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A Simplified Method for the Analysis of Interference from JTIDS Radio Networks to DME Aeronautical Radionavigation Systems

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Three distinct mechanisms of interference of JTIDS transmissions to DME are investigated: false interrogator triggering, transponder overloading and desensitization of either transponder or interrogator receivers. The effect of transponder overloading due to JTIDS transmissions is found the most serious. An analytical method is provided which relates the transponder reply efficiency to the load of the interrogations received in the presence of JTIDS transmissions.

KEY WORDS

1. JTIDS. 2. DME. 3. Reply Efficiency.

1. INTRODUCTION. Distance Measuring Equipment (DME) is a radionavigation system intensively used by aviation for the determination of distance within radio line of sight. It operates in the frequency band 960–1215 MHz. In NATO countries the same band is utilized for the operation, on a non-interference basis, of a military radio system known as Joint Tactical Information Distribution System (JTIDS).

As many NATO countries are intensifying the use of JTIDS or planning the deployment of JTIDS on a variety of platforms, the issue of interference avoidance takes on particular significance among the civil aviation community. Although NATO has provided guidance [1] for the avoidance of interference and promoted a set of elementary technical compatibility criteria, the issue is not considered settled for a variety of reasons. First, many countries have considered it necessary to apply stricter compatibility criteria for the operation of JTIDS. Second, contrary to other systems, the case of JTIDS interference has not passed through the scrutiny of civil standardization bodies. Third, the information available on the issue is limited, fragmented and mainly of an experimental nature.

The present paper constitutes an attempt to explain the mechanisms of JTIDS interference to DME on first principles. Drafted on the basis of unclassified material,

[1] The publication of this paper does not imply adoption of its content by the Hellenic Civil Aviation Authority.

approximate in its logic and not accompanied by measurement results, this paper can only be viewed as a first rough assessment of the interference in question. It is, however, hoped that it will bring to light interesting aspects of compatibility analysis and further the understanding of a subject that appears misty to many civil aviation professionals. Numerical results presented in this paper by no means cover all cases of interest; they have been included in order to exemplify the methodology.

The paper is organized as follows. First, the principles of operation of both DME and JTIDS are briefly revisited. Exposition is focused on the technical characteristics of the two systems that are pertinent to the analysis of interference. The subsequent presentation of possible interference mechanisms of JTIDS to DME constitutes the main body of the paper. Three distinct mechanisms of interference are successively discussed and analyzed in order to determine the conditions under which they could downgrade the performance of DME below acceptable levels. These are:

- false interrogator triggering,
- transponder overloading and
- desensitization of either transponder or interrogator receivers.

The findings of the analysis are summarized and commented in the last section of the paper.

2. OVERVIEW OF DME.

The DME system comprises two basic components, one fitted in the aircraft, the other installed on the ground. The aircraft component is known as the interrogator and the ground component as the transponder. Both components send and receive pairs of pulses. The emissions of the interrogator and the transponder are called interrogations and replies respectively.

The DME principle of operation is very similar to that of radar: the time required for signal propagation is translated to distance. In elementary terms, the DME mode of operation can be described as follows: An interrogation pair of pulses hits the transponder receiver after having covered the interrogator-transponder distance. The pair is subsequently decoded and re-emitted by the transponder with a predetermined delay. On its way back it covers approximately the same distance and ends up at the interrogator receiver. On measuring the total round-trip time and subtracting the processing delays, the interrogator-transponder distance is readily derived.

For the sake of generality, the technical characteristics of DME considered in the present study are gleaned from the respective ICAO standard [2]. It is furthermore noted that these characteristics belong to the so-called narrow spectrum system specification (DME/N), which is currently the commonest choice in DME installations.

Under normal operating conditions, the interrogators emit pairs of pulses at a repetition rate of 10 to 30 ppps (pairs of pulses per second) [3]. They are emitted in bursts (of 10 interrogations in 1/15 sec, for instance). It must be stressed, however, that the time between successive pairs of interrogation pulses is not kept constant. On the contrary, it is an essential characteristic of the system that this inter-pair time varies randomly. To understand why, it is necessary to look more closely at the processing of the reply pulse pairs by the interrogator. First of all it should be noted that a transponder does not serve exclusively a single aircraft but should be capable of handling a large number (nominally up to 100) of aircraft. The question therefore arises as to how an aircraft interrogator can discriminate the replies of its own

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Table 1. The pulse pair spacing specification for DME/N modes X and Y.

	Pulse pair spacing (μs)	
Mode	Interrogation	Reply
X	12	12
Y	36	30

interrogations among the multitude of replies received. This can be achieved thanks to the randomness in the time between successive interrogation pairs, which is intentionally introduced.

More specifically, following each interrogation and for a series of successive interrogations, the interrogator receiver records the delays between the arriving transponder replies and the interrogation emission. For interrogations spanning a fraction of a second, the replies of its own interrogations arrive with essentially constant delay because the speed of aircraft is much less than the speed of light. Indeed, within an interrogation burst of 1/15 sec, the displacement of an aircraft travelling at sonic speed will be roughly 22 metres, whose respective signal propagation time (0·147 μs) is a small fraction of the temporal resolution of interrogator measurements (on the order of 2 μs). Notably, the randomness in an aircraft's interrogation emissions contributes no delay to the replies of its own interrogations since the delay is recorded with respect to the time of the interrogation emission.

Consequently, while the replies of an aircraft's own interrogations arrive synchronously, the rest of the replies are dispersed in the listening time after each interrogation. In this account, the interrogator's processor, which has the listening time divided in elementary intervals, can readily identify the interval that includes the replies to its own interrogations. This is accomplished by comparing the number of hits in the elementary intervals over a series of successive interrogations and choosing the interval with the largest proportion of hits.

The total duration of a DME/N pulse is in the order of 6·2 μs (including rise and fall times). The time interval between the 50% amplitude points of a DME pulse is referred to in the following as the DME pulse width and is in the order of 3·5 μs. The temporal distance between the two pulses of a pair, known as pulse pair spacing, is an important characteristic of the system. On the basis of this spacing for interrogation and reply pairs of pulses, ICAO distinguishes four operating modes (X,Y,W,Z) for DME. The pulse pair spacing specification for DME/N modes X and Y, which are widely used in Europe, are shown in Table 1. The benefit of using a pair of pulses instead of a single pulse is first and foremost robustness in decoding. Additionally and under certain conditions [2, Attachment C], the use of different modes by two otherwise equivalent systems is advantageous in terms of mutual interference tolerance. The frequency channels for DME are spaced every 1 MHz.

