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WEALTH INEQUALITY AND OPTIMAL MONETARY POLICY

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We study the money-in-the-utility-function model in which agents are heterogeneous in their initial wealth. We show that the Friedman rule is not optimal even if the government uses nonlinear income taxation for redistribution. A positive nominal interest rate raises social welfare because it relaxes the incentive compatibility constraint for highly endowed agents. Although the setup is close to that of da Costa and Werning [*Journal of Political Economy* (2008) 116, 82–112], who investigate skill heterogeneity, the role of the nominal interest rate in this paper here differs from the one in their model.

Keywords: Monetary Policy, Private Information, Friedman Rule

1. INTRODUCTION

A number of authors study the Friedman rule of setting the nominal interest rate to zero in representative agent models. Chari and Kehoe (1999) show that the rule is optimal in most of the basic monetary models. On the other hand, Schmitt-Grohe and Uribe (2004) investigate a model with imperfect competition and show that a deviation from the rule raises welfare. Heer (2003) and Arseneau and Chugh (2008) consider models with search frictions and get similar results.

A recent paper by da Costa and Werning (2008) (henceforth DW) develops a heterogeneous agent model in which each agent has different skill and it is private information. DW show that if the government uses incentive-compatible nonlinear income taxation for redistribution, the Friedman rule is optimal. Hence redistributions by expansionary monetary policy lower welfare. Although DW make significant contributions to the analysis of heterogeneous-agent models, we do not know how monetary policy should respond to different kinds of heterogeneity, as Kocherlakota (2005) points out.

This paper investigates the money-in-the-utility-function model in which each agent is heterogeneous in his initial wealth and the level is private information. As in DW, the government uses nonlinear income taxation for redistribution. The setup is close to DW, but this paper mainly investigates wealth heterogeneity.[1]

The model has two types of agents, highly endowed agents and poorly endowed agents. They are ex ante identical, but receive idiosyncratic shocks on their wealth. The government maximizes expected utility of the ex ante homogeneous agents

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by choosing a nonlinear income tax (fiscal policy) and a nominal interest rate (monetary policy).

The paper shows that even if the government can use a redistributive income tax, deviations from the Friedman rule raise welfare. This result is contrary to DW's finding. A positive nominal interest rate is welfare-improving because it relaxes the incentive constraints for highly endowed agents. Although it generates a distortion between consumption and labor, this negative effect is negligible if the interest rate is small.

To see how the nominal interest rate affects the incentive constraint, let us compare a poor agent with a rich agent who mimics him. Although they receive the same labor income, the rich agent holds more money. Because the nominal interest rate makes money costly, it lowers the utility gain from deviating from the truth-telling strategy. Hence it relaxes the incentive constraint for the rich. Here the nominal interest rate plays the role of a penalty for misreporting.

In DW, the same mechanism does not work. In their model, the incentive constraints for skilled agents are binding. If a skilled agent understates his type, he holds the same amount of money as unskilled agents, whom he mimics. Therefore the nominal interest rate cannot relax the incentive constraint for him.

There is a growing list of papers that investigate the optimality of the Friedman rule in heterogeneous agent models. Bhattacharya et al. (2005, 2008) and Ireland (2005) analyze models with heterogeneity in monetary satiation levels or in endowment levels. Palivos (2005) studies an OLG model with heterogeneity in the degree of altruism. Antinolfi et al. (2007) consider an endowment economy in which there are cash agents who only use currency and credit agents who use currency and loans. All of them find that a positive nominal interest rate has distributional effects and it may improve welfare.

The limitation of these papers is that in their models, the government can use a lump-sum tax at best. If a redistributive tax is available, monetary redistributions may be unnecessary or even harmful. This paper shows that even if the government can use fiscal instruments for redistribution, the nominal interest rate is effective in a model with wealth heterogeneity. Ireland (2005) raises a question about which policies, monetary or fiscal, work effectively in redistributing income. In our model, monetary and fiscal policies are not substitutes and a mix of the two policies raises social welfare.

The result of this paper is in some sense relevant to practical policy making because it shows that the strict Friedman deflation as an average inflation target is not optimal. The main objective of the current central banks is price stabilization, although the Friedman rule implies a severe deflation. It is interesting that the framework in this model rationalizes a deviation from the rule and the result is robust to the introduction of tax policy instruments.

As Chari and Kehoe (1999) point out, the Friedman rule is related to the uniform commodity tax theorem. Cremer et al. (2001) show that in a model where agents differ in ability and endowment, commodity taxation or subsidy may improve welfare. However, their formula on the optimal commodity tax rates is complex and they do not show when the optimal tax is nonzero. Hence we cannot apply their results to our monetary model.

In the following, Section 2 describes the model. Section 3 characterizes the first-best allocation. Section 4 shows the nonoptimality of the Friedman rule. Section 5 provides several extensions. Section 6 concludes the paper. Proofs are in the Appendix.

2. THE MODEL

In this section, we set up the model. It follows DW and Ljungqvist and Sargent (2001).

2.1. Preferences

Time is discrete and denoted by $t = 0, 1, 2, \ldots \infty$. There is a continuum of agents with measure one. They are ex ante identical and their instantaneous utility function is

$$U(c, m, l) = u(c, m) - g(l), \tag{1}$$

where c is consumption, m is money in real terms, l is work time, u is the utility of consumption and money balance, and g is the disutility of labor. We assume that c and m are normal goods. The intertemporal preferences of each agent are

$$(1-\beta)\sum_{t=0}^{\infty} \beta^t U(c_t, m_t, l_t), \tag{2}$$

where $\beta < 1$ is a discount factor.

