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Magnetic viscosity due to collapsing flux cells

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Abstract. Reconnective annihilation of magnetic field leads to the formation of magnetic flux cells with small scales, followed by enhanced transverse plasmons occurring in a thin current sheet with a very small vertical extent. The analysis here focuses on the nonlinear interaction between the flux and plasmons. The transverse plasmon field is modulationally unstable in the Lyapunov sense. When the initial pumping wave amplitude attains the threshold of instability, this instability occurs with a high growth rate. Nonlinear development of modulational instability eventually results in self-similar collapse, due to nonlinear equilibrium, giving rise to a spatially intermittent, collapsing magnetic flux, very similar to a turbulent pattern. The Maxwell stress tensor from the turbulence flux determines the anomalous magnetic viscosity, i.e. the parameter α. It is shown that the instability is responsible for the alternation of outburst or quiescent states in astrophysical accretion disks. When the instability occurs, the parameter α is large. In the quiescent state, the instability is suppressed, leading to a smaller, collapse-quenching value of α.

1. Introduction

During the magnetic coalescence of multiple loops, the plasma and magnetic flux may be compressed towards a current sheet from both sides, driven by a Lorentz force, leading to a resistive instability. A reconnective annihilation of the magnetic field occurs, and magnetic energy is converted into kinetic energy of particles, thermal energy of the plasma, and radiation by Ohmic dissipation, followed by the formation of many small magnetic islands and enhanced turbulence plasmons, including Langmuir and transverse ones, occurring in the thin current sheet. Then the islands with very small scale and the strong turbulent plasmons can be intimately coupled by ponderomotive interaction. This nonlinear interaction will cause a instability of the magnetic islands in the current sheet. The instability can create more turbulent patterns, and lead to magnetic viscosity. Therefore, the ponderomotive effects within the current sheet are a very interesting problem in astrophysical plasmas.

An anomalous viscosity is usually assigned to accretion disks, since , in almost all cases, normal microscopic viscosities are too small to sustain an astrophysically significant accretion flow. Current α-models make the ad hoc assumption, by virtue of introducing a parameter α to parametrize our ignorance, that the 'turbulent viscosity' η_t satisfies $\eta_t = \alpha c_s H \rho$, ($\alpha$ = const), where H is the disk scale height, c_s the sound speed, and ρ the mass density. The problem is that plausible mechanisms that underlie the processes remain uncertain. The first calculation of magnetic viscosity resulting from Keplerian fluid shear was performed by Eardley and Lightman (1975). Coroniti (1981) and Torkelsson (1993) extended the model by including magnetic buoyancy. These models concentrate on viscosity as a direct result of field line stretching by the Keplerian mean flow. It seems to be difficult to find a pure hydrodynamic instability to initiate turbulence for a Keplerian disk. Fortunately, in the presence of a weak magnetic field, the magnetized disks are subject to the Chandrasekhar–Balbus–Hawley magnetorotational instability (Balbus and Hawley 1998). Such magnetohydrodynamic (MHD) turbulence can be developed by the instability and might explain the magnitude of the effective viscosity.

There is a wide range of systems containing accretion disks. The best studied ones are cataclysmic variable stars with apparently non-magnetic or very weak magnetic white dwarfs, which serve as one of the best laboratories to study accretion disks in astrophysics. An interesting, and for this paper important, subclass is the dwarf nova, which undergoes outbursts with amplitudes of about five magnitudes with intervals of some weeks. The outbursts that last for some days can be modelled as instabilities in the accretion disks (Osaki 1974). During the outbursts, the disk switches from a state with a low accretion rate to one with a high accretion rate. To reproduce the right time scales in the models, it is necessary to assume that α is smaller in the quiescent stage ($\alpha = 0.05$) than during the outbursts ($\alpha = 0.2$) (Cannizzo et al. 1988). Similar outbursts seem to occur in T-Tauri stars and X-ray transients. Black-hole candidates with high-mass companions, such as Cyg X-1, also have two states. The differences in luminosity are taken to be the result of different accretion rates within the disks. Typically, one finds short-term variability – flickering in cataclysmic variables and shot-noise variability in black-hole candidates – in the low state. It is expected (Schramkowski and Torkelsson 1996) that accretion disks to be magnetically active in their low-activity state, but the accretion rate is higher in the high-activity state and for that reason the disks seemingly require more MHD turbulence in the high-activity state. However, in view of current understanding of magnetic activity, a sort of instability within the disks would be responsible for the alternation of outburst or quiescent state. But this intrinsic instability is still unknown (Osaki 1993). In this paper, we will address this instability, which can switch between two states with different α values.

