Dynamics of Populations of Planetary Systems
Proceedings IAU Colloquium No. 197, 2005
Z. Knežević and A. Milani, eds.

DOI: 10.1017/S174392130400852X

Planetary orbits in double stars: influence of the binary's orbital eccentricity

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Abstract. Regularity and chaos of "quasi-stable" (i.e. appearing stable during a finite interval of time) planetary orbits around one component of binary stars is investigated for different values of the binary's mass ratio and orbital eccentricity e. The behavior of fictitious planetary orbits around 16 Cyg B-like stars is presented. Among the quasi-stable orbits we found that there exists a ("stability zone") for every values of e, but that the existence of nearly-circular planetary orbits is restricted to values of e less than 0.8. However, not all the quasi-stable orbits are regular: emergence of chaos when e increases is shown in two sets of quasi-stable orbits (each set has fixed initial conditions for the planet, but varying values for e from 0 to 0.99). In the first set, which lies in the "heart" of the stability zone, chaos appears only when e approaches 1. The second one is near the border of the stability zone, and chaos appears as soon as e reaches 0.7. The influence of the binary's orbital eccentricity on the limit between regularity and chaos is therefore stronger in the latter case (wider planetary orbits) than for the former one (closer planetary orbits).

Keywords. Celestial mechanics, methods: n-body simulations, stars: binaries: individual (16 Cyg B), planetary systems

1. Introduction

Cosmogonical theories as well as recent observations allow us to expect the actual existence of numerous exo-planets, including in binaries (for example around 16 Cyg B and in the 16 Cyg system; see Gorshanov *et al.* 2005). Then arises the dynamical problem of stability for planetary orbits in double star systems. Modern computations (Dvorak *et al.* 1989, 2004; Hale 1994; Holman & Wiegert 1999; Lohinger *et al.* 1993; Udry *et al.* 2004) have shown that many such stable orbits do exist, among which we consider orbits around one component of the binary (called S-type orbits; see Pilat-Lohinger & Dvorak 2002; Pilat-Lohinger 2005).

The model is the planar restricted three-body problem, where the parameters reduce to e, the orbital eccentricity of the binary, and μ, the reduced mass of the primary of the planet, and where the initial conditions of the massless body may be reduced to two: $X_o > 0$ and V_o. The equations of motion are written in a rotating-pulsating frame, and are integrated using a classical 4th order Runge-Kutta scheme with improved coefficients and with variable time step. We call an orbit stable – or, more rigorously, "quasi-stable" (but hereafter called nevertheless "stable" for simplification) – if there has been neither collision with a star nor escape during a given time, as stated in previous papers (see, e.g., Benest 1988a, which contains a large list of references).

Within this framework, the phase space of initial conditions for fictitious S-type planetary orbits in given binaries is systematically explored. The subset of stable initial conditions – the "stability zone" – is established, and easily visualised on the plane $(X_o > 0, V_o)$.

2. Stable planetary orbits in binaries

In previous papers (Benest 2003, and references therein), one of us showed that stable planetary orbits, around either the lightest (B) or the heaviest (A) component of Sirius and of 4 nearby binary systems for which $\mu = 0.45$ and 0.55, exist up to distances from their primary of the order of more than half the binary's periastron separation. The results obtained in the circular case ($e = 0$) were then confirmed in the more realistic elliptic case. Among these stable orbits, nearly-circular ones exist for the binaries having a not too high orbital eccentricity.

Besides, we use Lyapunov indicators as indicators of chaos, computed with a Bulirsch & Stoer integrator with variable time step, and we study the stochasticity of some such stable planetary orbits. Then, we have found an example of a chaotic orbit which, nevertheless, lasted during 10^{10} years in a quasi-stable state: a longer integration would be needed to determine its actual nature, bounded chaos (i.e. the motion is chaotic but stays inside a finite volume for an infinite time; see, e.g., Laskar 1990) or sticky orbit (i.e. the motion looks like a bounded chaos during a long, sometimes very long, time – which may be called "confined chaos" – but finally escapes out to infinity; see Dvorak *et al.* 1998).