3. OVERVIEW OF JTIDS.

JTIDS is a military digital radio system which:

- supports network operations,
- is organized on the basis of a Time Division Multiple Access (TDMA) architecture and
- employs frequency hopping in radio transmission.

Although access to information on JTIDS is restricted, the basic characteristics of the system can be found in a variety of sources [1, 4, 5].

In JTIDS [1], information is transmitted in the form of pulses whose carrier frequency varies, in accordance with a pseudorandom hopping code, from a set of 51 values in the 960–1215 MHz band; the spacing between the carrier frequencies is 3 MHz. Not the entire band is utilized because of the provision of ample guard bands around 1030 MHz and 1090 MHz for the protection of SSR/IFF operation.

Similarly to the DME, the duration of each pulse is 6·4 μs and the spacing between two consecutive pulses equals 13 μs (start-to-start). The spectrum, however, of the JTIDS pulse is significantly wider than that of DME, not only because it has sharper rise and fall but mainly because a JTIDS pulse carries a 32-bit modulated message symbol.

The use of transmission time is organized in time slots, each with duration of 7·8125 ms. Each second contains 128 time slots. Each terminal transmission within a time slot consists of a sequence of either 72, 258 or 444 pulses spaced by 13 μs. The remaining part of each time slot (at least 2 ms) is allocated to propagation delay and intentional jittering of the start of transmission. In the following, we will need to use the fraction λ of the active 13-μs intervals contained in each time slot. A time slot contains approximately 601 13-μs intervals. Given that transmissions in a time slot occur over a sequence of 72, 258 or 444 consecutive intervals, the fraction λ takes on the values $444/601=0{\cdot}74$, $258/601=0{\cdot}43$ or $72/601=0{\cdot}12$ respectively.

For a given JTIDS terminal or network, the Time Slot Duty Factor (TSDF) Q is defined as the percentage of the time slots occupied. The combined ratio of exploitation of the available time by a terminal or network is, thus, given by $Q\lambda/100$. If there are N networks operating with identical settings, the maximum exploitation ratio is $NQ\lambda/100$ (under no-overlap conditions).

4. FALSE INTERROGATOR TRIGGERING.

For DME transponders and interrogators the ICAO standard specifies that the receiver should reject pulse pairs with width and spacing deviating from the specifications set. Although the width of JTIDS pulses is roughly double that of DME and their frequency varies randomly, they have similar total duration and pulse spacing (with Mode X DME). Because JTIDS is not an internationally standardized system, doubts about the rejection of JTIDS pulses by DME receivers are perhaps justified, especially for old installations with limited decoding capability.

In the following we carry out a worst-case analysis for the effect of JTIDS transmissions on interrogators, assuming that a single JTIDS pulse can indeed trigger an interrogator. In particular, under this pessimistic, if not unrealistic, assumption, we are going to estimate the probability of false measurement of distance. In the calculation below, the following assumptions are made:

- The listening time after each interrogation is 2·5 ms, corresponding to a range of 200 nautical miles.
- From a total of 51 JTIDS carrier frequencies, only three are deemed potentially harmful to the operation of a DME interrogator. These are the carrier frequency that is co-channel with the DME plus the two adjacent ones. It is assumed that the effect of second adjacent carrier frequencies (6 MHz) and beyond is eliminated by the filtering action of the JTIDS transmitter and the

interrogator receiver. Hence, the probability of a potentially harmful pulse is $3/51 = 1/17$.

- Single network operation for JTIDS, where only one terminal transmits in a given time slot.
- Uniform distribution of the probability of appearance of the carrier frequencies and statistical independence of such appearances. This presupposes that the pseudorandom hopping code approximates sufficiently the ideal randomness.
- The interrogator's listening time is divided in elementary intervals of 4 μs. This complies with the specification [2, para. 3.5.5.4.1] that the interrogator should not contribute more than ±315 m to the overall system error, which entails a resolution of temporal measurements of 2·1 μs and, therefore, a maximum width of elementary intervals of 4·2 μs.

As mentioned in Section 2, a distance measurement by DME is based on the arrival of successive replies with approximately the same delay with respect to the time of emission of the interrogations. In what follows, such replies we call synchronous. Let us assume that a measurement is recorded if in ten consecutive interrogations, the interrogator receives at least five synchronous replies. We are next going to show that it is very unlikely to have at least five synchronous potentially harmful JTIDS pulses within the listening times corresponding to ten successive interrogations.

We can imagine that the listening time of 2·5 ms after each interrogation is divided in 625 elementary intervals of 4 μs. Let us concentrate on a random elementary interval whose centre is at time ΔT, indicating a delay ΔT from the time of emission of each interrogation. If the JTIDS transmissions were continuous, i.e. pulses were emitted continuously every 13 μs, the probability of a JTIDS pulse impinging on the interrogator within this elementary interval would be 4/13. Because JTIDS transmissions are not continuous, this probability should be multiplied by the temporal exploitation ratio ($Q\lambda/100$). But only a fraction (1/17) of the pulses are potentially harmful. Therefore, the probability p of a potentially harmful JTIDS pulse impinging on the interrogator within this elementary interval is:

$$p = \frac{4}{13}\,\frac{1}{17}\,\frac{Q\lambda}{100} = 1{\cdot}81\ 10^{-4} Q\lambda \qquad (1)$$

For $Q = 100$ and $\lambda = 0{\cdot}74$, $p = 0{\cdot}0134$.

The probability of m occurrences in a particular elementary interval out of ten interrogations is given by the familiar formula:

$$P(m, 10) = C(m, 10)p^m(1-p)^{10-m} \qquad (2)$$

where $C(m, N) = N!/m!(N-m)!$ denotes the number of combinations of N objects taken m at a time. The probability of $m \geqslant 5$ occurrences in a particular elementary interval is thus:

$$P(m \geqslant 5, 10) = \sum_{m=5}^{10} C(m, 10)p^m(1-p)^{10-m} \qquad (3)$$

Taking into account that there are 625 elementary intervals, it follows that the probability P of one group of at least five synchronous potentially harmful JTIDS pulses occurring within the listening times corresponding to 10 successive interrogations is bounded by:

$$P < 625\,P(m \geqslant 5, 10) \approx 625\,C(5, 10)p^5(1-p)^5 = 625(252)p^5(1-p)^5 \qquad (4)$$

For $p = 0{\cdot}0134$, we obtain $P < 6{\cdot}36\ 10^{-5}$. Thus, under the pessimistic assumption that single JTIDS pulses can trigger a DME interrogator, it follows that the probability P of false measurement of distance is very low, much lower than the 5% error margin set in the DME standard. Of course, the probability P would have come out even lower if we were to consider that triggering could only occur by the much rarer pairs of successive co-frequency JTIDS pulses.