It is well known that a plasma is a system with a large number of degrees of freedom; in such a highly unstable plasma, the tendency for energy equipartition over the different possible degrees of freedom can produce turbulent waves, that is to say, plasmons excited at a rather high level. In a plasma, there is a transverse mode, with frequency ω^p that is nearly the electron plasma frequency ω_{pe}:

$$\omega^p = \omega_{pe} + \frac{k^2c^2}{2\omega_{pe}} \qquad (\omega_{pe} \gg kc). \tag{1}$$

The group velocity, like that of Langmuir waves, is very small compared with the speed of light c. It is extremely difficult for the oscillations to escape the plasma, because the index of refraction for these waves is very nearly zero. Hence the Langmuir and the transverse modes are often grouped and called plasma oscillations. Thus, it is convenient to call the transverse mode of (1) transverse plasmons (EM wave). Due to the very small group velocities, the dominant interactions between the transverse plasmons and Langmuir waves are strong; the interactions are connected with scattering by electrons and ions ($l + e \rightleftharpoons p + e', l + i \rightleftharpoons p + i'$) and decay processes ($l + l' \rightleftharpoons p$). Numerical calculations show that there is a continuous transfer from Langmuir waves to the transverse plasmons and back, and their

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energy densities are approximately the same, averaged over time, $W^l \approx W^p$ (Kaplan and Tsytovich 1973). In physical terms, it is natural that the interactions will lead to a tendency towards equipartition of energy over both Langmuir and transverse plasmons with the same frequencies near ω_{pe} and similar dispersion laws. On the other hand, for a plasma in thermal equilibrium, there is also a finite level of plasma waves, which represents the degrees of freedom excited in thermal equilibrium. Langmuir plasmons are excited by the charged particles of the plasma as they move due to their thermal energy by the Čerenkov processes $e \rightarrow l + e'$(spontaneous emission) and they are then reabsorbed by the plasma due to Landau damping. A balance between spontaneous emission and induced absorption leads to a thermal level of Langmuir plasmons. The energy density of Langmuir plasmons in thermal equilibrium is (Kaplan and Tsytovich 1973) $W_T^l = n_e k_B T_e / 6\pi^2 N_D$. As a result, we may expect for a plasma in thermal equilibrium that

$$\overline{W}^p \equiv \frac{|\mathbf{E}^p|^2}{4\pi n_e k_B T_e} = \overline{W}_T^p = \frac{W_T^l}{n_e k_B T_e} = \frac{1}{6\pi^2 N_D}, \tag{2}$$

where $|\mathbf{E}^p|^2/4\pi$ is the energy density of transverse plasmons, N_D is the Debye number, and k_B is Boltzmann's constant. For excited levels , $\overline{W}^p \gg \overline{W}_T^p$.

In this paper, we will study the collapse instability of the magnetic islands induced just by the enhanced transverse plasmons, responsible for magnetic viscosity, within a thin current sheet with a very small vertical extent δ,which is small compared with the scale length ℓ of the flux cell. The basic ponderomotive effects on the flux cell are given in Sec. 2. Section 3 discusses the collapse of the flux cell caused by the transverse plasmons. On the basis of these results, we find the anomalous viscosity, resulting from the collapsing spatially intermittent magnetic flux, in Sec. 4 followed by some conclusions in Sec. 5.

2. Ponderomotive effects

For a plasma with electromagnetic oscillations, the basic equations relevant to our discussion are the fluid equations for a two-component plasma consisting of electrons and ions, supplemented by the Maxwell equations. Because of the large difference in electron and ion oscillation frequencies in an astrophysical plasma, the two-time-scale approximation is also relevant. In this case, all the field quantities, say density, velocity, pressure, electric, and magnetic fields, can be divided into fast-time-scale and slow-time-scale components, $A = (n_e, n_i; \mathbf{v}_e, \mathbf{v}_i; P_e, P_i; \mathbf{E}, \mathbf{B}) = A_f + A_s$, and it can be assumed that the ensemble average value of the fast-time-scale components over the slow time scale vanishes: $\langle A_f \rangle = 0$. On a slow time scale, the quasineutrality condition leads to $n_s^e = n_s^i \equiv n_s$. Under these circumstances, with the aim of estimating the relative magnitude of the various terms in the fast- and slow-component equations, we obtain the transfer equation for the fast oscillation of electrons,

$$\nabla \times \nabla \times \mathbf{v}_f^e + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{v}_f^e = -\frac{1}{c^2}\frac{4\pi e^2}{m_e} n_s \mathbf{v}_f^e + \frac{3v_{T_e}^2}{c^2}\frac{1}{n_s}\nabla[\nabla \cdot (n_s \mathbf{v}_f^e)], \tag{3}$$

and the slow component continuity and momentum equations for the electrons and ions (Li and Zhang 1997; Li et al. 1994)

$$\frac{\partial}{\partial t} n_s + \nabla \cdot (n_s \mathbf{v}_s^e) = 0, \tag{4}$$

