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INFLATION AND PATTERN OF TRADE IN A DYNAMIC SPECIFIC-FACTORS MODEL WITH MONEY

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This paper presents a dynamic specific-factors model with money introduced through a cash-in-advance constraint. Two types of consumption goods are produced, and three types of factors—labor, capital, and land—are used. The cash-in-advance constraint is imposed on different sets of goods. When the constraint is imposed exclusive of the investment, inflation affects the pattern (and volume) of trade through a commodity-substitution effect. When the constraint is imposed inclusive of the investment, inflation may affect the pattern of trade through both the commodity-substitution effect and the factor-supply effect. In each case, we examine and prove the dynamic stability property of the steady-state equilibrium.

Keywords: Cash in Advance, Dynamic Specific-Factors Model, Inflation, Pattern of Trade

1. INTRODUCTION

In standard models of international trade as surveyed by Jones and Neary (1984), factor supplies play a key role in determining trade patterns. Stockman (1985) has shown how inflation may affect the pattern of trade by affecting the supply of labor and capital. Using a Heckscher-Ohlin model with money introduced through a cash-in-advance constraint, he has demonstrated that a small change in the rate of inflation may have a drastic effect on the pattern of trade in a small open economy: if the rate of monetary expansion exceeds a critical value, the economy exports only, say, the labor-intensive goods and imports the capital-intensive goods; a reduction in the rate of monetary growth below that critical value would cause the

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economy to specialize completely in the production of the capital-intensive goods and to import the labor-intensive goods.

Stockman's results were reexamined in a specific-factors model by Roldos (1992), who concentrated on the effect of inflation on capital accumulation vis-à-vis the trade pattern. It is shown that the change in inflation would cause a smooth change in the volume of trade and could eventually cause a change in the pattern of trade.

Both Stockman (1985) and Roldos (1992) were confined to the steady-state analysis, and they emphasized the role of capital accumulation induced by a change in inflation, that is, the factor-supply effect. This paper makes two contributions: in addition to the factor-supply effect, we show that there is another channel through which inflation may affect the pattern of trade, and second, we supplement both Stockman's and Roldos's analyses. We verify that the steady state is saddle-path stable. Hence, the comparative static analyses they performed are valid indeed.

According to the literature on cash in advance, it is shown that the composition of the cash-in-advance constraint plays an important role in determining the effects of inflation.[1] We analyze the effects of inflation on the pattern of trade for different structures of cash-in-advance constraint and then compare within the results.

The paper is organized as follows. In the next section, we present the first cash-in-advance model in which only part of the consumption goods are cash constrained. We derive the first-order conditions and the steady-state characterizations. Dynamic stability conditions are then examined. In Section 3, we present the second cash-in-advance model in which part of the consumption goods and the investment are cash constrained. In Section 4, we present the third cash-in-advance model in which all consumptions and the investment are cash constrained. In the last section, we conclude with a brief summary.

2. FIRST CASH-IN-ADVANCE MODEL[2]

Consider a small open economy in which there is a representative infinitely lived household that maximizes an intertemporal utility function:

$$\int_0^\infty [u(c_{1t}) + v(c_{2t})]\, e^{-\theta t}\, dt,$$

where $\theta > 0$ is the rate of time discount, c_{it} is the consumption of good i at time t. The momentary utility functions $u(\cdot)$ and $v(\cdot)$ are strictly increasing, strictly concave, continuously differentiable, and satisfies Inada conditions, respectively.

The production side of the economy is described by two representative firms. One of them produces good 1 (the industrial goods), using labor l and capital k, according to a constant-returns-to-scale production function $F(l, k)$. The industrial goods can be either consumed or added to the existing capital stock. The capital does not depreciate. The other firm produces good 2 (the agricultural goods), using (fixed-quantity) land $\bar{n}$ and labor $1 - l$, according to the constant-returns-to-scale production function $G(1 - l, \bar{n})$. The agricultural goods can only be consumed.