3. Orbiting 16 Cygni B-like stars

More recently, we have extended this study to planetary orbits around 16 Cygni B-like stars, i.e. around the lightest component of binaries having same value of μ that the actual 16 Cygni B ($\mu = 0.485$) but varying e: the orbital eccentricity of the double star 16 Cyg is not well known, its value being generally evaluated between 0.54 and 0.96, and we have extended this interval to [0,1] for our study.

The first results (Benest & Gonczi 2004) have confirmed the existence of stable (during 100 revolutions of the binary) nearly-circular S-type planetary orbits (very abundant for low values of e, see Figure 1; fairly abundant for $e = 0.7$, see Figure 2), excluding the case of very high eccentricity of the binary (Figure 3, $e = 0.95$). Moreover, investigating the chaotic behaviour of two sets of stable planetary orbits (one of close orbits in the very "heart" of the stability zone, and the other of wide ones just inside its border), we have shown that the stability of the first set is not destroyed when the binary's eccentricity increases even to very high values: for $e = 0.95$, the planetary orbit of this first set is chaotic but stays confined up to 1 billion years. On the contrary, the stability of the second set is destroyed as soon as the eccentricity e reaches the value 0.8, and the planetary orbit of this second set for $e = 0.7$ stays in confined chaos up to 1 billion years as well.

In the next section, we put the light on these two chaotic orbits which lie very near the limit between regularity and chaos, on the chaotic side. Both stay confined during the computation time span cited above, but determining the kind of confinement (bound or stickiness) would need more computation.

4. Bounded or sticky?

Figures 4 and 5 show, for the two orbits mentioned above, the evolution with time of the greatest Lyapunov indicator (top) together with the two main planetary orbital elements – eccentricity `e` and semi-major axis `a` in a.u. – (bottom). The time span of the computation

