Macroeconomic Dynamics, **18**, 2014, 1726–1750. Printed in the United States of America.
doi:10.1017/S136510051300014X

BANKS' PRECAUTIONARY CAPITAL AND CREDIT CRUNCHES

Fabián Valencia
International Monetary Fund

This paper develops a bank model to study supply-driven contractions in credit or *credit crunches*. In the model, the bank is affected by financial frictions in raising external funds. These frictions imply that the bank repairs its balance sheet only gradually following a negative shock that weakens the bank's capital position. Consequently, there is persistency in the response of bank lending even when the original shock (productivity or interest rate) is i.i.d. The nonlinear nature of these financial frictions also generates (i) a precautionary motive even with risk-neutral shareholders: the bank increases its desired level of capital if risk increases; (ii) an asymmetric response of lending: negative disturbances can have a bigger impact than positive ones; and (iii) volatility clustering in risk spreads and the bank's share price.

Keywords: Financial Frictions, Credit Crunch, Precautionary Motive

1. INTRODUCTION

Prior to the recent global financial crisis, the predominant view in the field of macroeconomics was that financial frictions affecting borrowers mattered for business cycle fluctuations [Bernanke et al. (1999) and others]. But there was skepticism about the quantitative importance of financial frictions affecting lenders [Driscoll (2004) and others]. The crisis, however, has brought to the fore the need to better understand how financial frictions affecting lenders can influence their lending decisions and the impact of those decisions on the aggregate economy.

This paper advances our knowledge in this regard by looking specifically at the behavior of banks in imperfect capital markets and shedding light on bank behavior in periods of distress such as the recent one. Its contribution comes at a crucial juncture for research and policy discussions in this area, because the lessons from the recent crisis are shaping the new macroeconomic policy framework going forward.

I am especially grateful to Christopher Carroll for his guidance and encouragement throughout graduate school. I would also like to thank Lawrence Ball, William Barnett (the editor), Allen Berger, Markus Brunnermeier, Jon Faust, an anonymous referee, and seminar participants at the Johns Hopkins University, the Bank of Canada, the International Monetary Fund, the Board of Governors of the Federal Reserve System, and the 2009 Financial Intermediation Research Society Conference for helpful comments. The views expressed in this paper are solely those of the author and do not represent the views of the IMF or IMF policy. Address correspondence to: Fabían Valencia, International Monetary Fund, 700 19th Street N.W., Washington, DC 20431, USA; e-mail: fvalencia@imf.org.

 1365-1005/13

The main contribution of this paper is a quantitative theoretical framework for understanding how changes in the financial condition of banks, triggered by aggregate shocks, affect lending decisions. The paper shows that financial frictions affecting lenders can have powerful and persistent effects on the supply of loans and thus the traditional view that focuses only on the demand side is incomplete.

The model developed in this paper consists of borrowers and a monopolistic bank. The borrower–bank relationship is modeled as a risky debt contract subject to costly state verification [Townsend (1979)]. The bank faces a maturity mismatch between loans and deposits and it faces frictions in raising external funds as a result of credit market imperfections. The bank mitigates credit market frictions by accumulating internal funds, but it does not overcome them fully because equity is costly. The balance between costly equity, which induces the bank to distribute dividends, and credit market imperfections, which induce the bank to accumulate capital, generates a target level of bank capital. To focus attention entirely on the bank's optimal decisions, borrowers' creditworthiness is held constant over time—that is, the Bernanke et al. (1999) financial accelerator mechanism is shut down. There are two sources of aggregate fluctuations in the model, interest rates and aggregate productivity.

The model's main result is that an i.i.d. one-time shock (to interest rate or aggregate productivity) generates a persistent response in lending. Because aggregate shocks are assumed to be i.i.d. and financial accelerator effects on borrowers are shut down, this result is entirely the outcome of financial frictions affecting the bank. When a negative shock—aggregate productivity or interest rate—deteriorates bank capital, the presence of financial frictions makes it costly for the bank to raise external funds. Therefore, the bank can restore bank capital only gradually by reducing dividends. Meanwhile, a persistent credit crunch arises because the cost of external funds to finance new lending has increased.

An important characteristic of the model is its nonlinearity. This characteristic produces additional implications. First, the model generates a precautionary motive even when shareholders/managers are assumed to be risk-neutral. This behavior in and of itself can trigger a persistent credit crunch. A mean-preserving increase in aggregate risk raises the marginal value of bank capital and the bank's target level of capital. However, the bank can accumulate capital to reach this new target level only gradually. Second, the response of lending to shocks is asymmetric: negative shocks can have a much stronger impact on lending than positive ones, and the contraction in credit following a negative shock is nonlinear in the size of the shock. And third, the model generates volatility clustering in spreads and the bank's share price.

The related theoretical literature on the dynamic implications of financial frictions includes prominent contributions that focused first on frictions affecting borrowers only, such as Gertler (1992), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), and Bernanke et al. (1999). The effect of frictions on borrowers having been established, several, mostly recent, contributions allow for financial frictions on intermediaries, such as Chen (2001), Christiano et al. (2004), Hirakata et al. (2009), Gertler and Karadi (2010), Gertler and Kiyotaki (2010), Meh and Moran (2010), Sun (2011), Kollmann et al. (2012), and Sandri and Valencia (2012).[1] However, in all these models, substantial simplifications in banks' behavior are introduced for tractability purposes. These generally include ignoring sources of nonlinear behavior, neglecting mismatches in maturity, and assuming exogenous dividend distribution. This paper explicitly models all these elements.

Closely related work includes that of Brunnermeier and Sannikov (2011), who present a general equilibrium model in continuous time with a financial sector. In their model, financial frictions affect only the leveraged sector, which is interpreted as the financial sector. In their model, the financial sector invests in a production technology and thus can be seen as an investor, whereas there are no maturity mismatches. In contrast, financial frictions in the model presented in this paper affect both sides of the financial sector balance sheet and are subject to a maturity mismatch. These differences bring the model presented here closer to the real structure of a bank. Van Den Heuvel (2006) and Aliaga-Diaz and Olivero (2012) present a bank model similar to the one presented in this paper, but with the friction being capital requirements. Instead, the friction in this paper arises from limited liability and asymmetric information among the parties involved in a financial contract.

This paper is organized as follows: the next section presents the model; Section 3 presents the quantitative analysis, which starts with the calibration of the model, followed by a discussion of the optimal policy rules and some quantitative experiments; and Section 4 concludes.

2. THE MODEL

Credit markets include a monopolistic bank and a continuum of ex ante identical borrowers. The borrower–bank relationship is modeled as a risky debt contract with costly monitoring. The existence of a bank in this environment is justified by its efficiency in ex ante evaluation and ex post monitoring of borrowers' investment projects, relative to individual investors. Although a monopolistic bank is an unrealistic assumption, only a much more complicated industry structure would capture reality more closely. However, one may justify it by thinking of a regional monopoly with the bank having substantial informational advantages over some segment of the pool of borrowers.

2.1. The Loan Contract

The loan agreement takes the form of a risky debt contract as described in Gale and Hellwig (1985), and similar to the one in Carlstrom and Fuerst (1997) and Bernanke et al. (1999). The demand side of credit comes from a continuum of entrepreneurs whose individual size is negligible relative to that of the bank. To avoid keeping track of entrepreneurs' credit history, it is assumed that entrepreneurs live for only two periods. Alternatively, one could assume that there is enough anonymity among borrowers so that only their endowment at the moment of applying for a loan matters for credit decisions. Entrepreneurs receive a common endowment of resources at birth, which for simplicity is normalized to 1, and consume only in the second period of their lives. The endowment, together with loans from the bank, l_t, is used to purchase capital, k, at a unit price. It is assumed that entrepreneurs have access to a common production technology that uses only capital as an input. For simplicity it is assumed that production takes two periods, with capital depreciating fully at the end of production. An entrepreneur's production y, at time $t+2$, is given by

$$y_{t+2} = \alpha_{t+2}\Phi_{t+2}\xi k_{t+2}, \qquad (1)$$

where $k_{t+2} = l_t + 1$, with α and Φ denoting i.i.d. stochastic idiosyncratic and aggregate productivity shocks respectively, with $E[\Phi] = E[\alpha] = 1$, for all t, distributed over a non-negative support. ξ denotes the deterministic return on capital. It is assumed that there exists a minimum scale for entrepreneurs' projects, requiring a level of investment that is strictly larger than the endowment.

