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THE MACROECONOMIC IMPACT OF MONETARY-FISCAL POLICY IN A "FISCAL DOMINANCE" WORLD

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This paper focuses on the question of what monetary and fiscal policy can do and should do in a "fiscal dominance" world. I first highlight that both "amplification" and "fiscal cushion" effects are always at work jointly in determining the evolution of inflation. I find the threshold of maturity of government bonds beyond which more aggressive monetary policy dampens inflation volatility is three quarters. In addition, I conduct welfare analysis to quantitatively evaluate the costs and benefits brought by long-term debt. My results show that the threshold of government debt maturity above which an aggressive monetary policy improves welfare is eight quarters. More importantly, I characterize optimal monetary and fiscal policy using simple and implementable rules. My results indicate an optimal monetary and fiscal combination calls for an aggressive response in both rules. Finally, I find that optimized simple monetary-fiscal rule is significantly welfare inferior to the Ramsey optimal policy.

Keywords: Fiscal Theory of the Price Level (FTPL), Optimal Monetary and Fiscal Policy, Long-Term Government Debt

1. INTRODUCTION

Many advanced economies are entering into a period of severe fiscal stress in the aftermath of the global financial crisis. They have experienced a sharp increase in their government debt as a result of expansionary fiscal policy. For example, according to the International Monetary Fund (2016), the public debt as a share of gross domestic product (GDP) rose from around 60% in 2007 to about 105% in 2015 for the United States. The debt-to-GDP ratios are even higher for the southern tier European countries who are now mired in sovereign debt crises, such as Greece, Italy, and Portugal (see Figure 1). Figure 1 shows the evolution of debt-to-GDP ratios for a selected group of advanced countries. Over the decade, especially in the recession period, most countries have experienced a dramatic growing in their

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TABLE 1. General government debt as a percentage of GDP

Country	2007	2014
Canada (CAN)	83.9	107.2
France (FRA)	75.6	120.4
Germany (DEU)	64.1	82.0
Greece (GRC)	112.8	179.0
Ireland (IRL)	27.4	122.9
Italy (ITA)	110.7	156.0
Japan (JPN)	180.0	246.6
Portugal (PRT)	78.1	151.7
Spain (ESP)	41.7	119.0
United Kingdom (GBR)	51.4	113.3
United States (USA)	77.2	124.2
Average ratio	82.1	138.4
Average ratio of G7 countries	91.8	135.7

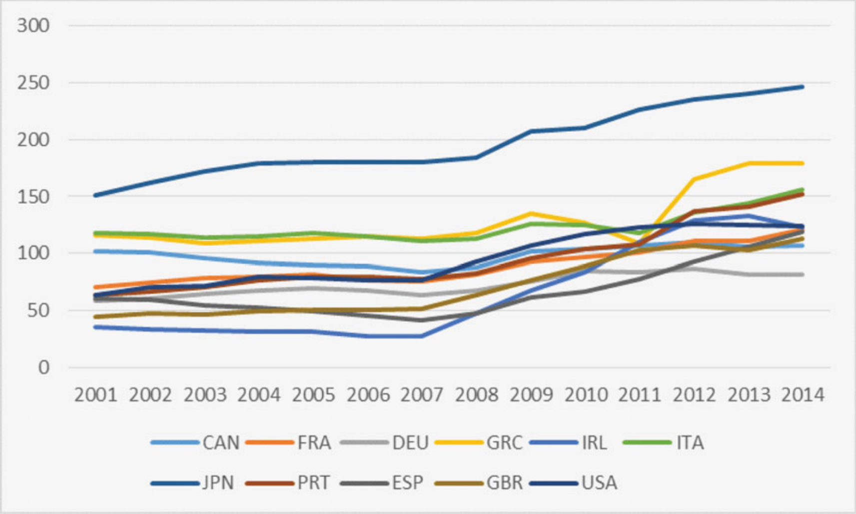

FIGURE 1. The evolution of debt-to-GDP ratios for selected countries.

government debt. Table 1 also reports that public debt expansions during the global recession were significant: the average debt-to-GDP ratio increased from 82.1% in 2007 to 138.4% in 2014 for the selected advanced economies. The average ratio for G-7 countries has increased to 135.7% in 2014.

Population aging is another major fiscal threat in industrial countries, forcing economies to approach their "fiscal limits".[1] For example, Japan is facing a significant societal aging problem. As in Hansen and İmrohoroğlu (2016), the

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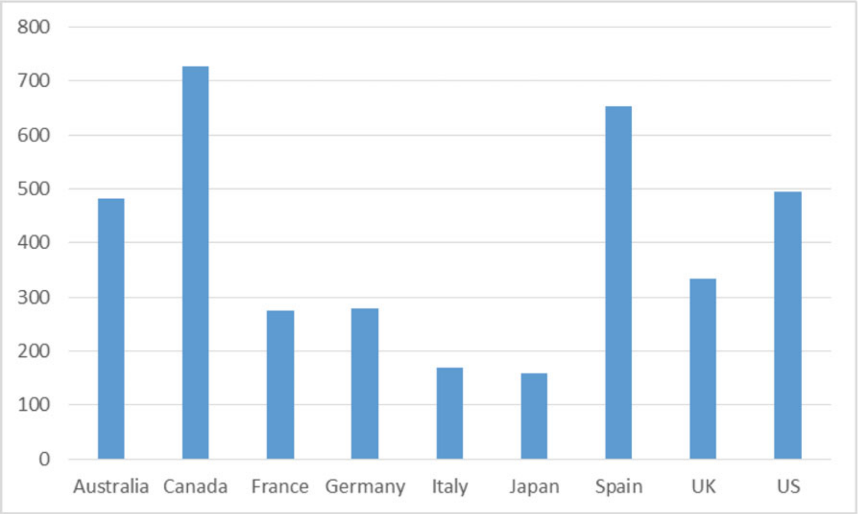

FIGURE 2. Net present value of impact on fiscal deficit of aging-related spending for selected countries.

ratio of the number of Japanese over the age 64 to those between 20 and 64 is projected to increase from 39% in 2010 to above 91% in 2070. The promised old-age benefits—including publicly-funded healthcare plans, long-term social securities, and income support programs for the elderly—are growing rapidly as a share of the economy. This in turn places substantial pressure on the sustainability of government finances [Castro et al. (2017)]. Figure 2 shows the International Monetary Fund's (2009) calculations of the net present value of aging-related spending as a share of GDP in several advanced economies. Canada ranks first among the group, with a long-term budget imbalance of 726% of GDP. The average share of GDP across G-20 countries exceeds 400% . Another study by the International Monetary Fund (2015) shows that the average ratio of pension and health spending as a percentage of GDP is 16.4 in 2015 for a group of 37 developed countries, and the number is projected to increase to 21.4 by 2050. In addition, due to political forces, tax collections do not necessarily increase for financing government expenditures. In fact, they tend to fall when economies are under fiscal stress.[2] Thus, in the absence of clear plans for financing these large expenditures, government primary surpluses would decrease.

The fiscal theory states that the price level is determined by the bond valuation equation [Cochrane (2001, 2005)]:[3]

$$\frac{\text{Nominal government liabilities}}{\text{Price level}} = \text{Expected present value of government primary surpluses.}$$

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Based on this equation, at some point (growing in government debt, together with declines in primary surpluses, just as described above), fiscal constraints must take hold, resulting in inflation. One can think of the above economy as a "fiscal dominance" world [Woodford (2001)]: fiscally dominant economies are normally under serious fiscal stress or approaching their fiscal limits; they are in episodes of "active fiscal policy, passive monetary policy" in Leeper's (1991) terminology. In a fiscally dominant regime, the price level is determined through the valuation equation; fiscal behavior places great pressure on the central bank to use monetary policy to maintain the market value of public debt. One classic example is the bond price-support program provided by the Fed in the 1940s, with the aim of maintaining relatively stable prices and yields for government securities [Woodford (2001)].

The bond-price support regime implies that the conventional "Ricardian Equivalence" does not obtain. Macroeconomic theories typically adopt the doctrine of "Ricardian Equivalence", leaving fiscal behavior irrelevant for the price level determination. However, it may not be plausible to assume agents would always form the expectations that the government budget itself will be subsequently adjusted to neutralize the effects of fiscal disturbances, especially in cases where the perceived constraints on fiscal policy have been severe. In a "fiscal dominance" world, government borrowing does not necessarily create an increase in the expected future primary surpluses by the same size, so that the price level has to adjust to maintain equilibrium.

This study focuses exclusively on the "fiscal dominance" regime since the nature of "monetary dominance" regime has already been thoroughly examined in the literature.[4] It is well-documented in the fiscal literature that monetary policy alone cannot control inflation in a "fiscal dominance" environment. However, the question of what monetary and fiscal policy can do and should do in a fiscal regime has received little attention. The overarching objective of my paper is to address this issue and explore the impact of monetary and fiscal policy on macroeconomy, albeit in the context of a richer fiscal environment. My model framework is based on a standard New Keynesian model with sticky prices, augmented with a fiscal block, a maturity structure for government debt, and distortionary taxation. Several studies along the literature have made it clear that, with those realistic features, we are widening the interactions between monetary and fiscal policy.

Specifically, I first evaluate how the aggressiveness of monetary policy affects macroeconomic variables in response to fiscal disturbances and how does this result depend on the maturity structure of government bonds. I further address the question of what would be the optimal monetary policy in a "fiscal dominance" world. Should the central bank set monetary policy aggressively in this environment? Does the effectiveness of an aggressive interest rate policy depend on the maturity structure of government debt? Then, I characterize optimized simple monetary and fiscal rules, and I wish to find a monetray-fiscal combination where social welfare is maximized. Finally, I solve the Ramsey problem, and quantitatively evaluate the welfare losses with optimized simple rules compared to the Ramsey optimal policy.

My paper contributes to the emerging literature on the impact of long-term government bonds on the effectiveness of monetary policy in a "fiscally dominant" environment. In the literature, using an endowment economy model augmented with long-term debt, Leeper and Leith (2016) find that more aggressive monetary policy (albeit passive) tends to stabilize inflation because of the "fiscal cushion" effect bought by long-term bonds. Using a New Keynesian model with one-period debt, their simulation result shows a different picture: an aggressive monetary policy increases inflation volatility. However, a rigorous comparison between the two sets of results is missing. I think it is potentially interesting to fully explore how the maturity structure of government debt affects the effectiveness of monetary policy.