The production functions F and G satisfy

$$F_1, F_2, G_1, G_2 > 0, \qquad F_{11}, F_{22}, G_{11}, G_{22} < 0,$$

and the Inada conditions, respectively.

In the setup, we assume that labor is mobile across firms. The household supplies its unit labor endowment inelastically, so that if l is the fraction of labor used in producing the industrial goods, $1 - l$ is the fraction of labor used in producing the agricultural goods. The economy faces given international prices of those goods, and units are chosen in such a way that allows normalization of both prices at unity. The exchange rate e_t converts them to domestic monetary units.

The cash is injected into the system through lump-sum transfers T_t (withdrawn by lump-sum taxes). The budget constraint of the household and the investment constraint can be written as follows:

$$e_t(c_{1t} + i_t) + e_t c_{2t} + \dot{M}_t = e_t F(l_t, k_t) + e_t G(1 - l_t, \bar{n}) + T_t, \tag{1}$$

$$\dot{k}_t = i_t, \tag{2}$$

with k_0, M_0 given. In the expressions, since c_{1t} can be converted to i_t on a one-to-one basis, they have the same nominal price at t. In this section, it is assumed that cash is needed to purchase the industrial goods; that is, a cash-in-advance constraint is imposed as follows:

$$e_t c_{1t} \leq M_t, \tag{3}$$

where M_t is the cash balance at time t.

Denoting by H the Hamiltonian of the problem, α, β, and γ the multipliers for (1)–(3), one can write H as[3]

$$H = \{u(c_1) + v(c_2) + \alpha[eF(l, k) + eG(1 - l, \bar{n}) + T - e(c_1 + i) - ec_2] + \beta i + \gamma(M - ec_1)\}e^{-\theta t}.$$

The first-order conditions for an interior optimal path are given by (1)–(3) and

$$u'(c_1)/e = \alpha + \gamma, \tag{4}$$

$$v'(c_2)/e = \alpha, \tag{5}$$

$$F_1(l, k) = G_1(1 - l, \bar{n}), \tag{6}$$

$$\alpha = \beta/e, \tag{7}$$

$$\dot{\alpha} = \alpha\theta - \gamma, \tag{8}$$

$$\dot{\beta} = \beta\theta - \alpha e F_2(l, k), \tag{9}$$

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and two transversality conditions:

$$\lim_{t\to\infty} \beta_t k_t e^{-\theta t} = 0, \tag{10}$$

$$\lim_{t\to\infty} \alpha_t M_t e^{-\theta t} = 0. \tag{11}$$

Equation (4) equates the marginal utility of the industrial goods per dollar spent to the sum of the marginal utility of income and the marginal utility of cash; equation (5) equates the marginal utility of the agricultural goods per dollar spent to the marginal utility of income; equation (6) equates the value of marginal product of labor in each sector; equation (7) equates the marginal utility of investment per dollar spent to the marginal utility of income. Equation (8) describes the dynamic motion of the marginal utility of income, while equation (9) describes the dynamic motion of investment. Equation (10) is used to rule out Ponzi-game behavior in trading physical capital, while equation (11) is used to rule out similar behavior in trading cash.

To simplify the system of (4)–(9), we first substitute (4) and (5) into (8) to obtain

$$\frac{\dot{\alpha}}{\alpha} = 1 + \theta - \frac{u'(c_1)}{v'(c_2)}. \tag{12}$$

Next, substitute (7) into (5) and (9) to obtain

$$v'(c_2) = \beta, \tag{13}$$

$$\frac{\dot{\beta}}{\beta} = \theta - F_2(l, k). \tag{14}$$

Combining (13) and (14), we obtain

$$\dot{c}_2 = \left[\frac{v'(c_2)}{v''(c_2)}\right][\theta - F_2(l, k)]. \tag{15}$$

