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HOW POWERFUL ARE TRADE UNIONS? A SKILL-BIASED TECHNOLOGICAL CHANGE APPROACH

Antonio Neto
University of Porto, Faculty of Economics, and CEF.UP

Oscar Afonso
University of Porto, Faculty of Economics, and CEFAGE-UBI

Sandra T. Silva
University of Porto, Faculty of Economics, and CEF.UP

This paper proposes a new theoretical framework aimed at understanding the link between technological change, skill premium, and employment. We build an endogenous growth model of directed technological change with vertical research and development (R&D) in which low-skilled workers might be organized in a trade union. This union can act as a monopoly seller of labor and decide unilaterally the low-skilled wage, or as a *managerial* union that bargains wage and employment with the employers' federation, i.e., firms. Our results suggest that (i) the impacts of trade unions on technological-bias and on the level of (un)employment crucially depend on their type and preferences; and (ii) trade unions can actually increase low-skilled wages and employment if they have some bargaining power and are employment-oriented. Furthermore, our framework provides some highlights to explain the relationship between wage dispersion and the deunionization process that occurred in the United Kingdom and the United States during the 1980s.

Keywords: Skill-Biased Technological Change, Bargaining Structure, Employment, Economic Growth

1. INTRODUCTION

Over the last few decades, the rise in income and wage inequality has been the rule rather than the exception in several advanced economies. According to Chusseau

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et al. (2008, p. 411), "this development is now well documented and it may be observed (...) for workers with similar skills and between skilled and less skilled workers." Skill-biased technological change (SBTC) is claimed to be one of the most important explanations for this pattern [Acemoglu et al. (2001)]. Violante (2008) defines it as "a shift in the production technology that favours skilled (...) labour over unskilled labour by increasing its relative productivity and, therefore, its relative demand. *Ceteris paribus*, SBTC induces a rise in the skill premium - the ratio of skilled to unskilled wages." On the other hand, Chusseau et al. (2008, p. 412) states that SBTC "occurs when technical progress increases the total relative demand for skill of the economy $\left(\frac{H}{L}\right)^D$ for given prices of skilled labour H and unskilled labour L." Assuming that the relative supply of skilled labor is constant, this effect will result in (i) a higher skill premium and/or (ii) unskilled labor unemployment.

Nevertheless, what explains SBTC in the first place? One of the most important explanations considers the possibility of an "endogenous" direction of technical change [Acemoglu (1998)].[1] In other words, the amount of research and development (R&D) activity conducted by firms might crucially influence the new type of arriving technology (i.e., "high" vs. "low"), which, in turn, may determine how high (or low)-skilled workers are affected. Following Violante (2008), the main determinants that can endogenously influence the type of conducted R&D activity are market size, relative prices, and institutions. We focus our analysis on the three effects and, in particular, on the relationship between labor market institutions and skill-biased technological change. Thus, there are several mechanisms by which unions can affect wages and, therefore, the type of research activity conducted by firms. In particular, (i) direct mechanisms, such as bargaining on behalf of covered workers to increase (or to maintain) wages, increasing the investment cost in technology that is complementary with these workers; and (ii) indirect mechanisms, such as rise workers' effort due to an increase in wages [see Bryson (2007)]. In this paper, we outline a novel analysis of the impacts of trade unions on SBTC through the first type—direct mechanisms.

Figure 1 provides us a rather interesting picture. First, we calculated the average trade union density among the OECD countries and divided them into two groups, depending on whether their trade union density is above or below the average. Then, we compared their performance taking into consideration two different variables: the decile $\frac{9\text{th}}{1\text{st}}$ ratio of gross earnings [Figure 1(a)]; and the ratio of high-/low-skilled workers [Figure 1(b)]. Interestingly, countries with higher trade union density (or, more precisely, above the average) seem to perform better in terms of lowering their wage dispersion and their ratio of high- over low-skilled workers (at least until 2011) which might be counterintuitive. According to Scheuer (2011), trade unions tend to represent more low-skilled workers rather than high skilled for a variety of reasons explained below. Hence, if one recalls the standard monopoly trade union framework, unions can indeed increase the low-skilled wage and thereby reducing wage dispersion but only through a decrease in the level of low-skilled employment, implying an increase in the employment ratio, which

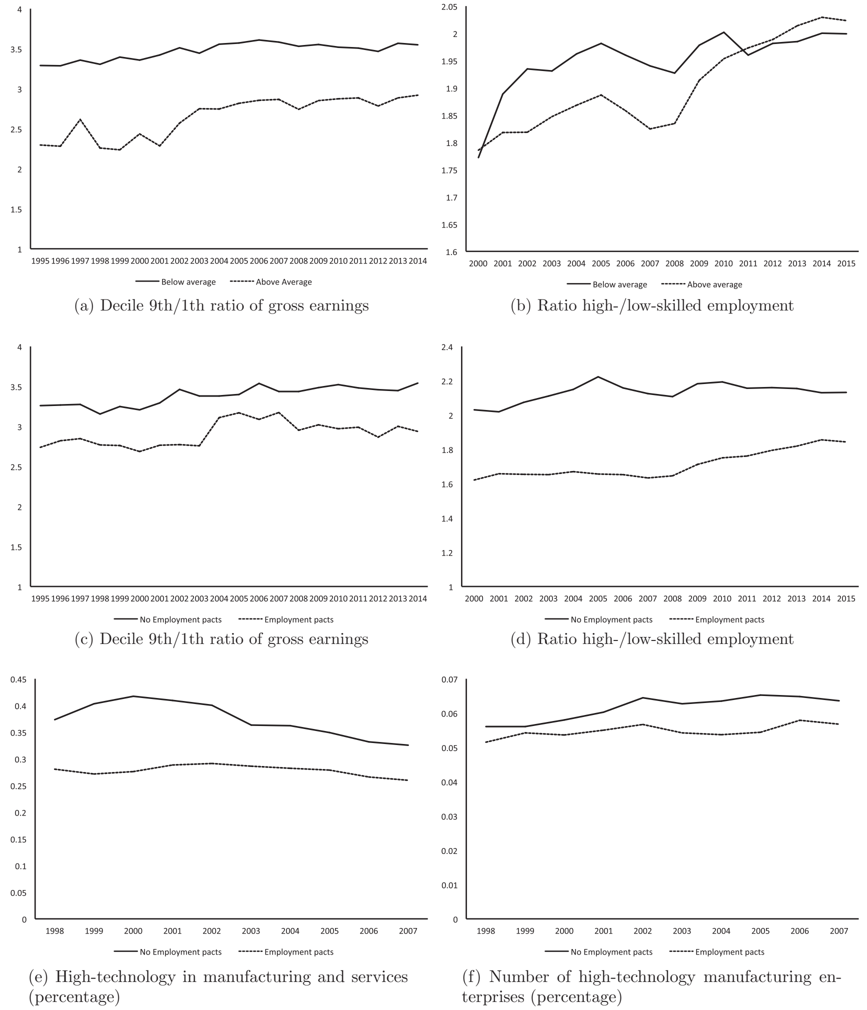

FIGURE 1. Empirical motivation. Source: Eurostat online database on Science, Technology, and Innovation—table Economic statistics on high-tech industries and knowledge-intensive services at the national level, available at http://ec.europa.eu/eurostat/data/database; and OECD online database—tables trade union density and decile ratio of gross earnings, available at http://stats.oecd.org. (1) We considered "low-skilled employment" as the number of 25–64 year olds who are employed and have less than primary and lower secondary education. (2) The average trade union density may differ from graph to graph depending on the available information for each country. For example, if we have information for Portugal regarding the decile ratio of gross earnings but not for the relative high-tech employment, the average trade union density would be different since Portugal would be used on the first graph but not on the last one. Panels (a) and (c) account for 28 OECD countries; panels (b), (d), (e), and (f) account for 23 OECD countries. (3) All calculations are available upon request.

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seems to be in contrast with the data. On the other hand, there is evidence that unions might not behave as a "monopolistic seller of work", but indeed bargain wages and employment with firms [Dobbelaere and Mairesse (2013), Dobbelaere (2004)]. This might help uncovering the impact of unions in wages.

Figures 1(c) and (d) plot the same indicators as Figures 1(a) and (b), respectively, but the countries are now separated into two different groups, depending on whether there had been at least one "employment pact" containing concessions regarding employment policies (i.e., job creation, subsidies, etc.) within the period under review.[2] In this case, in countries where unions show some concern regarding employment level, both the wage and employment dispersion are unambiguously lower throughout the considered period of time. Furthermore, this relationship also appears in technology, since both variables depicted in Figures 1(e) and (f), i.e., the percentage of high technology in manufacturing and services, and the number in percentage of high-technology manufacturing enterprises, are lower in countries with employment pacts.

The aforementioned data provides us the motivation to ask the following question: What is the *true* relationship between unions and SBTC? Furthermore, the literature combining SBTC with labor market imperfections is relatively scarce. In fact, Chang et al. (2007) and Lingens (2007, 2003) are among the most relevant references on this topic. Fadinger and Mayr (2014) presents a detailed analysis regarding SBTC and unemployment, but does not consider the role of trade unions and its impact on technology bias and unemployment. Indeed, the authors propose a novel extension of the standard direct technological change model by introducing skill-specific frictional unemployment and skill-premium migration. There are two main differences between Fadinger and Mayr (2014) and our model. First, Fadinger and Mayr (2014) introduces search and matching frictions, whereas we introduce trade unions with different types of preferences. Second, Fadinger and Mayr (2014, p. 405) assumes that "producers in the two [intermediate] sectors adopt technologies from the technological frontier (. . .) at a fixed cost", whereas we specifically analyze the role of R&D sector and the impacts of different labor market structure on the technological bias. Hence, our study also aims to complement their analysis.

Furthermore, according to Chusseau et al. (2008, p. 451):

> A number of works have already explored the interactions between labour market institutions and labour market adjustment on the one hand, and globalization and technical change on the other hand. Kreickemeier and Nelson (2006), however, remark that there is again much room for study in this respect. They suggest interactions between fair wage constraint and union bargaining, and their relation with NST [North-South trade] and SBTC as a potentially fruitful research programme.

Therefore, our motivation is straightforward, according to three main reasons. First, taking into account the recent economic developments worldwide, further insights into the relationship between wages and employment is crucial to promote an adequate economic policy toward a sustainable recovery [Blanchard (2007)].

Second, in the specific context of the European Monetary Union (EMU), additional knowledge on the linkage between SBTC and employment seems critical to assess the type of institutions that are more likely to ensure a steady economic growth rate. Finally, we try to fill the gap between the existent standard SBTC framework and the labor market models by combining both approaches into a generic benchmark model. In particular, as a departing point from the SBTC models, we follow Gil et al. (2013, 2016), Acemoglu (2003), and Aghion and Howitt (1992); and from the labor market framework Dunlop (1944), Ross (1948), and McDonald and Solow (1981).

Our main results conclude that the behavior of the trade union and its preferences regarding wages and employment crucially influence the firm's decisions to invest in high(low)-skill technology. On the one hand, by shifting the supply curve to the left, a monopoly union fails to decrease wage dispersion due to the result of three effects, namely (i) price effect since high-skilled workers are now relatively cheaper; (ii) market size effect since high-skilled workers are relatively more abundant; and (iii) relative demand effect since firms will adapt their decisions regarding workers, thereby increasing the relative demand for high-skilled workers which, due to the complementaries between workers and technology, will lead to an increase in the investment in high-skilled technology.

On the other hand, if the union has a stronger preference for employment and enroll in a bargaining process with firms, it can actually decrease wage dispersion, employment ratio, and technological bias, in line with Figures 1(c)–(f). Finally, our theoretical implications can also accommodate the impacts of deunionization that occurred in the United States and the United Kingdom during the 1980s on the wage premium.

This paper proceeds as follows: Section 2 provides a general description of the model; Section 3 describes the equilibrium and its main properties; Section 4 states the main results; Section 5 theoretically analyzes the antiunion phenomena that occurred in the United Kingdom and in the United States during the 1980s; Section 6 presents a brief empirical exercise; and Section 7 concludes.

2. THE MODEL

The model presented herein is based on Acemoglu and Zilibotti (2001) and Gil et al. (2013, 2016). We consider a closed economy, where a single competitively produced final good, Y, can be used in R&D, R, production of intermediate goods, X, and consumed, C, as $Y(t) = R(t) + X(t) + C(t)$. This final good is produced by a substantially large number of firms, each one using one of two substitute production technologies: a high(low)-technology that uses a combination of high(low)-specific quality adjusted intermediate goods indexed by $j_H \in [J, 1]$ ($j_L \in [0, J]$) and high(low)-skilled labor. The economy is populated by infinitely lived households that consume and collect income from labor and from investments in financial assets. Households elastically supply low-skilled labor, L, or high-skilled labor, H. The low-skilled workers might be organized in a labor

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union, which can act as a monopoly seller of labor and decide unilaterally the low-skilled wage, or as a *managerial* union that bargains wage and employment with the employers' federation, which represents firms. A potential entrant devotes resources to vertical R&D and directs them to either the low- or the high-skilled labor-specific technology. Vertical R&D increases the quality level of the goods of an existing industry, indexed by $k_m(j_m)$, $m = L, H$. The quality level $k_m(j_m)$ of each variety will translate into productivity through an efficiency effect from using the good produced by industry j_m, $q^{k_m(j_m)}$, where $q > 1$ is a parameter measuring the size of every quality upgrade. A successful R&D firm introduces the leading-edge quality $k_m(j_m)+1$ by improving on the current best-quality index k_m, thereby displacing the existing input. This successful innovator becomes a monopolist in j_m but only temporarily, since a new successful firm will replace the incumbent.

2.1. Households

Following Feng (2014) and Bertinelli et al. (2013), the individual's utility depends positively on its consumption and negatively on the amount of labor it supplies as follows:

$$U = \int_0^\infty \left[\frac{C_m(t)^{1-\phi} - 1}{1-\phi} - \frac{m(t)^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right] e^{-\rho t} dt, \tag{1}$$

subject to the following flow budget constraint

$$\dot{K}_m(t) = r(t)K_m(t) + W_m(t)m(t) - C_m(t), \tag{2}$$

where $m = H$ ($m = L$) for high-skilled (low-skilled) labor. $C_m(t)$ is the amount of final-goods consumption by each type of individual at time t; $\frac{1}{\phi}$ corresponds to the intertemporal elasticity of substitution; γ is the Frisch labor supply elasticity; $\rho > 0$ is the homogeneous subjective discount rate; $W_m(t)$ is the price paid for a unit of m-type labor; $K_m(t)$ is the real value of assets in form of equity shares in monopolistic intermediate goods firms, owned by each member of households; and $r(t)$ is the real interest rate. The optimality conditions for consumption and labor supply are, respectively,

$$\frac{1}{C_m(t)^\phi} = \mu(t),$$

$$m(t) = \left[\frac{W_m(t)}{C_m(t)^\phi} \right]^\gamma,$$

where $\mu(t)$ is the Hamiltonian co-state variable on (2). Hence, the relative labor supply curve is given by

$$\left[\frac{H(t)}{L(t)} \right]^S = \left\{ \frac{W_H(t)}{W_L(t)} \left[\frac{C_L(t)}{C_H(t)} \right]^\phi \right\}^\gamma. \tag{3}$$

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Finally, the standard Euler equation is

$$\hat{C} = \frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\phi}, \tag{4}$$

where $\hat{C}$ stands for the growth rate of C.

2.2. Final Goods Sector—Firms, Trade Unions, and Bargaining Structure

Firms. Following the contributions of Gil et al. (2013, 2016), Acemoglu (2015), Acemoglu and Zilibotti (2001), and Wälde (2000), firms are indexed by n over the range [0, 1]. There are two substitute production technologies: a high(low)-technology that uses a combination of high(low)-specific quality adjusted intermediate goods indexed by $j_H \in [J, 1]$ ($j_L \in [0, J]$) and high(low)-skilled labor. Normalizing the price of the composite final good at each time to one, the aggregate output at time t is defined as $Y(t) = \int_0^1 P_n(t)Y_n(t)dn = \exp[\int_0^1 \ln Y_n(t)dn]$, where $P_n(t)$ corresponds to the final goods price. The production function of firm n at time t is given by

$$Y_n(t) = A\left\{\int_0^J \left[q^{k_L(j_L,t)} x_{n,L}(j_L,t)\right]^\alpha dj_L\right\}[(1-n)\,lL_n(t)]^\beta$$
$$+ A\left\{\int_J^1 \left[q^{k_H(j_H,t)} x_{n,H}(j_H,t)\right]^\alpha dj_H\right\}[nhH_n(t)]^\beta . \tag{5}$$

Parameter A is an exogenous and positive variable representing the productivity level, which depends on several factors, such as the country's institutions—see, e.g., Acemoglu and Zilibotti (2001). $x_{n,m}(j_m,t)$ corresponds to the quantity of the intermediate goods j_m used by firm n, whereas $q^{k_m(j_m,t)}$ measures its quality level, with $q > 1$ and k_m as the highest quality rung at time t [Aghion and Howitt (1992)].[3] Parameter $\alpha \in]0, 1[$ and $\beta \in]0, 1[$ are the intermediate goods and labor share in production, respectively. Following Chang et al. (2007), we assume decreasing returns to scale, i.e., $0 < \alpha + \beta < 1$. This can be justified by the existence of other fixed factors (e.g., land). $h > l \geq 1$ captures an absolute productivity advantage of H over L. There is also a competitive equilibrium threshold $\overline{n}(t)$, endogenously determined, reflecting the idea that the production of final goods $n \in [0, \overline{n}]$ ($n \in [\overline{n}, 1]$) is more efficient using $L(H)$-technology. This implies that low-skilled workers are relatively more productive in final goods indexed by smaller ns, and vice-versa. As in Acemoglu and Zilibotti (2001, p. 568), "The key assumption is that some machines can only be used by unskilled workers, while some other machines can only be used by skilled workers."