I limit my analysis to the model economy when only long-term debt prevails and emphasize that the aggressiveness of monetary policy always has two conflicting effects.[5] On the one hand, more aggressive monetary policy tends to amplify the effects of fiscal disturbances on inflation, leading to a larger increase in inflation. One the other hand, long-term government debt is acting as a "fiscal cushion," the bond price decreases by more when monetary policy acts more aggressively, leaving less changes required for the aggregate price level. I argue that the two conflicting effects are always at work jointly in determining the evolution of inflation and the relative power of the two depends on the maturity structure of government bonds. In addition, I quantify that there is a threshold in the debt maturity structure above which an aggressive monetary policy dampens inflation volatility. In my calibration that is relevant to the United States, I find that the threshold is three quarters. My findings may contribute to improving our understanding on the effectiveness of monetary policy as we used to think tighter monetary policy leads to lower inflation. In my model environment with long-term debt, however, more aggressive monetary policy could also increase inflation so long as the "fiscal cushion" effect is dominated by the "amplification" effect.

In addition, my paper is one of the first to quantify the welfare gains associated with an aggressive monetary policy, in a "fiscal dominance" economy. Following the conventional literature under a "monetary dominance" regime, we used to know that tighter monetary policy is welfare-improving as higher interest rates tend to stabilize inflation and real activity. However, this argument can be overturned in a "fiscally dominant" economy when long-term government debt plays a crucial role.[6] This is because long-term nominal debt brings about a fiscal cushion that smooths out deficit-led inflation over time, but at the cost of inducing a higher future and hence total inflation. It is therefore not immediately obvious that the benefit of inflation smoothing outweighs the cost of higher overall inflation in welfare terms.

Indeed, I show that there exists a threshold beyond which tighter monetary policy is welfare enhancing. Quantitatively, I find that the threshold is 2 years, i.e., eight quarters. Interestingly enough, if one compares this threshold with the previous one for inflation dynamics, we see that with the maturity of long-term debt lying between three quarters and eight quarters, more aggressive monetary policy dampens inflation volatility but reduces welfare. I argue that this result may be interested from a policy maker's point of view: central banks need to consider the heterogeneous effects of long-term debt on inflation dynamics and social welfare when they implement an aggressive monetary policy.

More importantly, my paper also fits into the literature on optimal monetary policy and the literature on joint monetary-fiscal optimization. The former work, as exemplified by Woodford (2003), examines optimal monetary responses to shocks where there are lump-sum taxes available to continuously satisfy the government budget constraint.[7] However, in this paper, I aim at drawing the attention of the literature to a "fiscal dominance" environment whose role has been largely neglected in the normative analysis of optimal monetary policy. The latter literature, including Schmitt-Grohé and Uribe (2004, 2007) and Siu (2004), have looked at joint optimization of monetary and fiscal policy where there is a cost to surprise inflation.[8] My study differs from this strand of literature in that I consider both optimal simple and implementable monetary and fiscal rules as well as the Ramsey optimal policy, and in particular, I characterize optimized monetary and fiscal rules when our economy is restricted to be in a "fiscal dominance" world. It also allows us to quantitatively evaluate how close the optimized simple rules can get to the Ramsey policy. In addition, I conduct my analysis in a rich New Keynesian framework with sticky prices, long-term government bonds, and distortionary taxation.

In our benchmark economy with 5-year debt, the optimal monetary policy requires to set the inflation coefficient in the Taylor rule at its maximum value. The optimal monetary regime however depends on the maturity structure of government debt. In characterizing optimized simple rules, my results show that an optimal monetary and fiscal combination calls for an aggressive response to inflation in the interest rate rule and an aggressive response to lagged debt in the fiscal rule. In addition, more aggressive fiscal policy (albeit active) improves welfare as it introduces a strong fiscal feedback effect that stabilizes government debt as well as debt-led inflation. Compare the model dynamics to those under the Ramsey optimal allocation, I find that the key variables are much stabilized in the Ramsey economy. In particular, inflation is virtually zero under the Ramsey policy as it is costly to do so. Finally, I calculate the welfare losses associated with optimized simple rules to be around 2.16% in consumption unit, indicating that optimized simple monetary-fiscal rule is still significantly welfare inferior to the Ramsey optimal policy.

2. THE MODEL

My basic framework is based on a canonical textbook version of a New Keynesian model with sticky prices as in Woodford (2003) and Galí (2015), augmented with a fiscal block, a maturity structure for government debt, and distortionary taxes.

As emphasized by Sims (2011) and Leeper and Leith (2016), it is essential to incorporate inertial prices and long-term government debt into the model. Fiscal models with flexible prices and one-period debt can provide a useful benchmark for more complex and realistic analysis [e.g., Cochrane (2005, 2011)], it is, however, unrealistic in assuming perfectly flexible prices and all government debt is instantaneously short-term.

With sticky prices, monetary policy has influence over ex-ante real interest rate as well as nominal interest rate. This in turn means that the bond valuation equation could hold through a reduction in ex-ante real interest rate and not just ex-post real interest rate through inflation surprises. In this respect, we are widening the interactions between monetary policy and fiscal policy. With long-term government debt, the requirement that the real value of government debt equals the present value of future primary surpluses can be met by jumps in the bond price, which changes the value of outstanding debt, leaving less responses needed for the aggregate price level. Cochrane (2011) and Sims (2013) describe that long-term debt acts as a "fiscal cushion," like an equity.[9] With distortionary taxes, the model environment would neatly capture a trade-off in choosing the path of inflation [Schmitt-Grohé and Uribe (2004, 2007), Sims (2013)]. On the one hand, the government would like to use unexpected inflation as a nondistorting tax and minimize the need to vary distortionary taxes over the business cycle. On the other hand, changes in the rate of inflation come at a cost because of nominal rigidities. This feature is particularly important when I conduct welfare analysis later.

The model is a closed-economy Dynamic Stochastic General Equilibrium (DSGE) model. There are four agents in the economy: a representative household and firm, the government, and the central bank. The representative household consumes a consumption good, holds government bonds, and supplies labor to firms. The final good is a composite of differentiated products, each of which is produced using labor services as the sole input. A distorting tax is levied against firms' sales. The markets for differentiated goods are monopolistically competitive. Prices are sticky *à la* Calvo (1983). The economy is cashless and financial markets are complete.

2.1. Households

The economy is assumed to be populated by a continuum of infinitely lived households of size one. Households appreciate consumption and dislike labor. The representative household seeks to maximize a discounted sum of utilities of the following form:

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right), \tag{1}$$

where $\sigma, \varphi > 0$ are the inverse elasticities of intertemporal substitution of consumption and labor disutility, C_t is a consumption index defined across all differentiated goods, N_t is labor supply, E is the expectations operator and $0 < \beta < 1$.

The consumption index C_t is given by

$$C_t = \left[\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}},$$

where i denotes the good's type, $\epsilon > 1$ is the elasticity of substitution between types of differentiated goods.

The representative household is subject each period to a flow budget constraint of the form:

$$\int_0^1 P_t(i) C_t(i) di + P_t^S B_t^S + P_t^L B_t^L \leq B_{t-1}^S + (1 + \rho P_t^L) B_{t-1}^L + W_t N_t + \Phi_t,$$

where $P_t(i)$ is the price of type i good. Following Woodford (2001) and Bianchi and Ilut (2017), I assume that there are two types of government bonds: one-period government bonds, B_t^S, in zero net supply with a price P_t^S equal to the inverse of the gross nominal interest rate, and a more general portfolio of long-term government bonds B_t^L with beginning-of-period price P_t^L. P_t^L denotes the dollar-price of an asset at period t that pays coupons in the later periods. B_t^L can be interpreted as a portfolio of infinitely many bonds, with weights along the maturity structure given by ρ^j for $j \geq 0$. This debt instrument pays a declining coupon of ρ^j dollars $j + 1$ periods after they were issued, for each $j \geq 0$ and the decay factor $0 \leq \rho \leq 1$.[10] The value of such an instrument issued in period t in any future period $t + j$ is $P_{t+j}^{L-j} = \rho^j P_{t+j}^L$. The duration of the bond is given by $(1 - \beta\rho)^{-1}$, which allows us to vary as a means of changing the implicit maturity structure of government debt. Higher ρ raises the maturity of the bond portfolio.[11] A desirable feature of this structure is to model long-duration bonds without an increase in the dimensionality of the state space.[12] In the special case when $\rho = 1$, these bonds become infinitely lived consoles, as, for example, in Sims (2011); and when $\rho = 0$, the bonds reduce to the familiar one-period bonds typically studied in the literature.

I assume that a sufficient number of distinct types of bonds are traded for financial markets to be complete. Thus, any desired state-contingent value D_{t+1} of one's bond portfolio at the beginning of period $t + 1$ may be achieved through an appropriate choice of bond holdings B_t^L. $W_t N_t$ is nominal wage income and Φ_t denotes the nominal profits received by the household from the ownership of retail firms. P_t is the aggregate price level.

The household now must decide how to allocate its consumption expenditure among the different goods. Optimal behavior requires that the consumption index C_t be maximized for any given level of expenditures $\int_0^1 P_t(i) C_t(i) di$. The solution to the problem yields the following set of demand equations:

$$C_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} C_t,$$

where we have the aggregate consumer price index given by the following:

$$P_t = \left[\int_0^1 P_t(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}.$$

Furthermore, and conditional on such optimal behavior:

$$\int_0^1 [P_t(i)C_t(i)]di = P_t C_t,$$

i.e., total consumption expenditures can be written as the product of the price index times the quantity index. Plugging the expression into the budget constraint yields:

$$P_t C_t + P_t^S B_t^S + P_t^L B_t^L \leq B_{t-1}^S + (1 + \rho P_t^L) B_{t-1}^L + W_t N_t + \Phi_t. \tag{2}$$

Each household chooses optimal portfolio of assets, consumptions and labor supplies that maximize the life-time utility (1) subject to the budget constraint (2) for $t \geq 0$. Let us define the gross interest rate as the inverse of the price of short-term bonds, i.e., $R_t = 1/P_t^S$. The first-order conditions of the representative household are as follows:

$$\beta R_t E_t \left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \left(\frac{1}{\Pi_{t+1}}\right)\right] = 1, \tag{3}$$

$$\beta E_t \left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \left(\frac{1}{\Pi_{t+1}}\right)(1 + \rho P_{t+1}^L)\right] = P_t^L, \tag{4}$$

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi}, \tag{5}$$

where $\Pi_{t+1} \equiv \frac{P_{t+1}}{p_t}$ denotes the gross inflation rate from period t to $t+1$.

