Macroeconomic Dynamics, **21**, 2017, 259–277. Printed in the United States of America.
doi:10.1017/S1365100515000358

NONSEPARABLE PREFERENCES DO NOT RULE OUT AGGREGATE INSTABILITY UNDER BALANCED-BUDGET RULES: A NOTE

NICOLAS ABAD AND THOMAS SEEGMULLER
Aix-Marseille University (Aix-Marseille School of Economics)–CNRS–EHESS

ALAIN VENDITTI
Aix-Marseille University (Aix-Marseille School of Economics)–CNRS–EHESS and EDHEC Business School

We investigate the role of nonseparable preferences in the occurrence of macroeconomic instability under a balanced-budget rule where government spending is financed by a tax on labor income. Considering a one-sector neoclassical growth model with a large class of nonseparable utility functions, we find that expectations-driven fluctuations occur easily when consumption and labor are Edgeworth substitutes or weak Edgeworth complements. Under these assumptions, an intermediate range of tax rates and a sufficiently low elasticity of intertemporal substitution in consumption lead to instability.

Keywords: Indeterminacy, Expectations-driven Business Cycles, Labor Income Taxes, Balanced-Budget Rule, Nonseparable Preferences

1. INTRODUCTION

According to Schaechter et al. (2012), around 60 countries, mostly advanced, have adopted some kind of balanced-budget rule either at the national or at the supranational level in 2012. Balanced-budget rules can be specified as overall balance, structural or cyclically adjusted balance, or balance "over the cycle." Since the beginning of the Great Recession in 2008, the first type of rule has been widely discussed in particular in Europe and adopted by some countries

We thank two anonymous referees and an Associate Editor, together with Stefano Bosi, Raouf Boucekkine, Hippolyte d'Albis, Jean-Michel Grandmont, Cuong Le Van, Carine Nourry, Xavier Raurich, Chrissy Giannitsarou, and Roger Farmer for useful comments and suggestions. This paper also benefited from presentations at the "Conference in honor of Cuong Le Van," MSE-PSE-University of Paris 1, Paris, December 2011; OLG Days 2012, Marseille; PET 2012, Taipei and LAGV 2012, Marseille. Address correspondence to: Alain Venditti, GREQAM, 2 rue de la Charité, 13002 Marseille, France; e-mail: alain.venditti@univ-amu.fr.

 1365-1005/16

such as Germany or Switzerland as a "Golden Rule." But this type of balanced-budget rule does not have economic stabilization features. Following standard Keynesian arguments, such balanced-budget rules are not worthwhile, as they amplify business cycles. During booms, governments cut taxes and set higher public spending, whereas in recessions, taxes are increased and fiscal policies contract aggregate demand.

An additional argument along a similar line has been stressed in the seminal contribution of Schmitt-Grohé and Uribe (1997) (SGU). They show that a balanced-budget rule may be a source of aggregate instability, as it may generate a strong volatility of agents' expectations. In a standard Ramsey model with a constant stream of government expenditures financed by a distortionary tax on labor income, any increase in the expected tax rate implies a reduction in future labor supply and therefore in capital returns. Current investment decreases, so that households work less. Under a balanced budget, the tax rate is decreasing in the tax base. Facing a decreasing labor income, the government then has to increase the tax rate to maintain the balanced budget, and expectations are self-fulfilling. Further contributions, such as Guo and Lansing (1998), show that a progressive income tax stabilizes the economy by ruling out expectations-driven fluctuations generated by productive externalities. Assuming instead constant income taxes on capital and labor income financing endogenously determined government spending, Guo and Harrison (2004) show that aggregate instability never occurs. Hence, these contributions confirm that the destabilizing mechanism of balanced-budget rules strongly relies on the regressiveness of the labor income tax.

The main conclusions of SGU are established under the assumption of additively separable preferences with infinitely elastic labor and a Cobb–Douglas technology. In a subsequent work, however, Linnemann (2008) shows that the destabilizing effect of balanced-budget rules is not robust to the consideration of nonseparable preferences. He considers the nonseparable specification of King et al. (1988) (KPR), which is compatible with balanced growth and constant labor, and he shows that when the tax rate on wage income is calibrated to match realistic values, expectations-driven fluctuations and aggregate instability are ruled out. However, Linneman (2008) introduces in his analysis an additional restriction that strongly limits the set of KPR preferences he considers. He assumes that the elasticity of intertemporal substitution in consumption is lower than one, which implies that consumption and labor are strong Edgeworth complements.

In this paper, our aim is to prove that as long as a sufficiently large set of nonseparable preferences are considered, the destabilizing effect of balanced-budget rules under labor-income taxes exhibited by SGU is a robust property. We prove that aggregate instability under balanced-budget rules can easily be obtained with the nonseparable preferences commonly used in the macroeconomic literature, provided that the Edgeworth complementarity between consumption and labor is not too strong. This is explored in a neoclassical infinite-horizon growth model embedding most the popular nonseparable preferences used by macroeconomists. We consider two classes of nonseparable utility functions:

(i) a linearly homogeneous specification and (ii) the Jaimovich–Rebelo (2008) (JR) specification, where the degree of income effect can be controlled and which admits as polar cases the Greenwood–Hercowitz–Huffman (1988) (GHH) utility function (with no income effect) and the KPR formulation (with a maximal degree of income effect). A generalized production function describes the technology of the firms in order to cover a large range of substitutability between capital and labor.[1] The government is characterized by the same balanced-budget rule considered by SGU, for which the tax rate is countercyclical with respect to the tax base.

We prove that expectations-driven fluctuations due to a labor income tax and a balanced-budget rule are a robust outcome under nonseparable preferences. More precisely, local indeterminacy is likely to occur with preferences characterized by either Edgeworth substitutability or a low enough Edgeworth complementarity between consumption and labor. These properties are always satisfied if the utility function is linearly homogeneous, but require a high enough degree of income effect for JR preferences. We illustrate our results numerically for each set of preferences under a standard parameterization based on quarterly data. We show that the likelihood of indeterminacy increases as labor and consumption exhibit more Edgeworth substitutability and that a large set of OECD countries may experience aggregate instability under plausible parameterizations.

The rest of the paper is organized as follows. In the next section, we present the model. In Section 3, we prove the existence of a normalized steady state. In Section 4, we derive our main results, and we provide numerical illustrations in Section 5. The economic interpretations are discussed in Section 6. Section 7 concludes and an Appendix presents the proofs of our main results, whereas the proofs of technical results are given in a Technical Appendix available upon request.

2. THE MODEL

We consider an infinite-horizon model with three types of agents, consumers, producers, and the government, which fixes labor income taxes according to a balanced-budget rule.

2.1. Government

As in SGU, we assume that the government chooses a constant level of public spending G, which affects neither the preferences nor the technology, following a balanced-budget rule. The government expenditure is equal to the total tax revenue $\Omega(t)$ generated by a tax rate, $\tau(t)$, applied to labor income, $w(t)l(t)$, with $w(t)$ the wage rate and $l(t)$ the labor supplied:

$$G = \Omega(t) = \tau(t)w(t)l(t). \tag{1}$$

The balanced-budget rule can be written

$$\tau(t) = \frac{G}{w(t)l(t)}. \tag{2}$$

https://doi.org/10.1017/S1365100515000358 Published online by Cambridge University Press

Because government spending is constant, the tax rate decreases with respect to its tax base.