Given the production technology (5), the representative firm attempts to maximize its profits $\Pi_n(t)$ as follows:

$$\max_{x_{n,m}(j_m,t)} \Pi_n(t) = P_n(t)Y_n(t) - W_L(t)L_n(t) - W_H(t)H_n(t)$$
$$-\chi_L(t) - \chi_H(t), \tag{6}$$

where $\chi_L(t) = \int_0^J x_{n,L}(j_L,t)p_L(j_L,t)dj_L$ and $\chi_H(t) = \int_J^1 x_{n,H}(j_H,t)p_H(j_H,t)dj_H$. $p_m(j_{m,t})$ is the price of the intermediate goods j_m. Moreover, the demand of final goods firms for intermediate goods can be obtained by $\frac{\partial \Pi_n(t)}{\partial X_{n,m}(j_m,t)} = 0$ and described as follows:

$$x_{n,L}(j_L,t) = p_L(j_L,t)\left[\alpha A P_n(t)\right]^{\frac{1}{1-\alpha}}\left[(1-n)\, l L_n(t)\right]^{\frac{\beta}{1-\alpha}}\left[q^{k_L(j_L,t)}\right]^{\frac{\alpha}{1-\alpha}}, \quad (7)$$

$$x_{n,H}(j_H,t) = p_H(j_H,t)\left[\alpha A P_n(t)\right]^{\frac{1}{1-\alpha}}\left[nhH_n(t)\right]^{\frac{\beta}{1-\alpha}}\left[q^{k_H(j_H,t)}\right]^{\frac{\alpha}{1-\alpha}}. \quad (8)$$

Labor market framework: an introductory note. Taking into account the most recent empirical research on labor economics, there is mixed evidence regarding how labor markets work and, in particular, how trade unions affect the relationship between workers and firms [Dobbelaere et al. (2015), Booth (2014), Oesch (2010), Gartner et al. (2013)]. On the one hand, Dobbelaere and Mairesse (2013) and Dobbelaere (2004) report evidence for the efficient bargaining hypothesis in Belgian and France, respectively, i.e., unions and firms bargain both wages and working hours, a proxy of the employment level. On the other hand, Dobbelaere et al. (2015) finds that perfect competition or the right-to-manage bargaining model (i.e., unions and firms only bargain wages, leaving firms to decide the level of employment) prevails in Japan and in the Netherlands.[4] Furthermore, Donado and Wälde (2012) states that the role of trade unions can go beyond wage negotiation since they can also increase workplace safety and help provide the required information to insure occupational health and safety.[5]

Hence, focusing our analysis on the wage-setting framework, and taking into account that the monopoly union and efficient bargaining “represent the two most popular alternative economic representations of the wage-employment outcome of collective bargaining” [Lawson (2011, p. 289)], we analyze three types of situations in the labor market, namely the standard perfect competition and two types of trade unions: a monopoly union [Dunlop (1944), Kaufman (2002)] and a *managerial* trade union [Ross (1948), McDonald and Solow (1981), Chang et al. (2007)]. In both cases, (i) all low-skilled workers are members of the trade union and (ii) following Lingens (2007, 2003), the trade union does not bargain over the high-skilled wage and, therefore, the high-skilled wages always clear. Two main reasons support this last assumption. First, high-skilled workers usually have a higher bargaining power than low-skilled workers, which make them less keen on joining a trade union [Acemoglu et al. (2001)]. Second, according to Checchi et al. (2007, p. 3): “higher-earning people are more tolerant of inequality than those earning less and they are more likely to defend inequality as reward for effort or talent. Part of the explanation for a differential effect of relative earnings on the likelihood to join a trade union may therefore be found in different attitudes towards inequality.” Empirically, Scheuer (2011) provides us some interesting information regarding the membership of trade union by occupation status in Europe, indicating that low-skilled groups such as “operators/assemblers/laborers” and "technicians/semiprofessionals", if combined, correspond to the largest group of unionized workers in almost all the considered countries.

Perfect competition. In this case, wages of the low- and high-skilled workers are equal to their marginal productivity. Recalling (5) and taking into account (7) and (8), the marginal productivity of labour is given by

$$\bar{W}_L(t) = \beta L_n^{-1}(t) \underbrace{[(1-n)\, l L_n(t)]^{\beta} P_n(t) A \left\{ \int_0^J \left[q^{k_L(j_L,t)} x_{n,L}(j_L,t) \right]^{\alpha} dj_L \right\}}_{Y_L(t)}$$

$$= \beta L_n^{-1}(t)\, Y_L(t), \tag{9}$$

$$\bar{W}_H(t) = \beta H_n^{-1}(t) \underbrace{[nh H_n(t)]^{\beta} P_n(t) A \left\{ \int_J^1 \left[q^{k_H(j_H,t)} x_{n,H}(j_H,t) \right]^{\alpha} dj_H \right\}}_{Y_H(t)}$$

$$= \beta H_n^{-1}(t)\, Y_H(t), \tag{10}$$

where $\overline{W}_H(t)$ $\left[\overline{W}_L(t)\right]$ corresponds to the perfect competition high-skilled (low-skilled) wage. $Y_L(t)$ $[Y_H(t)]$ could be seen as the contribution to the production function of low-skill (high-skill) components, i.e., technology and workers. Note that the equilibrium high-skilled wage is higher than the equilibrium low-skilled wage since high-skilled workers are, in absolute terms, more productive than low-skilled workers, i.e., $h > l > 1$.

Trade unions—monopoly union. The monopoly trade union framework was first proposed by Dunlop (1944) and Ross (1948).[6] Within this setup, the union has monopoly power over some groups of workers and can be seen as a "monopoly seller of labour", the typical case for many European economies since most of them have a "nationwide union or cooperation/agreements among unions representing different industries" [Krusell and Rudanko (2016, p. 35)]. Theoretically, this is "equivalent [to] the union choosing their most preferred point on the firm's labour demand curve" [Lawson (2011, p. 283)]. In other words, the union decides unilaterally the level of wages, leaving firms to choose the level of employment afterward. We assume that the union makes a one shot offer to firms (i.e., a take-it-or-leave-it offer) as in Acemoglu et al. (2001) and Booth (1995). The monopoly trade union's utility function has the following Stone–Geary form:

$$U^M(t) = \left[W_L^M(t) - \overline{W}_L(t) \right]^{1-\upsilon} \left[L_n^D(t) \right]^{\upsilon}, \tag{11}$$

where $W_L^M(t)$ corresponds to the monopoly low-skilled wage and $L_n^D(t)$ corresponds to firm's demand curve for workers.[7] The value of $\upsilon =]0, 1[$ states whether the union is more employment-oriented or wage-oriented. Since wages are fixed by unions and firms decide the level of employment afterward, unions can anticipate the impact of wages on the employment level. Recalling (9), $L_n^D(t)$ is given by

$$L_n^D(t) = W_L(t)^{\frac{-1}{1-\beta}} \left(\beta P_n(t) A \left\{ \int_0^J \left[q^{k_L(j_L,t)} x_{n,L}(j_L,t) \right]^\alpha dj_L \right\} \right)^{\frac{1}{1-\beta}} \times [(1-n)\, l]^{\frac{\beta}{1-\beta}} . \quad (12)$$

Hence, replacing (12) into (11) and maximizing it in order to $W_L^M(t)$, taking into account (7), we get the standard monopoly wage

$$W_L^M(t) = \varphi \bar{W}_L(t), \quad (13)$$

where $\varphi = \frac{1}{(1-\frac{1-\upsilon}{\upsilon}\frac{1}{\varepsilon_{L,W_L}})}$ and $\varepsilon_{L,W_L} = |\frac{1}{(1-\beta)}|$ stands for the wage elasticity of the labor demand. Hence, the wage set by the monopoly union is a markup over the perfect competition wage, which depends negatively on υ and on the elasticity of wage supply.

Trade unions—efficient bargaining. The efficient bargaining model was first proposed by McDonald and Solow (1981).[8] To understand this framework, one can think of unions and firms deciding together the maximum amount of working hours, defining a contract that specifies the number of workers per machine, or even determining the number of workers each department should have within the organization [Johnson (1990)]. In this case, we assume that through a generalized Nash bargaining problem, both the union and the employers' federation negotiate simultaneously over wages and employment, taking into consideration the demand of final goods firms' for intermediate goods.[9] Formally, in line with Chang and Hung (2016) and Chang et al. (2007), the optimization problem can be written as

$$\max_{W_L^B(t),\, L_n(t)} \Omega(t) = \left[U^M(t) - \overline{U^M}(t) \right]^\theta \left[\Pi(t) - \overline{\Pi}(t) \right]^{1-\theta}, \quad (14)$$

$$s.t: x_{n,m} = \arg \max_{x_{n,m}(j_m,t)} \Pi_n(t),$$

where $\theta \in\,]0, 1[$ is the bargaining power of the union and $W_L^B(t)$ corresponds to the bargaining wage. $\bar{\Pi}(t)$ and $\overline{U^M}(t)$ are the disagreement points of the final good firm and union, respectively. Following Chang et al. (2007), we assume that, without reaching an agreement, the employment level regarding the low-skilled workers would be zero and, therefore, $\overline{U^M}(t) = 0$, and $\overline{\Pi}(t) = \Pi_H(t)$, where $\Pi_H(t)$ corresponds to the firm's profit using only high-skilled workers and high-specific quality adjusted intermediate goods.

Hence, by maximizing equation (14) with respect to $W_L^B(t)$ and $L_n(t)$ and taking into consideration (6)–(8), we get, with some manipulations, the optimal

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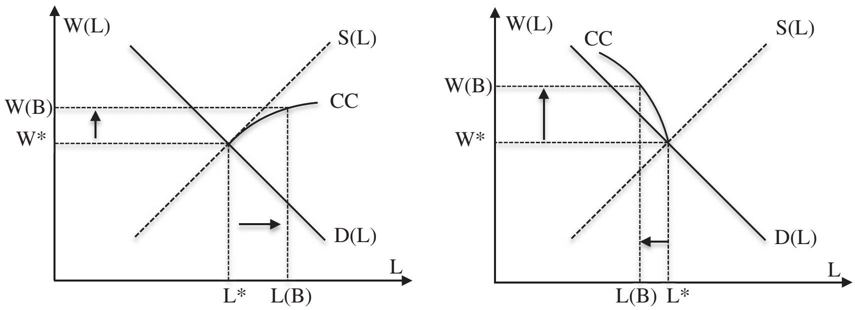

(a) Upward sloping contract curve (CC) (b) Downward sloping contract curve (CC)

FIGURE 2. Low-skilled labor market dynamics: Bargaining case. (a) Upward sloping contract curve (CC). (b) Downward sloping contract curve (CC). D(L) and S(L) correspond to demand and supply of low-skilled workers, respectively. W* is the perfect competition low-skilled wage and W(B) corresponds to the bargaining low-skilled wage. L* is the perfect competition low-skilled employment level and L(B) is the bargaining low-skilled employment level. CC corresponds to the contract curve.

conditions for wages and employment (see Appendix A)

$$\left[W_L^B(t) - \overline{W}_L(t)\right] = \frac{1-v}{v}\left[W_L^B(t) - \beta L_n^{-1}(t)\, Y_L(t)\right], \tag{15}$$

$$W_L^B(t) = \left[\beta + \frac{\theta v(1-\alpha-\beta)}{(1-\theta)+\theta v}\right] L_n^{-1}(t)\, Y_L(t). \tag{16}$$

Equation (15) corresponds to the contract curve in the (W_L, L) space, which establishes a relationship between wages and employment that both firms and union agreed with. On the other hand, equation (16) is *the rent division curve* or the low-skilled bargaining wage, where $\beta L_n^{-1}(t) Y_L(t)$ corresponds to the marginal productivity of low-skilled labor. In line with Chang et al. (2007), the contract curve is upward or downward sloping in the (W_L, L) plan if and only if the union is employment- or wage-oriented, i.e., $v > \frac{1}{2}$ or $v < \frac{1}{2}$.[10]

Figure 2 provides a graphical representation of the two possible types of contract curve. If the union is employment-oriented [Figure 2(a)], its members will agree to decrease their reservation wage, determined on the supply curve, to increase the level of employment beyond the perfect competition equilibrium. Intuitively, during the bargaining process, each worker is willing to give up "part" of his wage on the condition that the firm hires more workers.

On the other hand, if the union is wage-oriented [Figure 2(b)], its members will agree to demand a higher wage for the same amount of labor, but they would be willing to give up "part" of the employment level to achieve that wage increase. In this case, the bargaining solution would be placed on the left of the perfect competition equilibrium, but on the right of the demand curve. Interestingly, even though the union is wage-oriented, the employment level would be higher than in the monopoly case but lower than the perfect competition solution. This can be explained by the fact that in this scenario, (i) the union still has some preferences regarding employment since $v > 0$; and (ii) unions and firms decide jointly both the level of wage and employment.

Finally, it is worth noting that the contract curve replaces the supply curve, meaning no unemployment in the bargaining framework. Note also that, in both cases, the bargaining wage is a positive function of the union's bargaining power, θ, where $\frac{\theta v(1-\alpha-\beta)}{(1-\theta)+\theta v} > 0$ corresponds to the union's positive impact on low-skilled wages.[11]

Combining (6)–(8), (10), and (16), we get the profit function of the representative firm

$$\Pi_n(t) = (1-\alpha-\beta)\left[\frac{(1-\theta)}{1-\theta+\theta v}P_n(t)\,Y_L(t) + P_n(t)Y_H(t)\right] \geq 0. \quad \textbf{(17)}$$

Conditionally on the assumption of $0 < \alpha + \beta < 1$, the low-skilled part, Y_L, contributes positively to the firm's profit as long as the employers' federation has a positive bargaining power, i.e., $0 < \theta < 1$.[12] On the other hand, if $\theta = 1$, this contribution of the low-skilled part is reduced to zero.

2.3. Intermediate Sector

Intermediate-good $m-$technology sector consists of a continuum $j_m \in [0, 1]$ of industries. There is monopolist competition if we consider the whole sector. Hence, the monopolist firm in industry $j_m \in [0, 1]$ fixes the price $p_m(j_m, t)$ and faces an isoelastic demand curve given by (7) and (8). We assume that intermediate goods are nondurable [Acemoglu and Zilibotti (2001)] and entail a unit marginal cost of production, measured in terms of the aggregate final good, Y, whose price is taken as given (i.e., numeraire). Hence, each firm that holds the patent for the highest quality j_m at time t gets

$$\Pi_m(j_m, t) = [p_m(j_m, t) - 1]\, X_m(j_m, t). \quad \textbf{(18)}$$

Giving $X_L(j_L, t) = \int_0^J X_{n,L}(j_L, t)\, dj_L$ and $X_H(j_H, t) = \int_J^1 X_{n,H}(j_H, t)\, dj_L$, we can maximize (18) in respect to $p_m(j_m, t)$ to obtain the profit-maximization price of the m-monopolistic intermediate good

$$p_m(j_m, t) = \frac{1}{\alpha} > 1, m \in \{L, H\}. \quad \textbf{(19)}$$

Notice that this monopoly price is (i) a markup over the marginal cost of production, since $0 < \alpha < 1$; (ii) constant across intermediate goods and time invariant; and (iii) independent of the quality level of the intermediate good. Interestingly, if goods from quality rung below the highest quality, $k_m(j_m)$, are also available for production and the monopoly price is high enough, then it is possible that the producer of the next lowest grade could profit by simply producing. Hence, following Barro and Sala-i-Martin (2004, p. 344), we assume that “the quality leader employs a limit-pricing strategy; that is the leader sets a price that is sufficiently below the monopoly price so as to make it just barely unprofitable for the next best quality to be produced”, which is given by $p_m = q$.[13]

2.4. R&D Sector

Following Gil et al. (2013, 2016), R&D firms invest their resources in vertical R&D, targeting qualitative improvements of already existing intermediate-goods varieties. A patent is granted to each new design, and a successful innovator gets exclusive rights over the use of his product. Hence, by improving on the current top quality level, a successful R&D firm earns profits from introducing and selling the leading-edge quality $k_m(j_m, t) + 1$ to final-goods firms, thereby displacing the current best-quality level $k_m(j_m, t)$. We also assume free entry in the R&D business and perfect competition among entrants.