Equation (3) is the standard Euler equation while equation (4) is the Euler equation corresponding to the long-term government bonds. Equation (5) represents the optimal labor supply decision. That is, the real wage rate equals the marginal rate of substitution between consumption and leisure. I define $\Lambda_{t,t+1}$ as a stochastic discount factor, for discounting nominal returns D_{t+1} at date $t+1$ back to date t. Under my assumption of complete assets market, $\Lambda_{t,t+1}$ is uniquely defined. It is given by

$$\Lambda_{t,t+1} \equiv \beta \left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \left(\frac{1}{\Pi_{t+1}}\right). \tag{6}$$

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Combining equations (3) and (4) yields:[13]

$$P_t^L = R_t^{-1}(1 + \rho E_t P_{t+1}^L). \quad \textbf{(7)}$$

Iterating equation (7) forward connects current bond prices to expected paths of the short-term nominal interest rate:

$$P_t^L = \sum_{j=0}^{\infty} \rho^j E_t \left(\prod_{i=0}^{j} R_{t+i}^{-1} \right). \quad \textbf{(8)}$$

Besides these FOCs, necessary and sufficient conditions for household optimization also require the households' budget constraints to bind with equality. Defining household wealth (in nominal terms) brought into period t as

$$D_t \equiv (1 + \rho P_t^L) B_{t-1}^L + B_{t-1}^S. \quad \textbf{(9)}$$

The real value of the nominal assets is thus $d_t = \frac{D_t}{P_t} = \frac{(1+\rho P_t^L) B_{t-1}^L + B_{t-1}^S}{P_t}$. I also define the real interest rate as $r_t \equiv \frac{R_t}{E_t \Pi_{t+1}}$ for later use.

The household's wealth satisfies a transversality condition of the form:

$$\lim_{T \to \infty} E_t[\Lambda_{t,T} D_T] = 0. \quad \textbf{(10)}$$

To be precise, optimization behavior requires households plan to fully utilize their lifetime wealth: $\lim_{T \to \infty} E_t[\Lambda_{t,T} D_T] \leqslant 0$. On the other hand, no-Ponzi game condition imposes the constraint that households do not accumulate debt beyond their ability to pay back eventually: $\lim_{T \to \infty} E_t[\Lambda_{t,T} D_T] \geqslant 0$. The transversality condition is obtained by combining both conditions.

2.2. Firms

Firms set their prices subject to a Calvo (1983) price rigidity. Each firm may reset its price only with probability $1 - \theta$ in any given period, independent of the time elapsed since it last adjusted its price. Since the problem is symmetric, every firm faces the same decision problem and will choose the same optimal price P_t^*. This pricing behavior implies the law of motion for the aggregate price index:

$$P_t = [(1-\theta)(P_t^*)^{1-\epsilon} + \theta(P_{t-1})^{1-\epsilon}]^{\frac{1}{\epsilon-1}}. \quad \textbf{(11)}$$

A firm reoptimizing in period t will choose the price P_t^* that maximizes the current market value of the profits generated while that price remains effective. This corresponds to solving the problem:

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k}[(1-\tau_{t+k}) P_t^* Y_{t+k|t} - (1/\mu^s) \Psi_{t+k}(Y_{t+k|t})],$$

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subject to the sequence of demand constraints:

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k},$$

where $Y_{t+k|t}$ denotes output in period $t + k$ for a firm that last reset its price in period t, sales revenues are taxed at rate τ_t, Ψ_t is the nominal cost function, and $\mu^s = \frac{\epsilon}{\epsilon-1}$ is time-invariant employment subsidy, which can be used to eliminate the steady-state distortion associated with monopolistic competition. In addition, minimizing labor costs yields the expression for the real marginal cost: $mc_t = \frac{W_t}{AP_t}$, where A is aggregate technology, common across firms and taken to be constant.

The optimality condition associated with the problem above satisfies:

$$\left(\frac{P_t^*}{P_t}\right) = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j C_{t+j}^{-\sigma} (\frac{P_{t+j}}{P_t})^{\epsilon} mc_{t+j} Y_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j (1 - \tau_{t+j}) C_{t+j}^{-\sigma} (\frac{P_{t+j}}{P_t})^{\epsilon-1} Y_{t+j}} = \frac{K_t}{F_t}, \qquad (12)$$

where K_t and F_t are aggregate variables that satisfy the recursive relations:

$$K_t = C_t^{-\sigma} mc_t Y_t + \beta\theta E_t K_{t+1} \Pi_{t+1}^{\epsilon}, \qquad (13)$$

$$F_t = (1 - \tau_t) C_t^{-\sigma} Y_t + \beta\theta E_t F_{t+1} \Pi_{t+1}^{\epsilon-1}, \qquad (14)$$

Also, it follows from (11) and (12):

$$\left(\frac{1 - \theta \Pi_t^{\epsilon-1}}{1 - \theta}\right)^{\frac{1}{\epsilon-1}} = \frac{K_t}{F_t}. \qquad (15)$$

2.3. The Government

The government issues long-term government bonds B_t^L, collects taxes in the amount of $P_t \tau_t Y_t$, and faces an exogenous expenditure stream G_t. Imposing the restriction that one-period debt is in zero net supply, the government's flow budget constraint is given by:

$$P_t^L B_t^L = (1 + \rho P_t^L) B_{t-1}^L - P_t \tau_t Y_t + P_t G_t. \qquad (16)$$

Government purchases are assumed to follow a univariate autoregressive process of the form:

$$\ln(G_t/\tilde{G}) = \rho_G \ln(G_{t-1}/\tilde{G}) + \epsilon_t, \qquad (17)$$

where ρ_G is the first-order autocorrelation, $\tilde{G}$ is the steady-state value of government expenditures, and the standard deviation of ϵ_t is σ_ϵ.

Rewrite in real terms:

$$P_t^L b_t^L = (1 + \rho P_t^L) \frac{b_{t-1}^L}{\Pi_t} - \tau_t Y_t + G_t, \qquad (18)$$

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where real debt is defined as $b_t^L \equiv B_t^L/P_t$, debt-to-output ratio is given by $\omega_t \equiv (P_t^L B_t^L)/(P_t Y_t) = (P_t^L b_t^L)/Y_t$. Note that b_t^L is the real face value of outstanding debt; the real market value is $P_t^L B_t^L/P_t$.

Furthermore, I specify a fiscal feedback policy, relating real taxes to changes of real lagged outstanding debt and output from their steady state levels.[14] The reasons for choosing these variables in the fiscal rule are the following. First, the tax policy is meant to reflect the realistic feature that the fiscal authority raises (decreases) taxes in response to increases (decreases) in government indebtedness. It allows us to capture the automatic stabilization effects of fiscal policy on government debt. Second, I account for the response of fiscal policy to business cycles and recognize that actual tax policies have some degree of "pay-as-you-go" spending. The fiscal rule is therefore specified as

$$\frac{\tau_t Y_t}{\tilde{\tau}\tilde{Y}} = \left(\frac{b_{t-1}^L}{\tilde{b}^L}\right)^{\gamma_b} \left(\frac{Y_t}{\tilde{Y}}\right)^{\gamma_y}, \tag{19}$$

where $\tilde{\tau}\tilde{Y}$, $\tilde{b}^L$, and $\tilde{Y}$ are steady-state values of real taxes, real government bonds, and output, $\gamma_b, \gamma_y > 0$ characterize the relative weights to real debt deviations and output gap.

2.4. The Central Bank

I assume that monetary policy is conducted by means of an interest rate schedule following a standard Taylor rule:

$$\frac{R_t}{\tilde{R}} = \left(\frac{\Pi_t}{\tilde{\Pi}^T}\right)^{\phi_1} \left(\frac{Y_t}{\tilde{Y}}\right)^{\phi_2}, \tag{20}$$

where $\tilde{R}$ and $\tilde{Y}$ are steady-state values of nominal interest rates and output, and $\tilde{\Pi}^T$ is the central bank's headline inflation target, which is assumed to be one. $\phi_1, \phi_2 > 0$ are the relative weights measuring the response of interest rate to inflation deviations and output gap, respectively.

3. EQUILIBRIUM, DETERMINACY, AND EXPOSITIONS OF THE FISCAL THEORY

3.1. Equilibrium

The market clearing condition of goods market is summarized as follows:

$$Y_t = C_t + G_t, \tag{21}$$

and market clearing in labor market requires:

$$Y_t = \frac{A N_t}{\Delta_t}, \tag{22}$$

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where price dispersion $\Delta_t \equiv \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} di$ evolves according to

$$\Delta_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}} + \theta\Pi_t^{\epsilon}\Delta_{t-1} \qquad \textbf{(23)}$$

I define a rational expectations equilibrium is a collection of stochastic processes $\{C_t, R_t, P_t^L, \Pi_t, K_t, F_t, \Delta_t, b_t^L, \tau_t, Y_t, N_t\}_{t=0}^{\infty}$, satisfying each of the equilibrium conditions in equations (4),(7),(13)–(15),(18)–(23), consistently with the stochastic process for the exogenous disturbance $\{\epsilon_t\}$, and initial conditions.

3.2. Determinacy

As shown by Leeper (1991), one can distinguish four disjoint regions of the parameter space according to whether monetary and fiscal policies are active or passive. In general, these regions are a function of all parameters of the model. However, in practice, the two policy rules (19) and (20) are key in determining the existence and uniqueness of a stationary solution to the model.[15] There are two determinacy regions: Active Monetary/Passive Fiscal regime and Active Fiscal/Passive Monetary regime. The first one is the most familiar one in the literature: monetary policy is unconstrained and can actively pursue price stability by reacting strongly to inflation, and fiscal policy passively accommodates the behavior of the monetary authority ensuring debt stability. The second regime is the focus of this paper as it can generate perverse and surprising effects. In addition, when both authorities are active no stationary equilibrium exists, whereas when both of them are passive the economy is subject to multiple equilibria.[16]

I now turn to characterize the necessary and sufficient conditions for the equilibrium to be unique. For simplicity and illustrative purposes, let us consider the case with two simplified log-linearized policy rules: $\hat{\tau}_t + \hat{Y}_t = \gamma_b \hat{b}_{t-1}^L$, $\hat{R}_t = \phi_1 \pi_t$. If I then substitute the tax rule in the linearized law of motion for the real debt ratio and isolate the resulting coefficient for lagged debt, I get: $\hat{b}_t^L = (\frac{1}{\beta} - \kappa\gamma_b)\hat{b}_{t-1}^L + \ldots$, where $\kappa \equiv \frac{\tilde{\tau}\tilde{Y}}{\tilde{P}^L\tilde{b}^L}$. The Taylor principle is satisfied and the fiscal authority moves taxes in order to keep debt on a stable path when $\phi_1 > 1, \frac{1}{\beta} - \kappa\gamma_b < 1$. This is the Active Monetary/Passive Fiscal regime that I have discussed above. The "fiscal dominance" regime corresponds to the case in which the fiscal authority is not committed to stabilizing the process for debt: $\frac{1}{\beta} - \kappa\gamma_b > 1$.[17] Now it is the monetary authority that passively accommodates the behavior of the fiscal authority, disregarding the Taylor principle and allowing inflation to move in order to stabilize the process for debt: $\phi_1 < 1$.