The equilibrium conditions for the economy require that the money market clears and the trade is balanced. By Walras's law, we only need to impose the latter equilibrium condition:

$$\dot{k} + c_1 + c_2 = F(l, k) + G(1 - l, \bar{n}). \tag{16}$$

Following Stockman and Roldos, we will conduct the analysis under the assumption that the cash-in-advance constraint is binding. Multiply (3) by α and substitute (7) into it to obtain

$$\beta c_1 = \alpha M. \tag{17}$$

Taking logarithm of both sides of (17) and differentiating with respect to time,

$$\frac{\dot{\beta}}{\beta} + \frac{\dot{c}_1}{c_1} = \frac{\dot{\alpha}}{\alpha} + \frac{\dot{M}}{M}. \tag{18}$$

Money supply is assumed to follow a constant growth rate, μ,

$$\frac{\dot{M}}{M} = \mu. \tag{19}$$

Substituting (12), (14), and (19) into (18), we have

$$\frac{\dot{c}_1}{c_1} = 1 + \mu + F_2(l, k) - \frac{u'(c_1)}{v'(c_2)}. \tag{20}$$

The equilibrium motions of (c_1, c_2, l, k) are thus completely characterized by (6), (15), (16), and (20).

In a steady state, $\dot{c}_1 = \dot{c}_2 = \dot{l} = \dot{k} = 0$. This implies

$$F_1(\bar{l}, \bar{k}) = G_1(1 - \bar{l}, \bar{n}), \tag{21}$$

$$\theta = F_2(\bar{l}, \bar{k}), \tag{22}$$

$$\bar{c}_1 + \bar{c}_2 = F(\bar{l}, \bar{k}) + G(1 - \bar{l}, \bar{n}), \tag{23}$$

$$1 + \mu + F_2(\bar{l}, \bar{k}) = \frac{u'(\bar{c}_1)}{v'(\bar{c}_2)}, \tag{24}$$

where a bar over the variable denotes its steady-state value.

It is straightforward to verify that the system of (21)–(24) has a unique steady state. Equation (22) implies that the steady-state marginal productivity of capital is tied to the rate of time preference. The intuition underlying this result can be explained as follows. Since the cash-in-advance constraint (3) does not apply to either c_2 or $\dot{k}$, the household can reduce c_2 and, through (16), add to the investment at t directly. The increase in the production of the industrial goods can be used to increase c_{1s} for all $s > t$.[4] In the steady state, reducing c_{2t} by one unit incurs a loss of utility by

$$v'(\bar{c}_2),$$

whereas augmenting the capital stock by one unit forever incurs a gain of utility by

$$\int_0^\infty v'(\bar{c}_2) F_2(\bar{l}, \bar{k}) e^{-\theta t}\, dt.$$

Equating these two expression, one obtains the result that the steady-state real rate of return of capital is determined by (22).

We now study the effects of a perfectly anticipated inflation on the steady-state capital, the consumption goods and the trade pattern. Totally differentiate (21)–(24),

$$\frac{d\bar{k}}{d\mu} = 0, \tag{25}$$

$$\frac{d\bar{l}}{d\mu} = 0, \tag{26}$$

$$\frac{d\bar{c}_1}{d\mu} = \frac{v'}{u'' + (1 + \theta + \mu)v''} < 0, \tag{27}$$

$$\frac{d\bar{c}_2}{d\mu} = -\frac{v'}{u'' + (1 + \theta + \mu)v''} > 0, \tag{28}$$

where all derivatives are evaluated at the steady state.