There are no ex ante informational asymmetries; productivity is unknown when investment is made. However, following Townsend (1979), once idiosyncratic productivity, α, is realized, it is assumed to remain the private information of entrepreneurs. The bank may observe the productivity realization of an entrepreneur only after paying monitoring or bankruptcy costs $1 \geq u > 0$, expressed as a fraction of a borrower's project value. An entrepreneur's ex post return, π, is equal to the outcome of the investment minus the amount—principal plus interest—owed to the bank, $R_t l_t$:

$$\pi(R_t, l_t, \alpha_{t+2}, \Phi_{t+2}) = (l_t + 1)\xi\, \alpha_{t+2}\Phi_{t+2} - R_t l_t, \qquad (2)$$

where R_t and l_t denote the interest rate and loan amount agreed on the debt contract. Continuity of α implies that there exists a cutoff value $\underline{\alpha}$ such that $\pi = 0$, given by

$$\underline{\alpha} = \frac{R_t l_t}{(l_t + 1)\xi\Phi_{t+2}}. \qquad (3)$$

Under limited liability, the ex post return to an entrepreneur can be summarized by

$$\pi(\alpha_{t+2}, R_t, l_t, \Phi_{t+2}) = \left\{ \begin{matrix} \alpha_{t+2}\xi\Phi_{t+2}(1 + l_t) - R_t l_t & \text{if } \alpha_{t+2} \geq \underline{\alpha} \\ 0 & \text{if } \alpha_{t+2} < \underline{\alpha} \end{matrix} \right\}. \qquad (4)$$

The bank makes "take-it-or-leave-it" offers to entrepreneurs that include an amount l_t and an interest rate R_t. Because borrowers are identical ex ante, they are all offered the same contract. A borrower's decision is limited to accepting or rejecting the offer. If a borrower rejects the offer, he invests his endowment in the project and earns an expected return equal to the deterministic return on capital ξ. The

https://doi.org/10.1017/S136510051300014X Published online by Cambridge University Press

problem for an entrepreneur is given by

$$\underset{\{\text{accept, reject}\}}{\text{Max}} \{\mathrm{E}_t \pi(\alpha_{t+2}, R_t, l_t, \Phi_{t+2}), \xi\} \tag{5}$$

Participation of any entrepreneur is subject to a rationality constraint requiring that the rate of return from the project is at least as good as his opportunity cost; that is,

$$\mathrm{E}_t \pi(\alpha_{t+2}, R_t, l_t, \Phi_{t+2}) \geq \xi. \tag{6}$$

With the "take-it-or-leave-it" assumption, and the existence of an interior solution, constraint (6) holds with equality at the levels of lending and interest rate that solve the bank's problem—presented in the next section. If not, the bank could always charge a slightly higher interest rate and would still have a borrower accepting the offer. Thus, equation (6) implicitly defines the interest rate schedule $R(l)$ charged by the bank. The assumption of risk-neutral agents avoids the need to bring risk-sharing considerations into the picture.

Revenues for the bank are then given by

$$g(\alpha_{t+2}, l_t, \Phi_{t+2}) = \left\{ \begin{array}{ll} R_t l_t & \text{if } \alpha_{t+2} \geq \underline{\alpha} \\ \xi \alpha_{t+2} \Phi_{t+2}(1 + l_t)(1 - u) & \text{if } \alpha_{t+2} < \underline{\alpha} \end{array} \right\}. \tag{7}$$

The assumption of a continuum of borrowers and the law of large numbers imply that the bank can perfectly diversify the idiosyncratic component of risk. Denoting the mean of a variable across realizations of idiosyncratic productivity as $M[\cdot]$, ex post revenues for the bank are given by

$$G(l_t, \Phi_{t+2}) = M[g(\alpha_{t+2}, l_t, \Phi_{t+2})] \tag{8}$$

Notice that with a fixed endowment for borrowers, the Bernanke et al. (1999) financial accelerator effect is shut down. Furthermore, with i.i.d. shocks, ex post revenues for the bank are a function only of aggregate risk and the aggregate amount of lending.

2.2. The Bank's Optimization Problem

It is assumed that risk-neutral stockholder–managers simultaneously choose amounts of lending (l_t), dividends (d_t), and deposits (c_t) to maximize the market value of the bank, given by

$$\underset{\{l_s, d_s, c_s\}}{\text{Max}} \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{t-s} d_s, \tag{9}$$

where β denotes the discount factor. Decisions are made before uncertainty is revealed. Table 1 shows the sequence of events for the bank.

https://doi.org/10.1017/S136510051300014X Published online by Cambridge University Press

TABLE 1. Sequence of events

$t-1$		t		$t+1$
State variables after decisions are made: o_{t-1}, l_{t-1}, c_{t-1}				
	Uncertainty is realized: Φ_t, ρ_t			
		State variables: o_t, n_t Control variables: c_t, d_t, l_t		
			State variables after decisions are made: o_t, l_t, c_t	
				Uncertainty is realized: Φ_{t+1}, ρ_{t+1}

As seen in the table, the bank has two state variables: the book value of bank capital, n, and outstanding loans, o. Recall that loans are extended with a two-period maturity. Their respective transition equations are given by

$$n_{t+1} = G(o_t, \Phi_{t+1}) + l_t - (1 + i_{t+1})c_t, \tag{10}$$

$$o_{t+1} = l_t, \tag{11}$$

where (10) denotes the difference in book value of assets and liabilities. $G(o_t, \Phi_{t+1})$ denotes the bank's ex post revenues from loans made in the previous period shown in equation (8), l_t denotes loans made in the current period, and $(1 + i_{t+1})c_t$ denotes obligations to depositors. i is the interest rate on deposits at which funds are supplied infinitely elastically by depositors. Finally, it is assumed that credit market frictions affect the bank in two ways: (i) through the interest rate on deposits i, which is assumed to be given by $i = \rho + f$, where ρ is the risk-free rate and $f = f(\frac{c}{l+o})$ is a risk spread that is assumed to be a function of the bank's leverage, which is increasing and twice continuously differentiable in deposits c; and (ii) through a no-equity finance constraint, equivalent to assuming

https://doi.org/10.1017/S136510051300014X Published online by Cambridge University Press

that it is infinitely costly for the bank to issue equity

$$d_t \geq 0. \tag{12}$$

The assumption of having funding costs linked to the financial condition of the bank through f can be interpreted in several ways. One interpretation is that of deposit insurance premium, costs associated with intensified regulatory scrutiny as the bank's financial position weakens, or simply expected bankruptcy costs as in the case of the borrower–bank relationship. In Appendix A, I show how the costly state verification framework, used in the borrower–bank contract, can be used in the bank–depositor contract to generate f endogenously. Because the objective of this paper is not to argue in favor of some specific form of friction, it suffices to use the reduced-form modeling device introduced through f.

A final restriction requires deposits to be at least as large as the funding needs of the bank, given by the difference between the book value of assets and bank capital net of dividends. This restriction is summarized by

$$c_t \geq o_t + l_t - (n_t - d_t). \tag{13}$$

The preceding relation cannot hold with strict inequality because the excess over the right-hand side would not yield any returns if held in cash, or it would yield the risk-free return if held in government securities. Notice that $i > \rho$ as long as the bank has positive leverage, because in that case $f > 0$. This means that equation (13) always holds with equality, because otherwise the bank can always increase profits by reducing deposits.

For holdings of liquid assets, either government securities or cash, to be positive in this model, it would be necessary to introduce coordination problems as in Diamond and Dybvig (1983) or Kashyap et al. (2002), or alternatively, by introducing a liquidity shock into the problem in a second stage after loans have been made, but before they are collected. The latter would allow a role for liquid assets. Because the focus of this paper is entirely on solvency, this nontrivial modification of the model is left for future work.

The following equations summarize the bank's problem, written in Bellman's equation form:

$$V_t(n_t, o_t) = \underset{\{d_t, l_t\}}{\text{Max}} \{d_t + \text{E}_t \beta V_{t+1}(n_{t+1}, o_{t+1})\}, \tag{14}$$

s.t.

$$c_t = o_t + l_t - (n_t - d_t), \tag{15}$$

$$d_t \geq 0, \tag{16}$$

$$n_{t+1} = G(o_t, \Phi_{t+1}) + l_t - (1 + \rho_{t+1} + f_t)c_t, \tag{17}$$

$$o_{t+1} = l_t. \tag{18}$$

https://doi.org/10.1017/S136510051300014X Published online by Cambridge University Press

A final assumption, necessary to guarantee a well-defined solution to the preceding problem, is that stockholders are impatient. This assumption requires $1/\beta > 1 + \mathrm{E}[\rho]$, and it guarantees that the bank does not fund itself entirely with internal funds. f is assumed to be increasing in leverage because as leverage goes to zero, financial frictions in raising deposits lose relevance and funding costs approach the risk-free rate. At that point, it does not pay to accumulate capital further and shareholders prefer dividends. This assumption is consistent with alternative micro-founded modeling approaches that would yield the same outcome (i.e., positive leverage), for instance, if leaving too much capital at the bank increased agency problems between shareholders and managers, or if debt had a preferential tax treatment through deduction of interest expenses.