The function u is concave, twice differentiable, and increasing in consumption c. For each c, agents are satiated with money m if $m = \phi(c)$, where the function ϕ satisfies $\phi(0) = 0$ and $\phi'(c) > 0$. The marginal utility of money $u_m = \partial u/\partial m$ satisfies $u_m > 0$ if $m < \phi(c)$, $u_m(c, m) < 0$ if $m > \phi(c)$, and $u_m[c, \phi(c)] = 0$. Let $\bar{u}(c) = u[c, \phi(c)]$ denote the utility function under satiation (in money). The function $\bar{u}(c)$ is also concave.

The disutility of labor g is twice differentiable, increasing, and convex and satisfies $g(0) = 0$. Production technology is linear and one unit of labor produces one unit of a single good. Here each agent has identical skill. This assumption is different from DW.

2.2. Wealth Shock

The ex ante identical agent receives an idiosyncratic shock on his initial endowment of real bonds. De Nardi et al. (1999) and Cremer et al. (2001) study similar shocks, although their models are nonmonetary. We assume that there are informational frictions between the government and the agents and that the wealth shock is private information.[2]

The private information is described by a parameter θ. It can take two values, $\theta_H > 0$ and $\theta_L < 0$, and is independent across agents. Let $\Theta = \{\theta_H, \theta_L\}$ denote a set of θ. Also, let $\Pr(\theta = \theta_i) = \pi_i > 0$ $(i = H, L)$ denote a probability distribution. We assume that θ satisfies $\mathbf{E}[\theta] = \pi_H\theta_H + \pi_L\theta_L = 0$. By the law of large numbers, the probability π_i $(i = H, L)$ is equal to the ex post density of an agent with shock θ_i [see Albanesi and Sleet (2006)]. For each $i(= H,\ L)$, a type θ_i agent is called an *individual* i.

2.3. Government Policy

After an agent receives the wealth shock, he reports to the government on the shock according to the strategy $\sigma(\theta) : \Theta \to \Theta$. We determine σ later. The government then assigns an allocation $\{x_t(\hat{\theta}),\ y_t(\hat{\theta})\}_{t=0}^{\infty}$ to an agent who pretends to be a type $\hat{\theta}$, where x_t is income and y_t is labor in period t. Labor tax at time t is equal to $y_t - x_t$. The government also determines a nominal interest rate r_t as monetary policy.

DEFINITION 1. *Government policy is a sequence* $\{\mathbf{X}_t, \mathbf{Y}_t, r_t\}_{t=0}^{\infty}$ *where* $\mathbf{X}_t = [x_t(\theta_H), x_t(\theta_L)]$ *is income,* $\mathbf{Y}_t = [y_t(\theta_H), y_t(\theta_L)]$ *is labor, and* r_t *is the nominal interest rate at time* t*. The policy is stationary if income, labor, and the nominal interest rate are time-independent.*

2.4. Consumer's Problem

This section describes the problem of a type θ agent who pretends to be type $\hat{\theta}$. Given the initial condition $\{[M_0(\theta),\ B_0(\theta)] = (0, \theta)\}$, he solves the following problem:

$$V(\hat{\theta}, \theta) \equiv \max_{\{c_t, M_{t+1}, B_{t+1}, l_t\}_{t=0}^{\infty}} \left[(1-\beta)\sum_{t=0}^{\infty} \beta^t U(c_t, m_t, l_t)\right], \tag{3}$$

$$\text{s.t. } c_t + \frac{M_{t+1}}{p_t} + \frac{B_{t+1}}{R_t} = B_t + \frac{M_t}{p_t} + x_t(\hat{\theta}), \tag{4}$$

$$l_t = y_t(\hat{\theta}). \tag{5}$$

Here $V(\hat{\theta}, \theta)$ is his value function, M_{t+1} is the nominal balance held between times t and $t+1$, p_t is the price level, $m_t = M_{t+1}/p_t$ is the real balance, B_t is the real value of bond holdings that mature at the beginning of time t, and R_t is the real rate of return on the bonds. Equation (4) is the budget constraint and equation (5) is the labor assignment.

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Here the nominal interest rate r_t is expressed as $r_t = [R_t - (p_t/p_{t+1})]/R_t$, which shows the difference between return on bonds and the one on money.[3] We must have $r_t \geq 0$.

We impose the transversality conditions on money and bonds for every agent:

$$\lim_{t\to\infty}\left(\prod_{i=0}^{t} R_i^{-1}\right) B_{t+1}(\theta) = \lim_{t\to\infty}\left(\prod_{i=0}^{t} R_i^{-1}\right)\frac{M_{t+1}(\theta)}{p_t} = 0. \quad (\mathbf{6})$$

We use equation (6) and consolidate the budget constraints as

$$\sum_{t=0}^{\infty} q_t[c_t + r_t m_t - x_t(\hat{\theta})] = \theta,$$

where q_t is the price of the good at time t. It satisfies $q_0 = 1$ and $q_t = \prod_{i=0}^{t-1} R_i^{-1}$ if $t > 0$. If we let $[c_t(\hat{\theta},\theta), m_t(\hat{\theta},\theta)]_{t=0}^{\infty}$ denote the demand function, the value function is written as $V(\hat{\theta},\theta) = (1-\beta)\sum_{t=0}^{\infty}\beta^t U[c_t(\hat{\theta},\theta), m_t(\hat{\theta},\theta), l_t(\hat{\theta})]$.