The reasoning for the results in (25)–(28) can be explained as follows. Because cash is used to purchase c_1, but not to purchase c_2, an increase in the opportunity cost of holding money raises the price of c_1 relative to c_2, this induces a substitution of c_2 for c_1. If the economy is originally at the steady state, the domestic production is not affected by the increase in μ; yet, this will cause a once-and-for-all increase (decrease) in c_2 (c_1).[5] A cash-in-advance constraint is an effect very like a consumption tax, whose effective rate is increasing in the rate of inflation. Hence, if the economy is initially importing the industrial goods (good 1) and exporting the agricultural goods (good 2), it will import and export less of both goods as the inflation rate increases; that is, trade volume will decrease. With still higher inflation, it will eventually lead the home country to import the agricultural goods and export the industrial goods—a change in the pattern of trade.

The effect of an increase in the rate of monetary growth on the pattern of trade demonstrated above is thus an example of commodity substitution, which is different from the factor-supply effect emphasized by Stockman and Roldos. In the next section, we study another specification of the cash-in-advance constraint, which serves to illustrate both commodity-substitution and factor-supply effects.

3. SECOND CASH-IN-ADVANCE MODEL

In this section, the basic structure of the model in the preceding section is retained. However, instead of assuming (3), it is assumed that cash is needed to purchase the industrial and the investment goods:

$$e_t(c_{1t} + i_t) \leq M_t. \tag{29}$$

The first-order conditions for an interior optimal path are given by (1), (2), and (29), and are the same as (4)–(9), except that (7) becomes

$$\alpha + \gamma = \beta/e. \tag{7a}$$

Follow the same procedure as in Section 2 to simplify the system of the first-order conditions; we still have (12). Then, substitute (7a) into (4) and (5) into (9) to obtain

$$u'(c_1) = \beta, \tag{30}$$

$$\dot{\beta} = \beta\theta - v'(c_2)F_2(l, k). \tag{31}$$

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Combining (30) and (31), we obtain

$$\dot{c}_1 = \left[\frac{1}{u''(c_1)}\right][\theta u'(c_1) - v'(c_2)F_2(l, k)]. \tag{32}$$

For the equilibrium conditions, condition (16) must hold to keep trade balanced. When the cash-in-advance constraint is binding, we multiply (29) by α and substitute (5) and (16) into it to obtain

$$v'(c_2)(F + G - c_2) = \alpha M \tag{33}$$

Taking logarithm of both sides of (33) and differentiating with respect to time, and taking (6) into account,

$$\frac{v''(c_2)}{v'(c_2)}\dot{c}_2 + \frac{1}{(F + G - c_2)}(F_2\dot{k} - \dot{c}_2) = \frac{\dot{\alpha}}{\alpha} + \frac{\dot{M}}{M}. \tag{34}$$

As before, let the money supply follow a constant growth rate, μ. Substituting (12) and (16) into (34), we have

$$\dot{c}_2 = \frac{1}{A}\left[1 + \mu + \theta - \frac{u'(c_1)}{v'(c_2)} - \frac{F_2}{(F + G - c_2)}(F + G - c_1 - c_2)\right], \tag{35}$$

where

$$A = \frac{v''}{v'} - \frac{1}{F + G - c_2}.$$

The equilibrium motions of (c_1, c_2, l, k) are thus completely characterized by (6), (16), (32), and (35).

In the steady state, $\dot{c}_1 = \dot{c}_2 = \dot{l} = \dot{k} = 0$, which implies (21) and (23) and that

$$v'(\bar{c}_2)F_2(\bar{l}, \bar{k}) = \theta u'(\bar{c}_1), \tag{36}$$

$$1 + \theta + \mu = \frac{u'(\bar{c}_1)}{v'(\bar{c}_2)}. \tag{37}$$

Given the conditions imposed, it is straightforward to verify that a unique steady state exists. Before the formal comparative analysis, we note that equations (36) and (37) imply