3. QUANTITATIVE ANALYSIS

3.1. Calibration

The model's parameters to calibrate include the discount factor, β; the parameters a, b of the function $f = a[c/(l + o)]^b$; the parameters of the loan contract, which include the bankruptcy costs, μ; the deterministic return on capital, ξ; the parameters of the distribution of productivity shocks σ_α and σ_Φ; and interest rate shocks. The objective is to calibrate the stochastic steady state of the model, computed as the ergodic mean of the state variables. This point can be defined as the point where the bank chooses to stay at a given date if it expects future risk, but the realization of shocks today equals their unconditional mean.

Starting with the exogenous processes, it is assumed that aggregate productivity shocks are mean-one, lognormally distributed with standard deviation equal to 1.8%. This value corresponds to the unconditional standard deviation of an AR(1) process estimated on the log difference of total factor productivity series from the Congressional Budget Office over 1955–2009. It is assumed that the real risk-free interest rate is normally distributed with mean 1.5% and standard deviation 2.5%. These values correspond to the unconditional mean and standard deviation of an AR(1) process estimated on treasury bills yields deflated with the U.S. GDP deflator over the period 1955–2009.

The bankruptcy costs parameter, chosen from Bernanke et al. (1999), corresponds to a value $\mu = 0.12$. The deterministic return on capital, ξ, is set equal to 1.025 to approximate the net worth-to-assets ratio in the real sector to 0.5, which correspond to the Nonfarm Nonfinancial Corporate Business average net worth-to-assets ratio for 2002–2008 from the U.S. Flow of Funds. The model generates leverage in the real sector exactly equal to 0.5. Idiosyncratic productivity is assumed to be lognormally distributed, with mean one and standard deviation σ_α. The standard deviation is chosen to approximate the spread between the lending rate and the risk-free rate to the average spread between the prime bank loan rate and U.S. treasury bills. The data show an average spread of 2.4% in annual terms over the period 1955–2009. The model generates a steady state spread of 2.6%.

The lognormal distribution was chosen because it guarantees an interior solution for the loan contract [Bernanke et al. (1999)].

The parameter a of the financial frictions function, f, is chosen to approximate the average risk spread on bank debt to 76bps, which is the average spread on one-year BBB bonds issued by banking institutions in the United States over the period 2002–2008. The model generates a spread of 77bps. The parameter $b = 2$ corresponds to the minimum condition on f required to generate a decreasing marginal value of capital. Finally, the discount factor β is chosen to approximate the median capital-to-assets ratio of 10.3% observed among U.S. commercial banks during the period 2002–2008. A value $\beta = 0.975$ generates a steady state capital-to-assets ratio (after dividends) of 12.1%.

3.2. Optimal Policy Rules

The model is solved numerically using Carroll (2006)'s endogenous gridpoints method. Appendix B provides a detailed description of the solution algorithm. For intuition purposes, it is convenient to define end-of-period capital as $q_t = n_t - d_t$. Using equation (15) to substitute out deposits and equation (18) to substitute out outstanding loans, the problem can be written as

$$V_t(n_t, l_{t-1}) = \underset{\{d_t, l_t\}}{\text{Max}} \{n_t - q_t + \text{E}_t \beta V_{t+1}(n_{t+1}, l_t)\}, \quad \textbf{(19)}$$

s.t.

$$n_t - q_t \geq 0, \quad \textbf{(20)}$$

$$n_{t+1} = G(l_{t-1}, \Phi_{t+1}) + l_t - (1 + \rho_{t+1} + f_t)(l_{t-1} + l_t - q_t), \quad \textbf{(21)}$$

with first-order conditions

$$\text{E}_t[(1 + \rho_{t+1} + f_t) - (l_{t-1} + l_t - q_t)\partial f/\partial q]\partial V_{t+1}(n_{t+1}, l_t)/\partial n = 1/\beta, \quad \textbf{(22)}$$

$$\begin{aligned}&\text{E}_t\{[-\rho_{t+1} - f_t - (l_{t-1} + l_t - q_t)\partial f/\partial l]\partial V_{t+1}(n_{t+1}, l_t)/\partial n \\ &\quad + \partial V_{t+1}(n_{t+1}, l_t)/\partial l\} = 0\}.\end{aligned} \quad \textbf{(23)}$$

Equation (22) tells us that the amount of dividends distributed is such that their marginal value—given by $1/\beta$—equals the marginal value of bank capital. The left-hand side of the equation shows that the marginal value of bank capital is affected by the marginal costs of raising deposits, given by the expression within squared brackets, and changes in future profits. These terms affect the decision to distribute dividends because retained earnings reduce the costs of raising deposits today, but also generate dividends in the future. Notice also that the first term, the savings from raising less deposits, is not a constant because f links the funding costs of the bank to its leverage. This means that an extra dollar of capital is very valuable when leverage is high and much less valuable when leverage is low.

https://doi.org/10.1017/S136510051300014X Published online by Cambridge University Press

In the case of lending, equation (23) dictates that optimal lending is such that the marginal profit from lending equals the marginal cost of raising funds. This is better appreciated by rewriting equation (23) as

$$\begin{aligned} &\mathrm{E}_t[\rho_{t+1} + f_t + (l_{t-1} + l_t - q_t)\partial f/\partial l]\partial V_{t+1}(n_{t+1}, l_t)/\partial n \\ &\quad = \mathrm{E}t[\partial V_{t+1}(n_{t+1}, l_t)/\partial l]. \end{aligned} \tag{24}$$

In equation (24), the left-hand side corresponds to the marginal cost of lending, which is determined by the increase in funding costs—given by $[\rho_{t+1} + f_t + (l_{t-1} + l_t - q_t)\partial f/\partial l]$—and by the change in future profitability of the bank, $\partial V_{t+1}(n_{t+1}, l_t)/\partial n$. The latter term appears because an increase in lending today affects the bank's capital position tomorrow and thus tomorrow's lending opportunities. The right-hand side of equation (24) corresponds to the marginal benefit from lending, which incorporates the change in future profitability of the bank due to lending today because loans extended in the current period are collected in the next period. As was noted in the analysis of equation (22), the marginal cost of lending is not a constant unless $f = 0$. Because f increases with leverage, the higher the leverage, the more costly it is for the bank to grant new loans. Clearly, if $f = 0$, the link between the bank's lending decisions and its financial structure breaks and Modigliani and Miller (1958)'s theorem would be applicable to the bank.

The problem is solved recursively using equations (22) and (23) as described in Appendix B. Figure 1 shows the time-invariant optimal decision rules as a function of the state variables, with the arrowheads indicating the stochastic steady state for each variable, as defined in the calibration section, which are also referred to as the targets. In both cases one can notice a kink in the policy functions, which is precisely the point where the constraint on dividends is binding. Furthermore, the policy functions are nondecreasing in bank capital. Take the policy function on dividends. The constraint on equity financing dictates that they cannot be negative, but as long as the constraint is not binding, the bank finds it optimal to distribute any excess capital (capital above the target) in dividends. The lending policy function is nondecreasing in bank capital and decreasing in the amount of outstanding loans. For a given level of outstanding loans, the more capital the bank has, the lower the intensity of financial frictions, which implies a lower marginal cost of lending. However, once capital has increased above the level where the constraint on dividends binds, the bank finds it optimal to distribute dividends instead of increasing lending further. For a given level of capital, lending decreases as outstanding loans in the portfolio increase. This aspect is intuitive, as the size of the existing loans portfolio grows, leverage is increasing, which implies it is more costly for the bank to fund new loans.

3.3. Risk and the Target Level of Solvency

An additional implication of the presence of financial frictions can be appreciated in Figure 2, ignoring the dashed line for the time being. The figure plots the