3.3. Expositions of the Fiscal Theory

To provide further explanations particularly relevant to discussing the economics underlying the fiscal theory, it is useful to derive the bond valuation equation, which determines the aggregate price level when monetary policy is passively adjusted. Iterating forward on household's flow budget constraint (2) and imposing the transversality condition (10), I can derive the intertemporal budget constraint:[18]

$$E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}(P_t C_t) \leq E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}(W_t N_t) + D_t. \quad \textbf{(24)}$$

The household exhausts its intertemporal budget constraint, so the above condition holds with equality. Substitution of (6) and (9) into (24) and imposing the market-clearing condition (21) yield the equilibrium condition (in real terms):

$$\frac{(1+\rho P_t^L)B_{t-1}^L}{P_t} = E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}^R(\tau_t Y_t - G_t), \quad \textbf{(25)}$$

where the real stochastic discount factor is given by $\Lambda_{t,t+j}^R \equiv \beta^j (\frac{C_t}{C_{t+j}})^{\sigma}$.[19]

Equation (25) is the equilibrium bond valuation equation, as widely discussed in the fiscal theory literature [e.g., Cochrane (2001), Davig et al. (2011)]. This condition states that the real value of net government liabilities must equal the present value of expected future primary budget surpluses. The above equation works as the constraint of fiscal policy in the "monetary dominance" regime (i.e., fiscal policy should satisfy the equation) while it is an equilibrium condition in the "fiscal dominance" regime (i.e., the price-level changes keep the equation satisfied). As argued by Woodford (2001), among others, this relation necessarily obtains in a rational expectations equilibrium, not because we have assumed it as a constraint upon the government's fiscal policy, but rather it follows from private sector optimization, along with market clearing. It is an equilibrium condition, not a constraint on government behavior. In fact, the bond valuation equation imposes no restrictions on the government's choice of future surpluses. In a "fiscal dominance" regime that I consider in this paper, the price level adjustment is the key mechanism to satisfy this equilibrium condition.

4. PARAMETERIZATION

To study the properties of the model, I parameterize it using standard values in the literature–especially those focusing on the United States, for which a number of recent papers provide estimated parameter values. The model is parameterized at a quarterly frequency. Parameter values are summarized in Table 2. The discount factor β is set at 0.99, which gives a steady state annualized interest rate of 4%. The intertemporal elasticity of substitution is set to one half ($\sigma = 2$), based on evidence in Attanasio and Weber (1993, 1995). I calibrate $\varphi = \frac{1}{3}$, which implies a Frisch elasticity of labor supply of 3. This value is consistent with the macro-evidence of Peterman (2016) based on empirical work which matches volatilities of aggregate worked hours and of wages. The elasticity of substitution between intermediate goods among themselves, ϵ, is set equal to 10, which implies a value for the

TABLE 2. Benchmark parameterization: Key parameter values

Parameter	Value	Description
β	0.99	Discount factor
σ	2	Inverse elasticity of intertemporal substitution
φ	3	Inverse Frisch elasticity of labor supply
ϵ	10	Elasticity of substitution between differentiated goods
θ	0.66	Price stickiness parameter
ρ	0.9596	Coupon decay parameter
γ_b	0.136	Response of government primary surpluses to lagged real debt
γ_y	0.4596	Response of government primary surpluses to output gap
φ_1	0.5305	Response of nominal interest rate to inflation deviations
φ_2	0.0485	Response of nominal interest rate to output gap
ρ_G	0.886	Autoregressive coefficient of government spending shock
σ_ϵ	0.027	Standard deviation of innovation to government spending shock

steady-state mark-up rate, $\epsilon/(\epsilon - 1)$, of approximately 11%, consistent with the estimate reported by Basu and Fernald (1997). The price stickiness parameter, θ, is set at 0.66, the estimated value by Smets and Wouters (2007), which corresponds to the average duration of price contracts of about three quarters. The coupon decay parameter, $\rho = 0.9596$, corresponds to 5 years of government debt maturity, consistent with the data in many advanced economies [Eusepi and Preston (2013)]. In addition, I set the steady-state annualized debt-to-output ratio at 100%, which is currently around the average sovereign debt level in many advanced economies [Bai et al. (2017)].[20]

Regarding the parameters characterizing the fiscal rule, following Davig and Leeper (2006) estimated for the United States, I set γ_b and γ_y at 0.136, 0.4596, respectively. The share of government expenditures of final output $\tilde{G}/\tilde{Y}$ is fixed at 0.115, which is the mean value estimated by Davig and Leeper (2006). In the calibration of the monetary policy rule, I follow the Taylor estimates as in Davig and Leeper (2011) and set ϕ_1 and ϕ_2 equal to 0.5305 and 0.0485, respectively. Note that $\phi_1 < 1$, so the monetary policy rule makes the real interest rate respond less than proportionately to inflation in the long run. I verify that my parameter values reflect a "passive money, active fiscal" configuration of policy. Finally, I set the autoregressive coefficient of government spending shock ρ_G to 0.886 and the standard deviation of innovation to government spending shock σ_ϵ to 0.027. These two values are estimated by Leeper and Zhou (2013) using quarterly US data from 1948Q1 to 2013Q1.

5. MODEL ANALYSIS

5.1. Responses to Fiscal Shock

I start by describing the dynamic effects of an expansionary fiscal shock on a number of macroeconomic variables, as shown in Figure 3. Government purchases

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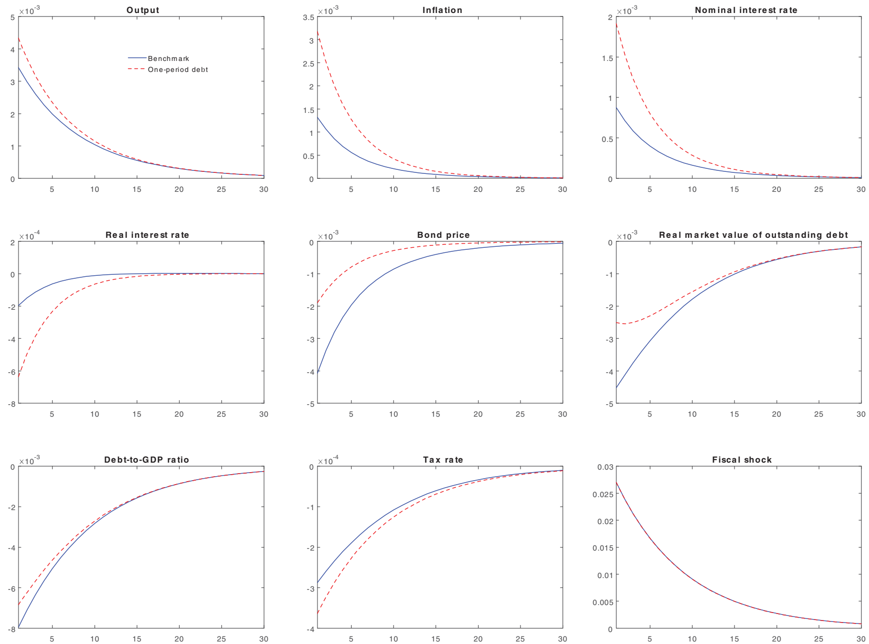

FIGURE 3. Benchmark case and one-period government debt.

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are assumed to increase by one standard deviation. The basic economic mechanism connecting fiscal developments and inflation dynamics is the wealth effect of fiscal disturbances upon private expenditure. At first, we see that the shock leads to an increase in government spending and a reduction in tax collections. The anticipation of lower primary surpluses makes households feel wealthier (through the private budget constraint), and thus leads households to demand more goods and services than the economy can supply, driving up the aggregate price level, as a result inflation occurs. A similar interpretation can be obtained from the bond valuation equation (25): the equation suggests that lowering future primary surpluses is inflationary, thereby the inflation rate rises. The increase in the price level also reduces the real market value of nominal assets held by the public, which restrains the effects of aggregate demand. An equilibrium is restored when prices rise to the point that the real value of those nominal assets no longer exceeds the present value of expected future primary surpluses.

Following the standard Taylor rule, the central bank increases the nominal interest rate. However, since monetary policy is passively adjusted, the nominal interest rate increases less than one-for-one of the response of inflation, the real interest rate actually falls on impact, and therefore output increases. At the same time, private agents revise upward (downward) their expectations about future short-term interest rates (future inflation), leading to a decline in the price of long-term bonds. Finally, the increase in growth and the drop in the real market value of long-term bonds determine a decline in the debt-to-GDP ratio.

Moreover, in order to illustrate the role that debt maturity structure plays in model dynamics, I compare the impulse responses of one-period debt (red-dashed lines) with an average of 5-year maturity debt (blue-solid lines, our benchmark case), also shown in Figure 3. With single period debt, the bond price drops much less than it does for 5-year government debt. This is because the bond price is no longer a function of inflation expectations, only the current price level matters. When $\rho = 0$, based on the bond valuation equation, the only way to respond expected future fiscal disturbances is through changes of the price level, since B_{t-1}^L is predetermined in this case. Therefore, as Figure 3 suggests, inflation increases much more than it does for 5-year debt. In this regard, long-term debt acts as a “fiscal shock absorber,” the changes of bond prices absorb much of the impact of fiscal disturbances, leaving less price changes needed when responding to a fiscal shock. This is the “fiscal cushion” effects of long-term bonds discussed in the fiscal literature [Sims (2013)].

With one-period debt, as inflation raises by more, following again the passive interest rate policy, nominal interest rates also increase by more. On impact, this leads to a larger drop in the real interest rate, therefore a larger increase in total output. Due to a larger response in the price level, the real market value of outstanding debt falls by less, but it still remains negative. The less drop in the real value of long-term bonds dominates the higher growth in output, on impact, the debt-to-GDP ratio declines by less. To sum up, in our context, introducing long-term government bonds helps to mitigate the effects of fiscal disturbances on inflation and many other macroeconomic variables.

5.2. The Aggressiveness of Monetary Policy

Noted by Sims (2011), even though the economy is in a "fiscal dominance" environment, it does not mean monetary policy is powerless. To account for the effects of monetary policy on macroeconomy, I consider how alternative monetary rules with different degrees of aggressiveness alter the impacts of a fiscal expansion. Figure 4 compares the impulse responses to a fiscal shock, as the high aggressiveness case being $\phi_1 = 0.8$, $\phi_2 = 0.0485$ and the low aggressiveness case being $\phi_1 = 0.2$, $\phi_2 = 0.0485$, same as below.[21] By reacting more aggressively to inflation, monetary policy ensures that the real interest rate falls by less, tempering the output increases.

The inflation dynamics are interesting and need to be highlighted. In Figure 4, I show that inflation increases by less when monetary policy is more aggressive while increases by more when monetary policy becomes less aggressive. This result is in line with Leeper and Leith (2016) who consider a simple endowment economy with long-term debt. However, using a New Keynesian model with one-period debt, their simulation result shows a different picture: an aggressive monetary policy increases inflation volatility. Interestingly enough, it is presumably the case that the average debt maturity may have important implications on the effectiveness of monetary policy. But, a rigorous comparison between the two sets of results is missing in their study.