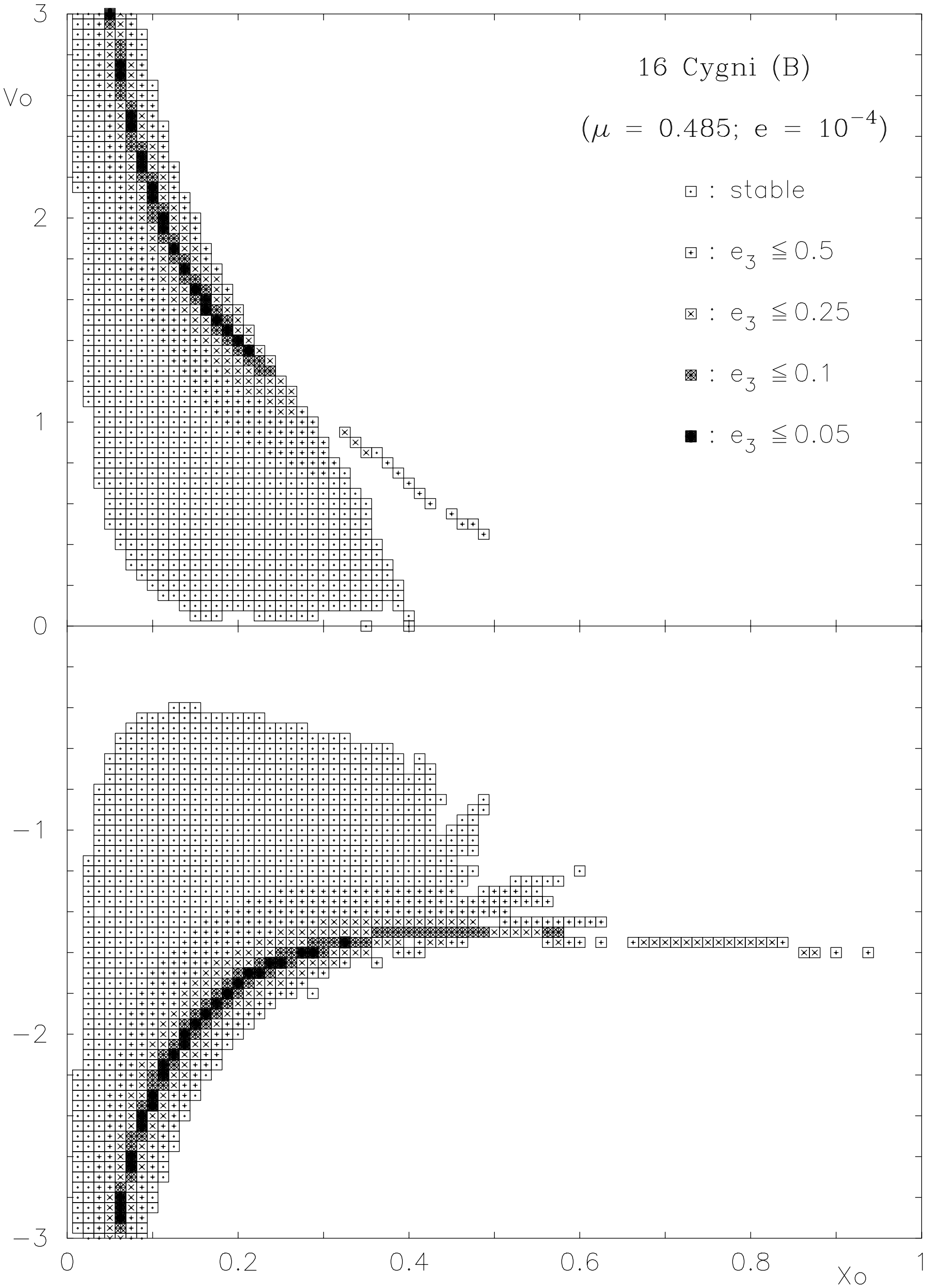

Figure 1. The subset of initial conditions in the (X_0,V_0) plane (dimensionless coordinates in the rotating-pulsating frame) for stable planetary orbits (stability zone) around a 16 Cyg B-like star when $e = 10^{-4}$: the pointed (resp. crossed, ×-ed, asterisked and filled) squares indicate planetary orbits whose eccentricity e_3 stays under the respective values given in the graph.

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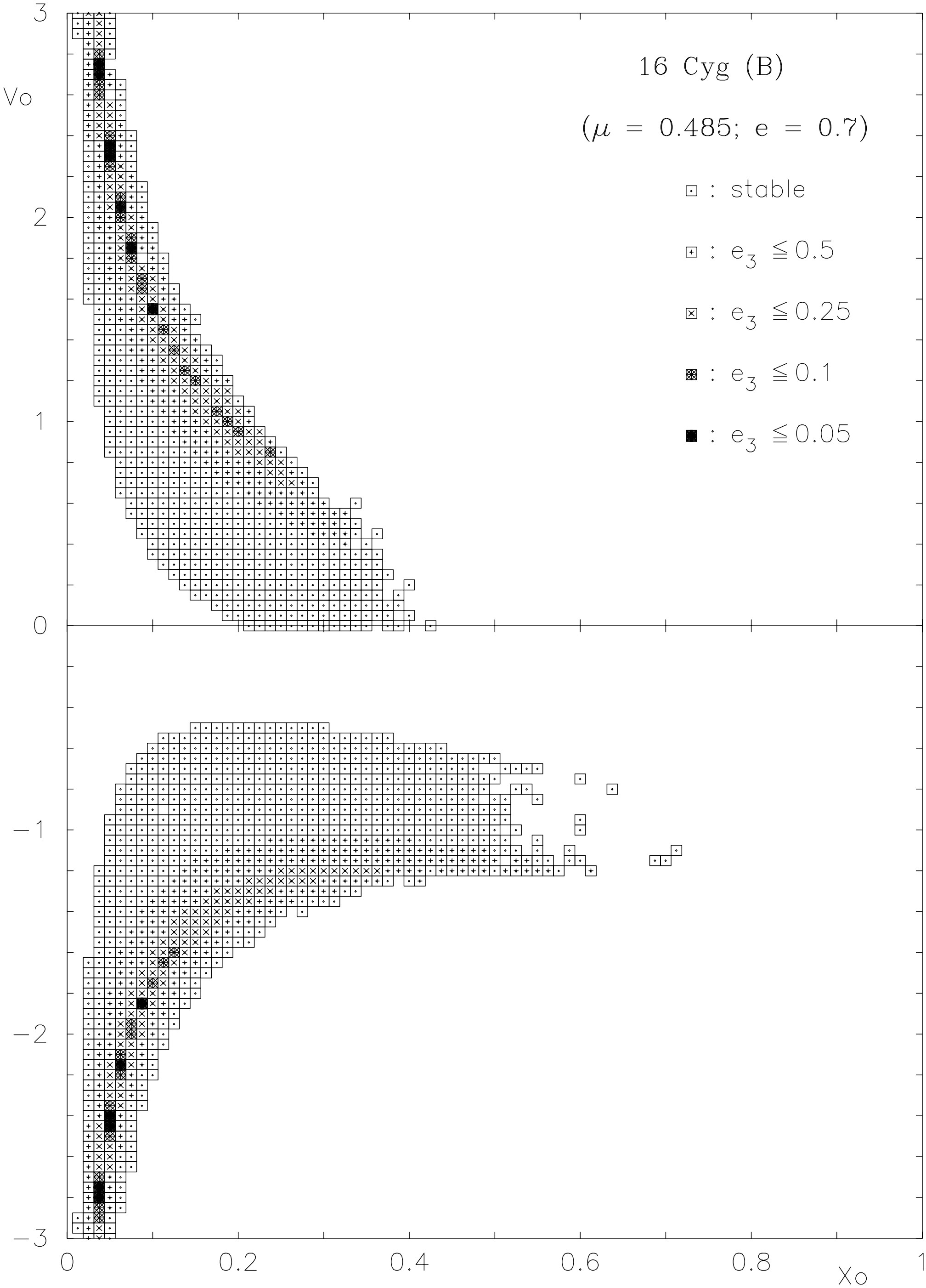