5. TRANSPONDER OVERLOADING.

5.1. *General.* Regardless of whether JTIDS pulses are rejected by DME transponder and interrogator receivers, their frequency of appearance should not overload the DME transponder in the sense of causing an unacceptable reduction of its rate of reply to received interrogations. An important specification for DME operation [2, para. 3.5.4.6.1] is that the reply efficiency of the transponder be at least 70% for all values of the foreseen transponder loading. In other words, a random interrogation should be replied with probability at least 70%. But the transponder cannot reply when it performs other functions.

First of all, the transponder cannot reply immediately after the reception of a pulse pair. According to the DME standard [2, para. 3.5.4.6.2], the transponder is rendered inactive for a period τ_D (dead time) not to exceed 60 µs.

The load of the transponder is described in terms of the rate i_r of impinging interrogations emitted by aircraft and having sufficient strength to excite it. Furthermore, the transponder is triggered spontaneously for monitoring purposes at a maximum rate of $m_r = 120$ times per second. In the following we use the term *event* to refer to the occurrence of either an interrogation or a spontaneous activation of the transponder. We also adopt the simplifying assumption that these events are randomly distributed with respect to time forming a Poisson process. Let e_r denote the total rate of events per second:

$$e_r = i_r + m_r. \tag{5}$$

An event, if not disregarded due to overlaps as described below, generates an interval of transponder inactivity. This is due to the afore-mentioned dead time τ_D, following the decoding of an event. The length of the induced inactivity interval is, however, larger than τ_D. Indeed, on considering the relative position of two events (see Figure 1) it can be verified that, in order for the later event to produce a reply, both pulses of this event should be outside the dead time τ_D corresponding to the earlier event. For this reason, the inactivity interval τ corresponding to an event is given by:

$$\tau = \tau_D + \tau_{SP} + \tau_{OD} + \frac{\tau_{PW}}{2} \tag{6}$$

where τ_{SP} is the pulse spacing (as in Table 1) and τ_{PW} is the pulse width between the 50% amplitude points (on the order of 3·5 µs). The time τ_{OD} represents the temporal offset of the start of the dead interval from the peak of the second pulse. The value of τ_{OD} is not specified in the DME standard; it cannot be less than $\tau_{PW}/2$ and probably it is slightly larger than the pulse decay time (on the order of 2·5 µs). In the subsequent numerical calculations we take $\tau = 77$ µs, which is appropriate for Mode X DME with $\tau_D = 60$ µs.

Another function of the transponder is the transmission of identification code groups [2, para. 3.5.3.6.3] at least once every 40 seconds. Again, during identification

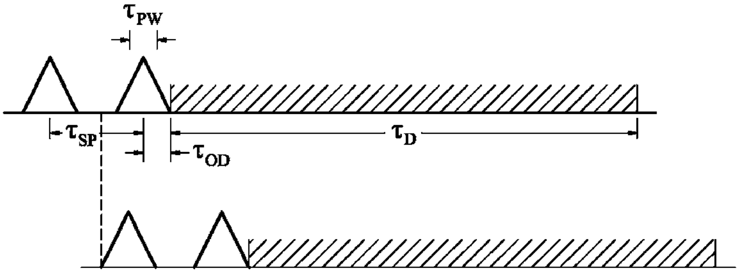

Figure 1. The structure and relative position of two events.

transmissions, reply pulses are suppressed. The maximum total key down time is 5 seconds. Therefore, $5/40 = 12{\cdot}5\%$ of the available time is consumed by the identification function. During the key down times, the transmission consists of the usual pairs of pulses at a repetition rate of 1350 pairs per second. Hence, the transmission rate d_r of identification pairs is $d_r = 0{\cdot}125 \times 1350 = 169$ ppps.

An interrogation obviously is not replied to when it arrives within an interval of transponder inactivity. In the following we adopt the simplified assumption that an interrogation is replied to when it arrives outside an interval of transponder inactivity. Under this assumption, the requirement for 70% transponder reply efficiency is equivalent to the requirement that over one second the transponder should not be rendered inactive for more than 300 ms on the average. It is on this basis that we will attempt to analyze the performance of the transponder and quantify the effect of the JTIDS transmissions. It must be pointed out that this basis of calculation leads to a slight overestimation of the transponder reply efficiency because it can happen that no reply is produced to an interrogation arriving at a time of transponder activity. For instance, if an interrogation is very closely followed by another event of comparable strength, both events and not just the latter can be rejected due to distortion of the pulse shape characteristics. Another such case is when an interrogation pulse is distorted by the second pulse of a preceding interrogation whose first pulse arrives close to the end of an interval of transponder inactivity.

Because an increase of the interrogation rate i_r results in greater inactivity time for the transponder, the reply efficiency r_e is expected to be a decreasing function of the rate i_r. In practical terms, it is desirable for the transponder to serve many aircraft; hence i_r should be as large as possible. For this reason, the value of i_r corresponding to $r_e = 0{\cdot}7$ can be thought of as the interrogation capacity of the transponder. The relationship between the quantities i_r and r_e will be elaborated in the following section. It is noted that the reply efficiency applies also to the spontaneous activations of the transponder.

Another significant quantity related to the performance of the transponder is its transmission rate. The transmission rate t_r of the transponder in pulse pairs per second (ppps) can be calculated as follows:

$$t_r = r_e e_r + d_r = r_e(i_r + m_r) + d_r \tag{7}$$

It is recommended [2, para. 3.5.4.1.5·5] that the transponder should be capable of continuous operation at a transmission rate $t_r = 2700$ ppps. In the calculations that follow, it is taken $m_r = 120$ times per second and $d_r = 169$ ppps.

5.2. *Transponder performance in the absence of JTIDS transmissions.* In this section, we will determine the interrogation capacity of the transponder in the absence of JTIDS transmissions. To this end, it is necessary to estimate the total inactivity time of the transponder. In this regard, due account should be taken of the possibility of overlap between intervals of inactivity. Because the probability of overlap is not negligible, the reduction of inactivity time due to the overlap is significant. As mentioned before, an event has a potential to generate an inactivity interval of duration τ. Two cases of overlap can be distinguished: between an event and an identification block or between two or more events.