I think it is potentially interesting to fully explore how the maturity structure of government debt affects the effectiveness of monetary policy. I limit my analysis to the model economy when only long-term debt prevails. Specifically, I consider two different experiments with 5-year debt (i.e., our benchmark case, as in Figure 4) and two-period debt, respectively. I choose two-period government debt as the point of comparison because it is short enough to generate opposite results, it is also long enough to have both "amplification" and "fiscal cushion" effects, as discussed below. Note that using a model with one-period debt as in the literature will limit the scope of our discussion as the "fiscal cushion" effect brought by long-term bonds is completely muted. I emphasize that the aggressiveness of monetary policy always has two conflicting effects. On the one hand, more aggressive monetary policy tends to amplify the effects of fiscal disturbances on inflation, leading to a larger increase in inflation. This is the "amplification" effect. One the other hand, long-term government debt is acting as a "fiscal cushion," the bond price decreases by more when the monetary policy rule becomes more aggressive, which absorbs much impact of the fiscal shock, leaving less changes required for the aggregate price level. I argue that the two conflicting effects are always in play jointly in determining the evolution of inflation. In Figure 4, the second effect dominates the first one, inflation actually raises by less when the interest rate rule becomes more aggressive.

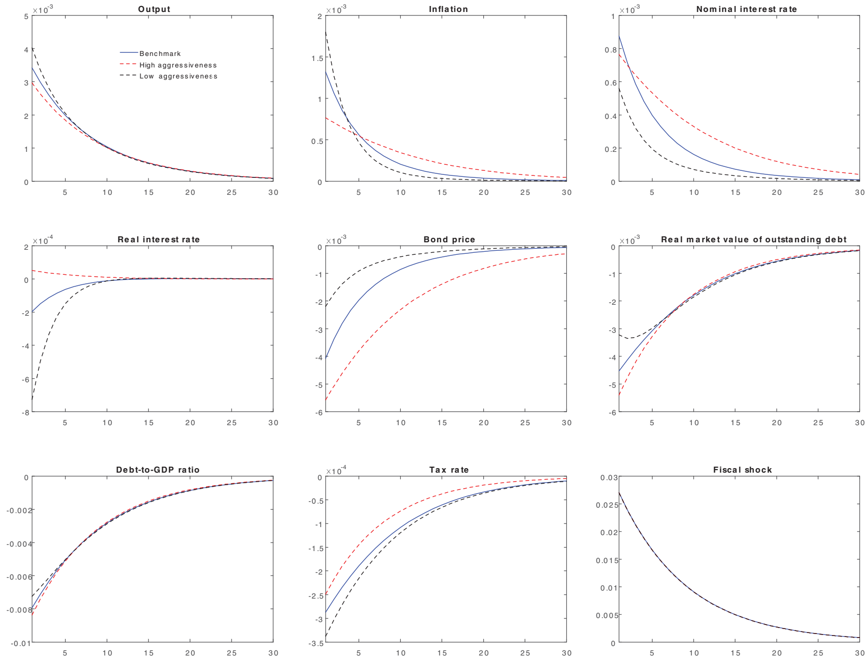

FIGURE 4. Benchmark case and different degrees of aggressiveness of monetary policy.

Figure 5 shows how the responses of macroeconomic variables to the same fiscal shock change when I now assume all government debt is two-period. Again, I consider two different scenarios: high aggressiveness monetary policy and low aggressiveness monetary policy. The results indicate that, compared to the base results in Figure 4, the responses are qualitatively similar for most of the variables, even though they are quantitatively more volatile due to a shorter maturity structure of government debt serving as "fiscal absorbers." The inflation dynamics are just the opposite: more aggressive monetary policy increases the aggregate price level by more, as it magnifies the effects on inflation from fiscal shocks as discussed earlier. This is the case where the "amplification" effect dominates the "fiscal cushion" effect.

In addition, I quantify that there is a threshold in the debt maturity structure above which an aggressive monetary policy dampens inflation volatility. In my current setting, I find the threshold is three quarters. This is to say, more aggressive monetary policy would amplify inflation volatility if the maturity structure of government debt is less than three quarters. My results may contribute to improving our understanding on the effectiveness of monetary policy: we used to think that tighter monetary policy leads to lower inflation, this may not, however, carry over in a "fiscal dominance" economy with long-term bonds. The effectiveness of monetary policy depends crucially on the maturity structure of government debt, and to be more precise, the two conflicting effects highlighted above. Thus, I show that tighter monetary policy could also increase inflation even with long-term debt, as long as the "fiscal cushion" effect is not strong enough.

6. OPTIMAL MONETARY-FISCAL POLICY AND WELFARE

Suppose that we are living in a "fiscal dominance" world, what should be the optimal monetary and fiscal policy for the government in response to fiscal disturbances? As noted in the introduction, this issue is of great practical concern to a number of advanced economies since more and more countries are entering into a "fiscally dominant" environment. Specifically, I begin by comparing the welfare performance of varying the response to inflation parameter in the Taylor rule. It allows us to address the important question that whether or not there are welfare gains if the monetary authority decides to use a relatively aggressive interest rate policy.[22] I then explore the impact of government debt maturity on the effectiveness of monetary policy in welfare terms. Does the previous result depend on the maturity structure of government debt? Is there a threshold in the debt maturity structure beyond which more aggressive monetary policy leads to a higher level of welfare? Next, I characterize optimal simple monetary and fiscal rules. In particular, I search numerically for a monetary-fiscal combination at which social welfare is maximized. By using implementable simple rules, my analysis takes into account the revealed behavior of policy makers as well as the institutional rigidities in policy making process. Finally, I perform a rigorous Ramsey optimal policy analysis, deriving the optimal allocation and price system,

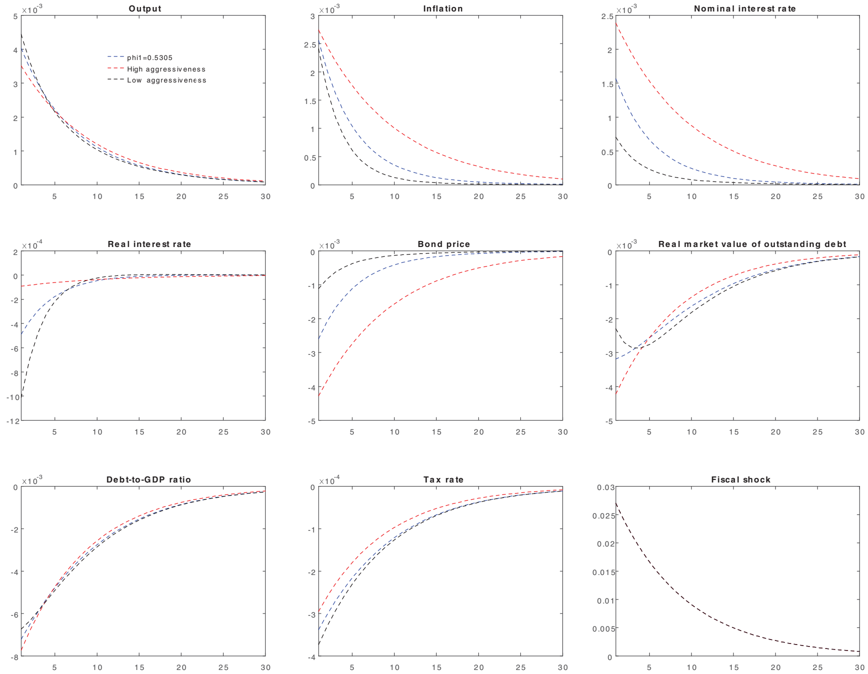

FIGURE 5. Different degrees of aggressiveness of monetary policy with two-period debt.

and then examine to what extent the optimized simple rules are able to replicate the implied dynamics. I also quantitatively evaluate the welfare losses with optimized simple monetary and fiscal rules compared to the Ramsey optimal policy.

Following Faia and Monacelli (2007) and Gertler and Karadi (2011), I assume the objective of the central bank is to maximize the average welfare of households. I begin by writing the household utility function in recursive form:

$$\Omega_t = U(C_t, N_t) + \beta E_t \Omega_{t+1}. \tag{26}$$

I then take a second-order approximation of this function around the deterministic steady state. I next take a second-order approximation of all model equations around the steady state, and then use this approximation to express the objective as a second-order function of the predetermined variables and shocks to the system.

For the convenience of comparing my welfare results with those obtained in the literature, I evaluate each policy specification by calculating the compensating variations in consumption, expressed in terms of the proportion of each period's consumption that a typical household would need to be compensated in the stochastic world in order to be indifferent from living in a deterministic risk-free world [e.g., Kolasa and Lombardo (2014), Lester et al. (2014)]. Specifically, I calculate λ that satisfies the following equation:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U[(1+\lambda)C_t, N_t] \right\} = \tilde{\Omega}, \tag{27}$$

where $\tilde{\Omega} = U(\tilde{C}, \tilde{N})/(1-\beta)$ is the value of Ω_t in the deterministic risk-free steady state.

Define two auxiliary value functions Ω_t^C, Ω_t^N :

$$\Omega_t^C = E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}, \Omega_t^N = E_0 \sum_{t=0}^{\infty} \beta^t \left(-\frac{N_t^{1+\varphi}}{1+\varphi} \right),$$

$$\Omega_t = \Omega_t^C + \Omega_t^N.$$

Under my specification of utility function one can solve for λ and obtain the following:

$$\lambda = \left(\frac{\tilde{\Omega} - \Omega_t^N}{\Omega_t^C} \right)^{1/(1-\sigma)} - 1. \tag{28}$$

If $\lambda > 0$, then the household would prefer to be in the risk-free regime, and vice versa. The higher the λ, the lower the welfare. I refer to Appendix A for more details on the computation of λ.

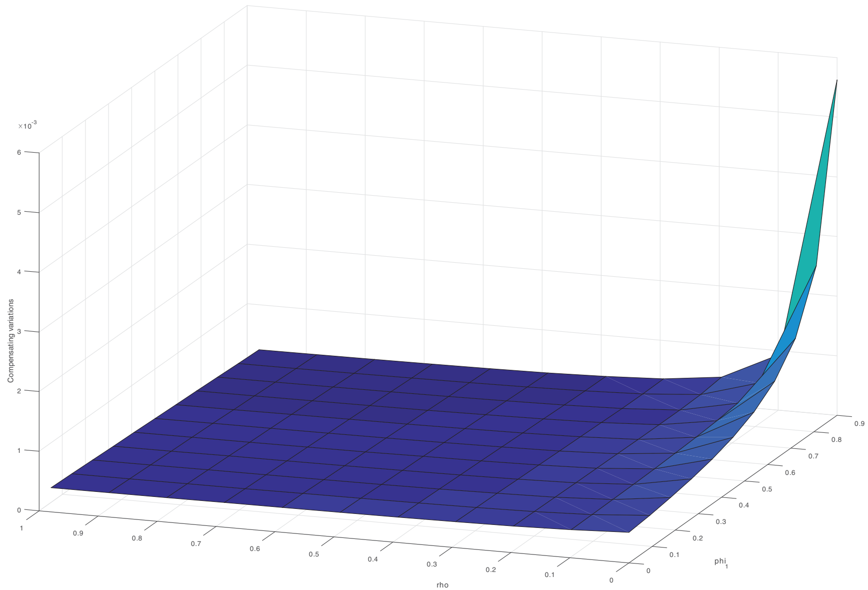

FIGURE 6. Effect on welfare of varying the response to inflation in the Taylor rule and to the average maturity structure of government bonds.