Three main arguments can be used to justify our formulation. First, some types of balanced-budget rules, either at the national or at the supranational level, have been adopted recently by many countries, mostly advanced [see Schaechter et al. (2012)]. Second, taxes on capital income are usually quite flat and thus not adjusted over the business cycle. Third, as shown by Lane (2003), government spending is countercyclical with respect to GDP in most OECD countries. As already discussed by SGU and the literature mentioned earlier, the likelihood of expectations-driven fluctuations increases as the labor income tax is more regressive. It is easy to check that this regressivity is reinforced if we include a constant tax on capital income and/or countercyclical government spending in the rule (2). Because the scope of this paper is investigation of the effect of nonseparable preferences on the main mechanism leading to indeterminacy, we focus on the simple rule considered in SGU.

2.2. Households

We consider an economy populated by a large number of identical infinitely lived agents. We assume without loss of generality that population is constant and normalized to one. At each point in time, a representative agent supplies an amount of labor $l \in [0, \bar{l}]$ elastically, with $\bar{l} > 1$ his time endowment. He then derives utility from consumption c and leisure $\mathcal{L} = \bar{l} - l$ according to the instantaneous utility function $U(c, \mathcal{L}/B)$, where B is a constant scaling parameter, which satisfies

Assumption 1. $U(c, \mathcal{L}/B)$ is twice differentiable, increasing with respect to each argument, and concave. Moreover, $\frac{U_{c\mathcal{L}}}{U_c}\frac{\mathcal{L}}{B} - \frac{U_{\mathcal{L}\mathcal{L}}}{U_{\mathcal{L}}}\frac{\mathcal{L}}{B} \neq 1$, $\lim_{X\to 0} XU_X(c, X)/U_c(c, X) = 0$, and $\lim_{X\to +\infty} XU_X(c, X)/U_c(c, X) = +\infty$, or $\lim_{X\to 0} XU_X(c, X)/U_c(c, X) = +\infty$ and $\lim_{X\to +\infty} XU_X(c, X)/U_c(c, X) = 0$.

This assumption is a sufficient condition for the existence of a normalized steady state. Within the class of utility functions satisfying these properties, we consider two different specifications of nonseparable preferences commonly used in the literature:

(i) A linearly homogeneous utility function $U(c, \mathcal{L}/B)$ characterized by the share of consumption within total utility $\alpha(c, \mathcal{L}/B) \in (0, 1)$ defined by

$$\alpha(c, \mathcal{L}/B) = \frac{U_c(c, \mathcal{L}/B)c}{U(c, \mathcal{L}/B)}, \tag{3}$$

whereas the share of leisure is given by $1 - \alpha(c, \mathcal{L}/B)$. As an illustration of such a function, we have the CES specification,

$$U(c, \mathcal{L}/B) = [\alpha c^{\phi} + (1 - \alpha)(\mathcal{L}/B)^{\phi}]^{1/\phi},$$

where $\phi < 1$ and $1/(1-\phi)$ is the elasticity of substitution between consumption and leisure. We derive in this case $\alpha(c, \mathcal{L}/B) = \frac{\alpha c^{\phi}}{[\alpha c^{\phi}+(1-\alpha)(\mathcal{L}/B)^{\phi}]}$.[2]

(ii) A Jaimovich–Rebelo (2008) formulation such that

$$U(c, \mathcal{L}/B) = \frac{[c + (\mathcal{L}/B)^{1+\chi} c^{\gamma}]^{1-\theta}}{1-\theta}, \tag{4}$$

with $\theta, \chi > 0$ and $\gamma \in [0, 1]$. The parameter γ controls the degree of income effect, which makes it possible to encompass two standard formulations. On one hand, in the absence of an income effect ($\gamma = 0$), the GHH formulation is obtained and yields a labor supply independent of consumption. On the other hand, when the income effect is the largest ($\gamma = 1$), the utility function corresponds to the KPR formulation, which is compatible with balanced growth and stationary hours worked.

Note that these utility functions satisfy normality of consumption and labor. In addition, the JR specification requires additional restrictions to satisfy the concavity conditions when $\gamma \neq 0$ (see Section 4.2 for further details). We also introduce the definition of Edgeworth substitutability between consumption and labor:

DEFINITION 1. *If the marginal utility of consumption is increasing in leisure such that $U_{c\mathcal{L}}(c, \mathcal{L}/B) > 0$, then consumption and labor are Edgeworth substitutes. Otherwise, if $U_{c\mathcal{L}}(c, \mathcal{L}/B) < 0$, consumption and labor are Edgeworth complements.*

We conclude that with linearly homogeneous preferences, consumption and labor are always Edgeworth substitutes, because $U_{c\mathcal{L}}(c, \mathcal{L}/B) > 0$, but with JR preferences, Edgeworth substitutability between consumption and labor requires $\gamma > \theta$ whereas Edgeworth complementarity is obtained when $\gamma < \theta$. Note that Linnemann (2008) restricts his attention to a KPR utility function with $\theta > 1$, which immediately implies strong Edgeworth complementarity between consumption and labor.

The maximization program of the representative agent is given by

$$\begin{aligned} \max_{c(t),l(t),K(t)} \quad & \int_{t=0}^{+\infty} e^{-\rho t} U\left(c(t), (\bar{l} - l(t))/B\right) \\ \text{s.t.} \quad & c(t) + \dot{K}(t) + \delta K(t) = r(t)K(t) + (1-\tau(t))w(t)l(t) \\ & K(0) = K_0 > 0 \text{ given}, \end{aligned} \tag{5}$$

where $r(t)$ is the rental rate of capital, $\rho > 0$ the discount rate, $K(t)$ the capital stock, and $\delta > 0$ the depreciation rate of capital. Moreover, we assume that each household considers as given the tax rate $\tau(t)$. Let us introduce the Hamiltonian in current value:

$$\mathcal{H} = U[c(t), (\bar{l} - l(t)]/B) + \lambda(t)\{r(t)K(t) + [1-\tau(t)]w(t)l(t) - c(t) - \delta K(t)\},$$

with $\lambda(t)$ the shadow price of capital $K(t)$. Considering the prices and the tax rate $\tau(t)$ as given, we derive the following first-order conditions:

$$U_c(c(t), (\bar{l} - l(t))/B) = \lambda(t), \tag{6}$$

$$(1/B)U_{\mathcal{L}}(c(t), (\bar{l} - l(t))/B) = \lambda(t)(1 - \tau(t))w(t), \tag{7}$$

$$\dot{\lambda}(t) = -\lambda(t)[r(t) - \rho - \delta], \tag{8}$$

Any solution with $K(0) = K_0$ should also satisfy the transversality condition

$$\lim_{t \to +\infty} e^{-\rho t} \lambda(t) K(t) = 0. \tag{9}$$

2.3. Firms

We consider a competitive economy with a continuum of firms of unit size that produce a single good Y. Firms use capital K and labor l through a constant-returns-to-scale production function $Y = AF(K, l)$, with A a scaling parameter. Let $a = K/l$, so that the production function can be written $Y/l = Af(a)$.