It is convenient to start the analysis with the assumption that there are no identification blocks. With the help of the Poisson model, we will first evaluate the inactivity time due to the events occurring at a rate e_r. If there were no overlap, these events would occupy a total of $e_r\tau\ 10^3$ ms per second. But overlap does occur. An event with no overlap means that no other event arrives within a temporal distance $\pm\tau$ of the arrival of the former. Considering the two intervals of length τ on either side of the event, the probability of an event with no overlap is calculated to be $P_0 = \exp(-2e_r\tau)$. As an example, for $\tau = 77$ μs and $e_r = 3000$ events per second, $P_0 = 0{\cdot}63$, meaning that 37% of the events are involved in overlaps. The contribution δ_0 of events with no overlap to the total inactivity time equals $\delta_0 = P_0 e_r \tau\ 10^3$ ms per second.

Let us next consider the isolated pairs of overlapping events. By this term we mean two events less than τ μs apart, but with no other event arriving within distance $\pm\tau$ of them. To calculate the probability P_1 of an event participating in such an isolated pair, we can distinguish the two equally likely possibilities where its counterpart event is on its right or on its left. Therefore, this probability equals:

$$P_1 = 2(1 - e^{-e_r\tau})e^{-2e_r\tau} \tag{8}$$

The average number of such pairs in a second equals $P_1 e_r/2$. More generally, we can think of an isolated set of (n+1) overlapping events, that is a sequence of (n+1) events for which the inter-arrival times are all less than τ with no other event arriving within distance τ of the first and the last event. Let $X_j, j = 1, \ldots, n$ denote the inter-arrival time between the j^{th} and the $(j+1)^{\text{st}}$ event. Note that X_j, $j = 1, \ldots, n$ are assumed to be independent identically distributed exponential random variables with mean e_r^{-1}. It will prove expedient to define the quantities α, β and γ as:

$$\alpha = e_r\tau, \quad \beta = e^{-e_r\tau}, \quad \gamma = 1 - e^{-e_r\tau} \tag{9}$$

The probability P_n of an event belonging to an isolated set of $(n+1)$ overlapping events is therefore:

$$P_n = (n+1)\beta^2\gamma^n. \tag{10}$$

Consequently, the average number of such sets of $(n+1)$ events over one second is $P_n e_r/(n+1)$. Let ζ_n be the mean inactivity time for an isolated set of $(n+1)$ overlapping events. Knowing ζ_n, the contribution of the sets of $(n+1)$ overlapping events to the total inactivity time is given by:

$$\delta_n = \zeta_n 10^3 P_n e_r/(n+1) = e_r\zeta_n\beta^2\gamma^n 10^3 \text{ ms per second.} \tag{11}$$

For $n = 1$, that is for a pair of overlapping events, $\zeta_1 = \tau$ because the second event arrives within time τ of the first, hence within the inactivity interval of the first event;

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therefore it is disregarded and brings about no additional inactivity to the transponder. For sets of $(n+1)$ overlapping events with $n \geqslant 2$, the calculation of the mean inactivity time per set ζ_n is less straightforward and is carried out in the Appendix.

With the use of the results of the Appendix and the application of formula (11), it is possible to calculate the inactivity times corresponding to the cumulative occurrence of sets of $(n+1)$ overlapping events, for $2 \leqslant n \leqslant 5$; higher order inactivity times are neglected. The results for δ_n, in ms per second, are as follows:

$$\delta_0 = \alpha\beta^2 10^3 \tag{12}$$

$$\delta_1 = \alpha\beta^2\gamma 10^3 \tag{13}$$

$$\delta_2 = \alpha\beta^2(2\gamma^2 - \gamma + \alpha\beta)10^3 \tag{14}$$

$$\delta_3 = \alpha\beta^2(2\gamma^3 - \gamma + \alpha\beta(1 + \alpha/2))10^3 \tag{15}$$

$$\delta_4 = \alpha\beta^2\left(2\gamma^4 + \beta^2(\alpha-\gamma)^2 + \alpha\beta\left(1 + \alpha/2 + \alpha^2/6\right) - \gamma\right)10^3 \tag{16}$$

$$\delta_5 = \alpha\beta^2\left(2\gamma^5 + \beta^2(\alpha-\gamma)(\alpha + \alpha^2 + \alpha\beta - 2\gamma - \gamma^2) + \alpha\beta\left(1 + \alpha/2 + \alpha^2/6 + \alpha^3/24\right) - \gamma\right)10^3 \tag{17}$$

Thus, for the total inactivity time, in ms per second, produced by the events, we can write:

$$\sum_{n=0}^{\infty} \delta_n \approx \sum_{n=0}^{5} \delta_n \tag{18}$$

Let us now consider the additional effect of identification emissions. These emissions occupy $\delta_{id} = p_{id} 10^3$ ms per second, where $p_{id} = 0{\cdot}125$ is the proportion of time consumed by the identification function. If one or more events fall within an identification emission, the corresponding inactivity time is absorbed by the inactivity time of the identification emission. Because this occurs with probability p_{id}, the total transponder inactivity time per second is given by the formula:

$$T_{in} = \delta_{id} + (1 - p_{id}) \sum_{n=0}^{\infty} \delta_n \tag{19}$$

The transponder reply efficiency r_e can then be expressed as:

$$r_e = 1 - T_{in} 10^{-3} \tag{20}$$

As mentioned before, a transponder reply efficiency of 70% corresponds to $T_{in} = 300$ ms per second. It follows from Equation (19) that the limit $T_{in} = 300$ ms per second corresponds in turn to $\sum_{n=0}^{\infty} \delta_n = 200$ ms per second. With the help of formulas (12–18), it is possible to solve the equation $\sum_{n=0}^{\infty} \delta_n = 200$ numerically for α. The result is $\alpha = 0{\cdot}25$, which, for $\tau = 77$ μs, corresponds to a rate $e_r = \alpha/\tau$ of 3247 events per second and, consequently, to $i_r = e_r - m_r = 3127$ interrogations per second, which can be considered as the interrogation capacity of the transponder. The respective transponder transmission rate equals $t_r = 0{\cdot}7e_r + d_r = 2442$ ppps. In Figure 2, graphs are shown of the transponder reply efficiency r_e and the transponder