Following Gil et al. (2013, 2016) and Connolly (2003), let $pb_m(k_m, j_m)$ denote the Poisson arrival rate of vertical innovations in industry j_m when the highest quality is k_m. This rate changes across firms, across industries, and over time, which can be described as follows:

$$pb_m(k_m, j_m, t) = rs_m(k_m, j_m, t)\, \varrho q^{k_m(j_m,t)} \zeta^{-1} q^{-(1-\alpha)^{-1} k_m(j_m,t)} m(t)^{-\frac{\beta}{1-\alpha}} f(j, t), \tag{20}$$

where

(i) $rs_m(k_m, j_m, t)$ corresponds to the total amount of aggregate final-goods resources devoted to R&D;

(ii) $\varrho q^{k_m(j_m,t)}$, $\varrho > 0$, is a learning effect relating past successful R&D in j_m with the current probability of success [Grossman and Helpman (1991, Ch. 12), Connolly (2003)]; ϱ corresponds to the coefficient on past successful R&D experience, where a great ϱ implies a better innovation capacity;

(iii) $\zeta^{-1} q^{-(1-\alpha)^{-1} k_m(j_m,t)}$, $\zeta > 0$, is an adverse effect resulting from the increasing complexity of quality improvements, i.e., the larger the level of quality, k_m, the costlier it is to introduce a further jump in quality [Etro (2008), Kortum (1997)];

(iv) $m(t)^{-\frac{\beta}{1-\alpha}}$ corresponds to the adverse effect of market size—the bigger the size of the market, measured by the labor employed, the more difficult it is to introduce new quality-adjusted intermediate goods and to replace old ones, due to, for example, coordination, organizational, and transportation cost [Şener (2008), Becker and Murphy (1992), and Dinopoulos and Thompson (1999)];

(v) $f(j, t)$ apprehends the absolute advantage of high-skilled labor over low-skilled labor to learn and work with advanced technology, i.e., it could be called a technological-knowledge-absorption effect [Nelson and Phelps (1966), Schultz (1975), Galor and Moav (2000)], where, as in Afonso (2006):

$$f(j,t) = \begin{cases} 1 & 0 \le j \le J; \quad m = L \\ \left[1 + \frac{H(t)}{H(t)+L(t)}\right]^{\sigma} & J \le j \le 1; \quad m = H \end{cases} \quad \text{and } \sigma = 1 + \frac{H(t)}{L(t)}.$$

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Finally, note that (ii) and (iii) are modeled to offset the positive influence of the quality rung on the profits of each intermediate goods leader [Afonso (2006)].

3. EQUILIBRIUM

We analyze the equilibrium of the model in three steps. First, we derive the equilibrium for a given technological-knowledge level. Second, we introduce the R&D activities and derive the aggregate spending in R&D as well as the law of motion of technological knowledge. Finally, we describe the transitional dynamics and the steady-state growth.

3.1. Equilibrium Given the Technological-Knowledge Level

Given (7), (8), and (19), one can write the final-goods output as

$$Y_n(t) = \begin{cases} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} \left[P_n(t)\right]^{\frac{\alpha}{1-\alpha}} \left[(1-n)\, l L_n(t)\right]^{\frac{\beta}{1-\alpha}} Q_L(t) & , 0 \leq n \leq \overline{n} \\ A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} \left[P_n(t)\right]^{\frac{\alpha}{1-\alpha}} \left[n h H_n(t)\right]^{\frac{\beta}{1-\alpha}} Q_H(t) & , \overline{n} \leq n \leq 1 \end{cases}, \quad \textbf{(21)}$$

where $Q_L = \int_0^J \left[q^{k_L(j_L,t)}\right]^{\frac{\alpha}{1-\alpha}} dj_L$ and $Q_H = \int_J^1 \left[q^{k_H(j_H,t)}\right]^{\frac{\alpha}{1-\alpha}} dj_H$ are aggregate quality indexes denoting the technological knowledge in each range of intermediate goods, adjusted by market power, which is the same for all monopoly producers. Hence, $\frac{Q_H}{Q_L}$ measures the technological-knowledge bias.

Regarding competitive final-goods producers, the economic viability of either type of technology depends on (i) the relative productivity, h/l, (ii) the price of the m-type labor, and (iii) the relative productivity and prices of the intermediate goods. The latter depends on complementarity with either m-type labor, on the technological knowledge embodied, or on the markup, all summarized in the quality indexes Q_L and Q_H.

Taking into account that L- and H-technology firms must break even at $\bar{n}$, the endogenous threshold $\overline{n}$ follows from the equilibrium in the input markets, and relies on the determinants of economic viability of the two technologies, such that (Appendix B)

$$\bar{n}(t) = \left(1 + \left\{\frac{h}{l}\frac{H(t)}{L(t)}\left[\frac{Q_H(t)}{Q_L(t)}\right]^{\frac{1-\alpha}{\beta}}\right\}^{\frac{1}{2}}\right)^{-1}. \quad \textbf{(22)}$$

$\bar{n}(t)$ can also be related to the ratio of price indexes of final goods produced with L- and H-technologies. Recalling that $\exp\int_0^1 \ln P_n dn = 1$ and taking into account that, in equilibrium, the marginal value product, $\frac{\partial}{\partial_{n,m}}\left[P_n(t)\, Y_n(t)\right]$, must be constant over n, $(1-n)^{\frac{\beta}{1-\alpha}} P_n^{\frac{1}{1-\alpha}}$ must be constant across $n \epsilon\, [0, \overline{n}]$, and $n^{\frac{\beta}{1-\alpha}} P_n^{\frac{1}{1-\alpha}}$ across $n \epsilon\, [\overline{n}, 1]$ [Gil et al. (2013, 2016)]. Hence, since at $\overline{n}(t)$ the L- and the

H-technology firms must break even, we have

$$\frac{P_H(t)}{P_L(t)} = \left[\frac{\bar{n}(t)}{1-\bar{n}(t)}\right]^{\beta}, \text{ where } \begin{cases} P_L(t) &= P_n(t)(1-n)^{\beta} = \exp(-\beta)\,\bar{n}(t)^{-\beta} \\ P_H(t) &= P_n(t)\,n^{\beta} = \exp(-\beta)\,[1-\bar{n}(t)]^{-\beta} \end{cases}. \quad \textbf{(23)}$$

Equation (22) shows that the labor market structure will crucially affect this threshold, which, in turn, will affect the direction of R&D activities. This relationship is analyzed in detail in the following section.

Finally, combining (21) with (22), we define the equilibrium aggregate resources devoted to intermediate-goods production, $X(t)$, and the equilibrium aggregate output, $Y(t)$, as (see Appendix C)

$$\begin{aligned} X(t) &= \int_0^1 \int_0^1 x_n(k,j,t)\,dj\,dn \\ &= \exp\left(-\frac{\beta}{1-\alpha}\right)\left(\frac{\alpha A}{\frac{1}{\alpha}}\right)^{\frac{1}{1-\alpha}} \\ &\quad \times \left(\left\{[lL_n(t)]^{\frac{\beta}{1-\alpha}} Q_L(t)\right\}^{\frac{1}{2}} + \left\{[hH_n(t)]^{\frac{\beta}{1-\alpha}} Q_H(t)\right\}^{\frac{1}{2}}\right)^2, \end{aligned} \quad \textbf{(24)}$$

$$Y(t) = \int_0^1 P_n(t)\,Y_n(t)\,dn = \left(\frac{\alpha}{\frac{1}{\alpha}}\right)^{-1} X(t). \quad \textbf{(25)}$$

From (24) and (25), we see that the dynamics of technological knowledge will drive economic growth.

Perfect competition. Taking into account (21), (22), and (23), we obtain the skill premium ω as a function of technological-knowledge bias,

$$\omega(t) = \frac{W_H(t)}{W_L(t)} = \left\{\left[\frac{H(t)}{L(t)}\right]^{\Psi}\left(\frac{h}{l}\right)^{\frac{\beta}{1-\alpha}} \frac{Q_H(t)}{Q_L(t)}\right\}^{\frac{1}{2}}, \quad \textbf{(26)}$$

where $\Psi = \frac{\beta-2(1-\alpha)}{(1-\alpha)}$. ω can also be considered as the relative demand wage by firms, $\omega^D(t)$, since it crucially depends on demand for low- and high-skilled labor. Furthermore, recalling (3), we can also write the relative supply wage as $\omega^S(t) = \frac{W_H(t)}{W_L(t)} = [\frac{H(t)}{L(t)}]^{\frac{1}{\gamma}}[\frac{C_H(t)}{C_L(t)}]^{\phi}$. Taking into account (4), the consumption ratio between high-skilled and low-skilled workers is constant along time, i.e., $\hat{C}_H(t) = \hat{C}_L(t)$. To simplify, we assume that $\frac{C_H}{C_L} = 1$, in line with Gil et al. (2016), which implies that differences in wages between low- and high-skilled workers are only reflected on the amount of equity shares in monopolistic intermediate goods firms owned by each type. Thus, at each t, the perfect equilibrium in the labor market, $\omega^S(t) = \omega^D(t)$, is given by

$$\left\{\left[\frac{\overline{H}(t)}{\overline{L}(t)}\right]\right\}^* = \left[\left(\frac{h}{l}\right)^{\frac{\beta}{1-\alpha}} \frac{Q_H(t)}{Q_L(t)}\right]^{\Theta}, \tag{27}$$

$$[\overline{\omega}(t)]^* = \left\{\left[\left(\frac{h}{l}\right)^{\frac{\beta}{1-\alpha}} \frac{Q_H(t)}{Q_L(t)}\right]^{\Theta}\right\}^{\frac{1}{\gamma}}, \tag{28}$$

where $\Theta = \frac{1}{\frac{2}{\gamma}-\Psi}$ and the superscript * denotes the equilibrium solutions for $\frac{\overline{H}(t)}{\overline{L}(t)}$ and $\overline{\omega}(t)$ at each period of time. (27) and (28) would be used as a baseline to determine the solutions for the monopoly and the bargaining cases.

Monopoly case. In what relates to the monopoly situation, from (13) and (28), the monopoly skill premium and the employment ratio $\left[\frac{H(t)}{L(t)}\right]^M$ are given by

$$\left[\omega^M(t)\right]^* = \frac{1}{\varphi}\left\{\left[\left(\frac{h}{l}\right)^{\frac{\beta}{1-\alpha}} \frac{Q_H(t)}{Q_L(t)}\right]^{\Theta}\right\}^{\frac{1}{\gamma}}, \tag{29}$$

$$\left[\left(\frac{H(t)}{L(t)}\right)^M\right]^* = \left(\frac{1}{\varphi}\right)^{\frac{2}{\Psi}} \left[\left(\frac{h}{l}\right)^{\frac{\beta}{1-\alpha}} \frac{Q_H(t)}{Q_L(t)}\right]^{\frac{\Theta}{\gamma}}. \tag{30}$$

Note that $[\omega^M(t)]^*$ is lower than $[\overline{\omega}(t)]^*$ and $[(\frac{H(t)}{L(t)})^M]^*$ is greater than $[(\frac{\overline{H}(t)}{\overline{L}(t)})]^*$ since low-skilled wages are higher under the monopoly agreement, but low-skilled employment is lower.

Bargaining case. Regarding the bargaining framework, from (16), we know that a low-skilled wage is also a markup over the perfect competition wage, i.e., $W_L^B(t) = \psi \overline{W}_L(t)$, where $\psi = \left[1 + \frac{1}{\beta}\frac{\theta v(1-\alpha-\beta)}{(1-\theta)+\theta v}\right]$. Hence, defining $\omega^B(t)$ as the wage ratio under the bargaining case, we have

$$\left[\omega^B(t)\right]^* = \frac{1}{\psi}\left\{\left[\left(\frac{h}{l}\right)^{\frac{\beta}{1-\alpha}} \frac{Q_H(t)}{Q_L(t)}\right]^{\Theta}\right\}^{\frac{1}{\gamma}}. \tag{31}$$

In this case, however, the ratio of high- over low-skilled workers will be determined on the contract curve. Combining (15), (26), and (28), we can define the relative contract curve and the equilibrium relative demand for the bargaining case as (Appendix D)

$$\left[\frac{W_L^B(t)}{\overline{W}_H(t)} - \frac{\overline{W}_L(t)}{\overline{W}_H(t)}\right] = \frac{1-v}{v}\left[\frac{\psi W_L^B(t)}{\overline{W}_H(t)} - \frac{\beta L_n^{-1}(t)\, Y_L(t)}{\beta H_n^{-1}(t)\, Y_H(t)}\right], \tag{32}$$

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$$\left\{\left[\frac{H(t)}{L(t)}\right]^B\right\}^* = \Phi^{\frac{2}{\Psi}}\left[\left(\frac{h}{l}\right)^{\frac{\beta}{1-\alpha}}\frac{Q_H(t)}{Q_L(t)}\right]^{\Theta}, \tag{33}$$

where $\Phi = \frac{1-v}{\psi(\frac{1-v}{v}-1)+1}$. As $\varphi > 1$, $[\omega^B(t)]^* < [\overline{\omega}(t)]^*$, since the low-skilled wage is higher under the bargaining scenario. Nevertheless, the relative demand for workers crucially depends on the union's preferences.

Result 1: If the union is employment- (wage) oriented, the low-skilled employment would be higher (lower) than the level achieved under the perfect competition scenario. Hence, the relative demand under the bargaining case would be lower (higher). In other words, $\{[\frac{H(t)}{L(t)}]^B\}^* < \{[\frac{\overline{H}(t)}{\overline{L}(t)}]\}^*$ ($\{[\frac{H(t)}{L(t)}]^B\}^* > \{[\frac{\overline{H}(t)}{\overline{L}(t)}]\}^*$) iff $v > \frac{1}{2}$ ($v < \frac{1}{2}$).

Proof. Recalling $\psi = [1 + \frac{1}{\beta}\frac{\theta v(1-\alpha-\beta)}{(1-\theta)+\theta v}]$ and taking into account $\Phi = \frac{\frac{1-v}{v}}{\psi(\frac{1-v}{v}-1)+1}$, a few steps of mathematical manipulations show that $\Phi = \frac{1}{1+\frac{1-2v}{1-v}\frac{1}{\beta}\frac{\theta v(1-\alpha-\beta)}{(1-\theta)+\theta v}}$. Since $\frac{\theta v(1-\alpha-\beta)}{(1-\theta)+\theta v} > 0$, we need only to take into consideration $\frac{1-2v}{1-v}$. Thus, if $v \gtrless \frac{1}{2}$, $\Phi \gtrless 1$. Finally, $\frac{2}{\Psi} < 0$, which implies that $\{[\frac{H(t)}{L(t)}]^B\}^* < \{[\frac{\overline{H}(t)}{\overline{L}(t)}]\}^*$ if $\Phi > 1$ and otherwise if $\Phi < 1$. ■

Growth path wages. Finally, in the three situations—perfect competition, monopoly union, or bargaining—the growth path of low-skilled and high-skilled wages are given by the same expression

$$\hat{W}_L(t) = \frac{1}{1-\alpha}\hat{P}_L(t) + \hat{Q}_L(t),$$

$$\hat{W}_H(t) = \frac{1}{1-\alpha}\hat{P}_H(t) + \hat{Q}_H(t).$$

3.2. Equilibrium with R&D

To determine the aggregate spending in R&D, we must understand how R&D is carried out in the intermediate-goods sector. We need to (i) determine which firms conduct R&D activities; (ii) determine the value of an innovation; and (iii) derive the laws of motion of $Q_L(t)$ and $Q_H(t)$.

Following Gil et al. (2013, 2016), Barro and Sala-i-Martin (2004), and Aghion and Howitt (1992), one can prove that, independently of j_m and the respective $q^{k_m(j_m)}$, it is more profitable to introduce a new quality of j_m by an outside firm rather than by the current monopolist. Indeed, note that the change in profits of an outside firm, $\triangle\Pi_{j_m}$, is given by

$$\triangle\Pi_{j_m} = \Pi_{j_m}(\tau) - 0 = \left[P_j(\tau) - 1\right] X_{j_m}$$
$$= \left(\frac{1}{\alpha} - 1\right)\left(\frac{P_m A\alpha}{\frac{1}{\alpha}}\right)^{\frac{1}{1-\alpha}} (\bar{m}m)^{\frac{\beta}{1-\alpha}} \left[q^{k_m(j_m,\tau)}\right]^{\frac{\alpha}{1-\alpha}}, \quad \textbf{(34)}$$

where $\bar{m} = h$ for $m = H$ and $\bar{m} = l$ for $m = L$.[14]

Let $\tau + d(\tau)$ be the time when a firm introduces the quality $q^{k_m(j_m)+1}\left[q^{k_m(j_m)}\right]$. The innovator that introduces $q^{k_m(j_m)}$ becomes the monopolist between τ and $\tau + d$ in j_m and earns a sum of profits given by $V(k_m, j_m, t) = \int_\tau^{\tau+d} \Pi(k_m, j_m, t)\exp[-r(t)]\,dt$.[15] Since, in equilibrium, the interest rate is constant between τ and $\tau + d$, the expected value of $V(k_m, j_m, t)$ is given by

$$E[V(k_m, j_m, t)] = \int_t^\infty \Pi(k_m, j_m, t)$$
$$\times \exp\left\{-\int_t^s [r(\tau) + pb(k_m, j_m, \tau)]\,d\tau\right\} ds = \frac{\Pi(k_m, j_m, t)}{r(t) + pb_m(k_m, j_m, t)}. \quad \textbf{(35)}$$

On the other hand, (35) can be seen as the no-arbitrage condition, where $V(k_m, j_m, t)\,r(t)$, the expected income generated by a successful innovation at time t on rung kth, equals the profit flow, $\Pi(k, j, t)$, minus the expected capital loss, $V(k_m, j_m, t) \times pb_m(k_m, j_m, t)$.[16] Regarding R&D activities, besides the assumption of free entry in R&D activities of the intermediate-goods sector, we also assume that when an innovation is introduced as a consequence of R&D efforts of many firms, the probability of a firm becoming the successful innovator is proportional to its share on aggregate R&D. Implicitly, the R&D spending to improve j_m should be equal to the expected payoff generated by the innovation, i.e., $rs_m(k_m - 1, j_m, t) = pb_m(k_m - 1, j_m, t)\,V_m(k_m, j_m, t)$. Thus,

$$rs_m(k_m, j_m, t) = pb_m(k_m, j_m, t)\,V_m(k_m + 1, j_m, t). \quad \textbf{(36)}$$

Given the equilibrium aggregate R&D spending

$$R(t) = \int_0^1 rs(k, j, t)\,dj = \int_0^J rs_L(k_L, j_L, t)\,dj + \int_J^1 rs_H(k_H, j_H, t)\,dj,$$

and combining it with (20), (35), and (36), we obtain (see Appendix E)

$$R(t) = \frac{\zeta}{\varrho} Q_L(t)\,L(t)\left\{\frac{\varrho}{\zeta} f(j)\,l^{\frac{\beta}{1-\alpha}}\left(\frac{q-1}{q}\right)[P_L(t)A\alpha]^{\frac{1}{1-\alpha}} - r(t)\right\}$$
$$+ \frac{\zeta}{\varrho} Q_H(t)\,H(t)\left\{\frac{\varrho}{\zeta} f(j)\,h^{\frac{\beta}{1-\alpha}}\left(\frac{q-1}{q}\right)[P_H(t)A\alpha]^{\frac{1}{1-\alpha}} - r(t)\right\}. \quad \textbf{(37)}$$

Finally, regarding the law of motion of $Q_m(t)$, suppose that a new quality of intermediate goods j_m is introduced. All else remaining equal, the change in the

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corresponding aggregate quality indexes is given by

$$\triangle Q_m = \left[q^{k_m(j_m)+1}\right]^{\frac{\alpha}{1-\alpha}} - \left[q^{k_m(j_m)}\right]^{\frac{\alpha}{1-\alpha}} = \left[q^{k_m(j_m)}\right]^{\frac{\alpha}{1-\alpha}} \left(q^{\frac{\alpha}{1-\alpha}} - 1\right). \quad \textbf{(38)}$$

Thus, combining (35), (36), and (37), the following equation arises (see Appendix F):

$$\hat{Q}_m(t) = \left\{\frac{\varrho}{\zeta} f(j)\, \bar{m}^{\frac{\beta}{1-\alpha}} \left(\frac{q-1}{q}\right) [P_m(t) A\alpha]^{\frac{1}{1-\alpha}} - r(t)\right\} \left(q^{\frac{\alpha}{1-\alpha}} - 1\right). \quad \textbf{(39)}$$

In (39), the term in large brackets corresponds to the equilibrium m-specific probability of successful R&D, $pb_m(t)$, given $P_m(t)$ and $r(t)$.