$$\theta(1 + \theta + \mu) = F_2(\bar{l}, \bar{k}). \tag{38}$$

To understand the intuition underlying (38), one can start from (36) and (37). Condition (36) describes the opportunity cost of purchasing c_1 in terms of c_2: μ represents the inflation tax, and θ represents the cost of converting current income into cash momentarily later in order to purchase c_1. Condition (37) can be understood as another expression of the opportunity cost of c_1 in terms of c_2. Since the cash-in-advance constraint (29) applies to both c_1 and $\dot{k}$ equally, the household can reduce c_1 and add to the investment at t directly. The increase in the production of the industrial goods can be used to increase c_{2s} for all $s > t$.[6] In the steady state, reducing c_1 by one unit incurs a loss of utility by $u'(\bar{c}_1)$, whereas augmenting the capital stock by one unit forever incurs a gain of utility by

$$\int_0^\infty v'(\bar{c}_2)F_2(\bar{l},\bar{k})e^{-\theta t}\,dt.$$

Equating these two expression, we obtain (36).

To study the effects of a perfectly anticipated inflation on the steady-state capital and the consumption goods, we have

$$\frac{d\bar{k}}{d\mu} = \frac{\theta(F_{11}+G_{11})}{F_{22}G_{11}} < 0, \tag{39}$$

$$\frac{d\bar{l}}{d\mu} = -\frac{\theta F_{12}}{F_{22}G_{11}} < 0, \tag{40}$$

$$\frac{d\bar{c}_1}{d\mu} = \frac{v' + F_2(1+\theta+\mu)v''(d\bar{k}/d\mu)}{u'' + (1+\theta+\mu)v''} < 0, \tag{41}$$

$$\frac{d\bar{c}_2}{d\mu} = \frac{F_2u''(d\bar{k}/d\mu) - v'}{u'' + (1+\theta+\mu)v''}. \tag{42}$$

The reasoning for the above results can be explained as follows. Because purchasing the investment goods requires cash, the inflation tax reduces the rate of return on investment and lowers the steady-state capital stock. Also, because l and k are substitutes ($F_{12} > 0$), the supply of labor in the first sector is reduced. Thus, $d\bar{k}/d\mu$ and $d\bar{l}/d\mu$ are both negative. These are the changes in the factor supplies due to inflation. The effects of inflation on c_1 and c_2 can be decomposed into the factor-supply effect and the commodity-substitution effect. In (41), the factor-supply effect of inflation on c_1 is

$$\left[\frac{F_2(1+\theta+\mu)v''}{u'' + (1+\theta+\mu)v''}\right]\left(\frac{d\bar{k}}{d\mu}\right),$$

and the commodity-substitution effect is

$$\frac{v'}{u'' + (1+\theta+\mu)v''}.$$

Since both effects are negative, higher inflation reduces c_1 unambiguously. The total effect of inflation on c_2 is uncertain though. This is because the positive commodity-substitution effect,

$$\frac{-v'}{u'' + (1+\theta+\mu)v''},$$

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and the negative factor-supply effect,

$$\left[\frac{F_2 u''}{u'' + (1+\theta+\mu)v''}\right)\left(\frac{d\bar{k}}{d\mu}\right),$$

offset each other.

The effect of inflation on the volume and pattern of trade is uncertain in this case. Higher inflation induces the economy to produce less of the industrial goods, but its (domestic) consumption is also less. Higher inflation induces the economy to produce more of the agricultural goods, but its effect on the domestic consumption is ambiguous. Hence, the trade volume, measured in terms of $c_1 - F$ or $G - c_2$, may either increase or decrease as the factor-supply effect works against the commodity-substitution effect. This ambiguity will not be present when the cash-in-advance constraint (29) is revised to include c_2. In the next section, we set out such a model and examine the equilibrium properties.[7]