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$$\frac{\partial}{\partial t} n_s + \boldsymbol{\nabla} \cdot (n_s \mathbf{v}_s^i) = 0, \tag{5}$$

$$\frac{D\mathbf{v}_s^e}{Dt} \equiv \frac{\partial}{\partial t}\mathbf{v}_s^e + (\mathbf{v}_s^e \cdot \boldsymbol{\nabla})\mathbf{v}_s^e = \frac{e}{m_e}\left(\mathbf{E}_s + \frac{1}{c}\mathbf{v}_s^e \times \mathbf{B}_s\right) - \frac{\boldsymbol{\nabla} P_s^e}{m_e n_s} + \mathbf{F}_p^e, \tag{6}$$

$$\frac{D\mathbf{v}_s^i}{Dt} \equiv \frac{\partial}{\partial t}\mathbf{v}_s^i + (\mathbf{v}_s^i \cdot \boldsymbol{\nabla})\mathbf{v}_s^i = -\frac{e}{m_i}\left(\mathbf{E}_s + \frac{1}{c}\mathbf{v}_s^i \times \mathbf{B}_s\right) - \frac{\boldsymbol{\nabla} P_s^i}{m_i n_s}, \tag{7}$$

where v_{Te} is the electron thermal velocity and $\mathbf{F}_p^e$ is the ponderomotive force describing the effect due to the high-frequency oscillations on the slow motion of the electron fluid,

$$\mathbf{F}_p^e = -\tfrac{1}{2}\boldsymbol{\nabla}\langle(\mathbf{v}_f^e)^2\rangle. \tag{8}$$

Eliminating $\mathbf{E}_s$ from (6) and (7), we have

$$m_e\frac{D\mathbf{v}_s^e}{Dt} + m_i\frac{D\mathbf{v}_s^i}{Dt} = \frac{\mathbf{j}_s \times \mathbf{B}_s}{cn_0} - \frac{1}{4}m_e\boldsymbol{\nabla}(|\mathbf{v}_{f0}|^2) - \frac{\boldsymbol{\nabla} P_s^e + \boldsymbol{\nabla} P_s^i}{n_0}, \tag{9}$$

where $\mathbf{j}_s$ is the current density. The fast oscillation velocity of the electron can be expressed as

$$\mathbf{v}_f^e = \tfrac{1}{2}[\mathbf{v}_{f0}(\mathbf{r}, t)e^{i\omega_0 t} + \text{c.c.}], \tag{10}$$

with ω_0 being the fast oscillation frequency of the plasmons, $\omega_0 \simeq \omega_{pe} = 4\pi e^2 n_0/m_e$, where n_0 is the density of the background plasma with large characteristic scale L_0 and c.c. denotes the complex conjugate of the first term. We recall the equation of the lowest order on the fast time scale, $\partial\mathbf{v}_j^e/\partial t \simeq (e/m_e)\mathbf{E}_f$, which, combined with the van der Pol types of complex electric vector in the waves, similar to (10), $\mathbf{E}_f^e = \frac{1}{2}[\mathbf{E}(\mathbf{r}, t)e^{i\omega_0 t} + \text{c.c.}]$, gives

$$\langle(\mathbf{v}_f^e)^2\rangle = \tfrac{1}{2}|\mathbf{v}_{f0}|^2, \qquad \mathbf{v}_{f0} \simeq \frac{-ie}{m_e\omega_0}\mathbf{E}. \tag{11}$$

For the purposes of this study, we have assumed that the turbulence parameter $\overline{W}$, which expresses the excitation of the plasmons as a fraction of the total thermal energy of the plasma, satisfies

$$\overline{W} = \frac{|\mathbf{E}|^2}{4\pi n_s T_e} \simeq \frac{|\mathbf{v}_{f0}|^2}{v_{Te}^2} < 1, \tag{12}$$

and that the electron plasma frequency ω_{pe} is much greater than the cyclotron frequency ω_{Be},

$$\omega_{pe} >> \omega_{Be}. \tag{13}$$

In this case, (3) becomes

$$2i\omega_{pe}\frac{\partial}{\partial t}\mathbf{v}_{f0} + c^2\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{v}_{f0} - 3v_{T_e}^2\boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{v}_{f0}) + \frac{\delta n}{n_0}\omega_{pe}\mathbf{v}_{f0} = 0, \tag{14}$$

with

$$\left|\frac{1}{\omega_0}\frac{\partial}{\partial t}\ln v_{f0}\right| \ll 1,$$

where $\delta n = n_s - n_0$, which is the slow disturbance density in the waves.

