to that of the poorest 10% countries was 41.5. In this paper, I round this ratio to the

nearest 10, and study how big the TFP difference needs to be to explain a 40-fold difference in real output. Throughout the paper, I shall use the phrase "TFP ratio" to refer to the TFP ratio between rich and poor countries required to generate a 40-fold difference in outputs. Moreover, the phrase "observed income differences" in the paper would mean the 40-fold income difference.

15. For international income comparisons, we are interested in the relative measured output, i.e., y_{mi}/y_{mj}. However, $y_{mi}/y_{mj} = y_i/y_j$. To see this, note that for country i, $y_{mi} = y_i - x_{zi} = ba_i^{\xi} - b_{xz}a_i^{\xi} = (b - b_{xz})a_i^{\xi}$. Likewise, for country j, $y_{mj} = (b - b_{xz})a_j^{\xi}$. Because I have assumed the same preferences and technology across countries, b and b_{xz} are the same in all countries. Hence $y_{mi}/y_{mj} = (a_i/a_j)^{\xi} = y_i/y_j$.

16. In general, the addition of intangible capital increases the income ratio by a factor of $(a_i/a_j)^{3.51-2.82} = (a_i/a_j)^{0.69}$. It means that if the TFP ratio is greater than 2.73 ($2.73^{0.69} = 2$), the addition of intangible capital to the model will more than double the model's ability to explain income differences.

17. In these papers, institutions are the prime cause of the different development experiences of countries. Although the papers are silent on the exact mechanism through which institutions affect development, they do suggest that the effects of institutions on development are multidimensional, and one such dimension is their effect on technology adoption. For example, Hall and Jones (1999) write, "In addition to its direct effects on production, a good social infrastructure (i.e., *quality of institutions*) may have important indirect effects by encouraging the adoption of new ideas and new technologies as they are invented throughout the world," (pp. 96–97; italics are mine). In a recent paper, Acemoglu et al. (2007) explicitly model the effects of institutions on technology adoption.

18. In all these cases y^R is higher than 90. That means the model can still explain more than twice as large an income difference as it could without intangible capital.

19. See the survey article by Le et al. (2003).

20. According to the Bureau of Labor Statistics, the total number of those who were employed and were 16 years of age or more in the United States at the end of the year 2000 was 137.3 million. The total number of those who were employed and were 55 years of age or more was 18.3 million. I assume that one-tenth of the 18.3 million retire from the labor force every year. Dividing the estimated number of retirees by the total number of employed gives 0.013 (1.83/137.3).

REFERENCES

Acemoglu, Daron, Philippe Aghion, and Fabrizio Zilibotti (2006) Distance to frontier, selection and economic growth. *Journal of the European Economic Association* 4, 37–74.

Acemoglu, Daron, Pol Antras, and Elhanan Helpman (2007) Contracts and technology adoption. *American Economic Review* 97, 916–943.

Acemoglu, Daron, Simon Johnson, and James A. Robinson (2005) Institutions as a fundamental cause of long-run growth. In Philippe Aghion and Steven N. Durlauf (eds.), *Handbook of Economic Growth*, pp. 385–472. Amsterdam: Elsevier.

Acemoglu, Daron and Fabrizio Zilibotti (2001) Productivity differences. *Quarterly Journal of Economics* 116, 563–606.

Aghion, Philippe, Peter Howitt, and David Mayer-Foulkes (2005) The effect of financial development on convergence: Theory and evidence. *Quarterly Journal of Economics* 120, 173–222.

Aizcorbe, Ana M., Carol E. Moylan, and Carol A. Robbins (2009) Towards better measurement of innovation and intangibles. *Survey of Current Business* 89, 10–23.

Barro, Robert J. and Jong-Wha Lee (2000) International Data on Educational Attainment: Updates and Implications. CID working paper 42, Center for International Development, Harvard University.

Barro, Robert J. and Xavier Sala-i-Martin (1992) Convergence. *Journal of Political Economy* 100, 223–251.

Beaudry, P. and P. Francois (2010) Managerial skills acquisition and the theory of economic development. *Review of Economic Studies* 77, 90–126.

Bils, M. and P. J. Klenow (2000) Does schooling cause growth? *American Economic Review* 90, 1160–1183.

Bontempi, Maria Elena and Jacques Mairesse (2008) Intangible Capital and Productivity: An Exploration on a Panel of Italian Manufacturing Firms. NBER working paper 14108.

Boss, J.W.B., C. Economidou, M. Koetter, and J.W. Kolari (2010) Do all countries grow alike? *Journal of Development Economics* 91, 113–127.