To study the dynamic behavior of the system in the neighborhood of the steady state, we can first solve for $l(k)$ from (6). Substitute $l(k)$ into (16), (32), and (35), and linearize around the steady state to obtain

$$\begin{bmatrix} \dot{c}_1/c_1 \\ \dot{k} \\ \dot{c}_2 \end{bmatrix} = \begin{bmatrix} \left(\frac{v'F_2}{c_1u'}\right) & \left(\frac{-v'}{c_1u''}\right)\left(\frac{F_{22}G_{11}}{F_{11}+G_{11}}\right) & \frac{-v''F_2}{c_1u''} \\ -1 & F_2 & -1 \\ \frac{1}{A}\left(\frac{-u''}{v'} + \frac{F_2}{c_1}\right) & \frac{1}{A}\left(\frac{-F_2^2}{c_1}\right) & \frac{1}{A}\left(\frac{u'v''}{(v')^2} + \frac{F_2}{c_1}\right) \end{bmatrix} \begin{bmatrix} \tilde{c}_1 \\ \tilde{k} \\ \tilde{c}_2 \end{bmatrix}, \qquad \mathbf{(43)}$$

and

$$\tilde{c}_1 = c_1 - \bar{c}_1,$$
$$\tilde{c}_2 = c_2 - \bar{c}_2,$$
$$\tilde{k} = k - \bar{k},$$

where all derivatives are evaluated at the steady state.

For the steady state to be a saddle point, it is necessary that the matrix in (43) have a unique negative characteristic root. The product of the characteristic roots of the system is given by the determinant

$$\frac{1}{A}\left(\frac{F_{22}G_{11}}{F_{11}+G_{11}}\right)\left(\frac{u''}{u'} + \frac{v''}{v'}\right) < 0.$$

This establishes that there are either three negative roots or one. To establish that there is only one negative root, we proceed to examine the trace of the matrix, which is given by

$$\frac{1}{A}\left[\frac{v'AF_2}{c_1u'} + \frac{v''F_2}{v'} + \frac{u'v''}{(v')^2}\right] > 0.$$

Since the trace is positive, this implies that there is at least one positive root. However, since we know it has either zero or two positive roots, it has two. There is therefore a unique negative characteristic root, and a unique perfect-foresight path satisfying (16), (32), and (35) that converges to the steady state.

4. THIRD CASH-IN-ADVANCE MODEL

In this section, we assume that the cash-in-advance constraint is imposed as follows:

$$e_t(c_{1t} + c_{2t} + i_t) \leq M_t, \tag{44}$$

where M_t is the cash balance at time t.

The first-order conditions for an interior optimal path are given by (1), (2), and (44), and are the same as (4)–(9) except here, (5) becomes

$$v'(c_2)/e = \alpha + \gamma. \tag{5b}$$

To simplify the system, we first rewrite (8) as

$$\frac{\dot{\alpha}}{\alpha} = \theta - \frac{\gamma e}{\alpha e}. \tag{45}$$

Define

$$q \equiv \alpha e; \tag{46}$$

then, (7) implies that

$$\gamma e = \beta - q \tag{47}$$

and (45) and (9) can be rewritten as

$$\frac{\dot{\alpha}}{\alpha} = 1 + \theta - \frac{\beta}{q}, \tag{48}$$

$$\frac{\dot{\beta}}{\beta} = \theta - \frac{q}{\beta} F_2. \tag{49}$$

Again, trade must be balanced; that is, condition (16) holds. When the cash-in-advance constraint is binding, we multiply (44) by α and substitute (46) and (16) into it to obtain

$$q(F + G) = \alpha M. \tag{50}$$

Taking the logarithm of both sides of (50) and differentiating with respect to time, after taking (6) into account, we obtain

$$\frac{\dot{q}}{q} + \frac{F_2}{(F + G)}\dot{k} = \frac{\dot{\alpha}}{\alpha} + \frac{\dot{M}}{M}. \tag{51}$$

Substituting (16) and (48) into (51), we have

$$\frac{\dot{q}}{q} = 1 + \mu + \theta - \frac{\beta}{q} - \frac{F_2}{(F + G)}(F + G - c_1 - c_2). \tag{52}$$