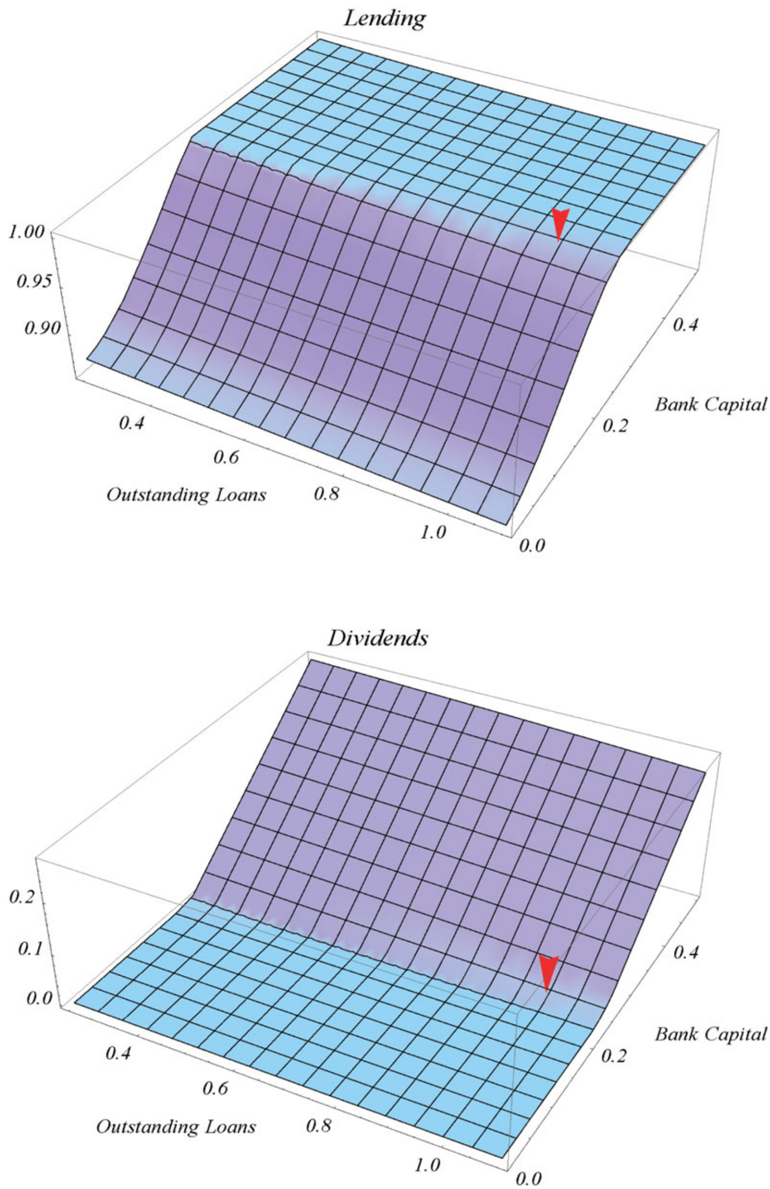

FIGURE 1. Optimal policy functions. Optimal solutions for the control variables as a function of the state variables of the system. The arrowheads show the location of the ergodic means for the corresponding variables.

https://doi.org/10.1017/S136510051300014X Published online by Cambridge University Press

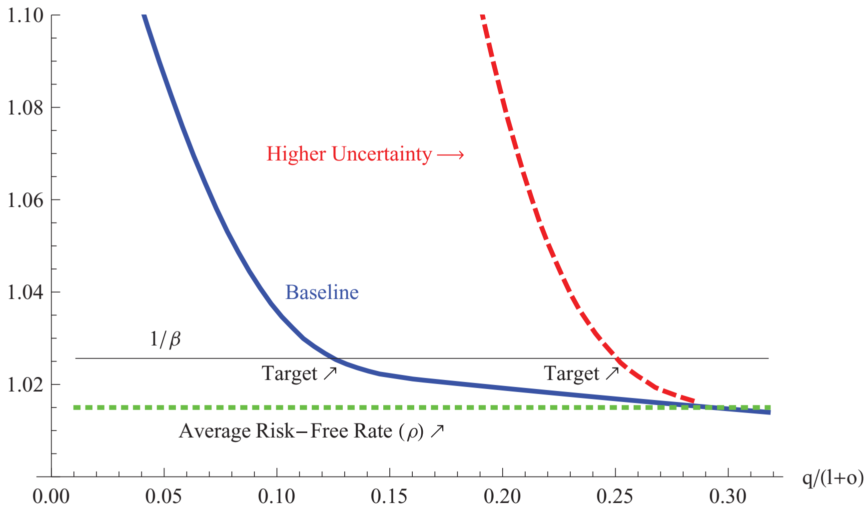

FIGURE 2. Marginal value of bank capital and target level of solvency. Baseline (solid) is the marginal value of bank capital evaluated at the steady state level of outstanding loans "o" and the optimal amount of new loans "l" for a range of values for bank capital net of dividends "q." High productivity and interest rate risk (dashed) is the marginal value of bank capital after the standard deviations of interest rate and aggregate productivity shocks are doubled relative to the baseline values.

marginal value of bank capital—left-hand side of equation (22)—as a function of the capital-to-assets ratio, or solvency, defined as $q/(l+o)$, setting outstanding loans equal to its steady state value and new loans to the optimal value that solves equation (24). For low values of bank capital—and therefore solvency—the marginal value of bank capital is high, because this is the region where financial frictions matter the most because of the bank's high risk of bankruptcy. Therefore, an extra dollar of capital has a powerful effect in reducing funding costs and thus it pays to retain earnings. On the other extreme, for sufficiently high levels of bank capital, its marginal value is low, because financial frictions in this region are negligible. If the bank were to accumulate large amounts of bank capital exceeding the point where the constraint on dividends binds, the envelope theorem implies that $\partial V_{t+1}(n_{t+1}, l_t)/\partial n = 1$ and the marginal value of bank capital would converge to the average risk-free rate. Because in this region the return of adding an extra dollar of capital is below the discount factor, it is optimal for the impatient stockholders to distribute dividends. The target level of solvency is the point where impatience and the desire to reduce the costs arising from financial frictions are exactly balanced.

Figure 2 also illustrates the relevance of the imposed assumptions on f and impatience. In the absence of credit market frictions, that is, $f = 0$, the value of an additional dollar of bank capital is simply given by the exogenous interest rate ρ. In that case, the bank would hold no capital because of the impatience assumption and would take infinite leverage. Suppose now that $f > 0$, but it is linear—recall that f is assumed to be twice continuously differentiable. In that case, the marginal value of bank capital would be a constant. Figure 2 would then show three parallel lines, one for the time preference rate, a second one for the average risk-free rate, and the third one for the marginal value of bank capital. The problem in that case does not have a well-defined solution because the bank would want infinite leverage (if the marginal cost of capital is below the time preference rate), or it would want to accumulate capital forever (if the marginal cost of capital is above the time preference rate), or any level of capital would be a solution (if the marginal cost of capital is exactly equal to the time preference rate). It is also easy to see in the chart that as impatience declines, the point where the time preference rate crosses the decreasing marginal value of capital moves to the right. In other words, as the shareholders' opportunity cost decreases, their willingness to increase retained earnings rises, and thus the target level of capital increases. However, if the discount factor becomes equal to the risk-free rate, the model has no well-defined solution again because the bank keeps accumulating retained earnings until it entirely self-finances.

It is useful to draw an analogy between the behavior just highlighted and that of a precautionary savings consumer. In the precautionary savings literature, the exact same behavior arises as long as the marginal utility of consumption is decreasing and the consumer is impatient [Carroll (2004)]. In the model presented in this paper, bank shareholders are risk-neutral, but the presence of nonlinear financial frictions generates curvature in the marginal value of bank capital and thus a precautionary motive. In Figure 2, this behavior is appreciated by comparing the solid and dashed lines. The dashed line corresponds to the marginal value of bank capital after a mean-preserving increase in interest rate and aggregate productivity risk equal to twice the baseline standard deviations of each variable. Higher risk makes capital more valuable, especially at low levels of solvency. This is because higher risk implies that hitting the no-equity financing constraint and thus the situation of a steep increase in funding costs becomes more likely. Consequently, the bank self-insures against this event by holding capital as a buffer. This analysis leads to the conclusion that the role of bank capital in this model is that of a cushion against unexpected shocks that would otherwise hinder the bank's lending operations.

3.4. Dynamic Simulations

This section presents some quantitative experiments using the optimal decision rules shown in Figure 1.

Changes in aggregate uncertainty. The first experiment consists of illustrating the implications of an unexpected increase in aggregate risk. Starting from the steady state, the exercise involves an increase in aggregate risk in period 3, which in our model takes the form of a mean-preserving increase in volatility

https://doi.org/10.1017/S136510051300014X Published online by Cambridge University Press

in both aggregate productivity and interest rate shocks equivalent to doubling the standard deviation of each process. One can think of this situation as the heightened uncertainty period that followed the collapse of Lehman Brothers in late 2008, when the vix reached the highest level since its inception.[2] Figure 3 shows the outcome of the simulation.

As discussed before, increased risk makes bank capital much more valuable than before because for a given level of bank capital, the likelihood of hitting the no-equity financing constraint and facing a steep rise in funding costs increases.[3] The bank responds by raising the target level of solvency as illustrated in the comparative statics exercise in Figure 2, but now in Figure 3 we can appreciate the dynamic adjustment toward the new equilibrium.

Because the bank cannot issue equity to bring capital to the new desired level instantaneously, it adjusts gradually by accumulating retained earnings. In the meantime, a credit crunch arises because although bank capital is below the target, it is costlier to issue new loans than in the steady state. Notice, however, that in the model, this sluggish adjustment is entirely driven by a more volatile economic environment and not by a negative realization of productivity and/or interest rate shocks. As the bank increases capital, its leverage position improves which implies lower financial frictions. This implies lower risk spreads for the bank, which reduces the marginal cost of lending. As can be seen in the figure, lending eventually recovers and settles at a higher level than the initial steady state. There is some additional inertia in the amount of total loans, which takes one period more to arrive at the new steady state than new loans. Quantitatively, this additional inertia is small because loans are only two-period contracts; however, if loans were granted for longer horizons this additional inertia would play a bigger role. Intermediation spreads also settle at a higher level because now loans are riskier—recall that borrower's net worth is fixed, so higher lending necessarily implies higher borrower leverage. But the higher riskiness of loans is mitigated through higher capital and thus risk spreads for the bank go down as I just discussed.