Figure 2. The stability zone for planetary orbits around a 16 Cyg B-like star when $e = 0.7$; same notations as in Fig. 1.

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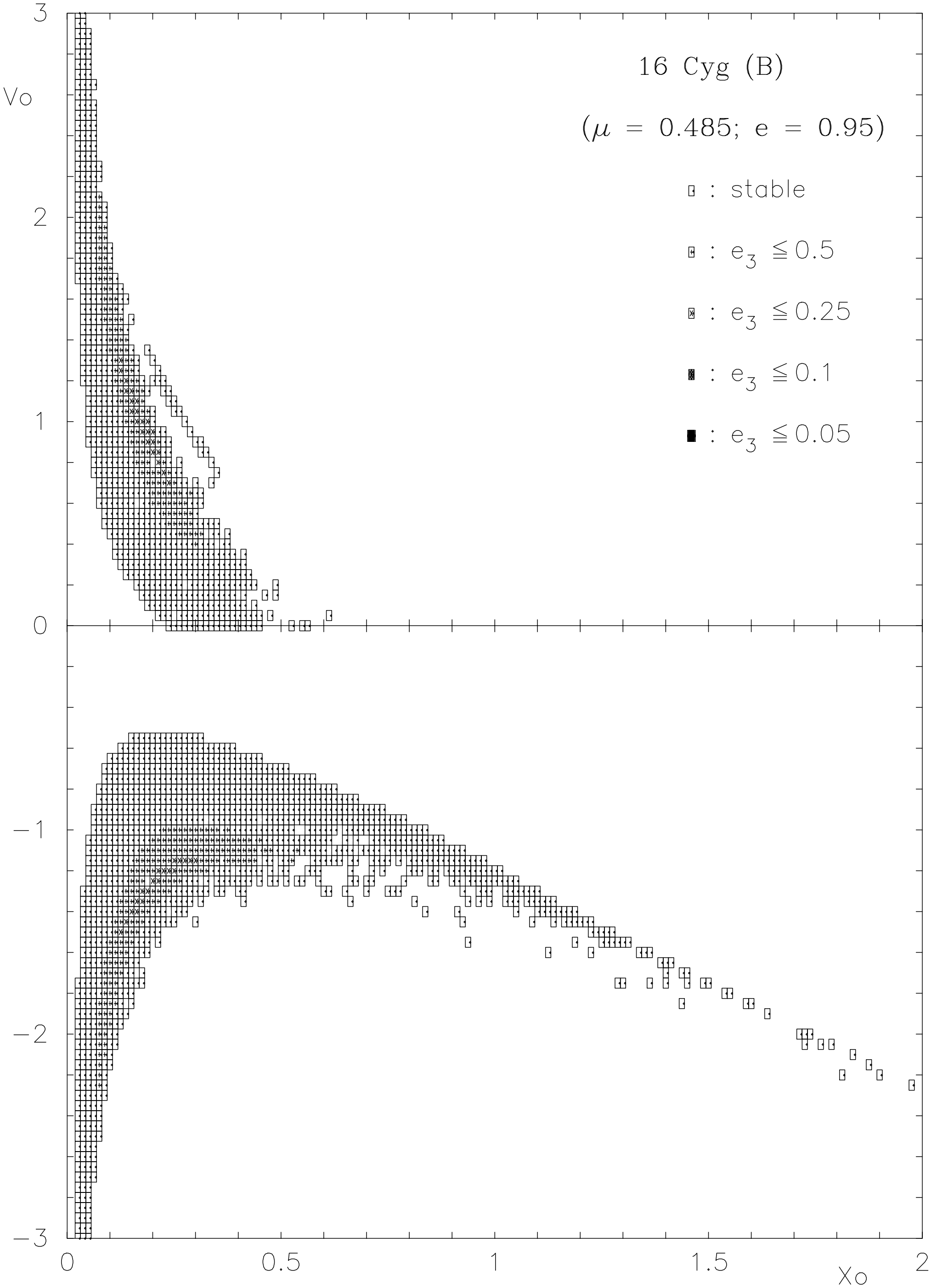