3.3. Transition Dynamics and Steady-State Growth

Taking into consideration (24), (25), (37) and recalling that $Y(t) = X(t) + R(t) + C(t)$, all macroeconomic aggregates can be expressed as multiples of the aggregate quality indexes, $Q_L(t)$ and $Q_H(t)$. This implies that the path of all relevant variables outside the steady state, including that of the wage ratio, depends on a single differential equation that governs the path of the technological-knowledge bias. Defining $Z(t) = \frac{Q_H(t)}{Q_L(t)}$, this single differential equation is given by $\hat{Z}(t) = \frac{\dot{Z}(t)}{Z(t)} = \frac{\dot{Q}_H(t)}{Q_H(t)} - \frac{\dot{Q}_L(t)}{Q_L(t)}$. Therefore, combining (23), (37), and (39), we obtain

$$\hat{Z}(t) = \left[\frac{\varrho}{\zeta}\left(\frac{q-1}{q}\right)(A\alpha)^{\frac{1}{1-\alpha}}\right]\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)\exp\left(-\frac{\beta}{1-\alpha}\right)$$
$$\times \left(h^{\frac{\beta}{1-\alpha}}\left[1 + \frac{H(t)}{H(t)+L(t)}\right]^{\sigma}\left\{1 + \left[Z(t)^{\frac{1-\alpha}{\beta}}\frac{hH(t)}{lL(t)}\right]^{-\frac{1}{2}}\right\}^{\frac{\beta}{1-\alpha}}\right.$$
$$\left. - \left\{1 + \left[Z(t)^{\frac{1-\alpha}{\beta}}\frac{h}{l}\frac{H(t)}{L(t)}\right]^{\frac{1}{2}}\right\}^{\frac{\beta}{1-\alpha}} l^{\frac{\beta}{1-\alpha}}\right). \quad \textbf{(40)}$$

Finally, in steady state, all variables grow at the same constant rate. Recalling $\hat{C} = \frac{r(t)-\rho}{\phi}$, we then know that

$$g* = \hat{Q}_H^* = \hat{Q}_L^*, \quad \textbf{(41)}$$

which allows us to find the steady-state threshold final goods and the technological-knowledge bias depending only on $\frac{H(t)}{L(t)}$ as

$$[\bar{n}(t)]^* = \frac{l}{f(h)^{\frac{1-\alpha}{\beta}} h + l}, \quad \textbf{(42)}$$

$$[Z(t)]^* = \left[\left(\frac{1-\bar{n}^*}{\bar{n}^*}\right)^2 \frac{l}{h}\frac{L(t)}{H(t)}\right]^{\frac{\beta}{1-\alpha}}, \quad \textbf{(43)}$$

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where $f(h) = [1 + \frac{H(t)}{H(t)+L(t)}]^{\sigma}$ and $\frac{H(t)}{L(t)}$ is given by (27), (30), or (33), for the perfect competition, monopoly, or bargaining case, respectively.[17]

4. RESULTS

In this section, we analyze and compare the three situations in the labor market regarding: (a) technological-knowledge bias, and (b) wage premium. Since $\frac{H(t)}{L(t)}$ crucially depends on the labor market situation, the direction of technological-knowledge progress and its repercussion on wage premium are expected to be different among the three possible cases.

Using the fourth-order Runge–Kutta classical numerical method, we analyze the behavior of the technological-knowledge bias for a set of baseline parameter values. Taking into account (5), we set $h = 1.20$ and $l = 1$ since high-skilled workers have an absolute productivity advantage over low-skilled workers [Afonso (2006)]. On the other hand, according to our theoretical assumptions regarding the R&D function (20), we set $\varrho = 1.60$ and $\zeta = 4.00$, in line with Gil et al. (2016) and Afonso (2006). Regarding α and β, Karabarbounis and Neiman (2014) report a decline of the labor share since the early 1980s. Hence, we set $\alpha = 0.7$ and $\beta = 0.2$, in line with Reis and Sequeira (2007) and Kwan and Lai (2003). In what relates to the utility function, we fix the inverse of the intertemporal elasticity of substitution to $\phi = 1.50$, as in Gil et al. (2016) and Attanasio and Weber (1993), and the homogeneous subjective discount rate to $\rho = 0.02$, as in Dinopoulos and Thompson (1999).[18] Regarding the Frisch labor supply elasticity, we follow Bertinelli et al. (2013) and set $\gamma = 0.5$ and $\gamma = 0.2$ to account for a standard and a weak responsive labor force, respectively.

Finally, to close the calibration, we also need to set values for the bargaining power of the employers' federation and the union, as well as the union's preferences regarding wages and employment. In what concerns the bargaining power, Mortensen and Pissarides (1994) assume $\theta = 0.5$, whereas Millard and Mortensen (1997) set $\theta = 0.3$. On the other hand, Bertinelli et al. (2013) consider three different situations, $\theta = 0.2$, $\theta = 0.6$, and $\theta = 0.9$. Hence, as a departing point, we also set different scenarios: $\theta = 0.4$, $\theta = 0.5$, and $\theta = 0.6$. In what relates to the union's preferences, v, Chu et al. (2016) only assume cases in which unions are wage-oriented. Nevertheless, Pencavel (1984) finds that larger unions are more "wage-oriented" than smaller ones. Furthermore, Faia and Rossi (2013) consider three different cases, i.e., a union can be wage-oriented, employment-oriented, or neutral. Thus, for the bargaining case, we also consider two different possibilities with $v = 0.4$ and $v = 0.6$ to account for the two type of contract curve. For the monopoly case, to ensure convergence, we consider the range of values to $v = [0.8; 1]$.

By replacing $\{[\frac{\overline{H}(t)}{\overline{L}(t)}]\}^{*}$, $\{[\frac{H(t)}{L(t)}]^{M}\}^{*}$, or $\{[\frac{H(t)}{L(t)}]^{B}\}^{*}$ into $\hat{Z}(t)$, we can find the steady-state values under perfect competition, the monopoly case, or the bargaining case.

TABLE 1. Perfect competition values in steady state

	γ	θ	v	$Z=\frac{Q_H}{Q_L}$	$\frac{W_H}{W_L}$	$\left(\frac{H}{L}\right)^D$	$\left(\frac{H}{L}\right)^S$
Perfect competition	0.5	1	1	9.6425	2.4483	1.5647	1.5647
	0.2	1	1	6.7686	2.4529	1.1966	1.9966

TABLE 2. Monopoly values in steady state

	γ	θ	v	$Z=\frac{Q_H}{Q_L}$	$\frac{W_H}{W_L}$	$\left(\frac{H}{L}\right)^D$	$\left(\frac{H}{L}\right)^S$
Monopoly case	0.5	–	0.8	40.7184	3.3617	2.8648	1.8335
	0.2	–	0.8	11.7383	2.5019	1.7555	1.2013

4.1. The Perfect Competition Case

Table 1 presents the main results regarding the dynamics of the model for the perfect competition scenario. Interestingly, the steady-state values crucially depend on the elasticity of labor supply, γ. A weak (strong) responsive labor force leads to a lower (higher) technological-knowledge bias and a lower (higher) relative demand. Intuitively, since the high-skilled workers have an absolute advantage over the low-skilled workers, firms have an a priori interest to invest in high-skill technology.

Hence, taking into account that a 1% increase in wages can attract less (more) workers under a lower (higher) elasticity of labor supply, firms would find it more profitable to hire relatively less (more) high-skilled workers under the former (latter) scenario.

Moreover, the relationship between $\frac{H}{L}$ and $\frac{Q_H}{Q_L}$, in which a higher employment ratio implies a higher technological-knowledge bias, is always present in all the considered cases. As stated before, this behavior is closely related with the existent complementaries between inputs—see equation (5).

4.2. The Monopoly Case

Regarding the monopoly case (Table 2), the wage ratio $\frac{W_H}{W_L}$ is higher than the perfect competition case under both standard and weak responsive labor supply. In this situation, unions fail to decrease the wage dispersion between high- and low-skilled workers, which can be explained by three main effects. On the one hand, unions shift the supply curve to the left, increasing low-skilled wages and lowering the employment level. This triggers two effects, namely the price effect since high-skilled workers are relatively cheaper, and the market size effect since high-skilled workers are also relatively more abundant. These two effects make it more profitable to hire high-skilled workers, thereby shifting the workers relative demand to the right (i.e., there is a relative demand effect). On the other hand,

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TABLE 3. Bargaining values in steady state

(a) Bargaining case—0.5

θ	υ	$Z=\frac{Q_H}{Q_L}$	$\frac{W_H}{W_L}$	$\left(\frac{H}{L}\right)^D$	$\left(\frac{H}{L}\right)^S$
0.5	0.4	11.4676	2.2862	1.7332	1.6544
	0.6	7.3203	1.8593	1.2819	1.4146
0.4	0.4	10.9277	2.3215	1.6869	1.6297
	0.6	7.7640	1.9751	1.3443	1.4478
0.6	0.4	12.1762	2.2502	1.7903	1.6850
	0.6	6.8890	1.7450	1.2160	1.3794

(b) Bargaining case—0.2

θ	υ	$Z=\frac{Q_H}{Q_L}$	$\frac{W_H}{W_L}$	$\left(\frac{H}{L}\right)^D$	$\left(\frac{H}{L}\right)^S$
0.5	0.4	7.3973	2.2321	1.2931	1.2173
	0.6	5.8032	1.9300	1.0184	1.1574
0.4	0.4	7.2214	2.2836	1.2673	1.2118
	0.6	6.0047	2.0359	1.0594	1.1666
0.6	0.4	7.6169	2.1761	1.3242	1.2238
	0.6	5.5986	1.8239	0.9740	1.1474

due to the complementaries between workers and technology, firms have now an incentive to invest more in high-skilled technology, which explains the changes in the technological-knowledge bias, i.e., $(\frac{Q_H}{Q_L})^M > (\frac{\overline{Q}_H}{\overline{Q}_L})$, where $(\frac{\overline{Q}_H}{\overline{Q}_L})$ corresponds to the technological-knowledge bias under the perfect competition scenario. It is also worth noting that the impact of trade unions is higher under a more responsive labor supply as the adjustments in the relative demand are higher in this scenario. In the end, the difference between the relative demand (column 3) and relative supply (column 4) corresponds to differences in low-skilled employment. In this case, $(\frac{H}{L})^D > (\frac{H}{L})^S$, meaning there is low-skilled unemployment.

4.3. The Bargaining Case

Table 3(a) and (b) present the main results regarding the two bargaining cases, i.e., $\upsilon = 0.4$ and $\upsilon = 0.6$, which corresponds to a downward [Figure 2(b)] and upward sloping [Figure 2(a)] contract curve for the low-skilled workers, respectively. Note that since unions and firms bargain wages and employment, the wage dispersion is actually lower in both cases, which is in line with Figure 1(c).

On the other hand, only when the union is employment-oriented ($\upsilon > 0.5$) is it possible to achieve a lower H/L ratio, in line with Figure 1(d), due to the prevalence of the upward sloping contract curve. Intuitively, it is more profitable now for firms to higher more low-skilled workers since for the same wage, firms can employ more low-skilled workers. Thus, all three effects identified above (i.e., price, market size, and relative demand effect) are at work in this scenario, but in an opposite direction. This change in the relative demand would be followed by a decrease in the technological-knowledge bias, where $(\frac{Q_H}{Q_L})^B \mid_{v=0.6} < (\frac{Q_H}{Q_L})^M < (\overline{\frac{Q_H}{Q_L}})$ is in line with Figures 1(e) and (f).

In what relates to the wage-oriented scenario, there is a decrease in the low-skilled workers market size, but this decrease is lower when compared with the monopoly case since the contract curve is on the right of the demand curve [Figure 2(b)]. This smaller reduction explains the smaller increase in the employment ratio, as $(\overline{\frac{H}{L}}) < (\frac{H}{L})^B \mid_{v=0.4} < (\frac{H}{L})^M$. Note also that, as in the monopoly case, the impact of trade unions is higher (lower) under a more (less) responsive labor supply.

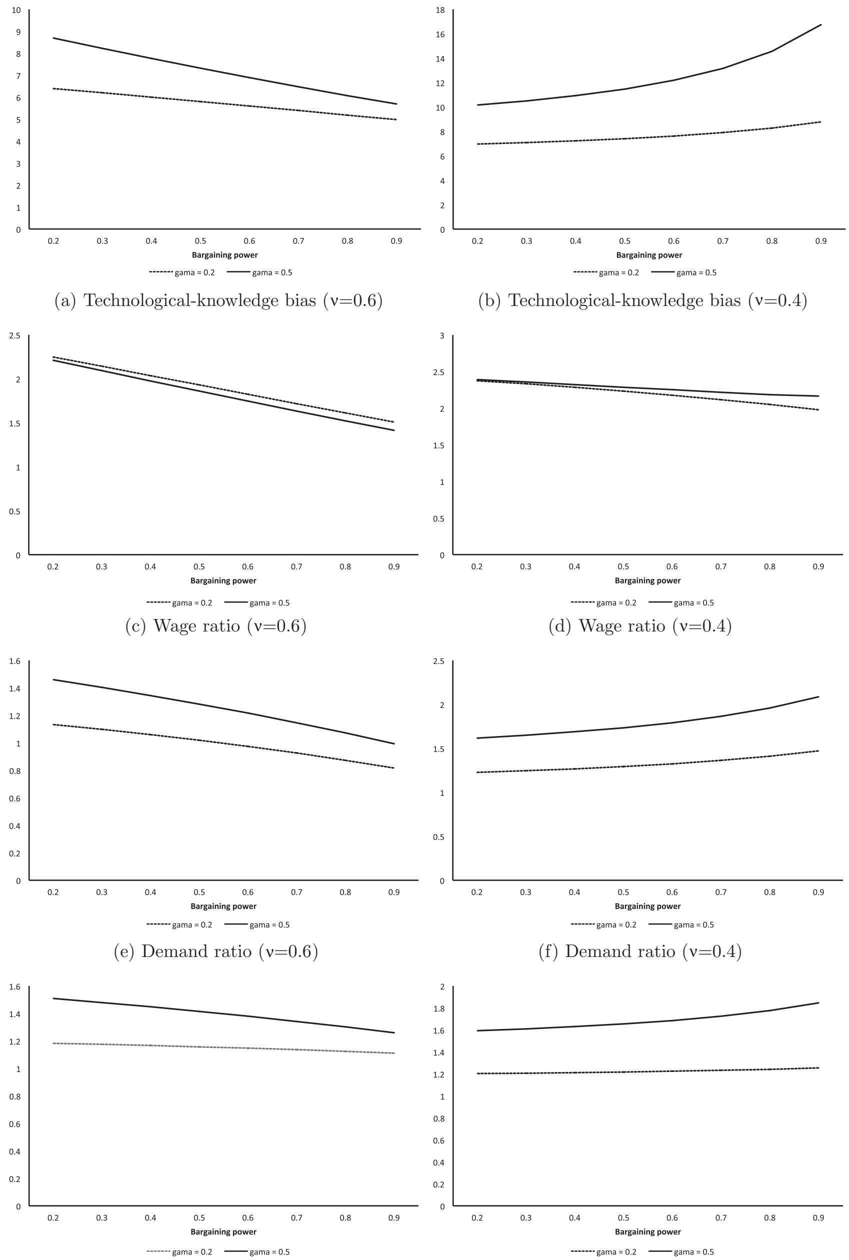
Hence, the key parameter that governs our main results is the union's preferences concerning employment and wages. Regarding Fadinger and Mayr (2014), the key parameter is the elasticity of the matching function. A relatively low (higher) elasticity of the matching function is related with a higher (lower) skill premium and relative employment.