2.5. Optimal Government Policy

In this section, we formalize the optimal government policy. The objective of the government is to maximize the expected utility of each agent. From the revelation principle, we can concentrate our attention on incentive-compatible government policies. The reporting strategy $\sigma(\theta)$ is defined as $\sigma(\theta) \in \operatorname{argmax}_{\hat{\theta}\in\Theta} V(\hat{\theta},\theta)$. The policy is incentive-compatible if

$$V(\theta,\theta) \geq V(\hat{\theta},\theta) \quad \text{for all } \theta \in \Theta. \quad (\mathbf{7})$$

In this case, the strategy is truth-telling and satisfies $\sigma(\theta) = \theta$. If $\sigma(\theta) = \theta$, the value function of a type θ agent is $V(\theta,\theta)$.

The budget constraint of the government in period t is given by

$$\sum_{i=H,L}\pi_i\left[\frac{M_{t+1}(\theta_i) - M_t(\theta_i)}{p_t} + \frac{B_{t+1}(\theta_i)}{R_t} - B_t(\theta_i) + y(\theta_i) - x(\theta_i)\right] \geq G, \quad (\mathbf{8})$$

where $G > 0$ is the government expenditure. The value G is exogenous. Substituting the agent's budget constraint (4) into (8) yields the following feasibility constraint:

$$\sum_{i=H,L}\pi_i\,[y_t(\theta_i) - c_t(\theta_i,\theta_i)] \geq G \quad \text{for all } t \geq 0. \quad (\mathbf{9})$$

DEFINITION 2. *The optimal government policy* $\{\mathbf{X}_t, \mathbf{Y}_t, r_t\}_{t=0}^{\infty}$ *is a policy that maximizes the expected utility of agents* $\mathbf{E}_\theta[V(\theta,\theta)]$ *subject to the incentive compatibility constraint* (7) *and the feasibility constraint* (9).

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For simplicity, we follow DW and assume that the intertemporal price sequence satisfies

$$q_t = \beta^t. \tag{10}$$

3. FIRST-BEST POLICY

To understand the importance of the incentive constraints, let us consider the first-best allocation without these constraints. It solves the following problem:

$$(P_{\mathrm{FB}}): \max_{\{\mathbf{X}_t, \mathbf{Y}_t, r_t\}_{t=0}^{\infty}} \sum_{i=H,L} \pi_i V(\theta_i, \theta_i) \quad \text{s.t. (9).}$$

To find a solution, we first consider the following "relaxed" problem, which ignores the agents' optimization problem and cares only about the feasibility:

$$(P_{\mathrm{FB}}^*): \max_{\{c_t(\theta_i), m_t(\theta_i), y_t(\theta_i)\}_{t=0}^{\infty}} \sum_{i=H,L} \pi_i \sum_{t=0}^{\infty} \beta^t U[c_t(\theta_i), m_t(\theta_i), y_t(\theta_i)] \tag{11}$$

$$\text{s.t.} \sum_{i=H,L} \pi_i [y_t(\theta_i) - c_t(\theta_i)] \geq G.$$

Let c^* be a constant such that $\bar{u}'(c^*) = g'(c^* + G)$. The next lemma finds a solution to (P_{FB}^*).

LEMMA 1. *The allocation Z in which $[c_t(\theta_i), m_t(\theta_i), y_t(\theta_i)] = (c^*, \phi(c^*), c^* + G)$ for all $i \in \{H, L\}$ and $t \geq 0$ solves the relaxed problem (P_{FB}^*).*

Proof. See the Appendix. ■

The next proposition shows that the allocation Z solves the original problem (P_{FB}).

PROPOSITION 1. *Under the stationary policy $\{\mathbf{X}, \mathbf{Y}, r\}_{t=0}^{\infty}$, in which $\mathbf{X} = [x(\theta_H), x(\theta_L)] = [c^* - (1-\beta)\theta_H, c^* - (1-\beta)\theta_L]$, $\mathbf{Y} = [y(\theta_H), y(\theta_L)] = (c^* + G, c^* + G)$, and $r = 0$, a competitive equilibrium coincides with Z. The Friedman rule holds at the first best.*

Proof. See the Appendix. ■

Here individual H has lower income than individual L, but they work the same. This is because the government redistributes their endowments through the income tax. Individual H now mimics individual L and his incentive constraint is violated.

4. SECOND-BEST POLICY

In this section, we take the following steps to show that the Friedman rule is not optimal when we care about the incentive constraints.

Step 1. By contradiction, suppose that the Friedman rule is optimal.
Step 2. Show that there exists a policy S_1 which is optimal and stationary.
Step 3. Find a stationary policy that deviates from the rule and is better than S_1.

4.1. Stationarity of the Friedman Rule Policy

Suppose by contradiction that the optimal monetary policy follows the Friedman rule. Let $S_0 = \{x_t(\theta_H), x_t(\theta_L), y_t(\theta_H), y_t(\theta_L), 0\}_{t=0}^{\infty}$ denote the optimal policy. When the nominal interest rate is r, the budget constraint is written as

$$\sum_{t=0}^{\infty} \beta^t [c_t + rm_t - (x_t(\hat{\theta}) + \alpha\theta)] = 0, \quad \textbf{(12)}$$

where $\alpha = 1 - \beta$. Equation (12) implies that agents spend in each period the annuity portion α of their initial wealth θ. If $r = 0$, agents are satiated with money and the utility of a type θ agent who mimics type $\hat{\theta}$ is $\bar{u}(x(\hat{\theta}) + \alpha\theta) - \alpha \sum_{t=0}^{\infty} \beta^t g(y_t(\hat{\theta}))$.[4] The social welfare is $\sum_{i=H,L} \pi_i [\bar{u}(x(\theta_i) + \alpha\theta_i) - \alpha \sum_{t=0}^{\infty} \beta^t g(y_t(\theta_i))]$. The policy satisfies the resource constraint

$$\sum_{i=H,L} \pi_i \, [y_t(\theta_i) - x(\theta_i)] \geq G \quad \textbf{(13)}$$

and the incentive constraints

$$\bar{u}[x(\theta) + \alpha\theta] - \alpha \sum_{t=0}^{\infty} \beta^t g[y_t(\theta)] \geq \bar{u}[x(\hat{\theta}) + \alpha\theta] - \alpha \sum_{t=0}^{\infty} \beta^t g[y_t(\hat{\theta})]. \quad \textbf{(14)}$$