To close (14), we need to give the nonlinear term proportional to $(\delta n/n_0)\mathbf{v}_{f0}$. In the case of a thin current sheet, where there is a flow and plasmons, linearizing with

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respect to $\mathbf{v}$, $D/Dt \simeq \partial/\partial t$, multiplying (9) by dz, and integrating from $z = -z_0$ to $z = z_0$ gives

$$\int_{-z_0}^{z_0} \frac{\partial}{\partial z}\left(\frac{1}{4}m_e|\mathbf{v}_{f0}|^2 + \frac{\delta n}{n_0}m_i c_s^2\right) dz = 0,$$

with $c_s^2 = (\gamma_e k_B T_e + \gamma_i k_B T_i)/m_i$, where we have neglected the following terms due to $z_0 \to 0$: $\int_{-z_0}^{z_0} \mathbf{v}_s\, dz = 2z_0\langle \mathbf{v}_s\rangle_z \to 0$ and $\int_{-z_0}^{z_0}(\mathbf{j}_s \times \mathbf{B}_s)\, dz = 2z_0\langle \mathbf{j}_s \times \mathbf{B}_s\rangle_z \to 0$. We then find that

$$\frac{\delta n(x,y,z)}{n_0} = -\frac{m_e/m_i}{4c_s^2}|\mathbf{v}_{f0}(x,y,z)|^2. \tag{15}$$

In fact, the above result in a magnetic field was also found by Sotnikov and Krasnoselskikh for the static limit (see Shapiro and Shevchenko 1984). By using (15), we can write (14) in the form

$$i\frac{\partial}{\partial \tau}\mathbf{v}'_{f0} + \alpha_{Te}\boldsymbol{\nabla}' \times \boldsymbol{\nabla}' \times \mathbf{v}'_{f0} - \boldsymbol{\nabla}'(\boldsymbol{\nabla}' \cdot \mathbf{v}'_{f0}) - |\mathbf{v}'_{f0}|^2\mathbf{v}'_{f0} = 0, \tag{16}$$

with

$$\mathbf{r}' = \tfrac{2}{3}\sqrt{\mu}k_d\mathbf{r}, \qquad \tau = \tfrac{2}{3}\mu\omega_{pe}t, \qquad \alpha_{Te} = \frac{c^2}{3v_{Te}^2}, \qquad \mu = \frac{m_e}{m_i}, \qquad \mathbf{v}'_{f0} = \frac{\sqrt{3}\mathbf{v}_{f0}}{4\sqrt{\mu}v_{Te}},$$

where k_d is the Debye wavenumber.

By using standard procedures we may obtain a set of equations for global coupling of MHD with the ponderomotive force by combining (4)–(7) (Li and Wu 1989). The relevant equations are

$$\frac{\partial}{\partial t}\rho + \boldsymbol{\nabla} \cdot (\rho\mathbf{U}) = 0, \tag{17}$$

$$\rho\left[\frac{\partial}{\partial t}\mathbf{U} + (\mathbf{U} \cdot \boldsymbol{\nabla})\mathbf{U}\right] = \frac{1}{4\pi}(\boldsymbol{\nabla} \times \mathbf{B}) \times \mathbf{B} - \boldsymbol{\nabla} P - \frac{1}{4}\frac{m_e}{m_i}\rho\boldsymbol{\nabla}(|\mathbf{v}_{f0}|^2), \tag{18}$$

where

$$\rho = n_s(m_i + m_e), \qquad \mathbf{U} = \frac{m_i\mathbf{v}_s^i + m_e\mathbf{v}_s^e}{m_i + m_e}, \qquad \mathbf{B} = \mathbf{B}_s,$$

and the pressure P is that at the center of mass of the system.

3. Magnetic flux cell collapse

It is known that reconnective annihilation of the magnetic field may create magnetic islands close to the surfaces of the current sheet. The field structure within a magnetic island is circular, as assumed by Coroniti (1981), which is prescribed by the simple vector potential

$$A_z = \frac{B_0(t)}{2\ell}(x^2 + y^2) \qquad (x^2 + y^2 \leqslant \ell^2), \tag{19}$$

where $B_0(t)$ is the characteristic magnetic field strength, which, in general, is dependent on time t; ℓ is the scale length of the flux cell ($L_0 \gg \ell \gg \delta$). The corresponding magnetic field components are

$$B_x = -B_0(t)\frac{y}{\ell}, \qquad B_y = B_0(t)\frac{x}{\ell}. \tag{20}$$

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Note that the field is not force-free. Usually, one can assume that the initial magnetic pressure is much less than the plasma thermal pressure P; on the other hand, the characteristic scale of the island, which is free to respond to local shear stresses of Keplerian differntial rotation, is too small to balance the thermal pressure force. Thus, the magnetic islands produced by reconnection may then interact with the plasmons, and this causes a *nonlinear equilibrium*. Hence, we have from (18)

$$\frac{1}{4\pi}(\boldsymbol{\nabla}\times\mathbf{B})\times\mathbf{B}=\frac{1}{4}\mu\rho\boldsymbol{\nabla}(|\mathbf{v}_{f0}|^2). \tag{21}$$

We will demonstrate later that this is the case.