Figure 3. The stability zone for planetary orbits around a 16 Cyg B-like star when $e = 0.95$; same notations as in Fig. 1.

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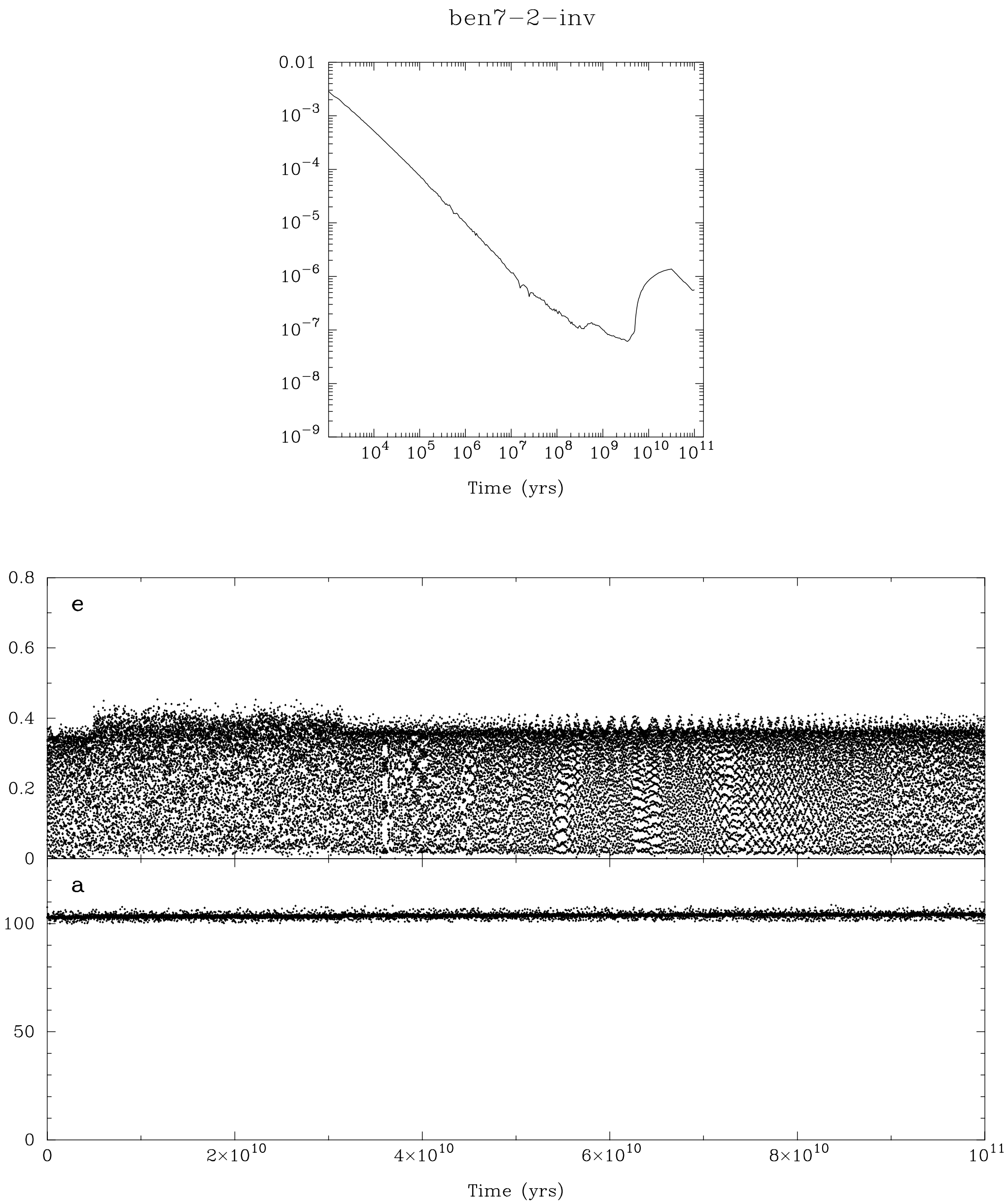