Formal evidence in support of this behavior can be found in Flannery and Rangan (2008), Valencia (2010), and Valencia and Verrier (2013). The former two studies document a strong cross-sectional relation between bank capitalization and asset risk. The third study examines the differential response of lending by U.S. commercial banks to changes in aggregate uncertainty. The study reports that an increase in aggregate uncertainty is associated with banks with weaker balance sheets reducing lending more than banks with stronger balance sheets. This evidence is consistent with the impulse responses shown in Figure 3.

Aggregate productivity shocks. We now simulate a negative realization of productivity for one period only, holding risk constant. Figure 4 plots the corresponding results for two sizes of the shock. As before, the initial conditions correspond to the stochastic steady state for each variable.

The transitory negative shock to productivity causes a decline in revenues because as shown in equation (3), the default threshold for borrowers rises, which

https://doi.org/10.1017/S136510051300014X Published online by Cambridge University Press

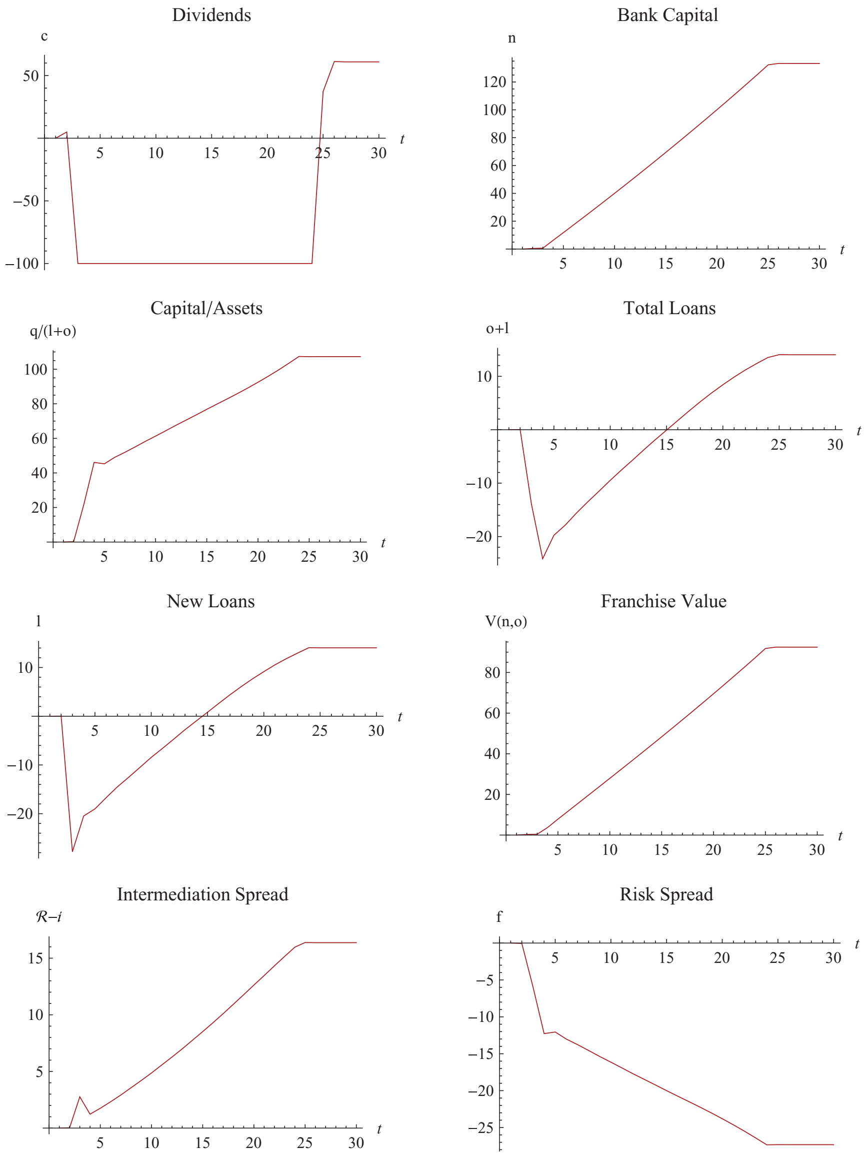

FIGURE 3. Responses to an increase in aggregate uncertainty. Responses to an unexpected two-standard-deviation increase in aggregate productivity and interest rate risk in period 3. The system is initialized at the stochastic steady state of each variable. Impulse responses are expressed as percent deviations from the original steady state.

https://doi.org/10.1017/S136510051300014X Published online by Cambridge University Press

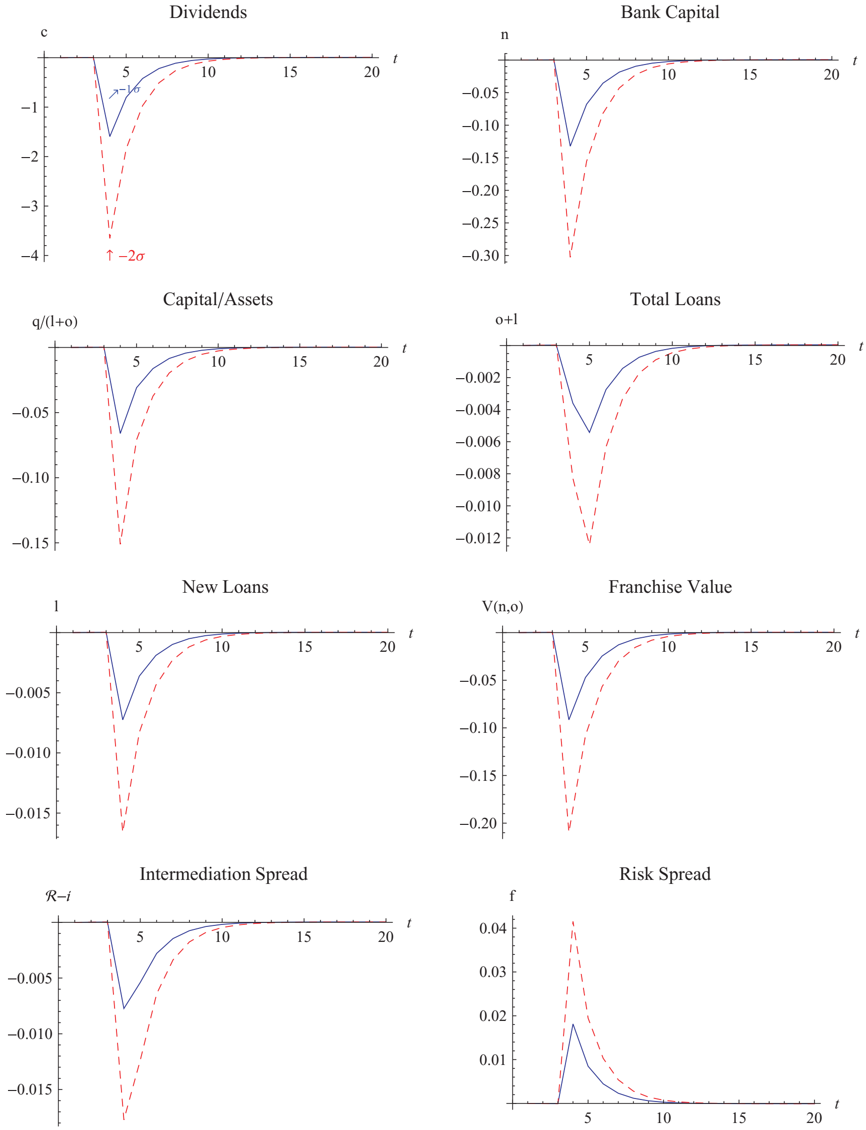

FIGURE 4. Responses to a productivity shock. Impulse responses to a one-standard-deviation (solid) and a two-standard-deviation (dashed) reduction in aggregate productivity that take place in period 3. The system is initialized at the stochastic steady state of each variable. Impulse responses are expressed as percent deviations from the stochastic steady state.

https://doi.org/10.1017/S136510051300014X Published online by Cambridge University Press

means that a larger fraction of borrowers goes bankrupt. Because of lower revenues bank capital falls below the target, to which the bank responds by cutting dividends. The same persistence effect described in the case of uncertainty shocks appears in this case. The bank cannot restore capital instantaneously and in the process of building up capital, higher costs for lending imply a persistent contraction in lending that lasts for several periods.

The intermediation spread decreases because with lower lending, and hence lower borrower leverage, the riskiness of loans goes down because leverage of borrowers goes down. However, the riskiness of the bank goes up because of the lower capital. This higher risk also implies higher spreads in deposit rates. As profitability is hurt, the bank's franchise value decreases as well. Because of the nonlinear features of the model, the contraction in lending is nonlinear in the size of the shock: The two-standard-deviation shock causes a decline in lending that is more than twice the decline following a one-standard-deviation shock.