As we have mentioned, many different types of wave may grow in a reconnecting magnetic field and produce a turbulent environment within the thin current sheet. We may assume that there are the plasmons oscillating along the z direction in the sheet : $\mathbf{v}_{f0}=v_{fz}\mathbf{z}$ and $v_{fz}\propto \operatorname{sech}(z\sigma_0)$ with $\sigma_0\gg 1$ (see below). Thus, the divergence of the fast oscillation velocity, $\boldsymbol{\nabla}\cdot\mathbf{v}_{f0}\propto -\operatorname{sech}(z\sigma_0)\tanh(z\sigma_0)$, vanishes near the surfaces of the sheet. Under this condition of transverse plasmons, (16) becomes

$$i\frac{\partial}{\partial\tau}v_{fz}+\frac{1}{2}\boldsymbol{\nabla}^2(v_{fz})+|v_{fz}|^2v_{fz}=0. \tag{22}$$

Here we have taken the complex-conjugate equation and omitted the asterisk '*' and prime '′'; and $\nabla^2=\partial^2/\partial\xi_x^2+\partial^2/\partial\xi_y^2+\partial^2/\partial\xi_z^2$ and $\boldsymbol{\xi}=\mathbf{r}'/\sqrt{2\alpha_{Te}}$. In the framework of (16), the transverse plasmon field is unstable in the Lyapunov sense to the finite-amplitude, monochromatic pumping wave $\mathbf{v}^0_{f0}$ (or $\mathbf{E}^p_0$, see (11)). It is found (Li 1989) that modulational instability occurs when and only when $k^2<2|\mathbf{v}^0_{f0}|^2$, or, in dimensional units,

$$\frac{|\mathbf{v}^0_{f0}|^2}{v^2_{Te}}>6\left(\frac{k}{k_d}\right)^2; \tag{23}$$

the maximum growth rate and corresponding wavenumber of the instability for the longitudinal perturbation ($\mathbf{k}\|\mathbf{v}^0_{f0}$) are

$$\frac{\gamma_{\max}}{\omega_{pe}}=\frac{1}{8}\frac{|\mathbf{v}^0_{f0}|^2}{v^2_{Te}},\qquad \frac{k^l_{\max}}{k_d}=\frac{1}{2\sqrt{3}}\sqrt{\frac{|\mathbf{v}^0_{f0}|^2}{v^2_{Te}}}. \tag{24}$$

Obviously, one can see from the condition (23) that this instability is a zero-threshold instability for the finite-amplitude monochromatic wave. In fact, one of the most important features of a turbulent field is that this field is always a relatively broad wavepacket. However, if the frequency spread $\Delta\omega$ of the packet is much less than $\gamma_{\max}$ and its wavenumber width Δk is much smaller than $k^l_{\max}$, then the perturbation will clearly have neither time nor space to realize that the initial wave is not monochromatic (Thornhill and ter Haar 1978). In this case, when its wavenumber width $\Delta k\ll k^l_{\max}$, we may find the *threshold* of the modulational instability for the turbulent field:

$$\frac{|\mathbf{v}^0_{f0}|^2}{v^2_{Te}}\gg\overline{W}_{\mathrm{MI}}\equiv 12\left(\frac{\Delta k}{k_d}\right)^2. \tag{25}$$

The modulation of the perturbed pumping field, occurring as a result of instability, leads to field localization. The nonlinear development of the modulational instability, for the two- or three-dimensional case, will lead to collapse (Li et al. 1995). This then causes an implosion of the waves and gives rise to spatially intermittent field structures with various intensities, very similar to a chaotic or more turbulent pattern. The numerical simulations illustrate a similar turbulent pattern.

Now let us seek the solution to (22) in the following forms with three dimensions:

$$v_{f0} = \sqrt{\sigma(R,z,\tau)}\,\exp[iS(R,\tau)], \tag{26}$$

$$\sigma = \sigma_0^2(R,\tau)\,\mathrm{sech}^2(z\sigma_0), \qquad R^2 = \xi_x^2 + \xi_y^2, \qquad z = \xi_z, \tag{27}$$

where the amplitude $\sigma_0(R,\tau)$ is a slowly varying function with respect to the phase function $S(R,\tau)$, i.e.

$$\left|\frac{\partial S}{\partial R}\right| \gg \left|\frac{\partial \sigma}{\partial R}\right|. \tag{28}$$