In this case we can find a stationary policy S_1 that is as good as S_0.

LEMMA 2. *Suppose the optimal policy follows the Friedman rule. Then there exists an optimal and stationary policy.*

Proof. See the Appendix. ■

4.2. Stationary Equilibrium under the Stationary Policy

This section shows that an equilibrium allocation under the stationary policy is also stationary, whether the nominal interest rate is zero or not. Define the stationary government policy by $\{\mathbf{X}, \mathbf{Y}, r\}_{t=0}^{\infty}$, in which $\mathbf{X} = \{x(\theta_H), x(\theta_L)\}$ and $\mathbf{Y} = \{y(\theta_H), y(\theta_L)\}$. If we let $[c(x, r), m(x, r)] = \text{argmax}_{c+rm=x} u(c, m)$, the optimal c_t and m_t satisfy $(c_t, m_t) = [c(x(\hat{\theta}) + \alpha\theta, r], m[x(\hat{\theta}) + \alpha\theta, r)]$. Therefore the equilibrium consumption–money allocation is also stationary.

Now let us characterize the incentive constraints. We can express the value function as $V(\hat{\theta}, \theta) = v[x(\hat{\theta}) + \alpha\theta, r] - g[y(\hat{\theta})]$, where $v(x, r) = \max_{(c,m):c+rm=x} u(c, m)$ is the indirect utility function. Hence the incentive constraint $V(\theta, \theta) \geq V(\hat{\theta}, \theta)$ holds if and only if

$$v[x(\theta) + \alpha\theta, r] - g[y(\theta)] \geq v[x(\hat{\theta}) + \alpha\theta, r] - g[y(\hat{\theta})] \quad \text{for all}\, \theta, \hat{\theta} \in \Theta. \quad \textbf{(15)}$$

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Define $x_i = x(\theta_i)$ and $y_i = y(\theta_i)$ $(i = H, L)$. If a stationary policy is incentive-compatible, the value function of a type θ agent is equal to $V(\theta, \theta) = v[x(\theta) + \alpha\theta, r] - g[y(\theta)]$ and the resource constraint is given by

$$\sum_{i=H,L} \pi_i \left[y_i - c(x_i + \alpha\theta_i, r)\right] \geq G. \qquad \textbf{(16)}$$

4.3. Problem of Finding the Best Stationary Policy

Because an optimal and stationary policy exists, it must be the best of all stationary policies. We show that this is not true. We first define the best stationary policy.

DEFINITION 3. *The best stationary policy* $\{\mathbf{X}, \mathbf{Y}, r\}_{t=0}^{\infty}$ *is the government policy that maximizes the expected utility* $E_\theta[v(x(\theta) + \alpha\theta, r) - g(y(\theta))]$ *subject to equations* (15) *and* (16).

Let $e(v, r) = \min_{(c,m):u(c,m)\geq v} (c + rm)$ denote the expenditure function. Also define the compensated demand functions by $[c^c(v, r), m^c(v, r)]$.[5] Following Mirrlees (1976) and Ebert (1992), we use the compensated demand functions and set up the problem of the best stationary policy:

$$(\mathbf{P}_{\text{ic}}) : \max \sum_{i=H,L} [v_i - g(y_i)]\, \pi_i,$$

$$\text{s.t.} \sum_{i=H,L} \{y_i - c^c(v_i, r)\}\pi_i \geq G, \qquad \textbf{(17)}$$

$$v_H - g(y_H) \geq v\left[e(v_L, r) - \alpha\theta_L + \alpha\theta_H, r\right] - g(y_L), \qquad \textbf{(18)}$$

$$v_L - g(y_L) \geq v\left[e(v_H, r) - \alpha\theta_H + \alpha\theta_L, r\right] - g(y_H), \qquad \textbf{(19)}$$

$$r \geq 0, \qquad \textbf{(20)}$$

where equation (17) is the resource constraint, equations (18) and (19) are respectively the incentive constraints for individuals H and L, and equation (20) is the arbitrage condition.

The left-hand side of the constraint (18) is the utility when individual H truthfully reports his type, whereas the right-hand side is the utility if he mimics individual L. Note that if individual j mimics individual k, then his expenditure is $x_k + \alpha\theta_j = e(v_k, r) - \alpha\theta_k + \alpha\theta_j$.