Assumption 2. $f(a)$ is twice differentiable, increasing, and concave.

From profit maximization, we obtain the wage rate $w(t)$ and the rental rate of capital $r(t)$ as

$$r(t) = Af'(a(t)), \tag{10}$$

$$w(t) = A[f(a(t)) - a(t)f'(a(t))]. \tag{11}$$

We also compute the share of capital in total income,

$$s(a) = \frac{af'(a)}{f(a)} \in (0, 1), \tag{12}$$

and the elasticity of capital–labor substitution,

$$\sigma(a) = -\frac{(1 - s(a))f'(a)}{af''(a)} > 0. \tag{13}$$

Our formulation admits as particular cases the technologies with a constant elasticity of capital–labor substitution, such as the Cobb–Douglas function with $\sigma(a) = 1$ or the CES specification with $\sigma(a) = \sigma \neq 1$ for all $a > 0$. As a result, our model encompasses the formulation of Ghilardi and Rossi (2014), where the results of SGU are extended to a CES technology. We introduce the following restriction:

Assumption 3. Capital and labor are sufficiently substitutes with $\sigma(a) > s(a)$.

This ensures that labor income is increasing with respect to the amount of labor.

3. EQUILIBRIUM

The following system of differential equations in K and λ, the shadow price of capital, describes the dynamics of the economy:[3]

$$\begin{aligned}\dot{K}(t) &= r[K(t),\lambda(t)]K(t) + \{1-\tau[K(t),\lambda(t)]\}w[K(t),\lambda(t)]l[K(t),\lambda(t)]\\ &\quad -\delta K(t) - c[K(t),\lambda(t)]\\ \dot{\lambda}(t) &= -\lambda(t)\{r[K(t),\lambda(t)]-\rho-\delta\}\end{aligned} \quad (\mathbf{14})$$

An intertemporal equilibrium is then a path $\{K(t),\lambda(t)\}_{t\geq 0}$, with $K(0)=K_0$, that satisfies equations (14) and the transversality condition (9).

A steady state is a solution (a^*, l^*, c^*, τ^*), with $a^* = K^*/l^*$, that satisfies the balanced-budget rule (2), equations (10) and (11), the first-order conditions of the households' program (6)–(8), and equations (14) with $\dot{K} = \dot{\lambda} = 0$. We use the scaling parameters A and B to ensure the existence of a normalized steady state (NSS), $a^* = 1$ and $l^* = 1$, which remains invariant with respect to preference and technological parameters.

PROPOSITION 1. *Let Assumptions 2.2 and 2.3 hold. Then there exist unique values A^* and B^* such that when $A = A^*$ and $B = B^*$, $(a^*, l^*) = (1, 1)$ is a NSS.*

Proof. See Section 2 of the Technical Appendix. ■

Let us introduce the following elasticities:

$$\begin{aligned}\varepsilon_{cc} &= -\frac{U_c(c,\mathcal{L})}{U_{cc}(c,\mathcal{L})c},\ \varepsilon_{lc} = -\frac{U_{\mathcal{L}}(c,\mathcal{L})}{U_{\mathcal{L}c}(c,\mathcal{L})c},\ \varepsilon_{cl} = -\frac{U_c(c,\mathcal{L})}{U_{c\mathcal{L}}(c,\mathcal{L})l},\\ \varepsilon_{ll} &= -\frac{U_{\mathcal{L}}(c,\mathcal{L})}{U_{\mathcal{L}\mathcal{L}}(c,\mathcal{L})l}.\end{aligned} \quad (\mathbf{15})$$

Normality of consumption and leisure requires that $\frac{1}{\varepsilon_{cc}} - \frac{1}{\varepsilon_{lc}} \geq 0$ and $\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}} \geq 0$ and holds for both utility functions we consider. Concavity requires $\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}} \geq 0$. This property is satisfied for linearly homogeneous preferences, but the JR formulation requires further restrictions in the neighboorhood of the NSS (see the proof of Proposition 3). According to Definition 1, note that when ε_{cl} and ε_{lc} are negative (positive), consumption and labor are Edgeworth substitutes (complements).

In the rest of the paper, we evaluate all the shares and elasticities previously defined at the NSS. From (3), (12), and (13), we denote $\alpha(c^*, (\bar{l}-1)/B^*) = \alpha$, $s(1) = s$, and $\sigma(1) = \sigma$.

4. INSTABILITY WITH NONSEPARABLE PREFERENCES

This section provides simple conditions for the existence of local indeterminacy with nonseparable preferences. Using the trace and the determinant of the Jacobian matrix associated with the linearized dynamic system evaluated at the NSS, we immediately derive a necessary condition for the existence of indeterminacy whatever the specification of the utility function:

LEMMA 1. *Let Assumptions 1–3 hold. A necessary condition for local indeterminacy of the NSS is* $\tau > \underline{\tau}$, *with*

$$\underline{\tau} = \frac{\frac{s}{\sigma} + \varepsilon_{cc}\left(\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}\right)}{1 + \varepsilon_{cc}\left(\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}\right)}. \tag{16}$$

Proof. See Section 3 of the Technical Appendix. ■

Given that the term $\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}$ measures the degree of concavity of the utility function, we conclude that stronger degrees of concavity imply that indeterminacy requires higher tax rates on labor income. Moreover, for any given specification of preferences, the lower bound $\underline{\tau}$ increases with the capital share of income and decreases with the elasticity of capital–labor substitution. In the particular case of additively separable preferences with infinitely elastic labor ($1/\varepsilon_{ll} = 1/\varepsilon_{cl} = 1/\varepsilon_{lc} = 0$), condition (16) becomes $\tau > s/\sigma$. The same necessary condition has been derived by Ghilardi and Rossi (2014) in this particular case with a CES technology.

4.1. Linearly Homogeneous Preferences

A linearly homogeneous specification is characterized by $\varepsilon_{lc}, \varepsilon_{cl} < 0$ such that consumption and labor are always Edgeworth substitutes. Moreover, notice that $\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}} = 0$ and we therefore obtain $\underline{\tau} = \frac{s}{\sigma}$ from (16). We derive the next proposition:

PROPOSITION 2. *Under Assumptions 2.2–2.3, let* $U(c, \mathcal{L}/B)$ *be linearly homogeneous and* $\underline{\tau} = \frac{s}{\sigma}$. *There exist* $\bar{\rho} \in (0, +\infty]$, $\bar{\tau} \in (\underline{\tau}, 1)$, *and* $\bar{\varepsilon}_{cc} > 0$ *such that the NSS is locally indeterminate if and only if* $\rho \in (0, \bar{\rho})$, $\varepsilon_{cc} < \bar{\varepsilon}_{cc}$, *and* $\tau \in (\underline{\tau}, \bar{\tau})$.