Interestingly, as stated in section *Trade unions—efficient bargaining*, there is no unemployment under the bargaining framework since all workers are on the contract curve. This question is particularly important if unions shift their attitudes from a monopolist to a bargaining perspective. According to Blanchard (2004, p. 20), this might be the case as in some European countries unions "speak of the need for a 'partnership between labor and capital.'" If this is true, our framework then predicts a decrease both in the unemployment level and in wage dispersion.

Finally, to account for the different possible values of θ present in the literature [Bertinelli et al. (2013)], Figure 3 provides a numerical simulation for $\theta = [0.2; 0.9]$ under the two bargaining scenarios, i.e., $v = 0.4$ and $v = 0.6$. As expected, the technological-knowledge bias governs the path of the relative demand and supply: under $v = 0.6$ ($v = 0.4$), an increase in the union's bargaining power, θ, leads to a(n) decrease (increase) in the $\frac{Q_H}{Q_L}$, which also implies a(n) decrease (increase) in $\left(\frac{H}{L}\right)^D$ and $\left(\frac{H}{L}\right)^S$. Nevertheless, the wage ratio is decreasing in both bargaining cases, in contrast with the monopoly situation.

5. DISCUSSION

In this section, we analyze the economic dynamics of changing the wage bargaining structure. In particular, we theoretically analyze the antiunion phenomena that occurred during Thatcher's (in the United Kingdom) and Reagan's (in the United States) period back in 1980s [Acemoglu et al. (2001)]. Hence, we consider three changes: (a) from the monopoly case to the competitive labor market; (b) the bargaining structure with an upward sloping contract curve to the competitive labor market; and (c) the bargaining structure with a downward sloping contract curve to the competitive labor market. In all three situations, we consider a weak responsive labor force, $\gamma = 0.2$, but all the dynamics and results can be generalized to the standard case.



(a) Technological-knowledge bias (ν=0.6)

(b) Technological-knowledge bias (ν=0.4)

(c) Wage ratio (ν=0.6)

(d) Wage ratio (ν=0.4)

(e) Demand ratio (ν=0.6)

(f) Demand ratio (ν=0.4)

(g) Supply ratio (ν=0.6)

(h) Supply ratio (ν=0.4)

FIGURE 3. Simulations.

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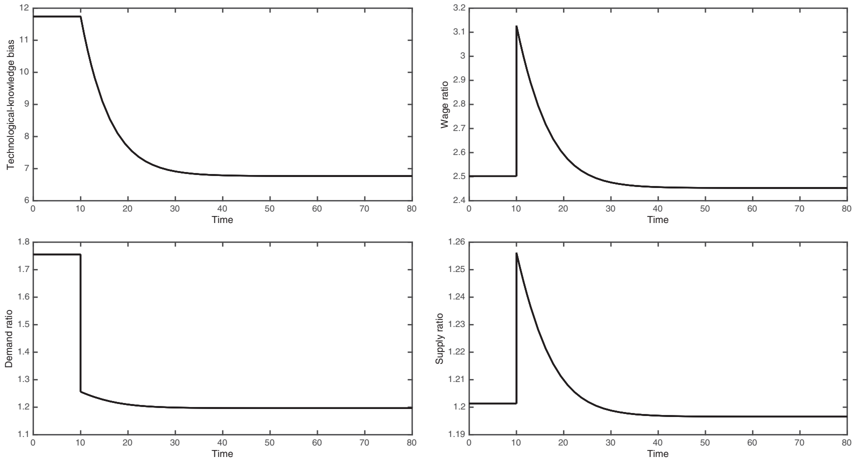

FIGURE 4. Monopoly case: Transition dynamics.

5.1. The Monopoly Case

Applying the same logic as in Section 4, we replace $\{[\frac{H(t)}{L(t)}]^M\}^*$ into $\hat{Z}(t)$ to study the equilibrium path and its dynamics under a monopolistic union. Figure 4 provides a graphical representation of the transition dynamics with $v = 0.8$.

From $t = [1, 10]$, the economy is under a monopoly union in the labor market. At $t = 10$, this monopoly union is dismantled, which immediately affects the steady-state paths of all four variables. Starting the analysis with H-premium, an immediate jump is verified, as one might expect. Indeed, under the competitive labor market scenario, the low-skilled wage is lower when compared with the monopoly case, which fully accounts for this jump in wage ratio and also explains the immediate rise (fall) in relative supply (demand).

Regarding the technological-knowledge bias, due to the fall in the relative demand, we observe a decrease in the technological-knowledge-absorption effect. In equation (20), $f(j)$ falls immediately from 3.8893 to 2.7144 as a result of the dismantling of the monopoly union.

However, from period 10 onward, we observe a decrease in all four variables, with special attention to the technological-knowledge bias and the wage ratio, both of which can be intuitively explained as follows: facing a fall in low-skilled wages (price effect), firms start increasing (decreasing) their demand for low(high)-skilled workers (relative demand effect), which, combined with the increase in the low-skilled market size, leads to an(a) increase(decrease) in the demand for low(high)-skilled technology. On the other hand, this decrease in the technological-knowledge bias leads to an increase in low-skilled wages—note the wage ratio depends on $\frac{Q_H}{Q_L}$. Indeed, an unexpected technological effect arises as a negative variation in $\frac{Q_H}{Q_L}$ leads to an increase in low-skilled wages, which accounts for the decrease in wage dispersion.

Moreover, we have already seen that changes in H-premium are closely related with changes in technological-knowledge bias—see (29). Indeed, a decrease in $\frac{Q_H}{Q_L}$ leads to a decrease in the supply of H-intermediate goods, thereby decreasing the number of final goods produced with H-technology and increase their relative price—see (22) and (23). Therefore, relative prices of final goods produced with H-technology decline continuously toward the new constant steady-state levels, implying that $\frac{Q_H}{Q_L}$ is falling, but at a decreasing rate toward its new lower steady-state value.

Furthermore, since labor market dynamics are endogenous to the model, this change in relative demand not only is an endogenous mechanism to the model [in contrast with Gil et al. (2016) and Afonso (2006)], but also enhances the relationship between $\frac{H}{L}$ and $\frac{Q_H}{Q_L}$ due to the relative demand effect. The new equilibrium is characterized not only by a lower relative wage but also by the absence of unemployment. Finally, it is interesting to note that, within this framework, the dismantling of the trade union actually contributes to decreasing wage dispersion.

5.2. The Bargaining Case

In this last case, we replace $\{[\frac{H(t)}{L(t)}]^B\}^*$ into $\hat{Z}(t)$. However, we have to take into consideration the value of υ, the union's preferences for wages or employment. Indeed, two cases arise.

Case 1: $\theta = 0.5$ and $\upsilon > 0.5$. Figure 5 describes this scenario. From $t = [0, 10]$, the economy is under a positive contract curve for low-skilled workers, with $\upsilon = 0.6$. This implies that firms and unions are negotiating both low-skilled wages and low-skilled employment. At $t = 10$, the union is dismantled, which immediately affects the steady-state paths of all four variables. Regarding H-premium, an immediate jump is verified, as one might expect—see Figure 2(a). Indeed, note that the low-skilled wage is no longer determined under the contract curve, which breaks down the unusual positive relationship between demanded wages and employment. Hence, we observe an interesting pattern: Both the wage ratio and the relative demand increase at the same time. This is due to the fact that, after the union is dismantled, firms are no longer better off with higher low-skilled employment, thereby increasing their demand for high-skilled workers, which is followed by an increase in their salary. This is in contrast with the dynamics of the monopoly case, and explains the immediate raise in the technological-knowledge-absorption effect, from 2.2808 to 2.5687.

Moreover, from period 10 onward, an increase in all four variables is verified, which is again in contrast with the previous situation. Under this type of efficient bargaining, firms agree to employ more low-skilled workers (in comparison with the perfect competition scenario) which, due to the existent complementaries between inputs, implies that it is more profitable to invest in low-skilled technology since $\frac{H}{L}$ is would be lower—see (5). However, from period 10 onward, firms are no longer obliged to be under the positive contract curve. Hence, firms can now

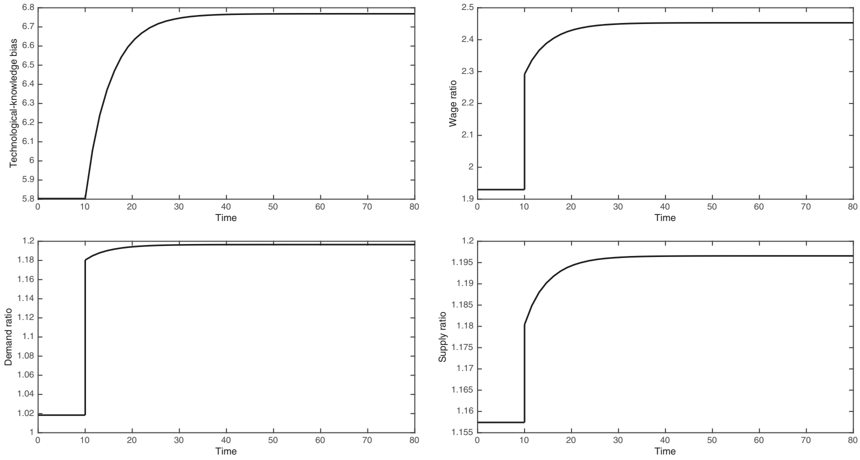

FIGURE 5. Bargaining case 1: Transition dynamics.

adjust freely their level of low-skilled workers toward the competitive market equilibrium with lower low-skilled wages and lower low-skilled employment. In this case, the increase in the high-skilled market size and the adjustment in the relative demand due to the technological complementaries more than compensates the increase in the wage ratio (price effect), which is followed by an increase in $\frac{H}{L}$. The new equilibrium is characterized by a higher wage dispersion and relative demand, which implies that, unions could, until period 10, actually contribute to lower the wage dispersion.

Case 2: $\theta = 0.5$ and $v < 0.5$. Figure 6 presents this case. Once again, firms and unions stop bargaining low-skilled wage and employment at $t = 10$, which instantly affects the paths of all four variables. Regarding the wage ratio, an immediate jump is once more verified since the low-skilled wage is now determined competitively—see Figure 2(b). This fully accounts for the fall in the relative demand, which is in contrast with the previous case, but in line with the monopoly situation.

Note that, in the previous bargaining situation firms were employing more low-skilled workers due to the upward sloping contract curve, whereas in this case we have a downward sloping contract curve and the bargaining solution is on the left of the perfect competitive equilibrium. This implies that, once the union is dismantled, firms will actually employ more low-skilled workers rather than lay off them. Hence, the technological-knowledge bias follows the decrease in the relative demand and a fall in $\frac{Q_H}{Q_L}$ is verified, due to the technological-knowledge-absorption effect, from 2.7883 to 2.6171.

From period 10 onward, a decrease in all four variables is observed, which contrasts with the previous bargaining case, but is in line with the monopoly dynamics with a small exception—although the wage ratio starts decreasing after

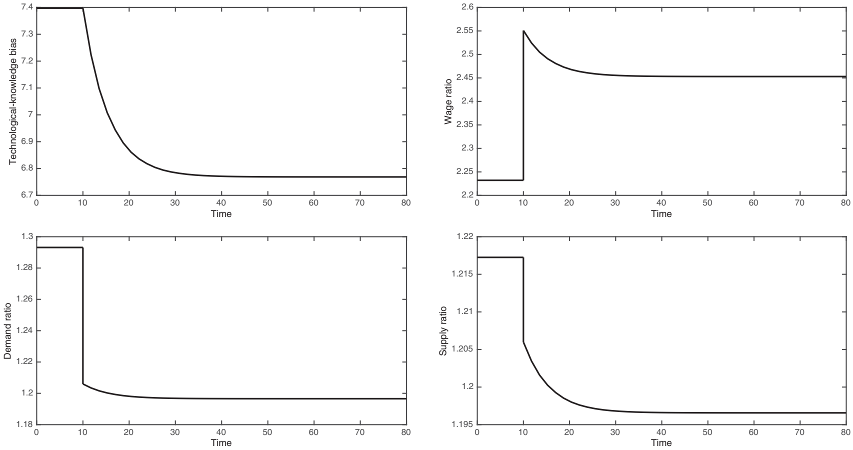

FIGURE 6. Bargaining, case 2: Transition dynamics.

$t = 10$, in the new steady-state it will still be higher when compared with the initial bargaining case. This is not the case with the monopolistic union since $(\frac{W_H}{W_L})^M > (\frac{\overline{W}_H}{\overline{W}_L}) > (\frac{W_H}{W_L})^B$. In what regards the technological-knowledge bias, the adjustment in the relative demand is followed by an adjustment in $\frac{Q_H}{Q_L}$, which is in line with Afonso (2006). Once again, this effect is enhanced by the relative demand effect, but it is not enough to return wages at their initial level. Therefore, the new equilibrium is characterized by a lower low-skilled wage and a higher wage dispersion, which is in line with the adjustments from the previous type of bargaining.

6. A BRIEF EMPIRICAL NOTE

Taking into account the variety of dynamics one can obtain by changing the bargaining structure in the labor market, in this section, we provide some empirical evidence to support our theoretical analysis. As we stated, depending on the initial wage structure, promoting competitive labor markets might decrease wage dispersion. Following Dobbelaere et al. (2015), we compare the impact of trade union density on wage dispersion under two different scenarios: (a) efficient bargaining and (b) perfect competition or right-to-manage.[19] Dowrick (1990) reports evidence for efficient bargaining in the United States and the United Kingdom.[20] On the other hand, Dobbelaere et al. (2015) identifie perfect competition or right-to-manage for Japan. Hence, we selected these three countries to test our theoretical implications of trade unions on wage dispersion. We propose the following model specification:

$$\text{Decile}\left(\frac{X}{Y}\right)_j = \beta_1 + \beta_2 \text{unions}_j + \beta_3 D1_j + \beta_4 \text{unions}_j D1_j + \varepsilon_j,$$

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TABLE 4. Econometric analysis

	Model 1 Decile $\frac{9th}{1st}$	Model 2 Decile $\frac{9th}{5th}$	Model 3 Decile $\frac{5th}{1st}$
C	5.1654***	2.4097***	2.1699***
	(0.0567)	(0.0172)	(0.0147)
Unions	−0.04824***	−0.0150***	−0.0093***
	(0.0021)	(0.0006)	(0.0005)
$D1_j$	−2.3491***	−0.5054***	−0.6992***
	(0.1989)	(0.0602)	(0.0514)
Unions × $D1_j$	0.0579***	0.0121***	0.0173***
	(0.0083)	(0.0025)	(0.0021)
	$F(1, 98) =$ 49.092[0.00]	$F(1, 98) =$ 23.565[0.00]	$F(1, 98) =$ 65.535[0.00]

Source: OECD online database—tables (a) trade union density and (b) decile ratio of gross earnings, available at http://stats.oecd.org (accessed on December 2015).
Notes: Annual data (1980–2013), $N = 102$. *, **, and *** indicate test statistic significance at 10%, 5%, and 1% levels, respectively.
Diagnostic tests:
Model 1: R-squared = 0.9131; S.E. of regression = 0.2014; log-likelihood value = 20.7433.
Model 2: R-squared = 0.8941; S.E. of regression = 0.0610; log-likelihood value = 142.666.
Model 3: R-squared = 0.9094; S.E. of regression = 0.052; log-likelihood value = 158.727.

where $j = n \times t$, n corresponds to a country, t to a year, unions to the trade union density, and $D1_j$ to a dummy variable to account for the different types of bargaining wage. $D1_j = 0$ for the United States and the United Kingdom, and $D1_j = 1$ for Japan. Hence

$$\text{for the United States and the United Kingdom: } E[\text{Decile }(\tfrac{X}{Y})_j | D1_j = 0]$$
$$= \beta_1 + \beta_2 \,\text{unions}_j.$$

$$\text{for Japan: } E[\text{Decile }(\tfrac{X}{Y})_j | D1_j = 1] = (\beta_1 + \beta_3) + (\beta_2 + \beta_4)\,\text{unions}_j.$$

The model is estimated by pooled OLS for (a) decile $\frac{9th}{1st}$, (b) decile $\frac{9th}{5th}$, and (c) decile $\frac{5th}{1st}$. Table 4 reports the main results.

Regarding the United States and the United Kingdom labor market framework $(D1_j = 0)$, a decrease in the level of unionisation leads to an increase in wage dispersion, which is in line with the efficient bargaining case. In what relates to Japan $(D1_j = 1)$, Unions × $D1_j$ has a positive sign. This extra term accounts for the different types of bargaining wage between (a) the United States and the United Kingdom, and (b) Japan, which decreases the negative impact of unions under the efficient bargaining case (i.e., $D1_j = 0$). Furthermore, the overall impact of unions on wage dispersion in Model 1 and Model 3 is in line with our results since $\hat{\beta}_4 > \hat{\beta}_2$. Hence, an increase in trade union density under competitive labor markets leads to an increase in the wage ratio.