4.4. Simplification of the Incentive Constraints

This subsection simplifies the problem $(\mathbf{P}_{\text{ic}})$ by showing that the incentive constraint for individual L [equation (19)] is not binding. Consider the following problem without Eq. (19):

$$(\mathbf{P}^*_{\rm ic}) : \max \sum_{i=H,L} [v_i - g(y_i)]\,\pi_i \quad \text{s.t. (17), (18), and (20).}$$

The Lagrangian of the relaxed problem ($\mathbf{P}^*_{\rm ic}$) is

$$L = \sum_{i=H,L} [v_i - g(y_i)]\,\pi_i + \lambda\left[\sum_{i=H,L} \pi_i\{y_i - c^c(v_i, r)\} - G\right]$$
$$+ \mu\{v_H - g(y_H) - v[e(v_L, r) - \alpha\theta_L + \alpha\theta_H, r] + g(y_L)\} + \tau r,$$

where λ, μ, and τ are the multipliers. The first-order conditions (FOCs) on y_i are

$$y_H : \lambda\pi_H - (\mu + \pi_H)g'(y_H) = 0, \quad \textbf{(21)}$$

$$y_L : \lambda\pi_L + (\mu - \pi_L)\,g'(y_L) = 0. \quad \textbf{(22)}$$

Equations (21) and (22) together imply the inequality[6]

$$\frac{g'(y_H)}{g'(y_L)} = \frac{\pi_H}{\pi_L}\frac{\pi_L - \mu}{\pi_H + \mu} = \frac{1 - \mu/\pi_L}{1 + \mu/\pi_H} < 1.$$

Note that the incentive constraint is not slack and $\mu > 0$. Since $g'() > 0$, we have

$$y_H < y_L. \quad \textbf{(23)}$$

Individual H works less hard than individual L since labor is costly if the government cares the incentive constraint. We have the following proposition.

PROPOSITION 2. *The two problems* ($\mathbf{P}_{\rm ic}$) *and* ($\mathbf{P}^*_{\rm ic}$) *coincide.*

Proof. See the Appendix. ■

In the following, we investigate ($\mathbf{P}^*_{\rm ic}$).

4.5. Nonoptimality of the Friedman Rule

This section shows that the Friedman rule is not optimal. If we let $\omega = \alpha(\theta_H - \theta_L) > 0$ denote the difference in wealth, the FOC on the nominal interest rate is written as

$$\frac{\partial L}{\partial r} = -\lambda \sum_{i=H,L} \left[c^c_r(v_i, r)\,\pi_i\right] - \mu\frac{d}{dr}v\,[e(v_L, r) + \omega, r] + \tau = 0. \quad \textbf{(24)}$$

The first, second, and third terms respectively denote the effect of r on the resource constraint (17), the incentive constraint (18), and and the arbitrage condition (20). To find the sign of $\partial L/\partial r$ around $r = 0$, we use the following lemma.

LEMMA 3. *The following hold:*

$$v_r(x, r) = -mu_c(x - rm, m) = -mv_x(x - rm, m), \quad (25)$$

$$e_r(v, r) = m^c(v, r) = m[e(v, r), r], \quad (26)$$

$$c_r^c(v, r) = -rm_r^c(v, r). \quad (27)$$

Proof. See the Appendix. ■

First of all, from equation (27), the first term $\sum_i (c_r^c \pi_i)$ of Eq. (24) is equal to zero if $r = 0$. Next, from equations (25) and (26), the second term dv/dr is expressed as

$$\begin{aligned}
&\frac{d}{dr} v[e(v_L, r) + \omega, r] \\
&\quad = e_r(v_L, r) v_x[e(v_L, r) + \omega, r] + v_r[e(v_L, r) + \omega, r] \\
&\quad = \{m[e(v_L, r), r] - m[e(v_L, r) + \omega, r]\} v_x[e(v_L, r) + \omega, r].
\end{aligned}$$

Because money is a normal good, by assumption, $m(e(v_L, r) + \omega, r) > m(e(v_L, r), r)$ and thus $dv/dr < 0$. This leads to the following proposition.

PROPOSITION 3. *In the second-best setting, the Friedman rule is not optimal.*

Proof. We have $c_r^c(v_i, r)\pi_i = 0$ and $dv/dr < 0$. Equation (24) implies that $\partial L/\partial r > 0$ if $r = 0$. ■

To understand Proposition 2, consider a welfare-maximizing government policy Z^0 under the Friedman rule. Let v_i denote individual i's utility from goods and money under Z^0. Also, let y_i denote the labor of individual i. The total utility is equal to $v_i - g(y_i)$.

Now define a policy Z^1 where individual i gets the same utility (v_i) from consumption and money, works the same (y_i), and the nominal interest rate r is positive. Consider individual H who mimics individual L. Because money is costly, Z^1 lowers his consumption by $r\phi(e_L + \omega)$, where $e_i = e(v_i, 0)$. Although it raises his income by $r\phi(e_L)$ to keep individual L's utility, the consumption loss dominates the income compensation. This is because he has more money than individual L. Thus Z^1 reduces his utility and relaxes the incentive constraint. Obviously the policy is feasible as long as r is small.[7]

Next consider a policy Z^2 that lowers individual L's labor by ϵ/π_L and raises individual H's labor by ϵ/π_H, where $\epsilon > 0$. The policy is feasible and is incentive-compatible as long ϵ is small. The labor reallocation is welfare-improving, because individual H works less than individual L under Z^1 and the reallocation contributes to equalize labor disutilities across agents. Hence Z^2 is better than Z^0.

Why is the result different from that of DW? In their setting, the incentive constraint for a skilled agent is binding. If a skilled agent underreports his or her type, his or her money holding is the same as that of unskilled agents. Hence the incentive constraint is not relaxed. The mechanism in this paper does not work.