Figure 4. Evolution vs. time of the greatest Lyapunov indicator (top) and of two orbital elements – eccentricity e and semi-major axis a – (bottom) for a fictitious planet widely orbiting a 16 Cyg B-like star when $e = 0.7$ (initial conditions given in the text).

is 100 billion years for orbit 1 (figure 4, $e = 0.7$, wide planetary orbit, $X_o = 0.325$ and V_o = -1.45 – see fig. 2), and only 25 billion years for orbit 2 (figure 5, $e = 0.95$, close planetary orbit, $X_o = 0.075$ and V_o = -2.0 – see fig. 3), the latter value being shorter due to the integration variable time step which is much more little for a close orbit than for a much wider one.

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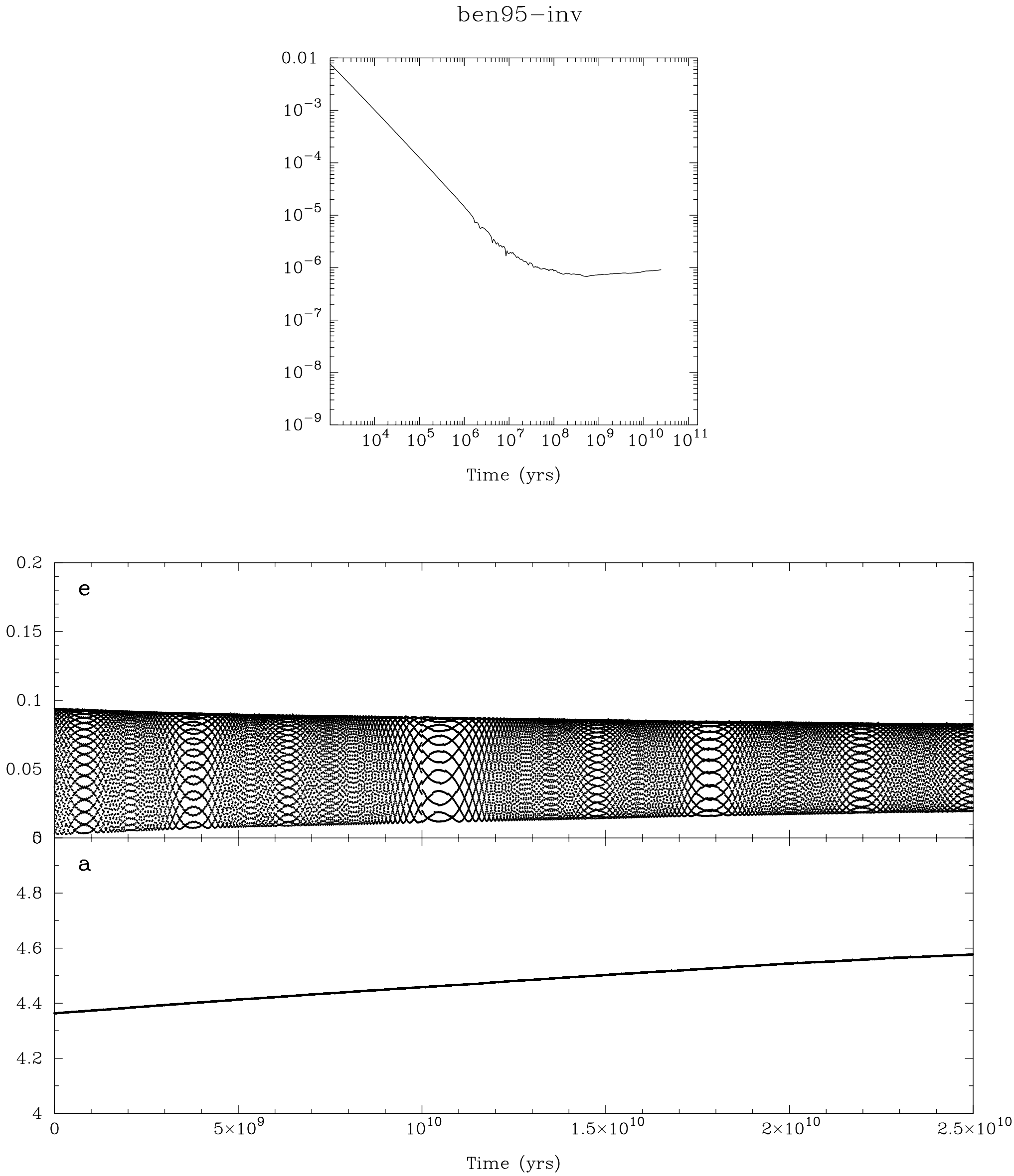