Caselli, Francesco (2005) Accounting for cross-country income differences. In Philippe Aghion and Steven N. Durlauf (eds.), *Handbook of Economic Growth*, pp. 679–741. Amsterdam: Elsevier.

Caselli, Francesco and Wilbur John Coleman (2001) Cross-country technology diffusion: The case of computers. *American Economic Review* 91, 328–335.

Caselli, Francesco and Wilbur John Coleman (2006) The world technology frontier. *American Economic Review* 96, 499–522.

Chari, V.V., Patrick J. Kehoe, and Ellen R. McGrattan (1996) The Poverty of Nations: A Quantitative Exploration. NBER working paper 5414.

Comin, D. and B. Hobijn (2004) Cross-country technology adoption: Making the theory face the facts. *Journal of Monetary Economics* 51, 39–83.

Corrado, Carol A. and Charles R. Hulten (2010) Measuring intangible capital: How do you measure a “technological revolution”? *American Economic Review* 100, 99–104.

Corrado, Carol A., Charles R. Hulten, and Daniel E. Sichel (2009) Intangible capital and U.S. economic growth. *Review of Income and Wealth* 55, 661–685.

Eaton, Jonathan and Samuel Kortum (2001) Trade in capital goods. *European Economic Review* 45, 1195–1235.

Erosa, Andres, Tatyana Koreshkova, and Diego Restuccia (2010) How important is human capital? A quantitative theory assessment of world income inequality. *Review of Economic Studies* 77, 1421–1449.

Gomme, Paul and Peter Rupert (2007) Theory, measurement and calibration of macroeconomic models. *Journal of Monetary Economics* 54, 460–497.

Hall, Robert E. and Charles I. Jones (1999) Why do some countries produce so much more output per worker than others? *Quarterly Journal of Economics* 114, 83–116.

Haveman, Robert and Barbara Wolfe (1995) The determinants of children’s attainments: A review of methods and findings. *Journal of Economic Literature* 33, 1829–1878.

Henderson, Daniel J. and R. Robert Russell (2005) Human capital and convergence: A production frontier approach. *International Economic Review* 46, 1167–1205.

Heston, Alan, Robert Summers, and Bettina Aten (2006) Penn World Tables, Version 6.2. Center for International Comparisons of Production, Income and Prices, University of Pennsylvania.

Hsieh, Chang-Tai and Peter J. Klenow (2007) Relative prices and relative prosperity. *American Economic Review* 97, 562–585.

Hsieh, Chang-Tai and Peter J. Klenow (2010) Development accounting. *American Economic Journal: Macroeconomics* 2, 207–223.

Hulten, Charles R. (2010) Decoding Microsoft: Intangible Capital as a Source of Company Growth. NBER working paper 15799.

Hulten, Charles R. and Anders Isaksson (2007) Why Development Levels Differ: The Sources of Differential Economic Growth in a Panel of High and Low Income Countries. NBER working paper 13469.

Jerzmanowski, M. (2007) Total factor productivity differences: Appropriate technology vs. efficiency. *European Economic Review* 51, 2080–2110.

Jones, Charles I. (2011) Intermediate goods, weak links and superstars: A theory of economic development. *American Economic Journal: Macroeconomics* 3, 1–28.

Klenow, Peter J. and Andres Rodriguez-Clare (1997) The neoclassical revival in growth economics: Has it gone too far? In Ben S. Bernanke and Julio J. Rotemberg (eds.), *NBER Macroeconomics Annual*, pp. 73–114. Cambridge, MA: The MIT Press.

Klenow, Peter J. and Andres Rodriguez-Clare (2005) Externalities and growth. In Philippe Aghion and Steven N. Durlauf (eds.), *Handbook of Economic Growth*, pp. 817–861. Amsterdam: Elsevier.

Kosempel, S. (2004) A theory of development and long run growth. *Journal of Development Economics* 75, 201–220.

Kumar, S. and R.R. Russell (2002) Technological change, technological catch-up and capital deepening: Relative contributions to growth and convergence. *American Economic Review* 92, 527–548.

Le, Trinh, John Gibson, and Les Oxley (2003) Cost- and income-based measures of human capital. *Journal of Economic Surveys* 17, 271–307.

Ljungqvist, Lars and Thomas J. Sargent (2004) *Recursive Macroeconomic Theory*. Cambridge, MA: MIT Press.

Lucas, Robert E., Jr. (1988) On the mechanics of economic development. *Journal of Monetary Economics* 22, 3–42.

Mankiw, N. Gregory, David Romer, and David Weil (1992) A contribution to the empirics of economic growth. *Quarterly Journal of Economics* 107, 407–437.