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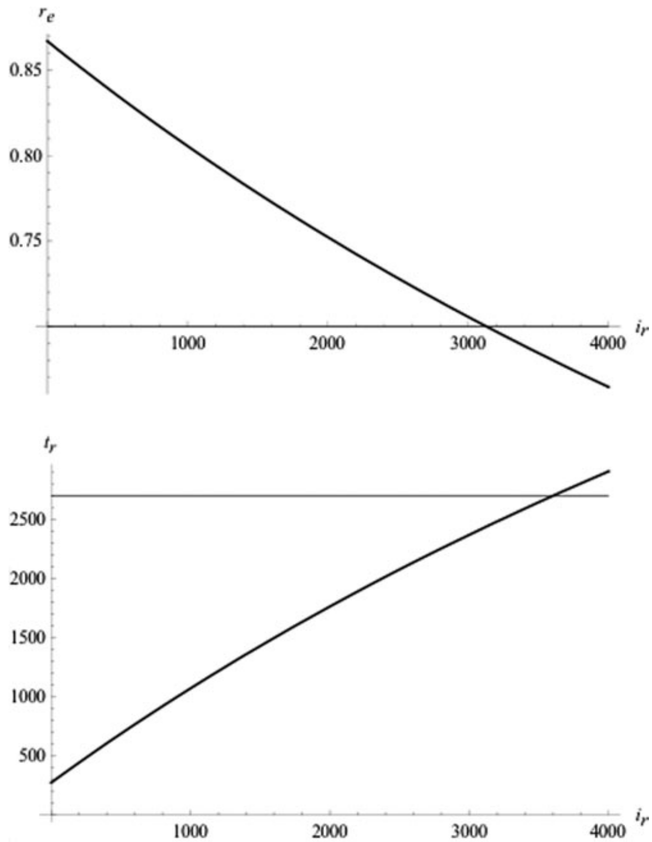

Figure 2. Performance curves for a Mode X DME as predicted by the model in Section 5.2.

transmission rate t_r as a function of the interrogation rate i_r. The calculation has been made on the basis of the above formulation for $\tau = 77$ µs. It is noted that the transmission rate takes on the recommended value $t_r = 2700$ ppps beyond capacity. The salient linearity of the above graphs suggests a linear approximation of the total transponder inactivity time T_{in} as a function of the parameter $\alpha = e_r \tau$. Numerical experimentation produces the following remarkably simple approximation:

$$T_{in} \approx 150 + 600\alpha. \tag{21}$$

Figure 3 is a graph of the function $V = T_{in} - 150 - 600\alpha$. For $0{\cdot}15 \leqslant \alpha \leqslant 0{\cdot}26$, it is verified that $|V| \leqslant 1$ ms per second. The error of approximation (21) is thus less than 0·5% for $0{\cdot}15 \leqslant \alpha \leqslant 0{\cdot}26$. If more precision is required, a parabolic correction can be easily adapted.

5.3. *Transponder performance in the presence of JTIDS transmissions.* It will be shown that in order for JTIDS transmissions to be tolerable, it will be required to compromise with a lower transponder load for the same value of transponder reply efficiency (70%). The method provided below permits the determination of the maximum transponder load. As for the JTIDS network organization, it is assumed throughout that only one terminal transmits in a given time slot.

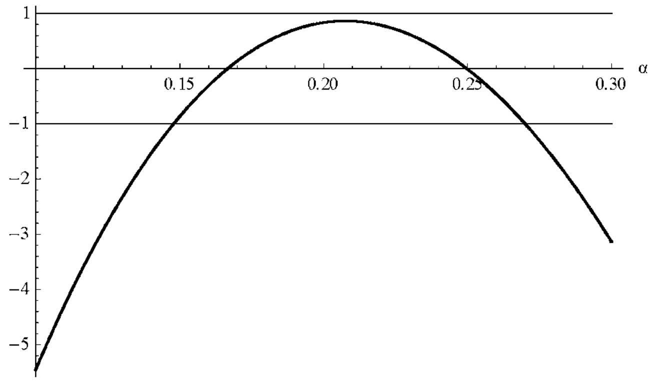

Figure 3. Graph of the function $V = T_{in} - 150 - 600\alpha$.

We start the analysis of the effect of the JTIDS transmissions with the remark that only a fraction $f_h = 1/17 \approx 6\%$ of the JTIDS pulses are considered potentially harmful. When a JTIDS pulse overlaps a DME pulse at the input of the transponder, it is expected, unless it is sufficiently weaker than the DME pulse, to cause the corruption of the shape characteristics of the DME pulse, its rejection by the decoder and eventually the loss of the corresponding reply. In particular, we assume that the separation between the peak of a DME pulse and the edge of a JTIDS pulse should be larger than $\tau_{PW}/2$; otherwise the DME pulse is corrupted. Therefore, a potentially harmful JTIDS pulse generates for the transponder a dead reception interval τ_J equal to the sum of the JTIDS pulse duration and one DME pulse width, that is $\tau_J = 6.4\ \mu s + \tau_{PW} \approx 10{\cdot}5\ \mu s$ (including a tolerance of 0·5 μs for τ_{PW}). In order for an event and in particular for an interrogation to be replied, neither of its two pulses should overlap a potentially harmful JTIDS pulse. It is furthermore noted that the probability of overlap with a potentially harmful JTIDS pulse is the same for the first and the second pulse of an event and equal to the proportion in the reception time of dead reception intervals of duration τ_J corresponding to potentially harmful JTIDS pulses.

Another issue that deserves mention is the relative strength of the JTIDS pulses with respect to the DME ones; for if the JTIDS pulse is sufficiently weaker it is not expected to distort appreciably the shape and hamper the decoding of the DME pulse. In order to keep the analysis simple, we assume that the separation of the JTIDS terminals from the transponder is much smaller than the DME service range. Hence the large majority of the co-channel JTIDS pulses are expected to corrupt overlapping DME pulses. Adjacent-channel JTIDS pulses must be less harmful since they are at least 10 dB weaker than the co-channel ones [1]. From the total population of potentially harmful JTIDS pulses, we assume in the following that a percentage $K = 0{\cdot}6 = 60\%$ have sufficient strength to corrupt an overlapping DME pulse.