Proof. See Appendix A.1. ■

We show that local indeterminacy easily arises under linearly homogeneous preferences for tax rates within a bounded interval. Note that in order to have $\underline{\tau} < \bar{\tau}$, we need a low enough elasticity of intertemporal substitution in consumption, i.e., $\varepsilon_{cc} < \bar{\varepsilon}_{cc}$, where $\bar{\varepsilon}_{cc}$ can be arbitrarily large when the share of consumption in total utility α is close to unity. Finally, the upper bound on the elasticity of intertemporal substitution in consumption has important implications for the wage elasticity of labor, ε_{ll}. It can be shown that $\varepsilon_{cc} < \bar{\varepsilon}_{cc}$ implies a sufficiently low wage elasticity of labor, $\varepsilon_{ll} < \bar{\varepsilon}_{ll}$.

4.2. Jaimovich–Rebelo Preferences

Considering now JR preferences, we show that local indeterminacy also arises for tax rates within a bounded interval. We get the following proposition:

PROPOSITION 3. *Under Assumptions 2.2–2.3, let* $U(c, \mathcal{L}/B)$ *be given by* (4). *There is a critical value* $\underline{\gamma} \in (0, 1)$ *such that for any given* $\gamma \in (\underline{\gamma}, 1]$, *there exist* $\bar{\rho} \in (0, +\infty]$, $\underline{\theta} \in (0, \gamma)$, $\bar{\theta} \in (\underline{\theta}, +\infty]$, $\underline{\sigma} \in (s, +\infty)$, $\underline{\tau} \in (0, 1)$, *and* $\bar{\tau} \in (\underline{\tau}, 1)$ *such that the NSS is locally indeterminate if and only if* $\rho \in (0, \bar{\rho})$, $\theta \in (\underline{\theta}, \bar{\theta})$, $\sigma > \underline{\sigma}$, *and* $\tau \in (\underline{\tau}, \bar{\tau})$.

Proof. See Appendix A.2. ■

Because $\underline{\theta} \in (0, \gamma)$, Proposition 3 shows that indeterminacy occurs when consumption and labor are either Edgeworth substitutes or weak Edgeworth complements. More precisely, local indeterminacy is ruled out with a GHH specification characterized by the absence of an income effect ($\gamma = 0$) and strong Edgeworth complementarity. In contrast, with KPR preferences ($\gamma = 1$), consumption and labor are Edgeworth substitutes if $\theta < 1$. In this case, the existence of a range of destabilizing tax rates is ensured provided that the elasticity of intertemporal substitution in consumption is not too high ($\theta < \bar{\theta}$). Otherwise, when consumption and labor become weak Edgeworth complements, indeterminacy may still hold but requires higher tax rates. This conclusion explains therefore the determinacy result of Linnemann (2008), because he assumes KPR preferences ($\gamma = 1$) with $\theta > 1$, i.e., strong Edgeworth complementarity between consumption and labor.

5. NUMERICAL ILLUSTRATIONS

To illustrate our results, we use the evidence on effective tax rates on labor income given by Trabandt and Uhlig (2011) by choosing countries representative of four levels of tax rates: The United States (28%) for the lowest tax rates, Germany (41%) and France (46%), respectively, for lower intermediate and upper intermediate rates, and Sweden (56%) for the highest tax rates. According to the empirical literature, there is no clear agreement on the size of the elasticity of capital–labor substitution. On one hand, the higher estimates of this elasticity belong to the range (1.24, 3.24) as shown in Duffy and Papageorgiou (2000) and Karagiannis et al. (2005). On the other hand, Klump et al. (2007, 2012), León-Ledesma et al. (2010), and McAdam and Willman (2013) give an interval of (0.4,0.9) for lower estimates. There is also no consensus on the elasticity of intertemporal substitution in consumption. Several contributions provide the range (0.2,0.8) [see Kocherlakota (1996) and Campbell (1999)], whereas Mulligan

Table 1. Calibration of the model

Parameter	Value
ρ	0.01
δ	0.025
s	0.3
ε_{cc}	[0.66,2.5]
ε_{ll}	2.5
σ	[0.8,1.4]

(2002), Vissing-Jorgensen and Attanasio (2003), and more recently Gruber (2013) show evidence for higher estimates with an interval (2,3). Finally, Rogerson and Wallenius (2009) investigate aggregate participation in the labor market at the macro level and find that the wage elasticity of labor belongs to the range (2.25, 3).

From this discussion preceding and considering calibrations based on quarterly data, Table 1 gives the values of parameters adopted.

Note that we consider values of the elasticity of capital–labor substitution σ higher than the share of labor $1 - s$. We do not cover the lower estimates of León-Ledesma et al. (2010) and McAdam and Willman (2013). Two reasons motivate this choice. First, these studies consider biased technological change, which is not present in our model. Second, as recently documented by Piketty and Zucman (2014), the share of capital in income is increasing with respect to the capital/income ratio, which implies that $\sigma > 1$. Although we do not reject the possibility of $\sigma < 1$, assuming that $\sigma > 1 - s$ ensures the property of increasing capital income, $r(t)K(t)$, with respect to the stock of capital.

The parameters α for linearly homogenous preferences and θ, χ, and γ for the JR preferences are chosen to match the elasticities ε_{cc} and ε_{ll}, taking as given the estimates of tax rate of the countries considered.

When the utility function is linearly homogeneous, all countries fall into the range of indeterminacy, with the exception of the United States, where σ is low, as shown in Figure 1. Considering Proposition 2 and that $\alpha \in (0.38, 0.6)$, we find a lower bound for local indeterminacy $\underline{\tau} \in (0.21, 0.38)$, whereas the interval of the upper bound is given by $\bar{\tau} \in (0.84, 0.95)$.

With the JR formulation, our conclusions are much more contrasted, as shown in Figure 2. On one hand, when $\varepsilon_{cc} = 0.66$, no country experiences indeterminacy. Note that in this case, all countries exhibit Edgeworth complementarity between consumption and labor because $\gamma < \theta$ holds. On the other hand, when $\varepsilon_{cc} = 2.5$, indeterminacy is plausible for countries with higher taxes. In this case, we find a critical value $\underline{\gamma} \in (0.48, 0.64)$, which implies Edgeworth substitutability, as $\theta = 0.5$.[4] Furthermore, because the values of γ implied by our calibration satisfy (0.89,0.97) when $\varepsilon_{cc} = 2.5$, we find an interval of the lower bound $\underline{\tau} \in (0.29, 0.5)$, whereas the upper bound satisfies $\bar{\tau} \in (0.78, 0.81)$.