6.1. Taylor Rule and the Maturity Structure of Government Bonds

I take as given that the policy maker implements a standard Taylor rule, then I search numerically for the values of ϕ_1 and ρ in a three-dimensional space that optimize Ω_t as a response to an expansionary fiscal shock. For illustrative purposes, I search the values of $\phi_1 \in (0, 0.9)$, and $\rho \in (0, 0.9848)$.[23] I first intend to quantify the welfare gains associated with an aggressive monetary policy in a "fiscal dominance" economy, and then examine how the maturity structure of government bonds affects the effectiveness of monetary policy in welfare terms. As noted in the introduction, long-term nominal debt brings about a fiscal cushion that smooths out deficit-led inflation over time, but at the cost of inducing a higher future, and hence total inflation. As in Leeper and Leith (2016), with long-term debt, bond prices reflect anticipated inflation rates further into the future, in essence spreading inflationary effects over longer horizons. The cost of doing so is to raise the long-run inflation impacts of fiscal policy. In an infinitely-lived dynamic model, this cost would matter for social welfare. Thus, if the costs of having long-term debt dominate the benefits, an aggressive monetary policy can be welfare-reducing. A careful welfare evaluation needs to take into account of both the benefits and the costs that long-term bonds bring about.

My results show that, as indicated in Figure 6, the maturity structure of government bonds, and thus the "fiscal cushion" channel brought by long-term debt have important welfare implications. First, indeed, if all government debt is instantaneously short-term (i.e., one-period), more aggressive monetary policy will amplify the effects of fiscal shocks, causing higher inflation and higher volatilities to macroeconomic variables, therefore leading to a welfare loss. Quantitatively, the costs of having an aggressive monetary policy are significant. For instance, if we move ϕ_1 from 0 to 0.9, the welfare losses turn out to be around 0.5% in consumption unit. However, the losses are decreasing dramatically when we allow for long-term government bonds, as the "fiscal cushion" channel starts to pick up. Second, in our benchmark economy with 5-year government bonds, more aggressive monetary policy yields a higher level of welfare. Thus, the optimal monetary policy requires to set the coefficient ϕ_1 at its maximum value. Intuitively, as monetary policy becomes more aggressive, bond prices drop by more, the "fiscal cushion" effects brought by long-term debt are magnified, current inflation responds by less. This is the case where the benefits dominate the costs.

Third and more interestingly, there seems to exist a threshold in the debt maturity structure beyond which more aggressive monetary policy leads to a higher level of welfare. The longer the maturity of government debt, the stronger the "fiscal cushion" effects for current inflation, and the higher benefits by long-term bonds. In my current setting, the threshold of government debt maturity is 2 years. This is also to say, when the average maturity of government debt is short enough (below eight quarters in this context), the costs of inducing higher and overall total inflation further in the future would outweigh the benefits of inflation smoothing in welfare terms, as a result, more aggressive interest rate policy would be welfare reducing. I argue that this result is important in shaping our understanding of monetary policy. Since following the conventional literature under a "monetary dominance" stance, we used to know that tighter monetary policy is welfare-improving as higher interest rates tend to stabilize inflation and real activity. However, this argument may be overturned in a "fiscally dominant" economy when long-term government debt can play a crucial role.

Fourth, interestingly enough, if one compares this threshold with the previous one for inflation dynamics, we see that with the maturity of long-term debt lying between three quarters and eight quarters, more aggressive monetary policy dampens inflation volatility but reduces social welfare. I claim that this result may be interested from a policy maker's point of view: we need to keep in mind the heterogeneous effects of long-term debt on inflation dynamics and social welfare when we implement monetary policy. Finally, as shown in Figure 6, the addition welfare gains of introducing an even higher maturity of government debt are economically negligible. For example, moving the average maturity from 5 years to 10 years, the welfare gains from an aggressive monetary policy (i.e., from 0 to 0.9 for ϕ_1) would only be 0.003% in consumption unit.

6.2. Optimized Simple Rules

In this section, I characterize optimized monetary and fiscal simple rules. I wish to find the monetary- and fiscal-policy-rule combination that is optimal and implementable within the simple family defined by (19) and (20). In order to search numerically for the optimal policy in a three-dimensional space, I set $\phi_2 = 0$ and $\gamma_y = 0$. Figure 7 depicts the effects on the conditional welfare surface of varying both ϕ_1 and γ_b in the policy rules.[24] Several interesting results are in order. First, it shows that, in general, an aggressive response to lagged debt in the fiscal rule is welfare improving. This means that it is important for welfare that fiscal policy allows for higher response in taxes to deviations of government liabilities from target. Intuitively, a large coefficient introduces a strong fiscal feedback effect that stabilizes government debt as well as debt-led inflation, leading to a higher level of welfare. For instance, increasing γ_b from 0 to 0.2 while fixing ϕ_1 at 0.63 would improve welfare by 0.0042% in consumption unit.

Second, as shown in the 3D graph, the benefits of introducing an aggressive fiscal policy are virtually the same as using more aggressive monetary policy. Notice that I am considering the case where the government issues 5-year maturity debt. I shall examine later how this result would change when I consider an economy with only one-period debt. Third, an optimal monetary and fiscal combination requires to set both of the coefficients at their maximum values. It therefore calls for an aggressive response to inflation in the interest rate rule and an aggressive response to lagged debt in the fiscal rule. As shown in Figure 7, my results also indicate that these two effects are complementary from a welfare perspective such that both are needed to yield the highest level of welfare.

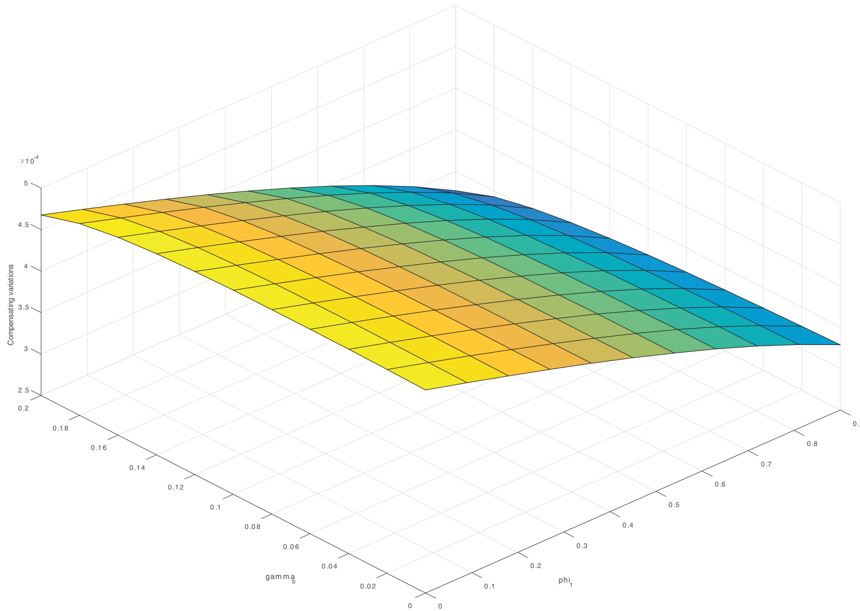

FIGURE 7. Welfare-maximizing monetary and fiscal rules.

To highlight the impact of maturity debt on model dynamics, it is also of interest to consider the case when assuming all government debt is single period. As shown in Figure 8, it is not surprising that more aggressive fiscal policy improves welfare, given the same reason discussed above. The gains are, however, much dominated by the large welfare losses from an aggressive monetary policy. In addition, since an aggressive interest rate rule reduces welfare, the optimal monetary and fiscal combination features $\phi_1 = 0$ and $\gamma_b = 0.2$.

6.3. Ramsey Optimal Policy

I have so far restricted attention to the case of implementable monetary and fiscal rules. But it is worthwhile asking to what extent the optimized simple rules that I have described are desirable. Thus, as a benchmark case, I compute a fully optimal policy, that is, joint monetary-fiscal optimization. The Ramsey policy is the process $\{R_t, \tau_t\}$ associated with the competitive equilibrium that yields the highest level of utility to the representative household, that is, that maximizes (1). In addition, I assume that the authorities have sufficient credibility to commit to the policy rules they announce at date 0.[25] In this study, I focus on optimal commitment policy, adopting Woodford's (2003) "timeless perspective."[26]

The Ramsey planner maximizes the welfare objective, taking as given all the private sector's optimizing decisions. The Lagrangian for the Ramsey planner's optimal policy problem is given by

$$\begin{aligned}
\mathcal{L}_t = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg[& \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\Delta_t^{1+\varphi} Y_t^{1+\varphi}}{1+\varphi} \right) \\
& + \lambda_{1t} \left\{ P_t^L - \beta \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{1}{\Pi_{t+1}} \right) (1 + \rho P_{t+1}^L) \right\} \\
& + \lambda_{2t} \{ K_t - \Delta_t^{\varphi} Y_t^{1+\varphi} - \beta\theta K_{t+1} \Pi_{t+1}^{\epsilon} \} \\
& + \lambda_{3t} \{ F_t - (1-\tau_t) C_t^{-\sigma} Y_t - \beta\theta F_{t+1} \Pi_{t+1}^{\epsilon-1} \} \\
& + \lambda_{4t} \left\{ \left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{1-\epsilon}} - \frac{K_t}{F_t} \right\} \\
& + \lambda_{5t} \left\{ \Delta_t - (1-\theta) \left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta} \right)^{\frac{\epsilon}{\epsilon-1}} - \theta \Pi_t^{\epsilon} \Delta_{t-1} \right\} \\
& + \lambda_{6t} \left\{ (1+\rho P_t^L) \frac{b_{t-1}^L}{\Pi_t} - P_t^L b_t^L - \tau_t Y_t + G_t \right\} \\
& + \lambda_{7t} \{ Y_t - C_t - G_t \} \Bigg].
\end{aligned}$$

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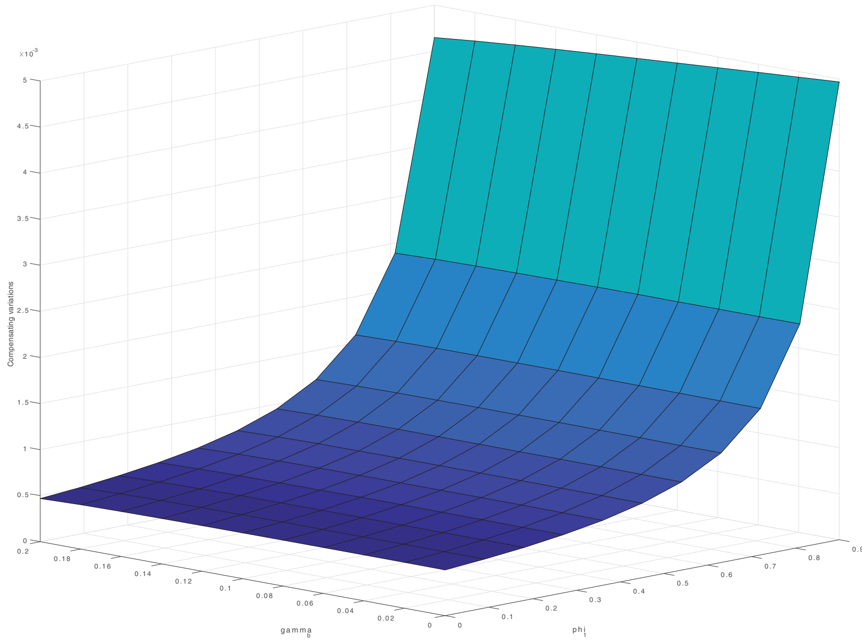

FIGURE 8. Welfare-maximizing monetary and fiscal rules with one-period debt.