4.6. Practical Implication

The result of this paper is relevant for practical policy making because it shows that the severe deflation implied by the Friedman rule does not maximize social welfare in an economy with wealth inequality.[8] The practical implication is close to that in Antinolfi et al. (2007), who also find that the optimal inflation rate can be positive in a heterogeneous-agent economy. However, their conclusion is limited in the sense that they do not consider fiscal policy instruments (e.g., income tax). In our model, the nonoptimality of the Friedman rule is robust to the consideration of fiscal policy instruments.

It is true that monetary policy is usually not thought to be aimed at distributional concerns. In his recent speech, Bernanke (2008) expresses his concern for the recent economic inequality, but he does not tell us how monetary policy should respond to this problem. Finding out the actual wealth distribution will be costly for the central banks. However, the paper at least suggests that if the wealth inequality is severe, then the popular price stabilization (0% inflation rate) policy may dominate the Friedman rule. This will be good news for the central bankers.

One application of this result to optimal taxation will be asset taxation. For example, let us consider an economy in which there exists two assets that differs in liquidity [see Allen and Gale (2004)]. In this case, taxation of the liquid asset (that reduces the transaction cost) may be welfare-improving.

5. EXTENSION

In this section, we consider two models: one in which agents are heterogeneous in skill and wealth, and one in which money holding bears an interest rate.[9]

5.1. Two-Dimensional Heterogeneity

This section considers an economy in which agents are heterogeneous in both skill and wealth. Let $n_i > 0$ denote individual i's skill level. The instantaneous utility function of individual i is given by $u(c, m) - g(l/n_i)$. The previous sections correspond to a case with $n_L = n_H = 1$. The optimal policy problem of the government is given by

$$(\mathbf{P}_{\text{ic2}}) : \max \sum_{i=H,L} [v_i - g\,(y_i/n_i)]\,\pi_i,$$

$$\text{s.t.} \sum_{i=H,L} \{y_i - c^c\,(v_i, r)\}\pi_i \geq G,$$

$$v_H - g\,(y_H/n_H) \geq v[e\,(v_L, r) + \omega, r] - g(y_L/n_H), \tag{28}$$

$$v_L - g\,(y_L/n_L) \geq v[e\,(v_H, r) - \omega, r] - g(y_H/n_L), \tag{29}$$

$$r \geq 0. \tag{30}$$

First we consider a case where the incentive constraint for individual H (wealthy agent) is binding but the one for individual L is not. The Lagrangian is written as

$$L = \sum_{i=H,L} [v_i - g(y_i/n_i)]\pi_i + \lambda \left[\sum_{i=H,L} \pi_i \{y_i - c^c(v_i, r)\} - G \right]$$
$$+ \mu [v_H - g(y_H/n_H) - v(e(v_L, r) + \omega, r) + g(y_L/n_H)] + \tau r.$$

Hence the FOC on the interest rate r is exactly the same as in Eq. (24), regardless of the value of n_i. Therefore the Friedman rule is not optimal.

Next we consider a case where only the incentive constraint for individual L is binding. This happens when the poorly endowed agents are high-skilled and the skill heterogeneity is more serious than the wealth heterogeneity. The Lagrangian is

$$L = \sum_{i=H,L} [v_i - g(y_i/n_i)]\pi_i + \lambda \left[\sum_{i=H,L} \pi_i \{y_i - c^c(v_i, r)\} - G \right]$$
$$+ \mu [v_L - g(y_L/n_L) - v(e(v_H, r) - \omega, r) + g(y_H/n_L)] + \tau r.$$

The FOC on r is written as

$$\frac{\partial L}{\partial r} = -\lambda \sum_{i=H,L} \left[c_r^c(v_i, r)\pi_i \right] - \mu \frac{d}{dr} v[e(v_H, r) - \omega, r] + \tau = 0. \quad \textbf{(31)}$$

The second term satisfies

$$\frac{d}{dr} v[e(v_H, r) - \omega, r] = \{m(e_H, r) - m(e_H - \omega, r)\} v_x(e_H - \omega, r) > 0,$$

where $e_H = e(v_H, r)$. Hence $\partial L/\partial r < 0$ if $r > 0$ and the Friedman rule is optimal. Intuitively, the positive nominal interest rate cannot relax the incentive constraint because the money demand of individual L, who claims to be individual H, is lower than that of individual H. These results are summarized as follows.

PROPOSITION 4. *The following hold:*

(a) *Suppose that only the highly endowed agents' incentive constraints are binding. Then the Friedman rule is nonoptimal.*
(b) *Suppose that only the poorly endowed agents' incentive constraints are binding. Then the Friedman rule is optimal.*

5.2. Interest Rate on Money

In this section, we consider a case in which the government pays an interest rate ($i_m > 0$) for money holding. The setting is similar to that of Mehrling (1995). First of all, the budget constraint of the type θ agent is given by

$$c_t + \frac{M_{t+1}}{p_t} + \frac{B_{t+1}}{R_t} = B_t + \frac{(1 + i_m)M_t}{p_t} + x_t(\theta). \quad \textbf{(32)}$$

Because the rate of return on money is $(1+i_m)p_t/p_{t+1}$, the nominal interest rate is $r_t^* = (R_t - (1+i_m)p_t/p_{t+1})/R_t$ and the intertemporal budget constraint is given by $\sum_{t=0}^{\infty} q_t[c_t + r_t^* m_t - x_t(\theta)] = \theta$. The budget constraint of the government now becomes

$$\sum_{i=H,L} \pi_i \left[\frac{M_{t+1}(\theta_i) - M_t(\theta_i)}{p_t} - \frac{i_m M_t}{p_t} + \frac{B_{t+1}(\theta_i)}{R_t} - B_t(\theta_i) + y(\theta_i) - x(\theta_i) \right] \geq G. \tag{33}$$

Substituting equation (32) into equation (33) yields the resource constraint (3). Therefore the setup coincides with the previous ones. We have the following conclusion.