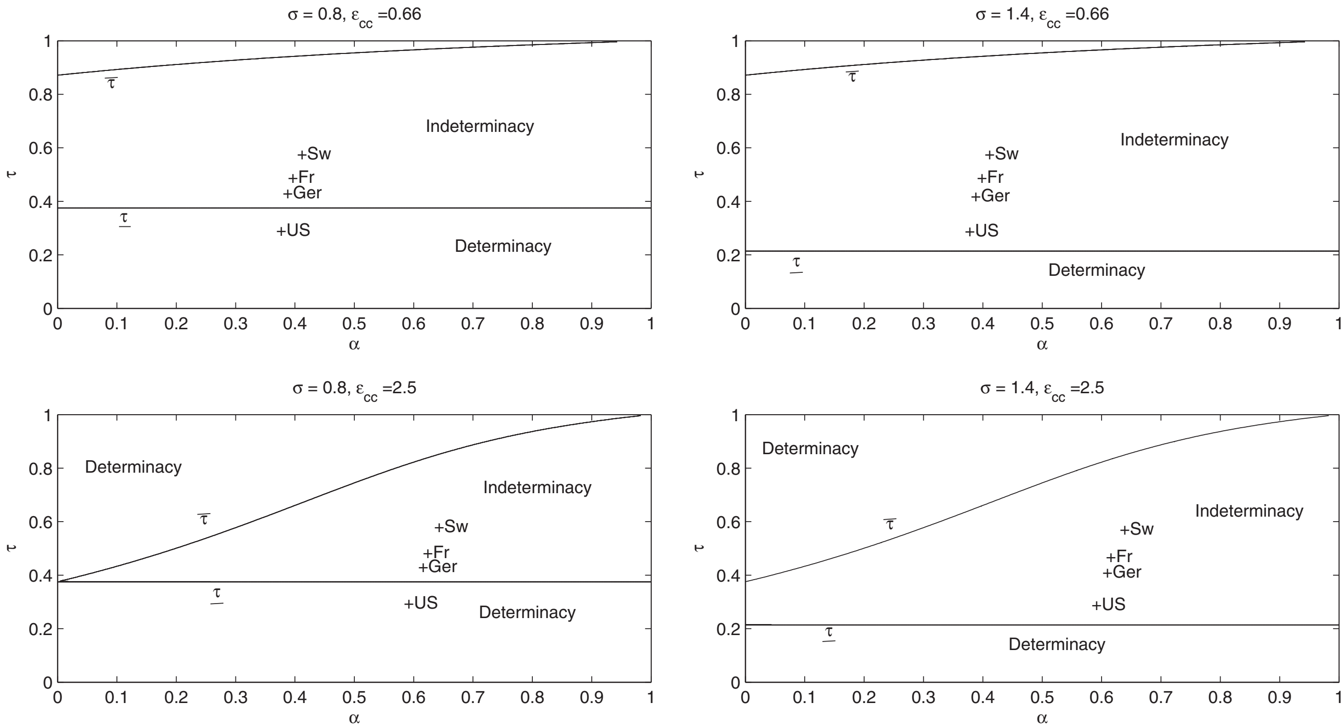

FIGURE 1. Destabilizing tax rates in the case with linearly homogeneous preferences.

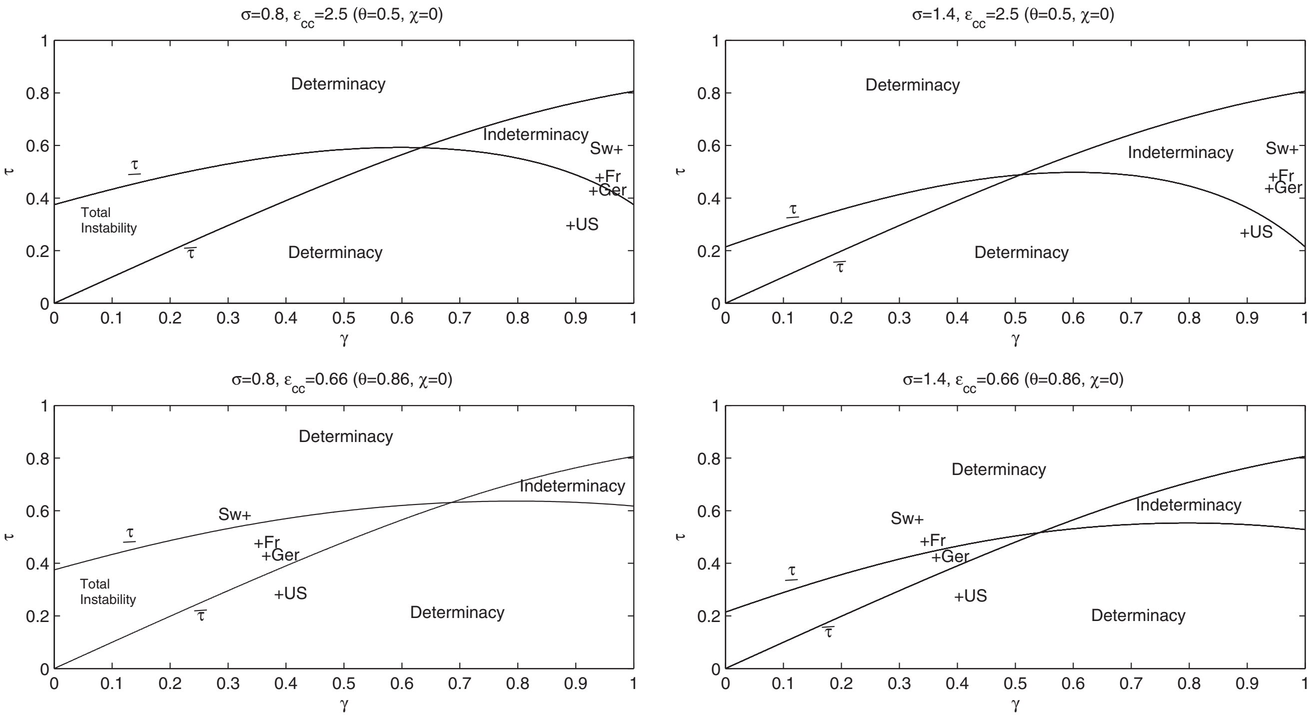

FIGURE 2. Destabilizing tax rates in the case with JR preferences.

Confirming the conclusions of SGU, our numerical exercises then show that most OECD countries may experience instability for a large set of nonseparable preferences with plausible values of structural parameters. Note, however, that we find local indeterminacy with significantly lower tax rates than SGU. It is also worth noting that although our formulation of the balanced-budget rule implies an elasticity of the tax rate with respect to wage income equal to -1, tax rates are not necessarily characterized by strong variation. Indeed, as shown in SGU, neither the first-order serial correlations, the contemporaneous correlations with output, nor the standard deviation relative to output of taxes, output, hours, and consumption is affected by the relative volatility of the sunspot shock or its correlation with the technology shock.

6. ECONOMIC INTUITION

To get an intuition for our results, let us start from the steady state and assume that agents expect an increase in the future tax rate. Following (7) and (10), future labor supply decreases and yields a lower interest rate. Consequently, current investment decreases and because households need to work less, they decrease their current labor supply below the steady state value. This generates a decline of income, leading agents to decrease their current consumption below their steady state value as well, and thus a decline of the tax base that causes the government to balance its budget by increasing tax rates. It follows that the initial agents' expectations are self-fulfilling. Nevertheless one question remains: Why do expectations-driven fluctuations occur under some classes of preferences when they are ruled out with others? The crucial point relies on the cross elasticity ε_{lc}, which needs to be negative or weakly positive; i.e., consumption and labor need to be Edgeworth substitutes or weak Edgeworth complements.