It is important to emphasize the role of two assumptions: i.i.d. shocks and fixed borrowers' endowment. If aggregate productivity or the risk-free rate were allowed a more realistic serially correlated process, a persistent response of lending to a productivity shock would not be a surprise. If productivity declined and only gradually returned to its unconditional mean, lending would also exhibit this pattern even if there were no frictions in the model. Although assuming i.i.d. shocks is not interesting per se, it strengthens transparency of the model because persistence comes from only one source, financial frictions. Similarly, if there were endogenous fluctuations in borrowers' net worth at the same time as in the banks', it would be difficult to isolate the contribution of the financial frictions affecting the bank.

The persistent credit crunch triggered by a negative bank capital shock because of financial frictions in credit markets has also policy implications. A social planner could improve welfare if it were to reallocate capital in the economy, in this case to the banks. The merit of this type of interventions is more formally analyzed theoretically in Gertler and Karadi (2010), Kollmann et al. (2012), Sandri and Valencia (2012), and others and empirically in Giannetti and Simonov (2013) and Laeven and Valencia (2013).

Interest rate shocks. Changes in the risk-free interest rate matter because the maturity of the bank's assets is longer than that of its liabilities. As before, the system is initialized at its stochastic steady state. In period 3, the risk-free rate increases by one or two standard deviations. Figure 5 plots the responses. An increase in the interest rate raises funding costs and deteriorates bank capital because of the mismatch in maturity between loans and deposits. The dynamics are similar to those in the previous cases. Although capital is below the target, the marginal cost of lending is higher than in steady state, and thus a persistent credit crunch arises while the bank replenishes capital, with the contraction being nonlinear in the size of the shock, as before.

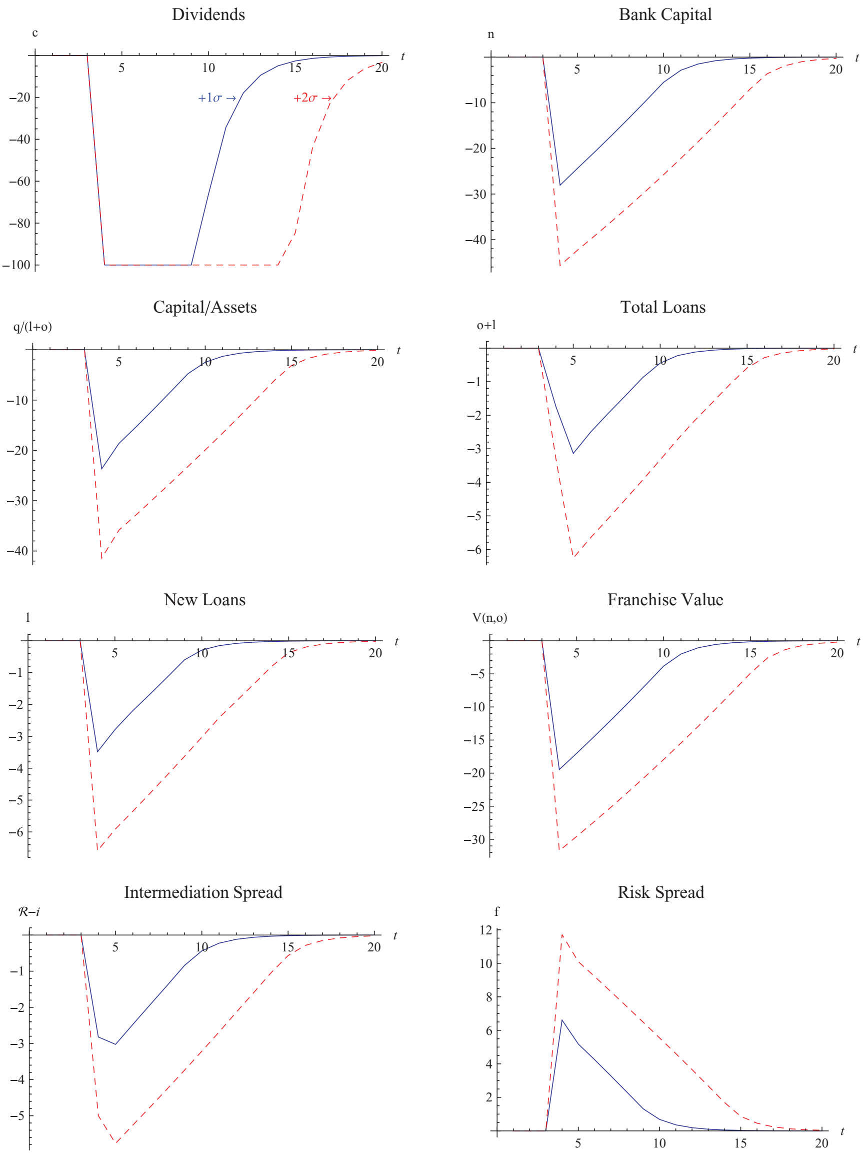

FIGURE 5. Responses to an interest rate shock. Impulse responses to a one-standard-deviation (solid) and a two-standard-deviation (dashed) increase in the risk-free rate that takes place in period 3. The system is initialized at the stochastic steady state of each variable. Impulse responses are expressed as percent deviations from the stochastic steady state.

https://doi.org/10.1017/S136510051300014X Published online by Cambridge University Press

It is important to discuss the implications of the assumption of mismatch in maturities of loans and deposits for the results presented in this and the previous subsections. Because loans have a longer maturity than deposits, the bank is exposed to interest rate risk because its funding costs can change after the bank has granted the loans, whereas the loan rate cannot. In the absence of maturity mismatches, the risk-free rate would be known at the moment of signing the loan contract and although the level of interest rates in a given period still influences how much is lent in that period, changes in interest rates would not affect bank capital. The latter implies that changes in interest rates would not generate a credit crunch and moreover, interest rate risk would bear no implications for bank decisions. The second role to highlight for maturity mismatches is that the bank cannot adjust the size of its assets (total loans) in a single period, generating additional inertia in the evolution of the total loans' portfolio of the bank.

Notice that the model suggests a larger effect in terms of contraction in lending and inertia coming from interest rate shocks than from aggregate productivity shocks. This is the consequence of calibration and the fact that a change in funding costs has a more direct impact on profits. Any increase in the risk-free rate causes a reduction in profits equal to the amount of deposits times the increase in the rate. An aggregate productivity shock, however, hits borrowers first. A large fraction of borrowers will mitigate the negative aggregate productivity shock with a positive realization of idiosyncratic productivity and their own net worth. The shock hits the bank only through the fraction of defaulting borrowers. However, a change in calibration could yield a different outcome if profitability of borrowers' project were reduced to make bankruptcies more sensitive to aggregate productivity.

3.5. Time-Series Implications

The simulations in the previous subsections have already highlighted some of the empirical implications of the model. The first result involves the persistence in the adjustment of the bank's balance sheet, which empirically has been documented by Hancock et al. (1995), Den Haan et al. (2011), and others. The second is that of a response of lending that depends on the level of capital, with a much larger elasticity at low levels of bank capital. Indeed, Carlson et al. (2011) find strong evidence of nonlinear effects in U.S. data in the form of a higher marginal effect for banks with capital ratios below the 25th percentile of the distribution than those above the 75th percentile. A third implication is that of asymmetry in the response of lending to shocks. This was already highlighted in the description of the optimal policy rules (Figure 1). When capital exceeds its steady state, the bank finds it optimal to distribute the excess capital in dividends instead of expanding the loans portfolio further, but when capital is below the steady state, a contraction in credit follows. Dell'Ariccia and Garibaldi (2005) estimate gross credit flows for the U.S. banking system between 1979 and 1999 and find that for any given rate of change in net credit, gross flows are larger in a recession than in a boom