7. CONCLUDING REMARKS

This paper presents an endogenous growth model of directed technological change with vertical R&D and different types of labor market frameworks to study the impact of trade unions on technology, employment, and wage dispersion. Generally, we conclude that the behavior of trade unions and their preferences regarding wages and employment crucially influence the direction of technology, which, in turn, determine the employment ratio and the skill premium.

On the one hand, by shifting the supply curve to the left, a monopoly union fails to decrease wage dispersion due to the result of three effects, namely (i) price effect since high-skilled workers are now relatively cheaper; (ii) market size effect since high-skilled workers are relatively more abundant; and (iii) relative demand effect since firms will adapt their decisions regarding workers, thereby increasing the relative demand for high-skilled workers which, due to the complementaries between workers and technology, leads to an increase in the investment in high-skilled technology.

On the other hand, if unions have a stronger preference for employment and enroll in a bargaining process with firms, they can actually decrease wage dispersion, employment ratio, and technological bias, in line with the empirical motivation provided above. In this case, an upward sloping contract curve prevails, which contrasts with the results obtained with the downward sloping curve. In the latter, the employment ratio and technological-bias are higher when compared with the perfect competition scenario, but lower in relation with the monopoly case. Finally, our theoretical implications are empirically verified and can also accommodate the impacts of deunionization that occurred in the United States and the United Kingdom during the 1980s on the wage premium. Starting with an efficient bargaining (perfect competition) framework, the deunionization process leads to a rise (fall) in wage dispersion essentially due to the combination of three effects, i.e., price effect, market size effect, and relative demand effect. This would be followed by a change in the technological bias toward the relatively cheaper and more abundant type of work. These results are important to (a) endogenously explain why an increase in high-skilled workers does not necessary lead to a decrease in wage premium; and (b) account for mixed evidence regarding the relationship between trade unions and wages.

In view of future research, we aim to apply our methodology to the European case through a "north-south" analysis, to introduce the possibility of education and training as an option to low-skilled workers, and to analyze the role of public R&D funding.

NOTES

1. For a survey of the most important literature see Acemoglu (2002), Aghion (2002), Hornstein et al. (2005), Chusseau et al. (2008), and Kurokawa (2014).

2. This information was gathered from the Data Base on Institutional Characteristics of Trade Unions, Wage Setting, State Intervention and Social Pacts, 1960–2014 (ICTWSS), available at http://www.uva-aias.net/208.

3. In equilibrium, only the top quality for every industry j_m is produced and used. Hence, $x_{n,m}(k_m, j_m, t) = x_{n,m}(j_m, t)$.

4. Dobbelaere et al. (2015) also state that efficient bargaining is the most prevalent labor market framework in France.

5. For a survey see Freeman and Medoff (1984).

6. For a survey of the literature see Lawson (2011) and Kaufman (2002).

7. Since unions do not have any preferences regarding the level of investment, we avoid the problem of underinvestment. For further information see, among others, Clark (1990) and Booth (1995).

8. For a critical review see Layard and Nickell (1990).

9. Again, since unions do not have any preferences regarding the level of investment, we avoid the problem of underinvestment.

10. Recalling (9), one can show that $\frac{dW}{dL} = \frac{1-v}{2v-1}(1-\beta)\beta L_n^{-2}(t)Y_L(t) \gtreqless 0$ iff $2v - 1 \gtreqless 0$.

11. Regarding the bargaining power, note that $\frac{\partial(W_L - \overline{W}_L)}{\partial\theta} = \frac{v(1-\alpha-\beta)[(1-\theta+\theta v)+(1-v)\theta]}{(1-\theta+\theta v)} > 0$.

12. Note that, as in Chu et al. (2016), the assumption of decreasing returns to scale allows the firm to have a positive profit, which facilitates the bargaining process.

13. Alternatively, one could follow Gil et al. (2013, p. 1461), where it is assumed that innovations are drastic, i.e., $\frac{1}{\alpha} < q_m$, such that "existing monopolies do not need to limit price and can instead charge the unconstrained monopoly price."

14. Note that in the scenario in which the latest innovation in intermediate good j is introduced by the current monopolist in that good, we have $\triangle\Pi^c_{jm} = \Pi_{jm}(\tau) - \Pi_{jm}(\tau - 1)$. Hence, $\triangle\Pi^c_{jm}(\tau) = ((\frac{1}{\alpha})^2 - 1)\{[\frac{P_m A\alpha}{(\frac{1}{\alpha})^2}]^{\frac{1}{1-\alpha}}(\bar{m}m)^{\frac{\beta}{1-\alpha}}[q^{k_m(j_m,\tau)}]^{\frac{\alpha}{1-\alpha}}\}$.

15. $V(k_m, j_m, t)$ can also be seen as the market value of the monopolistic firm or the value of the patent.

16. Alternatively, we could get the value for $V(k, j, t)$ from a dynamic programming equation, as in Thompson and Waldo (1994).

17. Note that, in steady state, $\hat{Q}_H(t) = \hat{Q}_L(t)$. From (39), and after some mathematical manipulations, we can get (42) and (43). Furthermore, note that it is not possible to obtain an analytic expression for $\left[\frac{H}{L}\right]^*$ in steady state because when (43) is combined with (27), its solution can only be found numerically.

18. The value of ρ implies that each period in our model represents a year.

19. Note that, empirically, it is difficult to distinguish between the perfect competition and the right-to-manage.

20. At least until 1984.

REFERENCES

Acemoglu, D. (1998) Why do new technologies complement skills? Directed technical change and wage inequality. *Quarterly Journal of Economics* 113(4), 1055–1090.

Acemoglu, D. (2002) Technical change, inequality, and the labor market. *Journal of Economic Literature* 40(1), 7–72.

Acemoglu, D. (2003) Patterns of skill premia. *Review of Economic Studies* 70(2), 199–230.

Acemoglu, D. (2015) Localised and biased technologies: Atkinson and Stiglitz's new view, induced innovations, and directed technological change. *Economic Journal* 125(583), 443–463.

Acemoglu, D., P. Aghion, and G. L. Violante (2001) Deunionization, technical change and inequality. *Carnegie-Rochester Conference Series on Public Policy* 55(1), 229–264.

Acemoglu, D. and F. Zilibotti (2001) Productivity differences. *Quarterly Journal of Economics* 116(2), 563–606.

Afonso, O. (2006) Skill-biased technological knowledge without scale effects. *Applied Economics* 38(1), 13–21.

Aghion, P. (2002) Schumpeterian growth theory and the dynamics of income inequality. *Econometrica* 70(3), 855–882.

Aghion, P. and P. Howitt (1992) A model of growth through creative destruction. *Econometrica* 60(2), 323–351.

Attanasio, O. P. and G. Weber (1993) Consumption growth, the interest rate and aggregation. *The Review of Economic Studies* 60(3), 631–649.

Barro, R. and X. Sala-i-Martin (2004) *Economic Growth*, 2nd ed. Cambridge, MA: The MIT Press.

Becker, G. and K. Murphy (1992) The division of labour, coordination costs, and knowledge. *Quarterly Journal of Economics* 107(4), 1137–1160.

Bertinelli, L., O. Cardi, and P. Sen (2013) Deregulation shock in product market and unemployment. *Journal of Economic Dynamics and Control* 37(4), 711–734.

Blanchard, O. (2004) The economic future of Europe. *Journal of Economic Perspectives* 18(4), 3–26.

Blanchard, O. (2007) Adjustment within the euro. The difficult case of Portugal. *Portuguese Economic Journal* 6(1), 1–21.

Booth, A. L. (1995) *The Economics of the Trade Union*. Cambridge: Cambridge University Press.

Booth, A. L. (2014) Wage determination and imperfect competition. *Labour Economics* 30, 53–58.

Bryson, A. (2007) The effect of trade unions on wages. *Reflects et perspectives de la vie économique* 46(2–3), 33–45.

Chang, J.-J. and H.-W. Hung (2016) Trade unions, unemployment, economic growth, and income inequality. *Macroeconomic Dynamics* 20(1), 404–428.

Chang, J.-J., M.-F. Shaw, and C.-C. Lai (2007) A "managerial" trade union and economic growth. *European Economic Review* 51(2), 365–384.

Checchi, D., J. Visser, and H. G. v. d. Werfhorst (2007) Inequality and Union Membership: The Impact of Relative Earnings Position and Inequality Attitudes. Discussion paper series IZA no. 2691.

Chu, A. C., G. Cozzi, and Y. Furukawa (2016) Unions, innovation and cross-country wage inequality. *Journal of Economic Dynamics and Control* 64, 104–118.

Chusseau, N., M. Dumont, and J. Hellier (2008) Explaining rising inequality: Skill-biased technical change and north–south trade. *Journal of Economic Surveys* 22(3), 409–457.

Clark, A. (1990) Efficient bargains and the Mcdonald–Solow conjecture. *Journal of Labor Economics* 8(4), 502–528.

Connolly, M. (2003) The dual nature of trade: Measuring its impact on imitation and growth. *Journal of Development Economics* 72(1), 31–55.

Dinopoulos, E. and P. Thompson (1999) Scale effects in Schumpeterian models of economic growth. *Journal of Evolutionary Economics* 9(2), 157–185.

Dobbelaere, S. (2004) Estimation of price-cost margins and union bargaining power for Belgian manufacturing. *International Journal of Industrial Organization* 22(10), 1381–1398.

Dobbelaere, S., K. Kiyota, and J. Mairesse (2015) Product and labor market imperfections and scale economies: Micro-evidence on France, Japan and the Netherlands. *Journal of Comparative Economics* 43(2), 290–322.

Dobbelaere, S. and J. Mairesse (2013) Panel data estimates of the production function and product and labor market imperfections. *Journal of Applied Econometrics* 28(1), 1–46.

Donado, A. and K. Wälde (2012) How trade unions increase welfare. *Economic Journal* 122(563), 990–1009.

Dowrick, S. (1990) Wage pressure, bargaining and price-cost margins in UK manufacturing. *The Journal of Industrial Economics* 38(3), 239–267.

Dunlop, J. (1944) *Wage Determination Under Trade Unions*. New York: Macmillan.

Etro, F. (2008) Growth leaders. *Journal of Macroeconomics* 30(3), 1148–1172.

Fadinger, H. and K. Mayr (2014) Skill-biased technological change, unemployment, and brain drain. *Journal of the European Economic Association* 12(2), 397–431.

Faia, E. and L. Rossi (2013) Union power, collective bargaining, and optimal monetary policy. *Economic Inquiry* 51(1), 408–427.

Feng, L. (2014) Taste shocks, endogenous labor supply, and equity home bias. *Macroeconomic Dynamics* 18(7), 1466–1487.

Freeman, R. B. and J. L. Medoff (1984) *What Do Unions Do?* New York: Basic Books.

https://doi.org/10.1017/S1365100516001346 Published online by Cambridge University Press

Galor, O. and O. Moav (2000) Ability-biased technological transition, wage inequality, and economic growth. *Quarterly Journal of Economics* 115(2), 469–497.

Gartner, H., T. Schank, and C. Schnabel (2013) Wage cyclicality under different regimes of industrial relations. *Industrial Relations: A Journal of Economy and Society* 52(2), 516–540.

Gil, P., O. Afonso, and P. B. Vasconcelos (2016) A note on skill-structure shocks, the share of the high-tech sector, and economic growth dynamics. *Macroeconomic Dynamics* 20(7), 1906–1923.

Gil, P. M., P. Brito, and O. Afonso (2013) Growth and firm dynamics with horizontal and vertical R&D. *Macroeconomic Dynamics* 17, 1438–1466.

Grossman, G. and E. Helpman (1991) *Innovation and Growth in the Global Economy*. Cambridge, MA: MIT Press.

Hornstein, A., P. Krusell, and G. Violante (2005) The effects of technical change on labor market inequalities. In P. Aghion and S. N. Durlauf (eds.), *Handbook of Economic Growth*, vol. 1, (suppl. part b), chap. 20, pp. 1275–1370. Amsterdam: Elsevier B.V.

Johnson, G. E. (1990) Work rules, featherbedding, and Pareto-optimal union-management bargaining. *Journal of Labor Economics* 8(1), S237–S259.

Karabarbounis, L. and B. Neiman (2014) The global decline of the labor share. *The Quarterly Journal of Economics* 129(1), 61–103.

Kaufman, B. (2002) Models of union wage determination: What have we learned since Dunlop and Ross? *Industrial Relations* 41(1), 110–158.

Kortum, S. (1997) Research, patenting, and technological change. *Econometrica* 65(6), 1389–1419.

Kreickemeier, U. and D. Nelson (2006) Fair wages, unemployment and technological change in a global economy. *Journal of International Economics* 70(2), 451–469.

Krusell, P. and L. Rudanko (2016) Unions in a frictional labor market. *Journal of Monetary Economics* 80, 35–50.

Kurokawa, Y. (2014) A survey of trade and wage inequality: Anomalies, resolutions and new trends. *Journal of Economic Surveys* 28(1), 169–193.

Kwan, Y. K. and E. L.-C. Lai (2003) Intellectual property rights protection and endogenous economic growth. *Journal of Economic Dynamics and Control* 27(5), 853 – 873.

Lawson, N. (2011) Is collective bargaining Pareto efficient? A survey of the literature. *Journal of Labor Research* 32(3), 282–304.

Layard, R. and S. Nickell (1990) Is unemployment lower if unions bargain over employment? *Quarterly Journal of Economics* 105(3), 773–787.

Lingens, J. (2003) The impact of a unionised labour market in a Schumpeterian growth model. *Labour Economics* 10(1), 91–104.

Lingens, J. (2007) Unions, wage setting, and economic growth. *Economic Modelling* 24(1), 167–188.

McDonald, I. M. and R. M. Solow (1981) Wage bargaining and employment. *American Economic Review* 71(5), 896–908.

Millard, S. P. and D. T. Mortensen (1997) *The Unemployment and Welfare Effects of Labour Market Policy: A Comparison of the USA and the UK*. Cambridge: Cambridge University Press.

Mortensen, D. and C. Pissarides (1994) Job creation and job destruction in the theory of unemployment. *Review of Economic Studies* 61(3), 397–415.

Nelson, R. R. and E. S. Phelps (1966) Investment in humans, technological diffusion, and economic growth. *American Economic Review* 56(1/2), 69–75.

Oesch, D. (2010) What explains high unemployment among low-skilled workers? Evidence from 21 OECD countries. *European Journal of Industrial Relations* 16(1), 39–55.

Pencavel, J. H. (1984) The tradeoff between wages and employment in trade union objectives. *The Quarterly Journal of Economics* 99(2), 215–231.

Reis, A. B. and T. N. Sequeira (2007) Human capital and overinvestment in R&D. *The Scandinavian Journal of Economics* 109(3), 573–591.

Ross, A. (1948) *Trade Union Wage Policy*. Berkeley: University of California Press.

Scheuer, S. (2011) Union membership variation in Europe: A ten-country comparative analysis. *European Journal of Industrial Relations* 17(1), 57–73.

Schultz, T. (1975) The value of the ability to deal with disequilibria. *Journal of Economic Literature* 13(3), 827–846.

Şener, F. (2008) R&D policies, endogenous growth and scale effects. *Journal of Economic Dynamics and Control* 32(12), 3895–3916.

Thompson, P. and D. Waldo (1994) Growth and trustified capitalism. *Journal of Monetary Economics* 34(3), 445–462.

Violante, G. L. (2008) Skill-biased technical change. In S. N. Durlauf and L. E. Blume (eds.), *The New Palgrave Dictionary of Economics*, 2nd ed. Basingstoke: Palgrave Macmillan. Retrieved 7 April 2017 from http://www.dictionaryofeconomics.com/article?id=pde2008_S000493

Wälde, K. (2000) Egalitarian and elitist education systems as the basis for international differences in wage inequality. *European Journal of Political Economy* 16(3), 445–468.