Manuelli, Rodolfo E. and Ananth Seshadri (2007) Human Capital and the Wealth of Nations. Working paper.

McGrattan, Ellen and Edward C. Prescott (2009) Openness, technology capital and development. *Journal of Economic Theory* 44, 2454–2476.

McGrattan, Ellen and Edward C. Prescott (2010) Unmeasured investment and the puzzling U.S. boom in the 1990s. *American Economic Journal: Macroeconomics* 2, 88–123.

McGrattan, E.R. and J.A. Schmitz (1999) Explaining cross-country income differences. In J. B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics*, pp. 669–737. Amsterdam: Elsevier.

Ngai, L. Rachel (2004) Barriers and the transition to modern growth. *Journal of Monetary Economics* 51, 1353–1383.

Parente, Stephen L. and Edward C. Prescott (1994) Barriers to technology adoption and development. *Journal of Political Economy* 102, 298–321.

Parente, Stephen L. and Edward C. Prescott (1999) Monopoly rights: A barrier to riches. *American Economic Review* 89, 1216–1233.

Parente, Stephen L. and Edward C. Prescott (2000) *Barrier to Riches*. Cambridge, MA: MIT Press.

Prescott, Edward C. (1998) Needed: A theory of total factor productivity. *International Economic Review* 39, 525–551.

Restuccia, Diego and Carlos Urrutia (2001) Relative prices and investment rates. *Journal of Monetary Economics* 47, 93–121.

Sadik J. (2008) Technology adoption, convergence and divergence. *European Economic Review* 52, 338–355.

Van Ark, Bart, Janet X. Hao, Carol Corrado, and Charles Hulten (2009) Measuring intangible capital and its contribution to economic growth in Europe. *EIB Papers* 14, 62–93.

Zeira, Joseph (1998) Workers, machines and economic growth. *Quarterly Journal of Economics* 113, 1091–1117.

APPENDIX: CALIBRATION STRATEGY

Let

$$D_{1i}(n, g, \beta, \delta_i) = \beta[(1+g)(1+n) - (1-\delta_i)],$$

$$D_{2i}(n, g, \beta, \delta_i) = [(1+g)(1+n) - \beta(1-\delta_i)],$$

where $i = \{h, k, z\}$. Also, let $\iota_i \equiv x_i^{SS}/y_m^{SS}$, where SS in the superscripts refers to steady-state values. I now describe the calibration strategy in some detail.

https://doi.org/10.1017/S136510051100040X Published online by Cambridge University Press

The targets of population growth rate and per capita consumption growth rate match one to one with parameters n and g. Parameter σ is chosen from the empirical literature. Parameter β, the modified discount rate, depends on n, g, σ, and $\tilde{\beta}$, where $\tilde{\beta} = 1/(1+\rho)$ and ρ is the implied discount rate. Specifically, $\beta = \tilde{\beta}(1+n)(1+g)^{1-\sigma}$.

I pick θ_k to match the target for ι_k. From the steady state solution of the model,

$$\theta_k = \frac{\iota_k}{1+\iota_z}\frac{D_{2k}}{D_{1k}}, \tag{A.1}$$

because

$$\frac{x_k^{\mathrm{SS}}}{y^{\mathrm{SS}}} = \frac{x_k^{\mathrm{SS}}/y_m^{\mathrm{SS}}}{y^{\mathrm{SS}}/y_m^{\mathrm{SS}}} = \frac{\iota_k}{1+\iota_z}.$$

I pick δ_k to match the target capital–output ratio (COR). Solving (4) for δ_k at the steady state gives

$$\delta_k = \frac{\iota_k}{\mathrm{COR}} + 1 - (1+g)(1+n),$$

where COR$= k^{\mathrm{ss}}/y_m^{\mathrm{ss}}$.

I pick ψ to match the steady state target for u, the fraction of time spent on accumulating human capital. From the steady state of the model,

$$\psi = u^{\mathrm{SS}}\frac{D_{2h}}{D_{1h}}. \tag{A.2}$$

I assume $\phi = 1 - \psi$. I pick θ_h to match the steady state target for x_h/y_m. The steady state of the model gives

$$\theta_h = \frac{\iota_h}{1+\iota_z}\frac{D_{2h}(1-u)}{D_{1h} - uD_{2h}}. \tag{A.3}$$

I pick δ_z to match the target depreciation rate of intangible capital. Parameter θ_z is picked to match the target investment in intangible capital. The steady state solution of the model gives

$$\theta_z = \frac{\iota_z}{1+\iota_z}\frac{D_{2z}}{D_{1z}}. \tag{A.4}$$