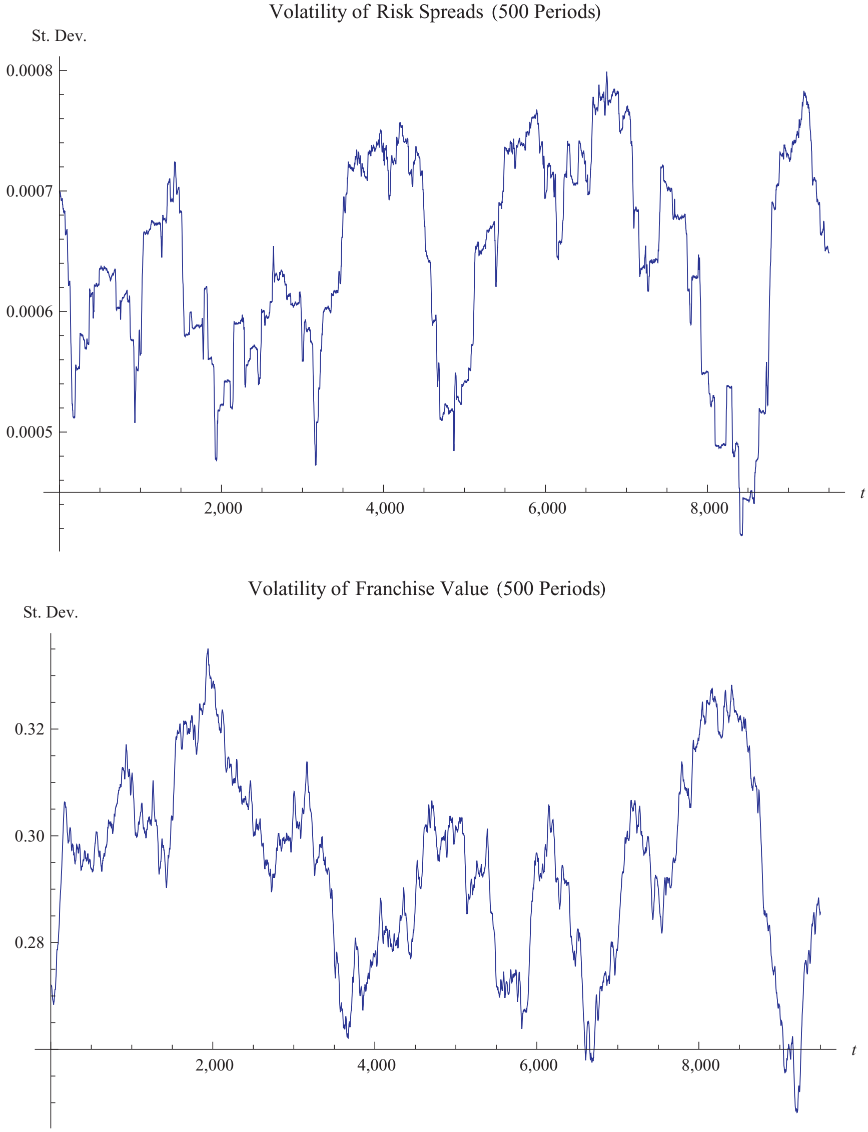

FIGURE 6. Volatility clustering. The model is simulated for 10,000 periods using the policy functions, the transition equations, and random draws from the assumed distributions for the aggregate shocks. Standard deviations of the bank's risk spread (f) and the bank's franchise value ($V(n, o)$) in rolling windows of 500 periods are plotted in the figures.

and that credit contractions are more volatile than credit expansions. Furthermore, this implication may also contribute to the asymmetric effects of monetary policy on output documented in Cover (1992).

https://doi.org/10.1017/S136510051300014X Published online by Cambridge University Press

A fourth implication is that second moments matter and changes in aggregate risk can trigger a persistent credit crunch. In this context, we conduct one last quantitative experiment with the model to point at an additional implication of its nonlinear features. Starting from the steady state, the bank's optimal decisions on lending and dividends are computed using the policy functions. After decisions are made, values for aggregate productivity and the risk-free rate are drawn randomly from their corresponding distributions. The realized stochastic shocks together with the transition equations for bank capital and loans generate the values for the state variables for the next period, which are used to compute the optimal values of loans and dividends for the next period. This routine is repeated for 10,000 periods.

Figure 6 shows the volatility of risk-spread, f, and the bank's franchise value, $V(n, o)$, which in this case is equivalent to the share price. The volatility is computed as the standard deviation of 500-period rolling windows. The model implies volatility clustering. Negative shocks that trigger sizable contractions in capital imply moving to the region of large marginal effects of bank capital on lending. This is the region where small changes in capital trigger large changes in spreads (i.e. the steep region of the marginal value of bank capital in Figure 2). Because the bank can only gradually move away from this region, large changes in spreads and in the share price will tend to cluster. Additional shocks hitting the bank while it is in this region exacerbate this result.

4. CONCLUSIONS

The recent global financial crisis has propelled research oriented toward better understanding how the financial sector's decisions can exacerbate or even originate aggregate economic fluctuations. This paper contributes to research in this area with a quantitative theoretical framework that highlights the consequences of financial frictions affecting the bank.

The paper's main result is that an i.i.d. one-time shock (interest rate or aggregate productivity) generates a persistent response in lending. Because aggregate shocks are assumed to be i.i.d. and financial accelerator effects on borrowers are shut down, this result is entirely the outcome of financial frictions affecting the bank. The nonlinear features of the model also have implications for the role of aggregate risk in bank decisions: First, the model generates a precautionary motive even when shareholders/managers are assumed to be risk-neutral. This self-insurance mechanism in and of itself can trigger a persistent credit crunch, because the bank can increase bank capital only gradually. Second, the response of lending to shocks is asymmetric: negative shocks can have a much stronger impact on lending than positive ones, and the contraction in credit following a negative shock is nonlinear in the size of the shock. And third, the model generates volatility clustering in spreads and the bank's share price.

The results derived from simulations of the model, and corroborated by the recent crisis, imply that the financial sector plays an important role in propagating and originating aggregate fluctuations.

NOTES

1. Bernanke and Gertler (1987) and Holmstrom and Tirole (1997) already incorporated a financial intermediary in a general equilibrium model, but in a static framework. Other related examples include Chiu and Meh (2011).

2. The vix reached 80.06 on October 27, 2008, and then 80.86 on November 20, 2008, from an average of around 23 for the first eight months of 2008.

3. It is important to note that the no-equity-financing restriction is not as restrictive as it may seem. As long as issuing equity is costly, the difference between assuming some finite cost or prohibitively high costs will only be a matter of magnitudes.

REFERENCES

Aliaga-Díaz, Roger and María Pía Olivero (2012) Do bank capital requirements amplify business cycles? Bridging the gap between theory and empirics. *Macroeconomic Dynamics* 16, 358–395.

Bernanke, Ben and Mark Gertler (1987) Banking and macroeconomic equilibrium. In William Barnett and Kenneth Singleton (eds.), *New Approaches to Monetary Economics,* pp. 89–112. New York: Cambridge University Press.

Bernanke, Ben, Mark Gertler, and Simon Gilchrist (1999) The financial accelerator in a quantitative business cycle framework. In John B. Taylor and Michael Woodford (eds.), *Handbook of Macroeconomics,* vol. 1, pp. 1341–1393. Amsterdam: Elsevier.

Brunnermeier, Markus and Yuliy Sannikov (2011) A Macroeconomic Model with a Financial Sector. Unpublished manuscript, Princeton University.

Carlson, Mark, Hui Shan, and Missaka Warusawitharana (2011) Capital Ratios and Bank Lending: A Matched Bank Approach. Working paper 2011-34, Federal Reserve Board of Governors.

Carlstrom, Charles and Timothy Fuerst (1997) Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *American Economic Review* 87, 893–910.

Carroll, Christopher (2004) Theoretical Foundations of Buffer Stock Saving. Working paper 10867, National Bureau of Economic Research.

Carroll, Christopher (2006) The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics Letters* 91, 312–320.

Chen, Nan-Kuang (2001) Bank net worth, asset prices and economic activity. *Journal of Monetary Economics* 48, 415–436.

Chiu, Jonathan and Césaire Meh (2011) Financial intermediation, liquidity, and inflation. *Macroeconomic Dynamics* 15, 83–118.

Christiano, Lawrence, Roberto Motto, and Massimo Rostagno (2004) The Great Depression and the Friedman–Schwartz hypothesis. Working paper 10255, National Bureau of Economic Research.

Cover, James (1992) Asymmetric effects of positive and negative money-supply shocks. *Quarterly Journal of Economics* 107, 1261–1282.

Dell'Ariccia, Giovanni and Pietro Garibaldi (2005) Gross credit flows. *Review of Economic Studies* 72, 665–685.

Den Haan, Wouter, Steven Sumner, and Guy Yamashiro (2011) Bank loan components and the time-varying effects of monetary policy shocks. *Economica* 78, 593–617.

Diamond, Douglas and Phillip Dybvig (1983) Bank runs, deposit insurance, and liquidity. *Journal of Political Economy* 91, 401–419.

Driscoll, John (2004) Does bank lending affect output? Evidence from the U.S. states. *Journal of Monetary Economics* 51, 451–471.

Flannery, Mark and Kasturi Rangan (2008) What caused the bank capital build-up of the 1990s? *Review of Finance* 12, 391–429.

Gale, Douglas and Martin Hellwig (1985) Incentive-compatible debt contracts: The one-period problem. *Review of Economic Studies* 52, 647–663.