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From (4), (5b), and (7),

$$u'(c_1) = v'(c_2) = \beta. \tag{53}$$

we substituting (53) into (52) and (49) to obtain

$$\frac{\dot{q}}{q} = 1 + \mu + \theta - \frac{u'(c_1)}{q} - \frac{F_2}{(F+G)}(F + G - c_1 - c_2), \tag{54}$$

$$\frac{\dot{c}_2}{c_2} = \left[\frac{v'(c_2)}{c_2 v''(c_2)}\right]\left[\theta - \frac{q}{v'(c_2)}F_2(l,k)\right]. \tag{55}$$

The equilibrium motions of (c_1, c_2, q, l, k) are thus completely characterized by (6), (16), (53), (54), and (55).

In the steady state, $\dot{c}_1 = \dot{c}_2 = \dot{q} = \dot{l} = \dot{k} = 0$. This implies (21) and (23) and that

$$\bar{q}F_2(\bar{l},\bar{k}) = \theta u'(\bar{c}_1), \tag{56}$$

$$u'(\bar{c}_1) = v'(\bar{c}_2), \tag{57}$$

$$1 + \theta + \mu = \frac{v'(\bar{c}_2)}{\bar{q}}. \tag{58}$$

Note that equations (56) and (58) imply that

$$\theta(1 + \theta + \mu) = F_2(\bar{l},\bar{k}), \tag{59}$$

which is identical to (38). To understand this result, one can start from (56) and (58). By definition, q is the marginal utility of income at t. Since current income cannot be used to purchase contemporary consumption goods, and it must be transformed into cash momentarily later to do the purchase, (58) expresses the opportunity cost of c_1 in terms of the current income: μ represents the inflation tax, and θ represents the time discount of income. Condition (56) can be understood as another expression of the opportunity cost of c_1 in terms of current income. Since the cash-in-advance constraint (44) applies to c_1 and $\dot{k}$ simultaneously, the household can reduce c_1 and add to the investment at t directly. The increase in the production of the industrial goods can be used to increase future income. In the steady state, reducing c_1 by one unit incurs a loss of utility by

$$u'(\bar{c}_1),$$

whereas augmenting the capital stock by one unit forever incurs a gain of utility by

$$\int_0^\infty \bar{q}F_2(\bar{l},\bar{k})e^{-\theta t}\,dt.$$

Equating these two expression, one obtains (56).

To study the effects of a perfectly anticipated inflation on the steady-state capital and the consumption goods, totally differentiate (21), (23), and (56)–(58), and we have

$$\frac{d\bar{k}}{d\mu} = \frac{\theta(F_{11} + G_{11})}{F_{22}G_{11}} < 0, \tag{60}$$

$$\frac{d\bar{l}}{d\mu} = -\frac{\theta F_{12}}{F_{22}G_{11}} < 0, \tag{61}$$

$$\frac{d\bar{c}_1}{d\mu} = \left(\frac{v'' F_2}{u'' + v''}\right)\left(\frac{d\bar{k}}{d\mu}\right) < 0, \tag{62}$$

$$\frac{d\bar{c}_2}{d\mu} = \left(\frac{u'' F_2}{u'' + v''}\right)\left(\frac{d\bar{k}}{d\mu}\right) < 0. \tag{63}$$

The reasoning for the results in (60)–(63) can be explained as follows. First, (60) and (61) are identical to (39) and (40). This implies that the inflation has the same effects on factor supplies, provided that the investment is cash constrained. Comparing (62) and (63) with (41) and (42), we see that only the factor-supply effect is present. This is because both c_1 and c_2 are cash constrained, the inflation does not change their relative prices, and hence the commodity substitution effect appeared in the preceding sections, but not here.

The effect of inflation on the volume and pattern of trade can be seen as follows. Higher inflation induces the economy to produce fewer industrial goods and more agricultural goods. The economy will consume fewer agricultural goods. Hence, if the economy is originally importing the industrial goods and exporting the agricultural goods, it will import and export more of both goods as the inflation rate increases; that is, trade volume will increase.