APPENDIX A: BARGAINING CASE—CONTRACT CURVE

$$\max_{W_L, L} \Omega = \left(U - \overline{U}\right)^{\theta} \left(\Pi - \overline{\Pi}\right)^{1-\theta}$$

$$s.t.: x_j = \text{argmax}_{x_{jn}} \Pi.$$

Notice that

$$\Pi - \overline{\Pi} = \Pi_L = P_n(t).Y_L(t) - W_L(t).L_n(t) - \int_0^J x_{jn}(t).P_j(t)dj.$$

- The first-order conditions are:

$$\frac{\partial \Omega}{\partial W_L} = 0$$

$$\theta U^{\theta-1} (1-v) \left(W_l - \overline{W}\right)^{-v} . (L_n)^v (\Pi_L)^{1-\theta} + U^{\theta} (1-\theta) (\Pi_L)^{1-\theta} (-L_n) = 0$$

$$\frac{\partial \Omega}{\partial L_n} = 0 \leftrightarrow \theta U^{\theta-1} v \left(W_l - \bar{W}\right)^{1-v} . (L_n)^{v-1} (\Pi_L)^{1-\theta} + U^{\theta} . \frac{\partial (\Pi_L)^{1-\theta}}{\partial L_n} = 0.$$

Therefore,

$$\frac{\partial (\Pi_L)^{1-\theta}}{\partial L_n} = (1-\theta) \Pi_L^{-\theta} . \Bigg(P_n A \left\{ \int_0^J \left[q^{k(j,t)} x_n (k, j, t)\right]^{\alpha} dj \right\} \times \beta \left[(1-n) l L_n\right]^{\beta-1} . (1-n) l - W_L(t) \Bigg).$$

Finally, we get

$$\frac{\partial \Omega}{\partial L_n} = 0 \leftrightarrow \theta U^{\theta-1} v \left(W_l - \bar{W}\right)^{1-v} . (L_n)^{v-1} \Pi_L^{1-\theta} + U^{\theta} . (1-\theta) \Pi_L^{-\theta} .$$

$$\left(P_n(t).A. \left\{ \int_0^J \left[q^{k(j,t)} x_n (k, j, t)\right]^{\alpha} dj \right\} \beta \left[(1-n) l L_n\right]^{\beta-1} . (1-n) l - W_L(t) \right) = 0.$$

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The next step is to find the optimal conditions for wage and employment

$$\frac{\frac{\partial \Omega}{\partial W_L}}{\frac{\partial \Omega}{\partial L_n}} = 0$$

$$\frac{\theta U^{\theta-1}(1-v)\left(W_l-\bar{W}\right)^{-v}.(L_n)^v\,\Pi_L^{1-\theta}}{\theta U^{\theta-1}v\left(W_l-\bar{W}\right)^{1-v}.(L_n)^{v-1}\,\Pi_L^{1-\theta}} =$$

$$\frac{U^{\theta}(1-\theta)\,\Pi_L^{-\theta}(L_n)}{-\left[U^{\theta}.(1-\theta)\,\Pi_L^{-\theta}.\left(P_n(t).A.\left\{\int_0^J\left[q^{k(j,t)}x_n(k,j,t)\right]^{\alpha}dj\right\}\beta\left[(1-n)\,lL_n\right]^{\beta-1}.(1-n)\,l-W_L(t)\right)\right]}$$

$$\frac{1-v}{v}\left(W_l-\bar{W}\right)^{-1}L_n =$$

$$-L_n.\left(P_n(t).A.\left\{\int_0^J\left[q^{k(j,t)}x_n(k,j,t)\right]^{\alpha}dj\right\}\right.$$

$$\left.\times\beta\left[(1-n)\,lL_n\right]^{\beta-1}.(1-n)\,l-W_L(t)\right)^{-1}.$$

Therefore, we get the contract curve:

$$\left(W_l-\bar{W}\right)=\frac{1-v}{v}\left(W_L-\beta L_n^{-1}\left[(1-n)\,lL_n\right]^{\beta}.P_n(t).A.\left\{\int_0^J\left[q^{k(j,t)}x_n(k,j,t)\right]^{\alpha}dj\right\}\right).$$

Note that we still have W_t in both sides of the previous equation. Thus, recalling that

$$\frac{\partial L}{\partial W_L}=0\leftrightarrow\theta U^{\theta-1}(1-v)\left(W_l-\overline{W}\right)^{-v}.(L_n)^v\,\Pi_L^{1-\theta}-U^{\theta}(1-\theta)\,\Pi_L^{-\theta}L_n=0$$

$$U^{\theta}\Pi_L^{1-\theta}\left[\theta U^{-1}(1-v)\left(W_l-\overline{W}\right)^{-v}.(L_n)^v-(1-\theta)\,L_n\Pi_L^{-1}\right]=0$$

$$\theta U^{-1}(1-v)\left(W_l-\overline{W}\right)^{-v}.(L_n)^v=(1-\theta)\,L_n\Pi_L^{-1}.$$

Notice that

$$U=\left[\left(W_t-\bar{W}\right)^{1-v}(L_n)^v\right]$$

$$\Pi_L=\left[P_n(t).Y_L(t)-W_L(t).L_n(t)-\int_0^J x_{jn}(t).P_j(t)dj\right],$$

where

$$\int_0^J x_{jn}(t).P_j(t)dj=\int_0^J x_{jn}(t)\left\{A.P_n\left[(1-n).l.L_n\right]^{\beta}\alpha\left(q^{k(j,t)}\right)^{\alpha}\left[\frac{1}{x_n(k,j,t)|j\epsilon[0,J]}\right]^{1-\alpha}\right\}dj.$$

To simplify, please note that

$$Y_L=\left[(1-n)\,lL_n\right]^{\beta}.P_n(t).A.\left\{\int_0^J\left[q^{k(j,t)}x_n(k,j,t)\right]^{\alpha}dj\right\}.$$

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Which implies that

$$\int_0^J x_{jn}(t).P_j(t)dj = \alpha Y_L.$$

Finally, we get

$$\frac{\theta\,(1-v)\left(W_l-\overline{W}\right)^{-v}.\,(L_n)^v}{\left[\left(W_t-\bar{W}\right)^{1-v}(L_n)^v\right]} = (1-\theta)\,L_n\,[Y_L - W_L L_n - \alpha Y_L]$$

$$\theta\,(1-v)\left(W_l-\overline{W}\right)^{-1} = (1-\theta)\,L_n\,[(1-\alpha)\,Y_L - W_L L_n]^{-1}$$

$$\left(W_l-\overline{W}\right)(1-\theta) = \theta(1-v)\,(1-\alpha)\,Y_L L_n^{-1} - W_L.$$

Recalling that

$$\left(W_l-\bar{W}\right) = \frac{1-v}{v}\left(W_L - \beta L_n^{-1}\,[(1-n)\,lL_n]^{\beta}\,.P_n(t).A.\left\{\int_0^J\left[q^{k(j,t)}x_n\,(k,j,t)\right]^{\alpha}dj\right\}\right).$$

And after some manipulations, we get

$$W_L = \left[\beta + \frac{\theta v\,(1-\alpha-\beta)}{(1-\theta)+\theta v}\right] L_n^{-1} Y_L.$$

APPENDIX B: THRESHOLD ANALYSIS

From the firm's maximization problem, we know that

$$Y_n(t) = A^{\frac{1}{1-\alpha}}\left[\frac{\alpha P_n(t)}{P(j,t)}\right]^{\frac{\alpha}{1-\alpha}}\left\{[(1-n)\,.l.L_n]^{\frac{\beta}{1-\alpha}}\,Q_L(t) + [nhH_n]^{\frac{\beta}{1-\alpha}}\,Q_H(t)\right\}.$$

And from the consumer's maximization problem, we know that $P_n\,(t)\,Y_n\,(t)$is equal across all n. In turn, this implies that for final goods $n = 0$ and $n = 1$, we can write

$$P_0\,(t)\,Y_0\,(t) = P_1\,(t)\,Y_1\,(t)\,.$$

Substitution in Y_n, we get

$$P_0\,(t)\,Y_0\,(t) = P_0\left\{A^{\frac{1}{1-\alpha}}\,(1-n)^{\beta}\,(lL_0)^{\frac{\beta}{1-\alpha}}\,Q_L(t)\left[\frac{\alpha P_L}{P(j,t)}\right]^{\frac{\alpha}{1-\alpha}}\right\}$$

$$P_1\,(t)\,Y_1\,(t) = P_1\left\{A^{\frac{1}{1-\alpha}}n^{\beta}\,(hH_1)^{\frac{\beta}{1-\alpha}}\,Q_H(t)\left[\frac{\alpha P_H}{P(j,t)}\right]^{\frac{\alpha}{1-\alpha}}\right\}.$$

Therefore,

$$P_0\left[(lL_0)^{\frac{\beta}{1-\alpha}}\,Q_L(t)\,(P_L)^{\frac{\alpha}{1-\alpha}}\right] = P_1\left[(hH_1)^{\frac{\beta}{1-\alpha}}\,Q_H(t)\,(P_H)^{\frac{\alpha}{1-\alpha}}\right].$$

Knowing that

$$P_L^{\frac{1}{1-\alpha}} = (1-n)^{\frac{\beta}{1-\alpha}} P_n^{\frac{1}{1-\alpha}} \quad ; \quad n=0 \rightarrow P_L = P_0$$
$$P_H^{\frac{1}{1-\alpha}} = n^{\frac{\beta}{1-\alpha}} P_n^{\frac{1}{1-\alpha}} \quad ; \quad n=1 \rightarrow P_H = P_1.$$

Thus,

$$P_L \left[(lL_0)^{\frac{\beta}{1-\alpha}} (P_L)^{\frac{\alpha}{1-\alpha}} Q_L(t) \right] = P_H \left[(hH_1)^{\frac{\beta}{1-\alpha}} (P_H)^{\frac{\alpha}{1-\alpha}} Q_H(t) \right]$$
$$P_L^{\frac{1}{1-\alpha}} (lL_0)^{\frac{\beta}{1-\alpha}} Q_L(t) = P_H^{\frac{1}{1-\alpha}} (hH_1)^{\frac{\beta}{1-\alpha}} Q_H(t).$$

In equilibrium, L_n is constant across $n \epsilon [0, \bar{n}]$, and H_n is constant across $n \epsilon [\bar{n}, 1]$. Thus,

$$L_n = \tfrac{L}{\bar{n}} \quad ; \quad \bar{n} \epsilon [0, \bar{n}]$$
$$H_n = \tfrac{H}{1-\bar{n}} \quad ; \quad \bar{n} \epsilon [\bar{n}, 1].$$

This can be proven as follows:

We know that $P_n(t) Y_n(t)$ is constant across n and that $P_n = P_L (1-n)^{-\beta}$. Then,

$$P_n Y_n = P_n Y_n \leftrightarrow P_0 Y_0 = P_n Y_n$$
$$P_L (1-0)^{-\beta} Y_0 = P_L (1-n)^{-\beta} Y_n \leftrightarrow P_L Y_0 = P_L (1-n)^{-\beta} Y_n.$$

Substituting Y_n by the previous equations:

$$P_L \left\{ A^{\frac{1}{1-\alpha}} (lL_0)^{\frac{\beta}{1-\alpha}} Q_L(t) \left[\frac{\alpha P_L}{P(j,t)} \right]^{\frac{\alpha}{1-\alpha}} \right\} =$$
$$P_L (1-n)^{-\beta} (1-n)^{\beta} \left\{ A^{\frac{1}{1-\alpha}} (lL_0)^{\frac{\beta}{1-\alpha}} Q_L(t) \left[\frac{\alpha P_L}{P(j,t)} \right]^{\frac{\alpha}{1-\alpha}} \right\}.$$

This hold only if $L_0 = L_n$ for all other $n \epsilon [0, \bar{n}]$. Thus, it must be that all firms $n \epsilon [0, \bar{n}]$ use the same amount of low-skilled labor:

$$L_n = \frac{L}{\bar{n}}.$$

This can also be proved for high-skilled workers.
Therefore, we have

$$P_L^{\frac{1}{1-\alpha}} \left(l \frac{L}{\bar{n}} \right)^{\frac{\beta}{1-\alpha}} Q_L(t) = P_H^{\frac{1}{1-\alpha}} \left(h \frac{H}{1-\bar{n}} \right)^{\frac{\beta}{1-\alpha}} Q_H(t)$$

$$\left(\frac{P_H}{P_L} \right)^{-\frac{1}{1-\alpha}} = \left(\frac{h}{l} \frac{H}{L} \frac{\bar{n}}{1-\bar{n}} \right)^{\frac{\beta}{1-\alpha}} \frac{Q_H(t)}{Q_L(t)} \leftrightarrow \left(\frac{P_H}{P_L} \right) = \left(\frac{h}{l} \frac{H}{L} \frac{\bar{n}}{1-\bar{n}} \right)^{-\beta} \left[\frac{Q_H(t)}{Q_L(t)} \right]^{-(1-\alpha)}$$

$$\left[\frac{\bar{n}^{\beta} P_n}{(1-\bar{n})^{\beta} P_n} \right] = \left(\frac{h}{l} \frac{H}{L} \frac{\bar{n}}{1-\bar{n}} \right)^{-\beta} \left[\frac{Q_H(t)}{Q_L(t)} \right]^{-(1-\alpha)}.$$

Therefore,

$$\left[\frac{\bar{n}}{(1-\bar{n})}\right]^{\beta} = \left(\frac{h}{l}\frac{H}{L}\right)^{-\beta}\left[\frac{Q_H(t)}{Q_L(t)}\right]^{-(1-\alpha)}\left(\frac{\bar{n}}{1-\bar{n}}\right)^{-\beta} \leftrightarrow \left(\frac{\bar{n}}{1-\bar{n}}\right)^{2\beta}$$

$$= \left(\frac{h}{l}\frac{H}{L}\right)^{-\beta}\left[\frac{Q_H(t)}{Q_L(t)}\right]^{-(1-\alpha)}$$

$$\left(\frac{\bar{n}}{1-\bar{n}}\right) = \left\{\left(\frac{h}{l}\frac{H}{L}\right)^{-1}\left[\frac{Q_H(t)}{Q_L(t)}\right]^{-\frac{1-\alpha}{\beta}}\right\}^{\frac{1}{2}} \leftrightarrow \left(\frac{1-\bar{n}}{\bar{n}}\right)$$

$$= \left\{\left(\frac{h}{l}\frac{H}{L}\right)^{1}\left[\frac{Q_H(t)}{Q_L(t)}\right]^{\frac{1-\alpha}{\beta}}\right\}^{\frac{1}{2}}.$$

Defining the r.h.s. by X, we get

$$\left(\frac{1-\bar{n}}{\bar{n}}\right) = X \leftrightarrow 1-\bar{n} = \bar{n}X \leftrightarrow 1 = \bar{n}X + \bar{n} \leftrightarrow 1 = \bar{n}(1+X) \leftrightarrow \bar{n} = (1+X)^{-1}.$$

Finally, we get the threshold:

$$\bar{n} = \left(1 + \left\{\frac{h}{l}\frac{H}{L}\left[\frac{Q_H(t)}{Q_L(t)}\right]^{\frac{1-\alpha}{\beta}}\right\}^{\frac{1}{2}}\right)^{-1}.$$

APPENDIX C: PRICES AND MACROECONOMIC AGGREGATES

Taking into consideration that (exp) $\int_0^1 lnp_n dn = 1$, we can rewrite the prices indexes P_L and P_H as follows:

$$\begin{cases} P_L = P_n(1-n)^{\beta} = \exp(-\beta)\,\bar{n}^{-\beta} \\ P_H = P_n(n)^{\beta} = \exp(-\beta)(1-\bar{n})^{-\beta}. \end{cases}$$

To prove this, notice that

$$(\exp)\int_0^1 lnp_n dn = 1 \leftrightarrow \int lnp_n dn = ln(1) = 0.$$

Replacing by the price indexes:

$$0 = \int_0^{\bar{n}} ln\left[P_L(1-n)^{-\beta}\right]dn + \int_{\bar{n}}^1 ln\left[P_H(n)^{-\beta}\right]dn,$$

$$0 = \int_0^{\bar{n}} [lnP_L - \beta ln(1-n)]\,dn + \int_{\bar{n}}^1 [lnP_H - \beta ln(n)]\,dn,$$

$$0 = \bar{n}lnP_L + (1-\bar{n})\,lnP_H - \beta\left[\int_0^{\bar{n}} ln(1-n)\,dn + \int_{\bar{n}}^1 ln(n)\,dn\right].$$

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Integrating by parts

$$(a)\int_0^{\bar{n}} ln\,(1-n)\,dn \rightarrow (\bar{n}-1)\,ln\,(1-\bar{n})-\bar{n}.$$

$$(b)\int_{\bar{n}}^1 ln\,(n)\,dn \rightarrow -1-ln\,(\bar{n})\,\bar{n}+\bar{n}$$

$$\int_0^{\bar{n}} ln\,(1-n)\,dn+\int_{\bar{n}}^1 ln\,(n)\,dn=-1+(\bar{n}-1)\,ln\,(1-\bar{n})-\bar{n}ln\,(\bar{n})\,.$$

This can be seen as follows:

$$\int_a^b u\,(x)\,v'\,(x)\,dx=[u\,(x)\,v\,(x)]_a^b-\int_a^b u'\,(x)\,v\,(x)\,dx.$$

In our case, if we denote $u(x)=ln\,(1-n)$ and $v\,(x)=n$, then,

$$(a)\int_0^{\bar{n}} \underbrace{(1-n)}_{u(x)}\underbrace{1}_{v'(x)}dn$$

Therefore,

$$\int_0^{\bar{n}} (1-n)\,dn=[ln\,(1-n)\,n]_0^{\bar{n}}-\int_0^{\bar{n}}\frac{-1}{1-n}ndn.$$

Notice that the last part of the previous expression is equal to

$$\int_0^{\bar{n}}\frac{-n}{1-n}dn=\int_0^{\bar{n}}\frac{-n+1-1}{1-n}=\int_0^{\bar{n}}\left(\frac{-n+1}{1-n}-\frac{1}{1-n}\right)dn$$
$$=\int_0^{\bar{n}} 1dn+\int_0^{\bar{n}}\frac{-1}{1-n}dn=\bar{n}+ln\,(1-\bar{n})\,.$$

Then,

$$\int_0^{\bar{n}} (1-n)\,dn=\bar{n}ln\,(1-\bar{n})-[\bar{n}+ln\,(1-\bar{n})]=(\bar{n}-1)\,ln\,(1-\bar{n})-\bar{n}.$$

Integrating by parts

$$(b)\int_{\bar{n}}^1 ln\,(n)\,dn$$

In this case, $u(x)=ln(n)$ and $v(x)=n$. Then,

$$\int_{\bar{n}}^1 ln\,(n)\,dn=[ln\,(n)\,dn]_{\bar{m}}^1-\int_{\bar{n}}^1\frac{1}{n}ndn=-[ln\,(\bar{n})\,\bar{n}]-[1-\bar{n}]=-1-ln\,(\bar{n})\,\bar{n}+\bar{n}.$$

Recalling that

$$0=\bar{n}lnP_L+(1-\bar{n})\,lnP_H-\beta\left[\int_0^{\bar{n}} ln\,(1-n)\,dn+\int_{\bar{n}}^1 ln\,(n)\,dn\right].$$