The scale length in the z direction, i.e. the thickness of the sheet, is $\delta = \sigma_0^{-1}$. The 'turbulence parameter' is given by

$$\overline{W}^p \approx \frac{|\mathbf{v}_{f0}|^2}{v_{Te}^2} = \frac{16}{3}\mu|\mathbf{v}'_{f0}|^2 < 1,$$

and the strong turbulence, $\overline{W}^p > \mu$, is, to order of magnitude, $|\mathbf{v}'_{f0}|^2 < \mu^{-1}$. Therefore the thickness δ is exceedingly small for strong plasmons with $\sigma_0 \gg 1$:

$$\delta \ll 1. \tag{29}$$

Substituting (26) in (22) and separating into real and imaginary parts, we find that

$$\frac{\partial \sigma}{\partial \tau} + \frac{\partial}{\partial \xi_x}\left(\sigma\frac{\partial S}{\partial \xi_x}\right) + \frac{\partial}{\partial \xi_y}\left(\sigma\frac{\partial S}{\partial \xi_y}\right) = 0, \tag{30}$$

$$\sigma\frac{\partial S}{\partial \tau} + \frac{\sigma}{2}\left[\left(\frac{\partial S}{\partial \xi_x}\right)^2 + \left(\frac{\partial S}{\partial \xi_y}\right)^2\right] - \left\{\frac{1}{4}\frac{\partial^2\sigma}{\partial \xi_x^2} + \frac{\partial^2\sigma}{\partial \xi_y^2} - \frac{1}{8\sigma}\left[\left(\frac{\partial \sigma}{\partial \xi_x}\right)^2 + \left(\frac{\partial \sigma}{\partial \xi_y}\right)^2\right]\right\} = \sigma^2 - \frac{1}{8\sigma}\left(\frac{\partial \sigma}{\partial z}\right)^2 + \frac{1}{4}\frac{\partial^2\sigma}{\partial z^2}. \tag{31}$$

Considering (28), (30) becomes

$$\frac{\partial \sigma}{\partial \tau} + \sigma\frac{\partial^2 S}{\partial \xi_x^2} + \sigma\frac{\partial^2 S}{\partial \xi_y^2} = 0. \tag{32}$$

In the thin sheet, we can ignore the terms in square brackets in (31) in view of (28) and (29); then integrating this equation and (32) over z from $z = -\infty$ to $z = \infty$ gives

$$\frac{\partial \sigma_0}{\partial \tau} + \frac{\sigma_0}{R}\frac{\partial}{\partial R}\left(R\frac{\partial S}{\partial R}\right) = 0, \tag{33}$$

$$\frac{\partial S}{\partial \tau} + \frac{1}{2}\left(\frac{\partial S}{\partial R}\right)^2 - \frac{1}{2}\sigma_0^2 = 0, \tag{34}$$

where we have used that $\int_0^\infty \mathrm{sech}^2 z\,dz = 1$ and $\int_0^\infty \mathrm{sech}^4 z\,dz = \frac{2}{3}$.

We can make an ansatz for self-similar collapse, which was first obtained by Gorev et al. (1976):

$$\sigma_0 = (\tau_0 - \tau)^{-2/3}V(\zeta), \qquad S = (\tau_0 - \tau)^{-1/3}\psi(\zeta), \qquad \zeta = \frac{R}{(\tau_0 - \tau)^{1/3}}. \tag{35}$$

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Substituting (35) into (33) and (34) yields

$$\psi + \zeta \frac{d\psi}{d\zeta} + \frac{1}{2}\left(\frac{d\psi}{d\zeta}\right)^2 = \frac{3}{2}V^2,$$

$$\frac{2}{3} + \frac{\zeta}{3}\frac{1}{V}\frac{dV}{d\zeta} + \frac{1}{\zeta}\frac{d\psi}{d\zeta} + \frac{d^2\psi}{d\zeta^2} = 0.$$

Putting $\psi = a + b\zeta^2$ and $3b(1+2b)/a = -\varepsilon^2$, and dropping the term

$$\frac{\zeta}{3}\frac{1}{V}\frac{dV}{d\zeta} \sim \frac{\varepsilon^2\zeta^2}{1-\varepsilon^2\zeta^2}$$

if $\varepsilon^2\zeta^2 \ll 1$, yields $a \approx 1/3\varepsilon^2$ and $b \approx -\frac{1}{6}$. As a result, we find

$$\sigma_0 = \frac{\sqrt{2}}{3\varepsilon}(\tau_0 - \tau)^{-2/3}\left[1 - \varepsilon^2\frac{R^2}{(\tau_0-\tau)^{2/3}}\right]^{1/2}, \tag{36}$$

$$S = \frac{1}{3\varepsilon^2}(\tau_0-\tau)^{-1/3}\left[1 - \frac{\varepsilon^2}{2}\frac{R^2}{(\tau_0-\tau)^{2/3}}\right]. \tag{37}$$