Gertler, Mark (1992) Financial capacity and output fluctuations in an economy with multi-period financial relationships. *Review of Economic Studies* 59, 455–472.
Gertler, Mark and Peter Karadi (2010) A model of unconventional monetary policy. *Journal of Monetary Economics* 58, 17–34.
Gertler, Mark and Nobuhiro Kiyotaki (2010) Financial intermediation and credit policy in business cycle analysis. In Benjamin M. Friedman and Michael Woodford (eds.), *Handbook of Monetary Economics*, vol. 3, pp. 547–599. Amsterdam: Elsevier.
Giannetti, Mariassunta and Andrei Simonov (2013) On the real effects of bank bailouts: Micro evidence from Japan. *American Economic Journal: Macroeconomics* 5, 135–167.
Hancock, Diana, Andrew Laing, and James Wilcox (1995) Bank capital shocks: Dynamic effects on securities, loans, and capital. *Journal of Banking and Finance* 19, 661–677.
Hirakata, Naohisa, Nao Sudo, and Kozo Ueda (2009) Chained Credit Contracts and Financial Accelerators. Discussion paper 09-E-30, Institute for Monetary and Economic Studies.
Holmstrom, Bengt and Jean Tirole (1997) Financial intermediation, loanable funds, and the real sector. *Quarterly Journal of Economics* 112, 663–691.
Kashyap, Anil, Raghuram Rajan, and Jeremy Stein (2002) Banks as liquidity providers: An explanation for the co-existence of lending and deposit-taking. *Journal of Finance* 57, 33–73.
Kiyotaki, Nobuhiro and John Moore (1997) Credit cycles. *Journal of Political Economy* 105, 211–248.
Kollmann, Robert, Werner Roeger, and Jan in't Veld (2012) Fiscal policy in a financial crisis: Standard policy vs. bank rescue measures. *American Economic Review* 102, 77–81.
Laeven, Luc and Fabián Valencia (2013) The real effects of financial sector interventions during crises. *Journal of Money, Credit and Banking* 45, 147–177.
Meh, Césaire and Kevin Moran (2010) The role of bank capital in the propagation of shocks. *Journal of Economic Dynamics and Control* 34, 555–576.
Modigliani, Franco and Merton Miller (1958) The cost of capital, corporation finance and the theory of investment. *American Economic Review* 48, 261–297.
Sandri, Damiano and Fabián Valencia (2012) Balance-Sheet Shocks and Recapitalizations. Working paper 12/68, *International Monetary Fund.*
Sun, Hongfei (2011) Money, markets, and dynamic credit. *Macroeconomic Dynamics* 15, 42–61.
Townsend, Robert (1979) Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory* 21, 265–935.
Valencia, Fabián (2010) Bank Capital and Uncertainty. Working paper 10/208, International Monetary Fund.
Valencia, Fabián and Jeanne Verrier (2013) Aggregate Uncertainty and the Supply of Credit. Unpublished manuscript, International Monetary Fund.
Van Den Heuvel, Skander (2006) The Bank Capital Channel of Monetary Policy. Unpublished manuscript, University of Pennsylvania.

APPENDIX A: AGENCY COSTS IN THE DEPOSITOR–BANK RELATIONSHIP

This appendix shows how financial frictions on the bank can be modeled endogenously. Following the same costly state verification framework used to model the bank-borrower relationship, the value of bank assets is subject to bank productivity shocks $\psi \in (0,1]$, assumed i.i.d. and continuously distributed. One can think of this shock as the efficiency of bank managers in collecting assets, or alternatively, fraud. Only the bank knows the realization of ψ, but depositors know its distribution. Therefore, there are no ex ante information asymmetries.

The bank is closed down in period $t+1$ if the book value of capital after the realization of ψ is $G(l_{t-1}, \Phi_{t+1}) + l_t\psi - (1+\rho_{t+1})c_t \leq 0$. From the continuity of ψ, there exists a value $\underline{\psi}$ such that the book value of equity is exactly zero: $\underline{\psi} = \frac{(1+\rho_{t+1})c_t}{G(l_{t-1}, \Phi_{t+1})+l_t}$. Realizations of ψ above this threshold imply that the bank pays depositors the agreed amount, whereas for realizations below this threshold, the bank is liquidated and depositors pay liquidation costs λ, expressed as a fraction of bank assets. For simplicity, in the event of bankruptcy, all depositors are paid a prorated amount of the bank's liquidation value. The return to a depositor is summarized by

$$\Omega(l_{t-1}, l_t, \Phi_{t+1}, c_t, r_t, \psi) = \left\{ \begin{matrix} r_t c_t & \text{if } \psi_{t+1} \geq \underline{\psi} \\ (G(l_{t-1}, \Phi_{t+1}) + l_t)\psi(1-\lambda) & \text{if } \psi_{t+1} < \underline{\psi} \end{matrix} \right\}. \quad \textbf{(A.1)}$$

Assuming depositors are risk-neutral, the risky deposit rate r_t solves the following arbitrage condition between the expected return on a bank deposit and the risk-free return on a government security:

$$\mathrm{E}_t[\Omega(l_{t-1}, l_t, \Phi_{t+1}, c_t, r_t)] \geq \rho_t c_t, \quad \textbf{(A.2)}$$

which in equilibrium holds with equality under the assumption of infinitely many and small—relative to the bank—price-taker depositors. The spread $r_t - \rho_t$ reflects two components: first, the riskiness of bank deposits given the uncertainty about ψ, and second, expected bankruptcy costs, because the interest rate will price what is lost when the bank is liquidated.

APPENDIX B: SOLUTION ALGORITHM

The starting point is to assume that the bank will be liquidated at some future time T. Therefore, as of time T, the optimal decisions involve setting $d_T = n_T$ and making no new loans. These decisions imply that $V_T(n_T, l_{T-1}) = n_T$. As of $T-1$, the problem becomes

$$V_{T-1}(n_{T-1}, l_{T-2}) = \underset{\{d_{T-1}\}}{\text{Max}} \{d_{T-1} + \mathrm{E}_{T-1}\beta n_T\}, \quad \textbf{(B.1)}$$

s.t.

$$d_{T-1} \geq 0, \quad \textbf{(B.2)}$$

$$c_{T-1} = l_{T-2} + (n_{T-1} - d_{T-1}), \quad \textbf{(B.3)}$$

$$n_T = G(l_{T-2}, \Phi_T) - (1+\rho_T)c_{T-1} - f_{T-1}c_{T-1}, \quad \textbf{(B.4)}$$

and the bank does not lend because it will be liquidated in the following period. The first-order condition for the preceding problem is given by

$$1/\beta = \mathrm{E}_{T-1}(f_{T-1} + 1 + \rho_T - c_{T-1}\partial f/\partial c). \quad \textbf{(B.5)}$$

This equation yields an optimal solution for dividends as a function of outstanding loans and beginning-of-the-period bank capital. From period $T-2$ and backward the bank makes

https://doi.org/10.1017/S136510051300014X Published online by Cambridge University Press

loans. With the value function obtained in the previous step, we can write the problem as of $T - 2$, which has a structure identical to the one shown in the main text—equations (15)–(19); therefore it can be generalized to period t. The continuous distributions are approximated using Gaussian quadrature with seven points.

Backwards induction is implemented using Carroll (2006)'s endogenous gridpoints method, which consists of starting with end-of-period state variables and constructing values for beginning-of-period state variable using the marginal value function. The algorithm involves first specifying values for q_t and l_{t-1} collected in Q and L, respectively. For each value $l_{t-1} \in L$, a root-finding procedure is used to determine the values of q_t and l_t that satisfy the first-order conditions (24) and (25). Define these values as q_t^* and l_t^*, for each value l_{t-1}. For increased numerical accuracy in the region where the constraint on dividends is binding, N is augmented with q_t^*. q_t^* is the optimal level of end-of-period bank capital. For every pair $\{q_t, l_{t-1}\}$ such that $q_t \in Q$ and $l_{t-1} \in L$, the solutions are obtained in the following way:

1. If $q_t \geq q_t^*$, the constraint on dividends is not binding; hence the optimal solutions are $d_t = q_t - q_t^*$ and $l_t = l_t^*$. The beginning-of-period capital is recovered using the definition $q = n$ - d.
2. If $q_t < q_t^*$, the constraint on dividends is binding; therefore $d_t = 0$. The beginning-of-period capital is simply $n_t = q_t$. The solution for lending is found using a root-finding procedure on equation (25), given q_t and l_{t-1}.

The previous steps generate triples $\{l_t, n_t, l_{t-1}\}$ and $\{d_t, n_t, l_{t-1}\}$. I approximate the continuous policy functions $d_t(n, l)$ and $l_t(n, l)$ by piecewise linear interpolation of these triples. With these interpolating functions on hand, the next step involves updating the marginal value functions $\partial V_{t+1}(n_{t+1}, l_t)/\partial n$ and $\partial V_{t+1}(n_{t+1}, l_t)/\partial l$, which are also constructed using linear interpolation. With the new marginal value functions on hand, I solve the problem from the perspective of one period earlier using the steps highlighted in the preceding. The algorithm is repeated until $||d_t(n, l) - d_{t-1}(n, l)|| \leq 0.001$ and $||l_t(n, l) - l_{t-1}(n, l))|| \leq 0.001$.