PROPOSITION 5. *When the government pays on interest for the money holding, the Friedman rule is not the optimal monetary policy.*

The nonoptimality of the Friedman rule is therefore robust to the way of injecting money.

6. CONCLUSIONS

In this paper we investigate the money-in-the-utility-function model where agents are heterogeneous in their initial wealth. We show that a deviation from the Friedman rule increases social welfare, because a positive nominal interest rate relaxes the incentive constraints. Kocherlakota (2005) raises the question of how monetary policy should respond to various kinds of agent-specific shocks. Although the wealth shock considered here is simple, this paper provides some insight into his question. This paper could lead to a better understanding of monetary policy analysis in a heterogeneous-agent model.

NOTES

1. Some authors have recently investigated the roles of economic policies in infinite horizon models with wealth heterogeneity. For example, Garcia-Penalosa and Turnovsky (2007) and Jin (2009) investigate endogenous growth models with wealth inequality and analyze the effect of fiscal and monetary policies on growth. However, they do not analyze social welfare, which is the main focus of this paper.
2. As an example, we can consider a government that cannot monitor the bank account of each agent.
3. The original definition of the nominal interest rate is $i_t = R_t/(p_t/p_{t+1}) - 1$, but there exists a one-to-one correspondence between r_t and i_t and $r_t = 0$ if and only if $i_t = 0$.
4. Note that the intertemporal price sequence is $q_t = \beta^t$.
5. By definition, $c^c(v, r) + rm^c(v, r) = e(v, r)$.
6. I thank an anonymous referee for the derivation of this inequality.

7. The policy Z^1 satisfies the incentive constraint for individual L as long as r is small, because it is not binding under Z^0.

8. It is important to note that monetary contraction (at a rate slower than the Friedman contraction) may still be optimal, depending on the parameters of the model.

9. I thank an anonymous referee for pointing out these issues.

REFERENCES

Albanesi, Stefania and Christopher Sleet (2006) Dynamic optimal taxation with private information. *Review of Economic Studies* 73, 1–30.

Allen, Franklin and Douglas Gale (2004) Financial intermediaries and markets. *Econometrica* 72, 1023–1061.

Antinolfi, Gaetano, Costas Azariadis, and James Bullard (2007) The Optimal Inflation Target in an Economy with Limited Enforcement. Federal Reserve Bank of St. Louis, Working Paper.

Arseneau, David M. and Sanjay K. Chugh (2008) Optimal fiscal and monetary policy with costly wage bargaining. *Journal of Monetary Economics* 55, 1401–1414.

Bernanke, Ben (2008) Remarks on Class Day 2008. The Federal Reserve Board.

Bhattacharya, Joydeep, Joseph Haslag, and Antoine Martin (2005) Heterogeneity, redistribution, and the Friedman rule. *International Economic Review* 46, 437–454.

Bhattacharya, Joydeep, Joseph Haslag, Antoine Martin, and Rajesh Singh (2008) Who is afraid of the Friedman rule? *Economic Inquiry* 46, 113–130.

Chari, V.V. and Patrick J. Kehoe (1999) Optimal fiscal and monetary policy. In John B. Taylor and Michael Woodford (eds.), *Handbook of Macroeconomics* 1C, pp. 1671–1745. Amsterdam: Elsevier.

Cremer, Helmuth, Pierre Pestieau, and Jean-Charles Rochet (2001) Direct versus indirect taxation: The design of the tax structure revisited. *International Economic Review* 42, 781–799.

da Costa, Carlos and Ivan Werning (2008) On the optimality of the Friedman rule with heterogeneous agents and non-linear income taxation. *Journal of Political Economy* 116, 82–112.

De Nardi, Mariacristina, Selahattin Imrohorŏglu, and Thomas Sargent (1999) Projected U.S. demographics and Social Security. *Review of Economic Dynamics* 2, 575–615.

Ebert, Udo (1992) A reexamination of the optimal nonlinear income tax. *Journal of Public Economics* 49, 47–73.

Garcia-Penalosa, Cecilia and Stephen Turnovsky (2007) Growth, income inequality, and fiscal policy: What are the relevant tradeoffs? *Journal of Money, Credit, and Banking* 39, 369–394.

Heer, Burkhard (2003) Welfare costs of inflation in a dynamic economy with search unemployment. *Journal of Economic Dynamics and Control* 28, 255–272.

Ireland, Peter (2005) Heterogeneity and redistribution: By monetary or fiscal means? *International Economic Review* 46, 455–463.

Jin, Yi (2009) A note on inflation, growth, and income inequality. *Macroeconomic Dynamics* 13, 138–147.

Kocherlakota, Narayana (2005) Optimal monetary policy: What we know and what we don't know. *International Economic Review* 46, 715–729.

Ljungqvist, Lars and Thomas Sargent (2001) *Recursive Macroeconomic Theory.* Cambridge, MA: MIT Press.

Mehrling, Perry G. (1995) A note on the optimum quantity of money. *Journal of Mathematical Economics* 24, 249–258.

Mirrlees, James A. (1976) Optimal tax theory: A synthesis. *Journal of Public Economics* 6, 327–358.

Palivos, Theodore (2005) Optimal monetary policy with heterogeneous agents: A case for inflation. *Oxford Economic Papers* 57, 34–50.

Schmitt-Grohe, Stephanie and Martin Uribe (2004) Optimal fiscal and monetary policy under imperfect competition. *Journal of Macroeconomics* 26, 183–209.