To study the dynamic behavior of the system in the neighborhood of the steady state, we first solve for $l(k)$ from (6) and $c_1(c_2)$ from (57), substitute $l(k)$ and $c(c_2)$ into (16), (54), and (55), and linearize around the steady state to obtain

$$\begin{bmatrix} \dot{c}_2/c_2 \\ \dot{q}/q \\ \dot{k} \end{bmatrix} = \begin{bmatrix} \frac{qF_2}{c_2 v'} & \frac{-F_2}{c_2 v''} & \frac{-q}{c_2 v''}\left(\frac{F_{22}G_{11}}{F_{11}+G_{11}}\right) \\ -\frac{v''}{q} + \frac{F_2}{F+G}\left(\frac{v''}{u''} + 1\right) & \frac{v'}{q^2} & -\frac{F_2^2}{F+G} \\ -\frac{v''}{u''} - 1 & 0 & F_2 \end{bmatrix} \begin{bmatrix} \tilde{c}_2 \\ \tilde{q} \\ \tilde{k} \end{bmatrix}, \tag{64}$$

and

$$\tilde{c}_2 = c_2 - \bar{c}_2,$$

$$\tilde{q} = q - \bar{q},$$

$$\tilde{k} = k - \bar{k},$$

where all derivatives are evaluated at the steady state.

Again, for the steady state to be a saddle point, it is necessary that the matrix in (64) have a unique negative characteristic root. The product of the characteristic

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roots of the system is given by the determinant

$$-\left(\frac{v'}{c_2 q}\right)\left(\frac{1}{u''}+\frac{1}{v''}\right)\left(\frac{F_{22}G_{11}}{F_{11}+G_{11}}\right)<0.$$

This establishes that there are either three negative roots or one. To establish that there is only one negative root, we proceed to examine the trace of the matrix, which is given by

$$\frac{qF_2}{c_2 v'}+\frac{v'}{q^2}+F_2>0.$$

Since the trace is positive, this implies that there is at least one positive root. And since we know it has either zero or two positive roots, it should have two. There is therefore a unique negative characteristic root, and a unique perfect-foresight path satisfying (16), (54), and (55) that converges to the steady state.

5. CONCLUSION

In this paper we have presented a dynamic specific-factors model with money introduced through a cash-in-advance constraint. Two types of consumption goods are produced, and three types of factors—labor, capital, and land—are used. The contribution of this paper is twofold. First, it shows that if the cash-in-advance constraint applies to only one good, then changes in the rate of monetary growth have a commodity-substitution effect in additition to the factor-supply effect examined by Stockman (1985) and Roldos (1992). This means that when the constraint is imposed on a subset of the consumption goods and is exclusive of the investment, inflation affects the pattern and volume of trade through the commodity-substitution effect, whereas, when the constraint is imposed inclusive of the investment, inflation may affect the pattern of trade through both the commodity-substitution effect and the factor-supply effect. Second, it explicitly checks for dynamic stability of the long-run equilibria. We have proved that, in each case, the steady-state equilibrium is saddle-path stable. This result allows us to perform the relevant comparative-static exercises.

NOTES

1. See Stockman (1981), Wang and Yip (1992), Palivos and Yip (1995), Huo (1997), and Mino (1997).
2. The basic model is a continuous-time formulation of one in Roldos (1992), which can be traced to Roldos (1991) and Jones (1971).
3. Time subscripts are omitted to conserve space.
4. This is because c_{1t} and c_{2t} have the same international price.
5. Note that there is no transition adjustment in the capital stock. Both c_1 and c_2 are nonpredetermined (jumping) variables.
6. This is becasue c_{1t} and c_{2t} have the same international price, and c_2 is not cash constrained.
7. This is also the constraint considered by Roldos (1992).

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