Figure 5. Greatest Lyapunov indicator, e and a for a fictitious planet closely orbiting a 16 Cyg B-like star when $e = 0.95$ (initial conditions given in the text).

For the two orbits, the greatest Lyapunov indicator, after having decreased regularly during almost 1 (Fig. 5) to 10 (Fig. 4) millions years, begins to deviate and even increase – and further decrease again for orbit 1 –, which is clearly an indicator of chaos; nevertheless, the planetary orbital elements keep a regular behaviour: e oscillates with a relative large amplitude but without any dangerous jump; besides, the amplitude of a is low – very low for orbit 2, which shows moreover a slow drift. Therefore, the chaos should be fairly weak and seems to stay confined during the integration time span. However, the question: do we have bounded chaos or stickiness? is still open dynamically speaking. But practically, after the death of the star, and its passage through the red giant stage, no interesting planet could probably survive (although the detection of exoplanets around pulsars could incline us to be cautious about such an assertion).

5. Conclusion

Our main result is that the influence of the binary's orbital eccentricity on the limit between regularity and chaos is therefore stronger in the latter case (wider planetary orbits) than for the first one (closer planetary orbits). Of course, the existence of stable orbits does not mean that there are actual planets on these trajectories, for this is the problem of formation of planets in a young binary's environment (see, e.g., Santos *et al.* 2004).

Let us conclude by two remarks. Firstly, our model is a planar one, but it has been shown since the seventies and by modern calculations (Pilat-Lohinger *et al.* 2003) that instability arises only for high values of the inclination. Finally, the existence of the gravitational field of other celestial bodies, such as stars or giant interstellar clouds, as well as galactic tides, may disturb this stability (regularity or confined chaos). But the present study is made within the frame of the three-body dynamical model and any more-body problem is, as Kipling said, another story.

Acknowledgements

This research is supported by “Action spécifique Planètes Extra-solaires” of the “Programme National de Planétologie” (C.N.R.S.: french “Centre National de la Recherche Scientifique”). Part of this work has been performed using the computing facilities provided by the program “Simulations Interactives et Visualisation en Astronomie et Mécanique (SIVAM)”. This research has made use of the SIMBAD database, operated at CDS, Strasbourg, France.

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