APPENDIX

A. PROOF OF LEMMA 1

The Lagrangian of the problem is

$$L = \sum_{i=H,L} \pi_i \sum_{t=0}^{\infty} \beta^t \{u[c_t(\theta_i), m_t(\theta_i)] - g[y_t(\theta_i)] + \lambda_t[y_t(\theta_i) - c_t(\theta_i)]\},$$

where λ_t is the multiplier. The FOCs are

$$u_c[c_t(\theta_i), m_t(\theta_i)] = g'[y_t(\theta_i)] = \lambda_t,$$
$$u_m[c_t(\theta_i), m_t(\theta_i)] = 0.$$

The solution to the conditions is type-independent. We can easily check that the allocation $\{\{(c_t(\theta_i), m_t(\theta_i), y_t(\theta_i))\}_{i=L,H}\}_{t=0}^{\infty} = \{\{c^*, \phi(c^*), c^* + G\}_{i=L,H}\}_{t=0}^{\infty}$ are the only allocation satisfying the FOCs. ■

B. PROOF OF PROPOSITION 1

Suppose $\mathbf{X} = (c^* - (1-\beta)\theta_H, c^* - (1-\beta)\theta_L)$, $\mathbf{Y} = (c^* + G, c^* + G)$, and $r = 0$. The budget constraint is given by $\sum_{t=0}^{\infty} \beta^t (c_t + 0m_t - c^*) = 0$. Hence the optimization problem of individual i is written as

$$\sum_{t=0}^{\infty} \beta^t U[c_t(\theta_i), m_t(\theta_i), c^* + G] \text{ s.t. } \sum_{t=0}^{\infty} \beta^t (c_t - c^*) = 0.$$

The solution is $[c_t(\theta_i), m_t(\theta_i), y_t(\theta_i)] = [c^*, \phi(c^*), c^* + G]$. ■

C. PROOF OF LEMMA 2

Let $\{\mathbf{X}_t, \mathbf{Y}_t, r_t\}_{t=0}^{\infty} = \{x_t(\theta_H), x_t(\theta_L), y_t(\theta_H), y_t(\theta_L), 0\}_{t=0}^{\infty}$ denote the optimal policy. Also, let $x^*(\theta_i) = (1-\beta)\sum_{t=0}^{\infty} \beta^t x_t(\theta_i)$ denote a weighted average of future income. When the price sequence is $q_t = \beta^t$ and the Friedman rule holds, consumption of individual i in each period is time-independent and is equal to $x^*(\theta_i)$. His money holding is $\phi[x^*(\theta_i)]$. The feasibility constraint is written as

$$\sum_{i=H,L} \pi_i y_t(\theta_i) \geq \sum_{i=H,L} \pi_i x_t(\theta_i) + G.$$

Now let $y^*(\theta_i)$ be a constant such that $g[y^*(\theta_i)] = (1-\beta)\sum_{t=0}^{\infty} \beta^t g[y_t(\theta_i)]$. Because g is convex and $(1-\beta)\sum_{t=0}^{\infty} \beta^t = 1$, we have

$$y^*(\theta_i) \geq (1-\beta)\sum_{t=0}^{\infty} \beta^t y_t(\theta_i).$$

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Hence the stationary policy $\{x(\theta_H), x(\theta_L), y(\theta_H), y(\theta_L), 0\}_{t=0}^{\infty}$ satisfies the feasibility

$$\sum_{i=H,L} \pi_i y^*(\theta_i) \geq \sum_{i=H,L} \pi_i x^*(\theta_i) + G,$$

and also incentive compatibility. The stationary policy is as good as the original nonstationary policy. ■

D. PROOF OF PROPOSITION 2

First we compare the labor income of individual H, $x_H = e(v_H, r) - \alpha\theta_H$, with that of individual L, $x_L = e(v_L, r) - \alpha\theta_L$. Because the constraint (18) is binding, we have

$$v\left(x_H + \alpha\theta_H, r\right) - v\left(x_L + \alpha\theta_H, r\right) = g\left(y_H\right) - g\left(y_L\right). \tag{A.1}$$

Because $y_H < y_L$, we get $x_L > x_H$. Individual H receives lower labor income than individual L. Because v is strictly concave, for any k and K such that $K > k \geq 0$, we have

$$\frac{\partial}{\partial\theta}\left[v\left(K + \theta, r\right) - v\left(k + \theta, r\right)\right] < 0. \tag{A.2}$$

Because $x_L \geq x_H$, equation (A.2) implies that

$$v\left(x_L + \alpha\theta_L, r\right) - v\left(x_H + \alpha\theta_L, r\right) \geq v\left(x_L + \alpha\theta_H, r\right) - v\left(x_H + \alpha\theta_L, r\right). \tag{A.3}$$

Substituting equation (A.1) into equation (A.3) yields

$$v\left(x_L + \alpha\theta_L, r\right) - v\left(x_H + \alpha\theta_L, r\right) \geq g\left(y_L\right) - g\left(y_H\right).$$

This is the same as (19). Therefore the constraint (19) is automatically satisfied if equation (A.3) is binding. ■

E. PROOF OF LEMMA 3

First, because $v(x, r) = \max_m u(x - rm, m)$, the envelope theorem implies equation (25). Next, we apply the envelope theorem to the expenditure function $e(u, r) = \min_{(c,m)}(c + rm)$ to get (26). Finally, differentiating $c^c(v, r) + rm^c(v, r) = e(v, r)$ with respect to r yields equation (27). ■