This solution with $\varepsilon = 1$ obtained by Gorev et al. (1976) is valid only for $\zeta < 1$. Now the solution above is also available for $\zeta > 1$, provided that $\varepsilon^2\zeta^2 \ll 1$. Hence, at the plane of the sheet,

$$|\mathbf{v}_{f0}|^2 = \sigma_0^2 = \frac{2}{9\varepsilon^2}(\tau_0-\tau)^{-4/3}\left[1 - \varepsilon^2\frac{R^2}{(\tau_0-\tau)^{2/3}}\right],$$

or, in dimensional units,

$$\frac{|\mathbf{v}_{f0}|^2}{v_{Te}^2} = \frac{16}{9}\left(\frac{3}{2}\right)^{1/3}\frac{\mu^{-1/3}}{\varepsilon^2}(\tilde{\tau}_0-\tilde{\tau})^{-4/3}\left[1 - \left(\frac{2}{81}\right)^{1/3}\mu^{1/3}\varepsilon^2\frac{3k_0^2(x^2+y^2)}{(\tilde{\tau}_0-\tilde{\tau})^{2/3}}\right], \tag{38}$$

with $\tilde{\tau} = \omega_{pe}t$ and $k_0 = \omega_{pe}/c$. Now one sees from (20), (21), and (38) that there is indeed *nonlinear equilibrium* between the ponderomotive force and the Lorentz force within the sheet. In this case, one has

$$\overline{B_y} = \frac{1}{2\ell}\int_{-\ell}^{\ell} B_y\,dx = \frac{1}{2}B_0(t) = \frac{2\pi^{1/2}}{3}(k_0\ell)\frac{P^{1/2}}{\tilde{\tau}_0-\tilde{\tau}}, \tag{39}$$

where $P = n_0k_BT$. It should be pointed that it is precisely the *nonlinear equilibrium* that causes the collapsed magnetic flux to be in a turbulent state, as turbulent transverse plasmons.

The collapse results from the development of modulational instabilities. Hence we may identify $\tau_{\rm coll} \equiv \tilde{\tau}_0 - \tilde{\tau}$ as the scale $(\tilde{\tau}_0 - \tilde{\tau}_{\rm min})$ to stop collapse, which corresponds to $\overline{W}^p \sim 1$. On the one hand, the derivation of our basic equations (16) and (18) breaks down as soon as $\overline{W}^p$ is no longer small compared with unity. On the other hand, the ideal situation would be that the parameter $\overline{W}^p$ was limited by unity: the strong field with $\overline{W}^p$ in excess thereof would make the nonlinear interactions very strong and the time scale of interactions very short; physically, an energy flow arises, which would lead to an increase in thermal energy n_eT_e during a very short period; and then $\overline{W}^p$ would quickly drop to below unity again after this period (Tsytovich 1977). Therefore the minimum time scale collapse motion could be found from the self-similar solution (38) as

$$(\tilde{\tau}_0 - \tilde{\tau})^2_{\min} \approx (\tfrac{16}{9})^{3/2}(\tfrac{3}{2})^{1/2}\mu^{-1/2}\varepsilon^{-3}. \tag{40}$$

4. Magnetic viscosity from collapsing flux cell

The magnetic viscous force due to the collapsed spatially intermittent flux is

$$f_i^m = \boldsymbol{\nabla}_j t_{ij}^m, \tag{41}$$

where the viscous stress tensor is

$$t_{ij}^m = \langle \delta B_i \delta B_j - \tfrac{1}{2}\delta_{ij}(\delta\mathbf{B})^2\rangle/4\pi. \tag{42}$$

The work done on the volume $d\mathbf{r}$, per unit time, by the stress is

$$-\frac{\partial t_{ij}^m}{\partial x_j} v_i \, d\mathbf{r},$$

which contributes to the thermal energy due to viscous dissipation, resulting in a change of entropy within the volume:

$$\dot{S} = \int \frac{1}{T}\left(-\frac{\partial t_{ij}^m}{\partial x_j} v_i\right) d\mathbf{r} = \int t_{ij}^m \frac{V_{ij}}{T}\, d\mathbf{r}, \tag{43}$$

where

$$V_{ij} \equiv \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right).$$

Equation (43) corresponds to that of Lifshitz and Pitaevskii (1981):

$$t_{ij}^m = \gamma_{ij;lk}\frac{V_{lk}}{T} = \eta_{ij,lk} V_{lk}, \tag{44}$$

where $\eta_{ij;lk}$ are kinetic coefficients. This above can be recast as

$$t_{ij}^m = \eta_m\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial v_k}{\partial x_k}\right), \tag{45}$$

with

$$\eta_{ij;lk} = \eta_m(\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl} - \tfrac{2}{3}\delta_{ij}\delta_{lk}), \tag{46}$$

where η_m is the magnetic viscosity defined through the kinetic coefficient as $\eta_m = \eta_{lk,lk}$ ($l \neq k$, with no summation over repeated lk).