Because capital is predetermined, we have $\dot{w}/w = -(s/\sigma)\dot{l}/l$. Taking the derivative with respect to time of the equation that describes the consumption/leisure trade-off, indeterminacy occurs if the following equality is satisfied:

$$\left(\frac{s}{\sigma} + \frac{1}{\varepsilon_{ll}}\right)\frac{\dot{l}}{l} - \frac{1}{\varepsilon_{lc}}\frac{\dot{c}}{c} = \frac{\dot{\lambda}}{\lambda}. \tag{17}$$

The decrease in interest rate involves $\dot{\lambda} > 0$ because $r < \delta + \rho$. As current consumption and labor have decreased with respect to their steady state values, we conclude that self-fulfilling expectations imply that $\dot{c}/c > 0$ and $\dot{l}/l > 0$ in order to guarantee convergence toward the long-run equilibrium. Therefore, expectations will be self-fulfilling if these different effects satisfy equation (17).

As $(\frac{s}{\sigma} + \frac{1}{\varepsilon_{ll}})$ is positive, this equation is satisfied if $1/\varepsilon_{lc}$ is negative or positive but sufficiently low so that the first term on the left-hand side dominates the second one. Obviously, this is always the case with linearly homogeneous preferences because $\varepsilon_{lc} < 0$. In contrast, with JR preferences, the sign of ε_{lc} is ambiguous and

https://doi.org/10.1017/S1365100515000358 Published online by Cambridge University Press

depends in particular on the size of τ and γ, namely

$$\frac{1}{\varepsilon_{lc}} = \frac{(\theta - \gamma)(1 + \chi) + \gamma(1 - \gamma)\mathcal{C}(\tau)}{1 + \chi - (1 - \gamma)\mathcal{C}(\tau)}, \tag{18}$$

where

$$\mathcal{C}(\tau) \equiv \frac{(1 - \tau)(\delta + \rho)(1 - s)}{(1 - \tau)(\delta + \rho)(1 - s) + s\rho}$$

is a decreasing function with respect to τ. The denominator in equation (18) being positive, the sign of ε_{lc} is given by the numerator. Consider first that $\gamma > \theta$. The numerator of (18) is then negative for γ close enough to one so that the JR utility function displays Edgeworth substitutability and equation (17) is always satisfied. In contrast, when $\gamma < \theta$, the numerator is positive and consumption and labor are therefore Edgeworth complements. It follows that the intertemporal mechanisms described in equation (17) can be satisfied only if $1/\varepsilon_{lc}$ is not too large. Because $\mathcal{C}$ is decreasing in τ, this requires higher tax rates. Assuming $\gamma = 1$ and $\theta > 1$, Linneman (2008) postulates a strong complementarity between consumption and labor that prevents the existence of local indeterminacy under realistic tax rates.

7. CONCLUDING COMMENTS

This paper contributes to the debate on the destabilizing properties of balanced-budget rules. We show that, contrary to the conclusion of Linneman (2008), there exist a large set of nonseparable preferences for which balanced-budget rules with labor income taxes can easily generate macroeconomic instability and expectations-driven fluctuations. We prove that the existence of self-fulfilling expectations requires Edgeworth substitutability or weak Edgeworth complementarity between consumption and labor. Under these conditions, an intermediate range of tax rates are destabilizing if the elasticity of intertemporal substitution in consumption is sufficiently low. We then confirm and extend the initial findings of SGU showing the plausibility of balanced-budget rules as a source of macroeconomic instability for a large sample of OECD countries.

NOTES

1. Under the same additively separable preferences as in SGU, Ghilardi and Rossi (2014) already depart from the Cobb—technology, considering a CES production function, and show that aggregate instability is less likely when capital and labor are weak substitutes.
2. Note that Assumption 2.2 is not satisfied in the Cobb–Douglas case with $\phi = 0$. However, the existence of a steady state can be proved analytically using the linear relationship between consumption and leisure in the first-order conditions directly.
3. For details, the interested reader can refer to Section 1 of a Technical Appendix available upon request.
4. The interval of γ with $\underline{\gamma} \in (0.48, 0.64)$ fits the upper estimates of Kahn and Tsoukalas (2011). Using Bayesian estimates, they report a distribution of γ with mean 0.81 and a 10th–90th percentile interval of [0.69, 0.95].

REFERENCES

Campbell, John (1999) Asset prices, consumption and the business cycle. In John B. Taylor and Michael Woodford (eds.), *Handbook of Macroeconomics*, pp. 1231–1303. Amsterdam: North-Holland.

Duffy, John and Chris Papageorgiou (2000) A cross-country empirical investigation of the aggregate production function specification. *Journal of Economic Growth* 5, 87–120.

Ghilardi, Matteo and Raffaele Rossi (2014) Aggregate stability and balanced-budget rules. *Journal of Money, Credit and Banking* 46, 1785–1807.

Greenwood, Jeremy, Zvi Hercovitz, and Gregory Huffman (1988) Investment, capacity utilization and the real business cycle. *American Economic Review* 78, 402–417.

Gruber, Jonathan (2013) A tax-based estimate of the elasticity of intertemporal substitution. *Quarterly Journal of Finance* 3, 1–20.

Guo, Jang-Ting and Sharon Harrison (2004) Balanced-budget rules and macroeconomic (in)stability. *Journal of Economic Theory* 119, 357–363.

Guo, Jang-Ting and Kevin Lansing (1998) Indeterminacy and stabilization policy. *Journal of Economic Theory* 82, 481–490.

Jaimovich, Nir and Sergio Rebelo (2008) Can news about the future drive business cycles? *American Economic Review* 99, 1097–1118.

Karagiannis, Giannis, Theodore Palivos, and Chris Papageorgiou (2005) Variable elasticity of substitution and economic growth: Theory and evidence. In Claude Diebold and C. Kyrtsou (eds.), *New Trends in Macroeconomics*, pp. 21–37. Heidelberg, Germany: Springer.

Khan, Hashmat and John Tsoukalas (2011) Investment shocks and the comovement problem. *Journal of Economic Dynamics and Control* 35, 115–130.

King, Robert, Charles Plosser, and Sergio Rebelo (1988) Production, growth and business cycles II: New directions. *Journal of Monetary Economics* 21, 309–341.

Klump, Rainer, Peter McAdam, and Alpo Willman (2007) The long-term success of the neoclassical growth model. *Oxford Review of Economic Policy* 23, 94–114.

Klump, Rainer, Peter McAdam, and Alpo Willman (2012) The normalized CES production function—Theory and empirics. *Journal of Economic Surveys* 26, 769–799.

Kocherlakota, Narayana (1996) The equity premium: It's still a puzzle. *Journal of Economic Literature* 36, 42–71.

Lane, Philip (2003) The cyclical behaviour of fiscal policy: Evidence from the OECD. *Journal of Public Economics* 87, 2661–2675.

León-Ledesma, Miguel, Peter McAdam, and Alpo Willman (2010) Identifying the elasticity of substitution with biased technical change. *American Economic Review* 100, 1330–1357.

Linnemann, Ludger (2008) Balanced budget rules and macroeconomic stability with non-separable utility. *Journal of Macroeconomics* 30, 199–215

MacAdam, Peter and Alpo Willman (2013) Medium run redux. *Macroeconomic Dynamics* 17 ,695–727.

Mulligan, Casey (2002) Capital Interest and Aggregate Intertemporal Substitution. NBER working paper 9373.