From these considerations it is deduced that the probability p_J of a reply loss due to collision with a JTIDS pulse is given approximately by:

$$p_J = 2Kd_J\, 10^{-3} \tag{22}$$

where $d_J\, 10^{-3}$ denotes the proportion in the reception time of dead reception intervals of duration τ_J corresponding to potentially harmful JTIDS pulses. The quantity d_J can thus be interpreted as the total duration in ms of the induced dead reception

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intervals in a second of JTIDS transmissions. For a JTIDS network with TSDF Q% and transmission rate p_{rs} pulses per time slot, the total duration d_J, in ms per second, of the dead reception intervals and the corresponding probability p_J are given by:

$$d_J = f_h(128\,p_{rs})(Q/100)\tau_J \tag{23}$$

$$p_J = f_h(128\,p_{rs})(Q/100)(2K\tau_J)10^{-3} \tag{24}$$

In these formulas use has been made of the fact that each second contains 128 JTIDS time slots. In the application of these formulas τ_J needs to be expressed in ms as well ($\tau_J = 10{\cdot}5\ 10^{-3}$ ms). It is also noted that formula (24) dictates that the effective duration of the inactivity interval produced by a potentially harmful JTIDS pulse equals $2K\tau_J$. It will prove useful to define the quantity $d_{J,eff}$:

$$d_{J,\,eff} = p_J\ 10^3 = f_h(128\ \mathrm{prs})(Q/100)(2K\tau_J). \tag{25}$$

This quantity is referred to in the sequel as the effective total duration of the potentially harmful JTIDS pulses and is expressed in ms per second. For $Q = 100$ and $p_{rs} = 444$ pulses per time slot, $d_{J,eff}$ equals 42 ms per second.

So far in this section we have considered the probability p_J of a reply loss as if there were solely JTIDS transmissions and have neglected the existence and the effect of the workload of the transponder. At this point we need to combine the above results with those of the previous section in order to calculate the transponder reply efficiency in the presence of JTIDS transmissions. Adhering to our previous formulation, we will express probabilities in terms of time proportions in ms per second. Let $T_{in}10^{-3}$ and $T_{in,\mathrm{comb.}}10^{-3}$ denote respectively the probability of a reply loss in the absence and in the presence of JTIDS transmissions. Since the two considered causes of reply loss (i.e. the execution of the transponder functions and the JTIDS transmissions) are statistically independent, it holds:

$$\begin{aligned} T_{in,\ comb}10^{-3} &= T_{in}10^{-3} + (1 - P(\textit{reply loss due to transponder functions}))p_j \\ &= T_{in}10^{-3} + (1 - T_{in}10^{-3})d_{J,\,eff}\,10^{-3} \end{aligned} \tag{26}$$

Hence:

$$T_{in,\ comb} = T_{in} + (1 - T_{in}10^{-3})d_{J,\,eff} \tag{27}$$

Clearly, the transponder reply efficiency r_e can then be expressed as follows:

$$r_e = 1 - T_{in,\ comb}\cdot 10^{-3} \tag{28}$$

It is perhaps worthy of mention that we could have arrived at the result (27), simply by regarding T_{in} and $d_{J,eff}$ as transponder inactivity times in a second and noting that inactivity intervals produced by potentially harmful JTIDS pulses do not have a degrading effect if they appear during intervals of intrinsic transponder inactivity. To demonstrate the utility of the above results, we present a numerical example for a JTIDS network with 100% TSDF and $p_{rs} = 444$ pulses per time slot, in which case $d_{J,eff} = 42$ ms per second.

In order to achieve 70% transponder effciency, $T_{in,comb}$ should not exceed 300 ms per second. To this value of $T_{in,comb}$ there corresponds:

$$T_{in} = \frac{T_{in,\ comb} - d_{J,\,eff}}{1 - d_{J,\,eff}10^{-3}} = 269{\cdot}3\ \mathrm{ms\ per\ second.} \tag{29}$$

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On using formula (19) in the opposite direction, we can then calculate the corresponding value of the sum $\sum_{n=0}^{\infty} \delta_n$:

$$\sum_{n=0}^{\infty} \delta^n = \frac{T_{in} - \delta_{id}}{1 - p_{id}} = 164{\cdot}9 \text{ ms per second.} \tag{30}$$

At this point, formulas (12–18) can be utilized again in order to solve numerically the above equation for α. The result comes out to be $\alpha = 0{\cdot}19748$. (Alternatively, approximation (21) can be applied for $T_{in} = 269{\cdot}3$ ms per second). For $\tau = 77$ μs, the corresponding value of the event rate is $e_r = \alpha/\tau = 2565$ events per second. Hence, the maximum permissible rate i_r of interrogations is $i_r = e_r - m_r = 2445$ interrogations per second. Compared to the maximum rate of 3127 interrogations per second in the absence of JTIDS, the value $i_r = 2445$ represents a 22% decrease of interrogation handling capability. The respective transponder transmission rate equals $t_r = 0{\cdot}7e_r + d_r = 1964$ ppps. Based on the above simplified analysis, it follows that the effect of JTIDS transmissions to transponder efficiency is not negligible in general.

6. DESENSITIZATION OF EITHER TRANSPONDER OR INTERROGATOR RECEIVERS.

In general, JTIDS pulses are by no means of negligible strength in comparison with the DME pulses. Interrogator receivers are protected from JTIDS pulses on account of the principle of their operation: nonsynchronous signals are disregarded. However, interrogator receivers and transponder receivers alike can be desensitized if operated in proximity to a JTIDS transmitter.

In the following, as indicative of the transponder and interrogator desensitization, points are taken the upper limits of their dynamic range, which are specified in the ICAO standard [2, paras. 3.5.4.2.3.3 and 3.5.5.3.2.3] in terms of impinging power flux. A transponder should be capable of handling signals of strength up to -35 dBW/m² (corresponding to 0·316 mW/m²). Likewise an interrogator should be capable of handling signals of strength up to -18 dBW/m² (corresponding to 16 mW/m²). Based on the aforementioned values, we are going to calculate below the respective estimates for the separation distance from a JTIDS transmitter.

If ρ (in pulses per second) is the rate of JTIDS transmissions and P (in W) is the average output power of the transmitter, the energy E_j of each pulse at the output of the transmitter equals:

$$E_j = \frac{P}{\rho} \tag{31}$$

Hence, at the output of the transmitter the mean power of each JTIDS pulse equals $E_j/(6{\cdot}4$ μs). Considering a 4 dB net gain of the transmission system [1] in the direction of maximum radiation, the mean equivalent radiated power flux Φ during a JTIDS pulse at distance D (in km) from its transmitter is calculated to be:

$$\Phi = \frac{10^{0{\cdot}4} E_j}{(6{\cdot}4\mu s)(4\pi 10^6 D^2)} = 31{\cdot}2 \frac{P}{\rho D^2} \text{ in mW/m}^2 \tag{32}$$

Solving for D we obtain:

$$D = \sqrt{\frac{31{\cdot}2P}{\rho\Phi}} \tag{33}$$

Table 2. Estimate of the required separation distance D from a JTIDS transmitter.