Note that, to keep the Ramsey problem tractable, I have reduced the set of private optimizing conditions by substituting out N_t. The bond pricing equation is not included because it is a nonbinding constraint to the Ramsey problem. The private optimizing conditions can be reduced to the seven equations shown above. The planner solves the optimal policy problem by choosing the nine endogenous variables summarized in the vector:

$$X_t \equiv [C_t, \Pi_t, P_t^L, Y_t, K_t, F_t, \Delta_t, b_t^L, \tau_t],$$

along with the seven Lagrangian multipliers λ_{it} for $i \in \{1, 2, \ldots, 7\}$. The Ramsey first-order conditions as well as the derivations of the Ramsey problem are shown in Appendix B.

Figure 9 shows the path of key variables following an expansionary fiscal shock for both the optimized simple rules and the Ramsey optimal allocation. For the optimized simple rules (that is, using the optimal monetary-fiscal combination that I have defined earlier), the model dynamics are very much the same as in Figure 3, albeit the volatilities are smaller. In response to an increase in government spending, inflation raises because of higher aggregate demand and the bond price reduces as long-term government debt brings in "fiscal cushion" effects. By doing so, government debt is stabilized, which, following a fiscal feedback rule, induces a drop in the tax rate. In the meanwhile, the monetary authority increases nominal interest rates using a simple interest rate rule, but not as much as the increase in inflation since the economy is in a "fiscal dominance" world. Thus, the real interest rate falls and output increases.

The allocation of the Ramsey optimal policy is however different from that of optimized simple monetary-fiscal rule. An immediate response is an increase in taxation due to an increase in government purchases and a higher level of government liabilities. Moreover, as shown in the figure, since inflation is costly from a welfare point of view, the Ramsey planner would tend to stabilize it as a priority. In my model, inflation actually increases very slightly at the beginning, it, however, looks like zero compared to the relatively large increase under optimized simple rules. The standard deviation of inflation is 0.2 basis point per quarter. This small jump in the inflation rate is mainly driven by a higher tax rate initially, given that I have distortionary taxation which affects the supply side of the economy through the New Keynesian Phillips Curve. Interestingly enough, the Ramsey planner uses higher interest rates to manipulate the optimal balance between monetary and fiscal policy. There is a tax-smoothing jump in taxation that would put upward pressure on inflation. But, it is offset by a tighter monetary policy that makes inflation zero after the initial period. In addition, bond prices also drop as the "fiscal cushion" effect picks up, but the drop is much smaller compared to the case in optimized simple rules. In general, as shown in Figure 9, the model economy is much stabilized under the Ramsey optimal policy.

Next, it would be interesting to quantitatively evaluate the welfare losses of optimized simple rules compared to the Ramsey optimal policy. Again, I compute

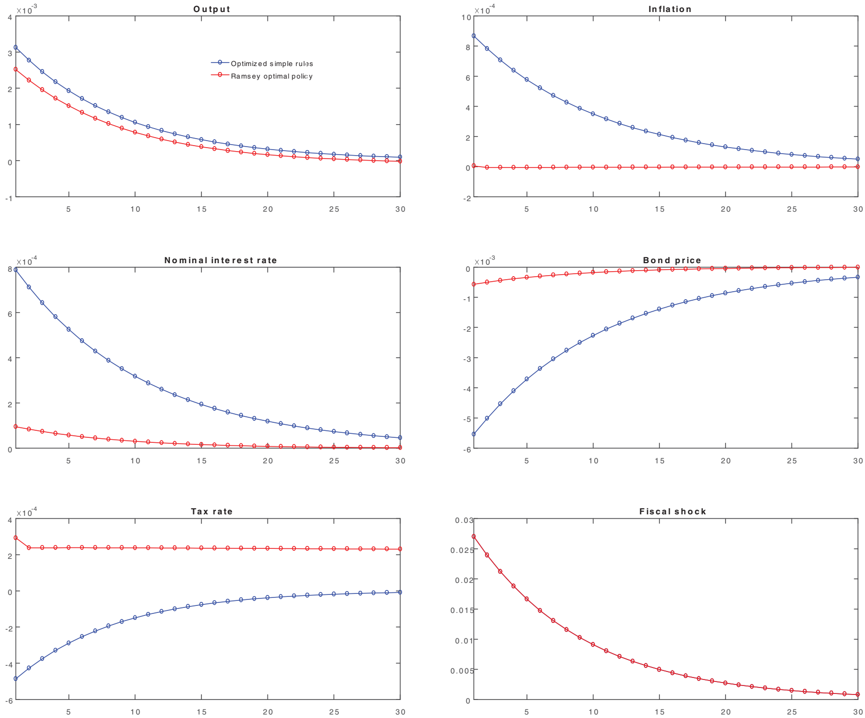

FIGURE 9. Optimized simple rules and the Ramsey optimal policy.

second-order accurate solutions to policy functions of the Ramsey problem and evaluate the welfare losses associated with optimal monetary-fiscal combination to the Ramsey policy by calculating the compensating variations in consumption. In my current setting, I compute the welfare losses to be around 2.16% in consumption unit. Compare this value to those typically obtained in the optimal policy literature, my result indicates that optimized simple monetary-fiscal rule is still significantly welfare inferior to the Ramsey optimal policy. The main reason for this non-trivial difference in the levels of welfare is that I restrict my analysis to the model economy of "fiscal dominance" world. Weak fiscal feedback in debt stabilization (active fiscal regime) damages the ability of monetary policy to reduce the social costs of macroeconomic shocks [Kirsanova and Wren-Lewis (2012)]. Indeed, using a model with sticky prices and one-period debt, Schmitt-Grohé and Uribe (2007) find that the optimal monetary/fiscal rule combination features an active monetary stance and a passive fiscal stance.

7. CONCLUSION

Many advanced economies are entering into a period of fast growing government debt and aging populations. Fiscal theory has made it clear that fiscal behavior can

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be a primary source for changes in the inflation rate and have sizeable effects on macroeconomy. This paper focuses on the question of what monetary and fiscal policy can do and should do in a "fiscal dominance" world. To address this issue, I develop a quantitative macroeconomic model with nominal rigidities, long-term debt, and distortionary taxation, accounting for explicitly and realistically the potential impacts of fiscal behavior on macroeconomy.

I find that even though the economy is under unresolved fiscal stress, monetary policy can still have powerful effects on both inflation and real activity. However, the effectiveness of monetary policy depends crucially on the average maturity structure of government bonds. I highlight that both "amplification" and "fiscal cushion" effects are always in play jointly in determining the evolution of aggregate price level. I find the threshold of maturity of government bonds beyond which more aggressive monetary policy dampens inflation volatility is three quarters. In addition, long-term debt brings about a fiscal cushion that smooths out deficit-led inflation over time, but at the cost of inducing a higher future, and hence total inflation. My welfare analysis quantitatively evaluates the costs and benefits brought by long-term bonds. My results show that the threshold of government debt maturity above which an aggressive monetary policy improves social welfare is eight quarters.

More importantly, I characterize optimal monetary and fiscal policy using simple and implementable rules. My results indicate that an optimal monetary and fiscal combination calls for an aggressive response to inflation in the interest rate rule and an aggressive response to lagged debt in the fiscal rule. In addition, I find that more aggressive fiscal policy improves welfare as it introduces a strong fiscal feedback effect that stabilizes government debt as well as debt-led inflation. Finally, I solve the Ramsey problem and evaluate to what extent the optimized simple rules are desirable. I find that the key variables are much stabilized under the Ramsey economy. I also calculate the welfare losses associated with optimized simple rules to be around 2.16% in consumption unit, indicating that optimized simple monetary-fiscal rule is still significantly welfare inferior to the Ramsey optimal policy.

NOTES

1. According to Davig and Leeper (2011a), a "fiscal limit" is a point beyond which tax collections can no longer increase and government expenditures can no longer be reduced.

2. Another argument for the impossibility of increasing taxes is that it is generally recognized economies face a natural limit to how much taxes a government can raise, a top of a Laffer curve, a fiscal limit [Davig and Leeper (2011a), Davig et al. (2011)].

3. For earlier discussions of the fiscal theory of the price level (FTPL), see Sargent and Wallace (1981), Aiyagari and Gertler (1985), Leeper (1991), Sims (1994, 1999), Woodford (1995, 1998, 2001), Cochrane (1998, 2001), Dupor (2000), Schmitt-Grohé and Uribe (2000), and Benhabib et al. (2001).

4. A complete account of empirical evidence on "fiscal dominance" regime is beyond the scope of my paper. In the literature, many studies have provided evidence that fiscal regime has prevailed in some historical periods, see Cochrane (1998, 2001), Woodford (2001), Kim (2003), Davig and Leeper (2006, 2011b), and Bianchi and Ilut (2017), among others.

5. In Leeper and Leith (2016), using a New Keynesian model with one-period debt, it shows that more aggressive monetary policy amplifies inflation volatility. However, this may limit the scope of our discussion as with only one-period debt the fiscal cushion effect is completely missing. My emphasis is on the relative power of amplification effect and fiscal cushion effect, as both effects matter for the price level determination.

6. One may think that since in general, more aggressive monetary policy with long-term bonds stabilizes inflation because of the "fiscal cushion" effect, and what is detrimental for welfare is inflation volatility, it is rather straightforward to conclude that tighter monetary policy is welfare-improving. I claim that it may be misleading, for the reasons stated below.

7. For optimal monetary policy analysis, see also Faia and Monacelli (2007) and Gertler and Karadi (2011) for close economy models, and Kolasa and Lombardo (2014) for an open economy.