Piketty, Thomas and Gabriel Zucman (2014) Capital is back: Wealth–income ratio in rich countries 1700–2010. *Quarterly Journal of Economics* 129, 1255–1310.

Rogerson, Richard and Johanna Wallenius (2009) Micro and macro elasticities in a life cycle model with taxes. *Journal of Economic Theory* 144, 2277–2292.

Schaechter, Andrea, Tidiane Kinda, Nina Budina, and Anke Weber (2012) Fiscal Rules in Response to the Crisis. IMF working paper 12/187.

Schmitt-Grohé, Stephanie and Martin Uribe (1997) Balanced-budget rules, distortionary taxes, and aggregate instability. *Journal of Political Economy* 105, 976–1000.

Trabandt, Mathias and Harald Uhlig (2011) The Laffer curve revisited. *Journal of Monetary Economics* 58, 305–327.

Vissing-Jorgensen, Annette and Orazio Attanasio (2003) Stock-market participation, intertemporal substitution and risk aversion. *American Economic Review Papers and Proceedings* 93, 383–391.

APPENDIX

A.1. PROOF OF PROPOSITION 2

Using the general expressions for the trace and the determinant, as given in Section 3 of the Technical Appendix, the trace and the determinant with linearly homogeneous preferences are given by

$$\mathcal{T} = \rho - \frac{(\rho+\delta)(1-s)\tau}{\sigma\tau - s}$$

and

$$\mathcal{D} = \frac{(1-\tau)(\delta+\rho)(1-s)}{(1-\alpha)s\sigma(\sigma\tau-s)}P(\tau),$$

with

$$P(\tau) = [(\rho+\delta)(1-s)+s\rho] + \frac{\alpha}{1-\alpha}(1-\tau)(\rho+\delta)(1-s) - \frac{\tau[(1-\tau)(\rho+\delta)(1-s)+s\rho](1-\alpha)\varepsilon_{cc}}{(1-\tau)}.$$

From the trace, we derive the lower bound on tax rate as given by (16):

$$\underline{\tau} = \frac{s}{\sigma}. \tag{A.1}$$

Under this condition, we conclude that $\mathcal{T} < 0$ when

(i) $\rho < \bar{\rho} = \frac{\delta(1-s)\tau}{\sigma\tau - s - (1-s)\tau}$ if $\sigma\tau - s - (1-s)\tau > 0$,
(ii) $\rho \in (0, +\infty)$ otherwise.

Moreover, we derive

$$\frac{\partial P(\tau)}{\partial \tau} = -\frac{\alpha}{1-\alpha}(\rho+\delta)(1-s) - \frac{(1-\alpha)\varepsilon_{cc}}{(1-\tau)^2}[(1-\tau)^2(\rho+\delta)(1-s)+s\rho] < 0.$$

The polynomial $P(\tau)$ is positive when $\tau = 0$ and negative when $\tau = 1$. Because $P(\tau)$ is monotonically decreasing in τ, there exists a unique solution $\bar{\tau} \in (0, 1)$ such that $P(\tau) > 0$ if $\tau < \bar{\tau}$. Because the denominator is positive when $\tau > \underline{\tau}$, the determinant is positive if and only if $\tau \in (\underline{\tau}, \bar{\tau})$. We finally need to check that $\underline{\tau} < \bar{\tau}$. Substituting $\underline{\tau} = \frac{s}{\sigma}$ into $P(\tau)$, the interval $(\underline{\tau}, \bar{\tau})$ is nonempty if and only if $P(\underline{\tau}) > 0$, i.e., ε_{cc} is low enough so that

$$\varepsilon_{cc} < \bar{\varepsilon}_{cc} = \frac{(1-s/\sigma)\left[(\rho+\delta)(1-s)+s\rho+(1-s/\sigma)(\rho+\delta)(1-s)\frac{\alpha}{(1-\alpha)}\right]}{(1-\alpha)\frac{s}{\sigma}[(1-s/\sigma)(\rho+\delta)(1-s)+s\rho]}. \tag{A.2}$$

■

A.2. PROOF OF PROPOSITION 3

Using the general expressions for the trace and the determinant as given in Section 3 of the Technical Appendix, the trace and the determinant with JR preferences are given by

$$\mathcal{T} = \rho - \frac{(\rho+\delta)(1-s)\tau}{\sigma\tau - s - (1-\tau)\sigma\varepsilon_{cc}\left[\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}\right]}$$

and

$$\mathcal{D} = \frac{(\rho+\delta)(1-s)\varepsilon_{cc}\left\{\frac{\gamma(1-\tau)[1+\chi-(1-\gamma)\mathcal{C}(\tau)]}{1+\chi}[(\rho+\delta)(1-s)+s\rho]+[(1-\tau)(\rho+\delta)(1-s)+s\rho][\gamma(1-\tau)\mathcal{C}(\tau)+\chi-\tau(1+\chi)]\right\}}{s\sigma\left[\sigma\tau-s-(1-\tau)\sigma\varepsilon_{cc}\left(\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}}-\frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}\right)\right]},$$

with

$$\varepsilon_{cc}\left(\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}}\right) = \frac{\theta(1+\chi)^2\left\{\chi+\gamma\mathcal{C}(\tau)\left[2-\frac{(1-\gamma)\mathcal{C}(\tau)}{1+\chi}\right]\right\}-\gamma\mathcal{C}(\tau)(\gamma+\chi)[1+\chi-(1-\gamma)\mathcal{C}(\tau)]}{\theta(1+\chi)^2-\gamma(1-\gamma)\mathcal{C}(\tau)[1+\chi-(1-\gamma)\mathcal{C}(\tau)]},$$

$$\frac{1}{\varepsilon_{cc}} = \theta\frac{1+\chi}{1+\chi-(1-\gamma)\mathcal{C}(\tau)} - \gamma(1-\gamma)\frac{\mathcal{C}(\tau)}{1+\chi},$$

and

$$\mathcal{C}(\tau) \equiv \frac{(1-\tau)(\delta+\rho)(1-s)}{(1-\tau)(\delta+\rho)(1-s)+s\rho}.$$

Note that local concavity of the utility function is ensured when $\frac{1}{\varepsilon_{cc}} \geq 0$ and $\frac{1}{\varepsilon_{cc}}\frac{1}{\varepsilon_{ll}} - \frac{1}{\varepsilon_{cl}}\frac{1}{\varepsilon_{lc}} \geq 0$. These two inequalities are satisfied if and only if

$$\theta \geq \underline{\theta}(\tau,\gamma,\chi) \equiv \frac{\gamma\mathcal{C}(\tau)(\gamma+\chi)[1+\chi-(1-\gamma)\mathcal{C}(\tau)]}{(1+\chi)^2\left\{\chi+\gamma\mathcal{C}(\tau)\left[2-\frac{(1-\gamma)\mathcal{C}(\tau)}{1+\chi}\right]\right\}}. \quad \textbf{(A.3)}$$

Note that Edgeworth substitutability is possible if $\gamma \geq \underline{\theta}(\tau,\gamma,\chi)$. This inequality is satisfied when $\gamma = 0$ and $\gamma = 1$. It follows that it will be satisfied for any $\gamma \in (0,1)$ if

$$1 \geq \frac{\mathcal{C}(\tau)(\gamma+\chi)[1+\chi-(1-\gamma)\mathcal{C}(\tau)]}{(1+\chi)^2\left\{\chi+\gamma\mathcal{C}(\tau)\left[2-\frac{(1-\gamma)\mathcal{C}(\tau)}{1+\chi}\right]\right\}}.$$

Straightforward computations show that this inequality holds for any $\gamma \in (0,1)$.