	For $\Phi = 0{\cdot}316$ mW/m² (Transponder)	For $\Phi = 16$ mW/m² (Interrogator)
P = 200 W, ρ = 56832 pps	D = 0·59 km	D = 0·083 km
P = 200 W, ρ = 33024 pps	D = 0·77 km	D = 0·109 km
P = 200 W, ρ = 9216 pps	D = 1·46 km	D = 0·206 km

where D, P and Φ are expressed in km, W and mW/m², respectively. Table 2 gives numerical results for the separation distance D for the aforementioned desensitization values.

Note that for the transmission rate ρ, under the assumption of full time slot exploitation (100% TSDF), the formula $\rho = 128p_{rs}$ was used, where $p_{rs} = 444$, 258 or 72 pulses per time slot.

It follows from Table 2 that a separation distance from a JTIDS transmitter to avoid desensitization of DME receivers is easily attainable. The above values should be contrasted to the distance $D = 295$ km, calculated for $P = 200$ W and $\rho = 56832$ pps, at which the mean power flux of a JTIDS pulse equals the minimum usable peak power density of a DME reply pulse (-89 dBW/m², or equivalently, $1{\cdot}26\ 10^{-6}$ mW/m²).

7. SUMMARY.

In the present paper three distinct mechanisms of interference of JTIDS transmissions to DME have been investigated in succession:

- false interrogator triggering,
- transponder overloading and
- desensitization of either transponder or interrogator receivers.

Concerning the first mechanism, analysis has shown that, under the pessimistic assumption that single JTIDS pulses can trigger a DME interrogator, the ensuing probability of false measurement of distance is negligible. As for the third interference mechanism, by using the upper limit of the dynamic range of an interrogator or a transponder, it has been found that with moderate distance separation DME receivers can be protected from desensitization by JTIDS pulses.

On the other hand, the effect of transponder overloading due to JTIDS transmissions has been found to be more serious. For this reason, it is deemed appropriate to summarize below the basic assumptions of the simplified analysis, which has led to such finding. The transponder is supposed to be rendered incapable of responding to an aircraft's interrogation during the execution of some of its functions (reception of other interrogations, transmission of identification blocks and spontaneous activations) as well as at the occurrence of potentially harmful JTIDS pulses. The calculation of the transponder reply efficiency has been based on the analysis of the cumulative effect of the induced inactivity intervals, under the assumption that their occurrence is random. JTIDS transmissions tend to increase the total inactivity time of the transponder and thus degrade its reply effciency.

From the side of JTIDS operation, this detrimental effect can be avoided by slowing down the transmission. The transmission rate is controlled by the TSDF and the number p_{rs} of pulses per time slot (or, equivalently, by the fraction λ).

From the side of DME operation, we may consider the option of reducing the transponder load, that is the number of interrogations received per second. For Mode X DME, the required decrease of the interrogation handling capability of the transponder is calculated at 22% (for TSDF = 100% and $p_{rs} = 444$ pulses per time slot), a figure which may be problematic in congested air-space environments. Another possibility is to allow a reduction of the time reserved for the identification transmissions.

A quasi-linear model has been derived for the probability of a reply loss, which is applicable also in the absence of JTIDS transmissions. It is recognized that the numerical results produced, which remain to be supported by experimental evidence, depend strongly on the assumptions employed. For instance, considerable performance improvement may result if the dead time after each interrogation is less than 60 μs.

More notably and beyond the discussion on the various parameter values, a method is developed in this paper which permits the analytical assessment of the performance of the transponder and which relates in particular the transponder reply efficiency to the load of the interrogations received in the presence of JTIDS transmissions.

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REFERENCES

[1]. NATO, Frequency Clearance Criteria Documentation for JTIDS/MIDS Operations, December 1996.

[2]. International Civil Aviation Organization, Annex 10 to the Convention on International Civil Aviation, Volume I, Sixth Edition incorporating Amendment 82.

[3]. C. Alari, Instruments de Radionavigation, Ecole Nationale de l'Aviation Civile, 6eme Edition, 1992.

[4]. C. Kopp, Network Centric Warfare Fundamentals -Part 3: JTIDS/MIDS, Defence Today Magazine, September 2005.

[5]. R.C. Dixon, Spread Spectrum Systems with Commercial Applications, John Wiley & Sons, 1994.

APPENDIX

Calculation of the Mean Inactivity Times ζ_n. *The case* n = *2 starts getting interesting because there are two possibilities: either* $X_1 + X_2 \leqslant \tau$, *in which case the time of inactivity is* τ, *or*, $X_1 + X_2 > \tau$, *in which case the time of inactivity is* 2τ. *To calculate the mean inactivity time* ζ_2, *it is necessary to determine first the probability of the event* $X_1 + X_2 \leqslant \tau$ *on condition that* $X_1 < \tau$ *and* $X_2 < \tau$:

$$\begin{aligned} P\{X_1 + X_2 \leqslant \tau / X_1 < \tau,\ X_2 < \tau\} &= \gamma^{-2} P\{X_1 + X_2 \leqslant \tau\} \\ &= \gamma^{-2} \int_0^{\tau} dx_1 e_r e^{-e_r x_1} \int_0^{\tau - x_1} dx_2 e_r e^{-e_r x_2} \qquad (A1) \\ &= \frac{\gamma - \alpha\beta}{\gamma^2}. \end{aligned}$$

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Therefore, the mean inactivity time ζ_2 *is given by:*

$$\zeta_2 = \tau \frac{\gamma - \alpha\beta}{\gamma^2} + 2\tau\left[1 - \frac{\gamma - \alpha\beta}{\gamma^2}\right] = \tau\left[2 - \frac{\gamma - \alpha\beta}{\gamma^2}\right]. \tag{A2}$$