8. It is worth mentioning that there are two distinct branches of earlier studies on optimal monetary and fiscal policy which deliver opposed policy recommendations. One branch includes Barro (1979) who shows that debt and taxes should follow martingale processes to minimize the discounted value of tax distortions. Another branch follows the theoretical framework laid out in Lucas and Stokey (1983) and includes Chari et al. (1991, 1994) and Calvo and Guidotti (1993). A key result of this part of literature is that it is optimal for the government to make the inflation rate highly volatile and serially uncorrelated. Thus, it allows the government to keep tax rates remarkably stable over the business cycle. However, all the above studies depend on surprise inflation and deflation being costless. In a Keynesian model with sticky prices or wages, the optimal policy can be strikingly different [see Sims (2013)].

9. Note also that central banks can rearrange the timing of inflation by changing the maturity structure of outstanding government debt [Cochrane (2011)]. For example, central bank's purchases of long-term debt, in exchange for short-term debt would result in more inflation today, less inflation in the future. This action makes sense of the "quantitative easing" plans for long-term debt purchases by the Fed after the financial crisis.

10. Same maturity structure of long-duration debt has also been adopted by Eusepi and Preston (2013), Leeper and Zhou (2013), and Bai et al. (2017), among others. Moreover, note that my structure is equivalent to assuming there are zero-coupon government bonds, which decline at a constant geometric decay rate each period, as examined by Cochrane (2001) and Leeper and Leith (2016).

11. As noted by Woodford (1998), the longer the duration of the asset, the more sharply its value will decline with increases in inflation, since expected future price levels increase even more than does the current price level.

12. Similar approaches of modeling long-term debt are developed in the literature. Notable examples include Rudebusch and Swanson (2008), Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), and Krause and Moyen (2016).

13. Note that, without loss of generality, I implicitly assume that the covariance between the stochastic discount factor for pricing nominal contingent claims and the gross nominal return on the bond portfolio is zero.

14. Unlike monetary policy, there is no widely accepted specification for fiscal policy in the literature. The simple fiscal rule that I adopt here is similar to the fiscal specifications in Davig and Leeper (2006), Schmitt-Grohé and Uribe (2007), and Sims (2011).

15. Noted also by Evans and Honkapohja (2007), whether the fiscalist solution emerges depends crucially on the joint fiscal and monetary policy regime.

16. Loyo (1999) discusses the Brazilian hyperinflation in the 1980s, due to the active monetary/fiscal policy mix.

17. Note that one version of the "fiscal dominance" regime emerges as the special case $\gamma_b = 0$, $\phi_1 < 1$, i.e., taxes are described as an exogenous process, see, for example, Woodford (2001), among others.

18. I shall also stipulate that the household's planned expenditure has a finite present value.

19. One can also derive the same equation by iterating forward on the government's budget constraint and imposing the transversality condition.

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20. Note that in my case, the steady-state annualized debt-to-GDP ratio is equal to $(\tilde{P}^L\tilde{B}^L)/(4\tilde{P}\tilde{Y})$.

21. Here, I set ϕ_1 arbitrarily at a relatively high value at 0.8 and a relatively low value at 0.2 for illustrative purposes, while keeping $\phi_2 = 0.0485$ unchanged. I verify that both cases are in a "fiscal dominance" environment.

22. Notice that even though I have shown in the foregoing discussion that long-term nominal debt brings about a fiscal cushion that smooths out deficit-led inflation over time when monetary policy responds aggressively to inflation, it is at the cost of inducing a higher future, and hence total inflation.

23. Note that $\rho = 0.9848$ corresponds to 10 years of government debt maturity.

24. I limit my attention to policy coefficients γ_b in the interval [0, 0.2]. In my model, the maximum value for γ_b to ensure local uniqueness of the rational expectations equilibrium is around 0.25. Although the size of this interval is arbitrary, I feel it is appropriate for my illustrative purposes. My results, however, are robust to expanding the size of the interval as long as the equilibrium is well-defined.

25. Many authors have claimed that central banks have either described their current monetary policy as policy under commitment, or come very close to doing so, see, for example, Svensson (2009) and Adolfson et al. (2011).

26. This approach is widely adopted in the literature, see, for instance, Schmitt-Grohé and Uribe (2004), Kirsanova and Wren-Lewis (2012), and Leeper and Zhou (2013), among others.

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APPENDIX A: COMPUTATION OF THE COMPENSATING VARIATION PARAMETER

Following Lester et al. (2014), I describe the calculation of compensating variations for welfare evaluations. For the case of additively separable preferences in utility, as in my case, the value function evaluated at a particular point in the state space, Ω_t, can be written as

$$\Omega_t = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right),$$

I then define the following two auxiliary value functions:

$$\Omega_t = \Omega_t^C + \Omega_t^N,$$

$$\Omega_t^C = E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma},$$

$$\Omega_t^N = E_0 \sum_{t=0}^{\infty} \beta^t \left(-\frac{N_t^{1+\varphi}}{1+\varphi} \right).$$

The risk-free deterministic steady state $\tilde{\Omega} = \left(\frac{\tilde{C}^{1-\sigma}}{1-\sigma} - \frac{\tilde{N}^{1+\varphi}}{1+\varphi} \right) / (1-\beta)$, where $\tilde{C}$ and $\tilde{N}$ are the steady-state values of consumption and employment.

The conditional compensating variation λ for the regime Ω_t is defined by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{[(1+\lambda)C_t]^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right\} = \tilde{\Omega}$$

Using the definitions above and simplifying:

$$\tilde{\Omega} = (1+\lambda)^{1-\sigma} E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} + E_0 \sum_{t=0}^{\infty} \beta^t \left(-\frac{N_t^{1+\varphi}}{1+\varphi} \right)$$
$$= (1+\lambda)^{1-\sigma} \Omega_t^C + \Omega_t^N.$$

Solving for λ :

$$\lambda = \left(\frac{\tilde{\Omega} - \Omega_t^N}{\Omega_t^C} \right)^{1/1-\sigma} - 1.$$

APPENDIX B: DERIVATIONS OF THE RAMSEY OPTIMAL POLICY PROBLEM

The planner maximizes the welfare objective:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right),$$

taking as given all the private sector's optimizing decisions summarized as follows:

$$\beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{1}{\Pi_{t+1}} \right) \left(1 + \rho P_{t+1}^L \right) \right] = P_t^L,$$

$$K_t = \Delta_t^{\varphi} Y_t^{1+\varphi} + \beta\theta E_t K_{t+1} \Pi_{t+1}^{\epsilon},$$

$$F_t = (1-\tau_t) C_t^{-\sigma} Y_t + \beta\theta E_t F_{t+1} \Pi_{t+1}^{\epsilon-1},$$

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$$\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{1-\epsilon}} = \frac{K_t}{F_t},$$

$$\Delta_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}} + \theta\Pi_t^{\epsilon}\Delta_{t-1},$$

$$P_t^L b_t^L = \left(1+\rho P_t^L\right)\frac{b_{t-1}^L}{\Pi_t} - \tau_t Y_t + G_t,$$

$$Y_t = C_t + G_t.$$

Note that, to keep the Ramsey problem tractable, I have reduced the set of private optimizing conditions by substituting out N_t. The bond pricing equation is not included because it is a nonbinding constraint to the Ramsey problem. The private optimizing conditions can be reduced to the seven equations shown before. The planner solves the optimal policy problem by choosing the nine endogenous variables summarized in the vector:

$$X_t \equiv [C_t, \Pi_t, P_t^L, Y_t, K_t, F_t, \Delta_t, b_t^L, \tau_t],$$

along with the seven Lagrangian multipliers λ_{it} for $i \in \{1, 2, \ldots, 7\}$. The Ramsey first-order conditions are summarized as follows:

$$C_t : C_t^{-\sigma} - \lambda_{1t} E_t \left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma-1}\left(\frac{\beta\sigma}{C_{t+1}}\right)\left(\frac{1+\rho P_{t+1}^L}{\Pi_{t+1}}\right)\right]$$

$$+\lambda_{1t-1}\sigma\left(\frac{C_{t-1}}{C_t}\right)^{\sigma-1}\left(\frac{1}{C_t}\right)^2 (C_{t-1})\left(\frac{1}{\Pi_t}\right)(1+\rho P_t^L)$$

$$+\lambda_{3t}(1-\tau_t)\sigma C_t^{-\sigma-1} Y_t - \lambda_{7t} = 0$$

$$\Pi_t : \lambda_{1t-1}\left(\frac{C_{t-1}}{C_t}\right)^{\sigma}\left(\frac{1}{\Pi_t}\right)^2 (1+\rho P_t^L) - \lambda_{2t-1}\theta\epsilon K_t \Pi_t^{\epsilon-1}$$

$$-\lambda_{3t-1}\theta(\epsilon-1)F_t\Pi_t^{\epsilon-2} - \lambda_{4t}\frac{1}{1-\epsilon}\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{1-\epsilon}}\left[\frac{\theta(\epsilon-1)\Pi_t^{\epsilon-2}}{1-\theta}\right]$$

$$-\lambda_{5t}\frac{\epsilon(1-\theta)}{1-\epsilon}\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{\epsilon-1}}\left[\frac{\theta(\epsilon-1)\Pi_t^{\epsilon-2}}{1-\theta}\right]$$

$$-\lambda_{5t}\theta\epsilon\Pi_t^{\epsilon-1}\Delta_{t-1} - \lambda_{6t}(1+\rho P_t^L)\frac{b_{t-1}^L}{(\Pi_t)^2} = 0$$

$$P_t^L : \lambda_{1t} - \lambda_{1t-1}\left(\frac{C_{t-1}}{C_t}\right)^{\sigma}\left(\frac{\rho}{\Pi_t}\right) + \lambda_{6t}\left(\rho\frac{b_{t-1}^L}{\Pi_t} - b_t^L\right) = 0$$

$$Y_t : -\Delta_t^{1+\varphi}Y_t^{\varphi} - \lambda_{2t}\Delta_t^{\varphi}(1+\varphi)Y_t^{\varphi} - \lambda_{3t}(1-\tau_t)C_t^{-\sigma} - \lambda_{6t}\tau_t + \lambda_{7t} = 0$$

$$K_t : \lambda_{2t} - \lambda_{2t-1}\theta\Pi_t^{\epsilon} - \lambda_{4t}\frac{1}{F_t} = 0$$

$$F_t : \lambda_{3t} - \lambda_{3t-1}\theta\Pi_t^{\epsilon-1} + \lambda_{4t}\frac{K_t}{(F_t)^2} = 0$$

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$$\Delta_t : -\Delta_t^{\varphi} - \lambda_{2t} Y_t^{1+\varphi} \varphi \Delta_t^{\varphi-1} + \lambda_{5t} - E_t \lambda_{5t+1} \beta \theta \Pi_{t+1}^{\epsilon} = 0$$

$$b_t^L : E_t \lambda_{6t+1} \beta (1 + \rho P_{t+1}^L) \frac{1}{\Pi_{t+1}} - \lambda_{6t} P_t^L = 0$$

$$\tau_t : \lambda_{3t} C_t^{-\sigma} - \lambda_{6t} = 0$$