We now study the indeterminacy conditions given that equation (A.3) is satisfied. When $\gamma = 0$ (GHH case), we get

$$\mathcal{T} = \rho - \frac{\tau(\rho+\delta)(1-s)}{\sigma\tau - s - (1-\tau)\sigma\chi}, \quad \textbf{(A.4)}$$

$$\mathcal{D} = \frac{(\delta+\rho)(1-s)\varepsilon_{cc}[(1-\tau)(\rho+\delta)(1-s)+s\rho]}{\sigma s[\sigma\tau - s - \sigma(1-\tau)\chi]}[\chi - \tau(1+\chi)]. \quad \textbf{(A.5)}$$

From (A.4), a necessary condition for $\mathcal{T} < 0$ is $\tau > \underline{\tau}^0$ with

$$\underline{\tau}^0 = \frac{\frac{s}{\sigma}+\chi}{1+\chi}.$$

In equation (A.5), the condition $\tau > \underline{\tau}^0$ implies a positive denominator. The sign of the determinant is therefore determined by the second factor of (A.5), i.e., $\chi - \tau(1+\chi)$. This expression is positive if and only if $\tau < \bar{\tau}^0$ with

$$\bar{\tau}^0 = \frac{\chi}{1+\chi},$$

which is lower than $\underline{\tau}^0$. Because $\bar{\tau}^0 < \underline{\tau}^0$, indeterminacy is ruled out.

When $\gamma = 1$ (KPR case), we now get

$$\mathcal{T} = \rho - \frac{(\rho+\delta)(1-s)\tau}{\sigma G(\tau)}$$

and

$$\mathcal{D} = \frac{(\rho+\delta)(1-s)\varepsilon_{cc}}{s\sigma G(\tau)} P(\tau),$$

with

$$G(\tau) = \tau - \frac{s}{\sigma} - (1-\tau)\chi - (1-\tau)\mathcal{C}(\tau)\left(2 - \frac{1}{\theta}\right)$$

and

$$P(\tau) = [(1-\tau)(\rho+\delta)(1-s) + s\rho][\chi - \tau(1+\chi)] \\ + (1-\tau)^2(\rho+\delta)(1-s) + (1-\tau)[(\rho+\delta)(1-s) + s\rho].$$

Indeterminacy requires that both $G(\tau)$ and $P(\tau)$ are positive. In order to get $G(\tau) > 0$, one can use Lemma 1. The lower bound $\underline{\tau}^1$ as given in (16) is implicitly given by $\underline{\tau}^1 = h(\underline{\tau}^1)$ with

$$h(\tau) = \frac{\frac{s}{\sigma} + \chi + (2 - \frac{1}{\theta})\mathcal{C}(\tau)}{1 + \chi + (2 - \frac{1}{\theta})\mathcal{C}(\tau)}. \tag{A.6}$$

We derive that $h(0) > 0$, whereas $h(1) = \frac{s/\sigma + \chi}{1+\chi} < 1$. There exists therefore $\underline{\tau}^1 \in (0,1)$ such that $G(\tau) > 0$ if and only if $\tau > \underline{\tau}^1$. Moreover, considering the expression of $G(\tau)$, we can show that $\underline{\tau}^1$ is unique. We directly observe that when ρ tends to zero, the trace is negative if $\tau > \underline{\tau}^1$. There exists therefore $\bar{\rho}^1 > 0$ such that $\mathcal{T} < 0$ if and only if $\tau > \underline{\tau}^1$ and $\rho \in (0, \bar{\rho}^1)$.

For the determinant, $P(\tau)$ is positive when $\tau = 0$ and negative when $\tau = 1$. Because $P(\tau)$ is strictly decreasing in $\tau \in (0,1)$, there exists a unique $\bar{\tau}^1 \in (0,1)$ such that $P(\tau) > 0$ if and only if $\tau < \bar{\tau}^1$. The determinant is therefore positive if and only if $\tau \in (\underline{\tau}^1, \bar{\tau}^1)$. The condition $\underline{\tau}^1 < \bar{\tau}^1$ is satisfied if and only if $P(\underline{\tau}^1) > 0$. Note that substituting $\theta = \underline{\theta}^1 \equiv \underline{\theta}(\tau, 1, \chi)$ in (A.6), we get

$$\underline{\tau}^1 = \frac{s}{\sigma}.$$

When P(τ) is evaluated at $\tau = \frac{s}{\sigma}$, we obtain

$$P(\frac{s}{\sigma}) = [(1 - \frac{s}{\sigma})(\rho+\delta)(1-s) + s\rho][\chi - \frac{s}{\sigma}(1+\chi)] + (1 - \frac{s}{\sigma})^2(\rho+\delta)(1-s) \\ + (1 - \frac{s}{\sigma})[(\rho+\delta)(1-s) + s\rho].$$

https://doi.org/10.1017/S1365100515000358 Published online by Cambridge University Press

On one hand, if $\sigma{=}s$, we get $P(\frac{s}{\sigma}) = -s\rho < 0$. On the other hand, when σ tends to $+\infty$, $P(\frac{s}{\sigma})$ is positive. There exists therefore $\underline{\sigma}^1 \in (s, +\infty)$ such that $\underline{\tau}^1 < \bar{\tau}^1$ if and only if $\theta = \underline{\theta}^1$ and $\sigma > \underline{\sigma}^1$. By a continuity argument, there exist therefore $\bar{\theta}^1 \in (\underline{\theta}^1, +\infty]$ and $\underline{\sigma}^1 \in (s, +\infty)$ such that $\underline{\tau}^1 < \bar{\tau}^1$ if and only if $\theta \in [\underline{\theta}^1, \bar{\theta}^1)$ and $\sigma > \underline{\sigma}^1$.

Given that indeterminacy occurs when $\gamma = 1$, $\tau \in (\underline{\tau}^1, \bar{\tau}^1)$, $\theta \in [\underline{\theta}^1, \bar{\theta}^1)$, $\rho \in (0, \bar{\rho}^1)$, and $\sigma > \underline{\sigma}^1$, but is ruled out when $\gamma = 0$, there exists therefore $\underline{\gamma} \in (0, 1)$ such that for any $\gamma \in (\underline{\gamma}, 1]$, there exist $\underline{\tau} \in (0, 1)$, $\bar{\tau} \in (\underline{\tau}, 1)$, $\bar{\rho} \in (0, +\infty]$, $\bar{\theta} \in (\underline{\theta}, +\infty]$, and $\underline{\sigma} \in (s, +\infty)$ such that the NSS is locally indeterminate if and only if $\tau \in (\underline{\tau}, \bar{\tau})$, $\theta \in [\underline{\theta}, \bar{\theta})$, $\rho \in (0, \bar{\rho})$, and $\sigma > \underline{\sigma}$. ■