The case n=3 *is similar. The time of inactivity equals* τ *when* $X_1 + X_2 + X_3 \leqslant \tau$ *and* 2τ *otherwise. The probability that the time of inactivity equals* τ *is provided by:*

$$\begin{aligned} &P\{X_1 + X_2 + X_3 \leqslant \tau / X_1 < \tau,\ X_2 < \tau,\ X_3 < \tau\} \\ &\quad = \gamma^{-3} P\{X_1 + X_2 + X_3 \leqslant \tau\} \\ &\quad = \gamma^{-3} \int_0^{\tau} dx_1 \int_0^{\tau - x_1} dx_2 \int_0^{\tau - x_1 - x_2} dx_3 e_r^3 e^{-e_r(x_1 + x_2 + x_3)} \\ &\quad = \frac{\gamma - \alpha\beta(1 + \alpha/2)}{\gamma^3}. \end{aligned} \tag{A3}$$

Thus, for the mean inactivity time ζ_3 *we obtain:*

$$\zeta_3 = \tau \frac{\gamma - \alpha\beta(1 + \alpha/2)}{\gamma^3} + 2\tau\left[1 - \frac{\gamma - \alpha\beta(1 + \alpha/2)}{\gamma^3}\right] = \tau\left[2 - \frac{\gamma - \alpha\beta(1 + \alpha/2)}{\gamma^3}\right]. \tag{A4}$$

In the case n=4 *there are three possible values of the inactivity time for an isolated set of 5 overlapping events:*

(*a*) τ, *when* $X_1 + X_2 + X_3 + X_4 \leqslant \tau$ *with probability:*

$$P\left\{X_1 + X_2 + X_3 + X_4 \leqslant \tau / \bigcap_{i=1}^{4} X_i < \tau\right\} = \frac{\gamma - \alpha\beta(1 + \alpha/2 + \alpha^2/6)}{\gamma^4}, \tag{A5}$$

(*b*) 3τ, *when* $X_1 + X_2 > \tau$ *and* $X_3 + X_4 > \tau$ *with probability:*

$$\begin{aligned} &P\left\{X_1 + X_2 > \tau,\ X_3 + X_4 > \tau / \bigcap_{i=1}^{4} X_i < \tau\right\} = P^2\left\{X_1 + X_2 > \tau / \bigcap_{i=1}^{2} X_i < \tau\right\} \\ &\quad = [1 - P\{X_1 + X_2 \leq \tau / X_1 < \tau,\ X_2 < \tau\}]^2 \\ &\quad = \left[1 - \frac{\gamma - \alpha\beta}{\gamma^2}\right]^2 = \frac{\beta^2(\alpha - \gamma)^2}{\gamma^4}, \end{aligned} \tag{A6}$$

(*c*) 2τ *otherwise. Consequently:*

$$\zeta_4 = 2\tau + \tau \frac{\beta^2(\alpha - \gamma)^2 + \alpha\beta(1 + \alpha/2 + \alpha^2/6) - \gamma}{\gamma^4}. \tag{A7}$$

Finally, we consider the case n=5. *As in the previous case, the possible values of the inactivity time are* τ, 2τ *and* 3τ. *The time of inactivity is* τ *with probability:*

$$P\left\{\sum_{i=1}^{5} X_i \leqslant \tau / \bigcap_{i=1}^{5} X_i < \tau\right\} = \frac{\gamma - \alpha\beta(1 + \alpha/2 + \alpha^2/6 + \alpha^3/24)}{\gamma^5}. \tag{A8}$$

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On the other hand, there are two possibilities for the inactivity time to be 3τ. First, when $X_1+X_2 \leqslant \tau$, $X_1+X_2+X_3>\tau$ *and* $X_4+X_5>\tau$ *with probability:*

$$\begin{aligned}
&P\left\{\sum_{i=1}^{2} X_i \leqslant \tau,\ \sum_{i=1}^{3} X_i>\tau,\ \sum_{i=4}^{5} X_i>\tau \Big/ \bigcap_{i=1}^{5} X_i<\tau\right\} \\
&\quad = P\left\{\sum_{i=1}^{2} X_i \leqslant \tau,\ \sum_{i=1}^{3} X_i>\tau \Big/ \bigcap_{i=1}^{3} X_i<\tau\right\} P\left\{\sum_{i=4}^{5} X_i>\tau \Big/ \bigcap_{i=4}^{5} X_i<\tau\right\} \\
&\quad = \gamma^{-5}\beta(\alpha-\gamma) P\{X_1+X_2 \leqslant \tau,\ X_1+X_2+X_3>\tau,\ X_3<\tau\} \\
&\quad = \gamma^{-5}\beta(\alpha-\gamma) \int_0^{\tau} dx_1 \int_0^{\tau-x_1} dx_2 \int_{\tau-x_1-x_2}^{\tau} dx_3 e_r^3 e^{-e_r(x_1+x_2+x_3)} \\
&\quad = \gamma^{-5}\beta^2(\alpha-\gamma)(\alpha\beta-\gamma+\alpha^2/2).
\end{aligned} \tag{A9}$$

Second, when $X_1+X_2>\tau$ *and* $X_3+X_4+X_5>\tau$ *with probability:*

$$\begin{aligned}
&P\left\{\sum_{i=1}^{2} X_i>\tau,\ \sum_{i=3}^{5} X_i>\tau \Big/ \bigcap_{i=1}^{5} X_i<\tau\right\} \\
&\quad = \left(1-P\left\{\sum_{i=1}^{2} X_i \leqslant \tau \Big/ \bigcap_{i=1}^{2} X_i<\tau\right\}\right)\left(1-P\left\{\sum_{i=3}^{5} X_i \leqslant \tau \Big/ \bigcap_{i=3}^{5} X_i<\tau\right\}\right) \\
&\quad = \gamma^{-5}\beta(\alpha-\gamma)(\gamma^3-\gamma+\alpha\beta+\alpha^2\beta/2) \\
&\quad = \gamma^{-5}\beta^2(\alpha-\gamma)(\alpha+\alpha^2/2-\gamma^2-\gamma).
\end{aligned} \tag{A10}$$

Hence:

$$P\{\textit{Inactivity Time}=3\tau\} = \gamma^{-5}\beta^2(\alpha-\gamma)(\alpha+\alpha^2+\alpha\beta-2\gamma-\gamma^2). \tag{A11}$$

It can thus be deduced that:

$$\zeta_5 = 2\tau+\tau\frac{\beta^2(\alpha-\gamma)(\alpha+\alpha^2+\alpha\beta-2\gamma-\gamma^2)+\alpha\beta(1+\alpha/2+\alpha^2/6+\alpha^3/24)-\gamma}{\gamma^5}. \tag{A12}$$