In many practical cases, including accretion disks and the Galaxy, the shear stress tensor V_{lk} has a dominant $r\varphi$ component, and then (45) becomes

$$t_{ij}^m = \eta_{ij,r\varphi}\frac{1}{2}\left(\frac{\partial v_\varphi}{\partial r} - \frac{v_\varphi}{r}\right) = \eta_{ij,r\varphi}\frac{1}{2}r\frac{\partial\Omega(r)}{\partial r}. \tag{47}$$

Furthermore, it yields from (42), (46), and (47)

$$\frac{1}{4\pi}|\langle\delta B_r \delta B_\varphi\rangle| = \eta_m \frac{1}{2} r\left|\frac{\partial\Omega(r)}{\partial r}\right|. \tag{48}$$

In this circumstance of turbulent flux cells, which are in various phases, we may assume that the collapsed magnetic field of the transverse modes is statistically isotropic on the scales of interest :

$$\tfrac{1}{2}\langle(\delta\mathbf{B})^2\rangle \approx \langle\delta B_r \delta B_\varphi\rangle. \tag{49}$$

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Up to an uncertainty of order unity, one has $\langle(\delta\mathbf{B})^2\rangle \approx \langle\delta\mathbf{B}^2_{\max}\rangle$. For a Keplerian-rotation thin disk, $\Omega(r) = \Omega_K = \sqrt{GM}r^{-3/2}$, (48) becomes

$$\frac{\delta\mathbf{B}^2_{\max}}{8\pi} = \tfrac{3}{4}\eta_m\Omega_K; \tag{50}$$

and, putting $\eta_m \approx \eta_t = \alpha c_s \rho H$ and $\Omega_K H \approx c_s$, we get formally

$$\frac{\delta\mathbf{B}^2_{\max}}{8\pi} = \tfrac{3}{4}\alpha\rho c_s^2 = \tfrac{3}{4}\alpha P, \tag{51}$$

with

$$\alpha = \frac{2}{27}\frac{k_0^2\ell^2}{(\tilde{\tau}_0 - \tilde{\tau})^2}. \tag{52}$$

It is rather difficult to give the width Δk for the transverse plasmons in (25), because their spectrum is unknown. However, as the thermal equilibrium state corresponds physically to the lowest energy level, obviously one has $\overline{W}_{\mathrm{MI}} > \overline{W}^p_T$ for a turbulent field with relatively broad wavepacket. The modulational instability is therefore suppressed in this low-level state of $\overline{W}^p < \overline{W}_{\mathrm{MI}}$, which could be considered as an indicator of the quiescent state of the accretion disk. Under these circumstances, this magnetic collapse is also quenched; then we get the parameter α for collapse-quenching from (52) as $\alpha = \alpha_q = \frac{2}{27}k_0^2\ell^2/\tilde{\tau}_0^2$. On the other hand, during outbursts, the accretion disk is magnetically more active and wave–wave and wave–particle interactions involving the transverse plasmons happen at higher levels. Thus, the modulational instability is induced by the enhanced plasmons with $\overline{W}^p > \overline{W}_{\mathrm{MI}}$; then the flux cell undergoes collapse up to a time limited by (40). Hence we have from (52) and (40) that $\alpha = \alpha_b = 0.2(n_0/10^{14})(\varepsilon^3\ell^2)$. Taking $ct_0 \sim \ell$ and $\varepsilon^3\ell^2 \sim 1$ yields that $\alpha = \alpha_q = 0.07$ in the quiescent stage of the disk and $\alpha = \alpha_b = 0.2$, with $(1 - t/t_0)_{\min} = 0.6$, during outbursts in a dwarf nova.

5. Conclusions

Reconnective annihilation of the magnetic field leads to the formation of small magnetic islands, followed by enhanced transverse plasmons occurring in a thin current sheet with very small vertical extent. In an island (i.e. a cell), the magnetic field lies in the plane of the disk, as assumed by Coroniti (1981). The flux cell interacts subtly with the plasmons in the plane of the disk, as described by (21).

The transverse plasmon field is modulationally unstable in the Lyapunov sense to the pump wave. When the initial pump wave amplitude satisfies the condition (25), instability occurs and attains the largest growth rate (24). Nonlinear development of the modulational instability results eventually in self-similar collapse, due to *nonlinear equilibrium*, giving rise to a more spatially intermittent, collapsing magnetic flux, very similar to a turbulent pattern, which is illustrated also by numerical simulation (Ma and Li 2002).

The Maxwell stress tensor of the intermittent flux determines the anomalous magnetic viscosity, i.e. the parameter α (see (52)). During outbursts, the plasmons are probably in high levels with $\overline{W}^p > \overline{W}_{\mathrm{MI}}$; then the flux undergoes collapse, leading to a large α value, say of 0.2. On the other hand, in low levels with $\overline{W}^p < \overline{W}_{\mathrm{MI}}$, the modulational instability is therefore suppressed and this magnetic collapse is also quenched. Then we get a smaller value of the parameter α for the quiescent stage of the disk.

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