We get

$$0=\bar{n}lnP_L+(1-\bar{n})\,lnP_H-\beta\,[-1+(\bar{n}-1)\,ln\,(1-\bar{n})-\bar{n}ln\,(\bar{n})]\,,$$

$$0 = \bar{n} ln P_L + (1-\bar{n}) ln P_H + \beta \left[1 + (1-\bar{n}) ln (1-\bar{n}) + \underbrace{\bar{n} ln (\bar{n}) - ln (\bar{n})}_{(1-\bar{n}) ln (\bar{n})} + ln (\bar{n}) \right],$$

$$0 = \bar{n} ln P_L + (1-\bar{n}) ln P_H + \beta \left[1 + (1-\bar{n}) ln \left(\frac{1-\bar{n}}{\bar{n}} \right) + ln (\bar{n}) \right].$$

Notice that

$$\frac{P_H}{P_L} = \frac{P_n \bar{n}^{\beta}}{P_n (1-\bar{n})^{\beta}} = \left(\frac{\bar{n}}{1-\bar{n}} \right)^{\beta}.$$

This implies that

$$ln P_H - ln P_L = \beta ln \left(\frac{\bar{n}}{1-\bar{n}} \right),$$

where $\beta ln \left(\frac{\bar{n}}{1-\bar{n}} \right) = \beta [ln (\bar{n}) - ln (1-\bar{n})] = -\beta [ln (1-\bar{n}) - ln (\bar{n})] = -\beta ln \left(\frac{1-\bar{n}}{\bar{n}} \right)$.
Solving for

$$ln P_L = ln P_H + \beta ln \left(\frac{1-\bar{n}}{\bar{n}} \right).$$

Thus, replacing

$$0 = \bar{n} \left[ln P_H + \beta ln \left(\frac{1-\bar{n}}{\bar{n}} \right) \right] + (1-\bar{n}) ln P_H + \beta \left[1 + (1-\bar{n}) ln \left(\frac{1-\bar{n}}{\bar{n}} \right) + ln (\bar{n}) \right],$$

$$0 = \bar{n} ln P_H + \bar{n} \beta ln \left(\frac{1-\bar{n}}{\bar{n}} \right) + ln P_H - \bar{n} ln P_H + \beta + \beta ln \left(\frac{1-\bar{n}}{\bar{n}} \right) - \beta \bar{n} ln \left(\frac{1-\bar{n}}{\bar{n}} \right) + \beta ln (\bar{n}),$$

$$0 = ln P_H + \beta + \beta ln (1-\bar{n}),$$

$$ln P_H = -\beta - \beta ln (1-\bar{n}),$$

$$P_H = \exp(-\beta) (1-\bar{n})^{-\beta}.$$

Similarly, one can prove that

$$P_L = \exp(-\beta) (\bar{n})^{-\beta}.$$

Therefore, knowing that $\bar{n} = (1 + \{ \frac{h}{l} \frac{H}{L} [\frac{Q_H(t)}{Q_L(t)}]^{\frac{1-\alpha}{\beta}} \}^{\frac{1}{2}})^{-1}$, we can write the prices in a more complex form:

$$P_L = \exp(-\beta) \left(1 + \left\{ \frac{h}{l} \frac{H}{L} \left[\frac{Q_H(t)}{Q_L(t)} \right]^{\frac{1-\alpha}{\beta}} \right\}^{\frac{1}{2}} \right)^{\beta}.$$

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Since $(1-\bar{n}) = \bar{n}\{\frac{h}{l}\frac{H}{L}[\frac{Q_H(t)}{Q_L(t)}]^{\frac{1-\alpha}{\beta}}\}^{\frac{1}{2}}$, this implies that

$$P_H = \exp(-\beta)\left(\bar{n}\left\{\frac{h}{l}\frac{H}{L}\left[\frac{Q_H(t)}{Q_L(t)}\right]^{\frac{1-\alpha}{\beta}}\right\}^{\frac{1}{2}}\right)^{-\beta}.$$

Therefore,

$$P_H = \exp(-\beta)\left[\left(1+\left\{\underbrace{\frac{h}{l}\frac{H}{L}\left[\frac{Q_H(t)}{Q_L(t)}\right]^{\frac{1-\alpha}{\beta}}}_{Z}\right\}^{\frac{1}{2}}\right)^{-1}\left\{\underbrace{\frac{h}{l}\frac{H}{L}\left[\frac{Q_H(t)}{Q_L(t)}\right]^{\frac{1-\alpha}{\beta}}}_{Z}\right\}^{\frac{1}{2}}\right]^{-\beta}$$

$$P_H = \exp(-\beta)\left[\left(1+Z^{\frac{1}{2}}\right)^{-1}Z^{\frac{1}{2}}\right]^{-\beta}.$$

Notice that $\frac{1+Z^{-\frac{1}{2}}}{Z^{-\frac{1}{2}}} = \frac{1}{Z^{-\frac{1}{2}}} + 1 = Z^{\frac{1}{2}} + 1$

$$P_H = \exp(-\beta)\left(1+\left\{\frac{h}{l}\frac{H}{L}\left[\frac{Q_H(t)}{Q_L(t)}\right]^{\frac{1-\alpha}{\beta}}\right\}^{-\frac{1}{2}}\right)^{\beta}.$$

Finally, combining these two price indexes, we can also rewrite X and Y as follows:

$$X = \left(\frac{\alpha A}{\frac{1}{\alpha}}\right)^{\frac{1}{1-\alpha}}\left[P_L^{\frac{1}{1-\alpha}}(lL_n)^{\frac{\beta}{1-\alpha}}Q_L + P_H^{\frac{1}{1-\alpha}}(hH_n)^{\frac{\beta}{1-\alpha}}Q_H\right],$$

$$X = \left(\frac{\alpha A}{\frac{1}{\alpha}}\right)^{\frac{1}{1-\alpha}}\left[\exp(-\beta)\left(1+\left\{\frac{h}{l}\frac{H}{L}\left[\frac{Q_H(t)}{Q_L(t)}\right]^{\frac{1-\alpha}{\beta}}\right\}^{\frac{1}{2}}\right)^{\beta}\right]^{\frac{1}{1-\alpha}}(lL_n)^{\frac{\beta}{1-\alpha}}Q_L$$

$$+\left(\frac{\alpha A}{\frac{1}{\alpha}}\right)^{\frac{1}{1-\alpha}}\left[\exp(-\beta)\left(1+\left\{\frac{h}{l}\frac{H}{L}\left[\frac{Q_H(t)}{Q_L(t)}\right]^{\frac{1-\alpha}{\beta}}\right\}^{-\frac{1}{2}}\right)^{\beta}\right]^{\frac{1}{1-\alpha}}(hH_n)^{\frac{\beta}{1-\alpha}}Q_H,$$

$$X\left(\frac{\alpha A}{\frac{1}{\alpha}}\right)^{-\frac{1}{1-\alpha}}\exp\left(-\frac{\beta}{1-\alpha}\right)^{-1}$$

$$= \left(1+\left\{\frac{h}{l}\frac{H}{L}\left[\frac{Q_H(t)}{Q_L(t)}\right]^{\frac{1-\alpha}{\beta}}\right\}^{\frac{\beta}{2(1-\alpha)}}\right)(lL_n)^{\frac{\beta}{1-\alpha}}Q_L$$

$$+\left(1+\left\{\frac{h}{l}\frac{H}{L}\left[\frac{Q_H(t)}{Q_L(t)}\right]^{\frac{1-\alpha}{\beta}}\right\}^{-\frac{\beta}{2(1-\alpha)}}\right)(hH_n)^{\frac{\beta}{1-\alpha}}Q_H,$$

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$$X\left(\frac{\alpha A}{\frac{1}{\alpha}}\right)^{-\frac{1}{1-\alpha}}\exp\left(-\frac{\beta}{1-\alpha}\right)^{-1}$$
$$= (lL_n)^{\frac{\beta}{1-\alpha}} Q_L + (lL_n)^{\frac{\beta}{1-\alpha}} Q_L \left[hHQ_H(t)^{\frac{1-\alpha}{\beta}}\right]^{\frac{\beta}{2(1-\alpha)}}\left[lLQ_L(t)^{\frac{1-\alpha}{\beta}}\right]^{-\frac{\beta}{2(1-\alpha)}}$$
$$+ (hH_n)^{\frac{\beta}{1-\alpha}} Q_H + (hH_n)^{\frac{\beta}{1-\alpha}} Q_H \left[hHQ_H(t)^{\frac{1-\alpha}{\beta}}\right]^{-\frac{\beta}{2(1-\alpha)}}\left[lLQ_L(t)^{\frac{1-\alpha}{\beta}}\right]^{\frac{\beta}{2(1-\alpha)}},$$
$$X\left(\frac{\alpha A}{\frac{1}{\alpha}}\right)^{-\frac{1}{1-\alpha}}\exp\left(-\frac{\beta}{1-\alpha}\right)^{-1}$$
$$= (lL_n)^{\frac{\beta}{1-\alpha}} Q_L + (lL_n)^{\frac{\beta}{2(1-\alpha)}} Q_L^{\frac{1}{2}} \left[hHQ_H(t)^{\frac{1-\alpha}{\beta}}\right]^{\frac{\beta}{2(1-\alpha)}}$$
$$+ (hH_n)^{\frac{\beta}{1-\alpha}} Q_H + (hH_n)^{\frac{\beta}{2(1-\alpha)}} Q_H^{\frac{1}{2}} \left\{lL\left[Q_L(t)\right]^{\frac{1-\alpha}{\beta}}\right\}^{\frac{\beta}{2(1-\alpha)}},$$
$$X = \exp\left(-\frac{\beta}{1-\alpha}\right)\left(\frac{\alpha A}{\frac{1}{\alpha}}\right)^{\frac{1}{1-\alpha}}\left\{\left[(lL_n)^{\frac{\beta}{1-\alpha}} Q_L\right]^{\frac{1}{2}} + \left[(hH_n)^{\frac{\beta}{1-\alpha}} Q_H\right]^{\frac{1}{2}}\right\}^2.$$

Similarly, we get

$$Y = \exp\left(-\frac{\beta}{1-\alpha}\right) A^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\frac{1}{\alpha}}\right)^{\frac{\alpha}{1-\alpha}}\left\{\left[(lL_n)^{\frac{\beta}{1-\alpha}} Q_L\right]^{\frac{1}{2}} + \left[(hH_n)^{\frac{\beta}{1-\alpha}} Q_H\right]^{\frac{1}{2}}\right\}^2.$$

APPENDIX D: RELATIVE CONTRACT CURVE

$$\left(W_L - \overline{W}_L\right) = \frac{1-v}{v}\left(W_L - \text{MPL}\right).$$

Since $\text{MPH} = W_H$, we have

$$\left(\frac{W_L}{W_H} - \frac{\overline{W}_L}{W_H}\right) = \frac{1-v}{v}\left(\frac{W_L}{W_H} - \frac{\text{MPL}}{\text{MPH}}\right).$$

Recalling $W_L^B = \psi W_L^{PC} = \psi \overline{W}_L$, we obtain the relative contract curve:

$$\left(\frac{W_L}{W_H} - \frac{\overline{W}_L}{W_H}\right) = \frac{1-v}{v}\left(\frac{\psi\overline{W}_L}{W_H} - \frac{\text{MPL}}{\text{MPH}}\right).$$

Hence,

$$(\psi - 1)\frac{\overline{W}_L}{W_H} = \frac{1-v}{v}\left(\frac{\psi\overline{W}_L}{W_H} - \frac{\text{MPL}}{\text{MPH}}\right)$$
$$(\psi - 1) = \frac{1-v}{v}\left(\psi - \frac{\text{MPL}}{\text{MPH}}\frac{W_H}{\overline{W}_L}\right)$$

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$$\frac{\text{MPL}}{\text{MPH}}\frac{W_H}{\overline{W}_L} = \psi - \frac{(\psi - 1)}{\frac{1-v}{v}}$$
$$\frac{\text{MPH}}{\text{MPL}} = \Phi\omega^{PC},$$

where $\Phi = \frac{\frac{1-v}{v}}{\psi(\frac{1-v}{v}-1)+1}$.

Recalling (26) and replacing it in the previous equation, we obtain the equilibrium relative demand for the bargaining case:

$$\left[\left(\frac{H}{L}\right)^{\Phi}\right]^{*} = \Phi^{\frac{2}{\Psi}}\left[\left(\frac{h}{l}\right)^{\frac{\beta}{1-\alpha}}\frac{Q_H}{Q_L}\right]^{\Theta}.$$

APPENDIX E: EQUILIBRIUM AGGREGATE R&D SPENDING

Taking into consideration that

$$pb\,(k, j, t)\,V\,(k+1, j, t) = r\delta\,(k, j, t)\,,$$

where the r.h.s. are the resources devoted to R&D, and recalling that

$$pb\,(k, j, t) = rs\,(k, j, t)\,.\varrho q^{k(j,t)}\zeta^{-1}q^{-(1-\alpha)^{-1}k(j,t)}.m^{-\frac{\beta}{1-\alpha}}.f\,(j)\,,$$

we get

$$\frac{\Pi\,(k+1, j, t)}{r\,(t) + pb\,(k+1, j, t)}rs\,(k, j, t)\,.\varrho q^{k(j,t)}\zeta^{-1}q^{-(1-\alpha)^{-1}k(j,t)}.m^{-\frac{\beta}{1-\alpha}}.f\,(j) = rs\,(k, j, t)$$
$$\Pi\,(k+1, j, t)\,.rs\,(k, j, t)\,.\varrho q^{k(j,t)}\zeta^{-1}q^{-(1-\alpha)^{-1}k(j,t)}.m^{-\frac{\beta}{1-\alpha}}.f\,(j) = r\,(t) + pb\,(k+1, j, t)\,.$$

Notice that

$$pb\,(k+1, j, t) = rs\,(k+1, j, t)\,.\varrho q^{k+1(j,t)}\zeta^{-1}q^{-(1-\alpha)^{-1}k+1(j,t)}.m^{-\frac{\beta}{1-\alpha}}.f\,(j)\,.$$

Therefore,

$$rs\,(k+1, j, t) = \frac{\varrho q^{k(j,t)}\zeta^{-1}q^{-(1-\alpha)^{-1}k(j,t)}.m^{-\frac{\beta}{1-\alpha}}.f\,(j)\,.\Pi\,(k+1, j, t) - r\,(t)}{\varrho q^{k+1(j,t)}\zeta^{-1}q^{-(1-\alpha)^{-1}k+1(j,t)}.m^{-\frac{\beta}{1-\alpha}}.f\,(j)},$$

$$rs\,(k+1, j, t) = \frac{q^{k(j,t)}q^{-(1-\alpha)^{-1}k(j,t)}.\Pi\,(k+1, j, t)}{q^{k+1(k,j)}q^{-(1-\alpha)^{-1}k+1(j,t)}}$$
$$-\frac{r(t)}{\varrho q^{k+1(j,t)}\zeta^{-1}q^{-(1-\alpha)^{-1}k+1(j,t)}.m^{-\frac{\beta}{1-\alpha}}.f\,(j)},$$

$$rs\,(k+1, j, t) = \frac{\Pi\,(k+1, j, t)}{q.q^{-\frac{1}{1-\alpha}}} - \frac{r(t)}{\varrho q^{k+1(j,t)}\zeta^{-1}q^{-\alpha^{-1}k+1(j,t)}.m^{-\frac{\beta}{1-\alpha}}.f\,(j)},$$

$$rs\,(k+1,j,t) = q^{\left(\frac{\alpha}{1-\alpha}\right)}\Pi\,(k+1,j,t) - \frac{r(t)}{\varrho q^{k+1(j,t)}\zeta^{-1}q^{-(1-\alpha)^{-1}k+1(j,t)}.m^{-\frac{\beta}{1-\alpha}}.f\,(j)}.$$

Recalling that

$$\Pi\,(k,j,t) = \left[P_j\,(\tau) - 1\right]\left\{\left[\frac{\alpha P_m A}{q\,(\tau)}\right]^{\frac{1}{1-\alpha}}(\bar{m}m)^{\frac{\beta}{1-\alpha}}\left[q^{k(j,t)}\right]^{\frac{\alpha}{1-\alpha}}\right\}$$

$\bar{m} = h$ for $m = H$ and $\bar{m} = l$ for $m = L$.
Therefore,

$$rs\,(k,j,t) = q^{\left(\frac{\alpha}{1-\alpha}\right)}\left[P_j\,(\tau) - 1\right]\left\{\left[\frac{\alpha P_m A}{q\,(\tau)}\right]^{\frac{1}{1-\alpha}}(\bar{m}m)^{\frac{\beta}{1-\alpha}}\left[q^{k(j,t)}\right]^{\frac{\alpha}{1-\alpha}}\right\}$$
$$-\frac{r(t)}{\varrho.f\,(j)}\zeta m^{\frac{\beta}{1-\alpha}}.q^{k(k,j)\frac{\alpha}{1-\alpha}}.$$

Since $p(j) = q(j)$, we then have

$$rs\,(k,j,t) = \left\{q^{\left(\frac{\alpha}{1-\alpha}\right)}(q-1)\left[\left(\frac{\alpha P_m A}{q}\right)^{\frac{1}{1-\alpha}}\bar{m}^{\frac{\beta}{1-\alpha}}\right] - r(t)\frac{\zeta}{\varrho.f\,(j)}\right\}m^{\frac{\beta}{1-\alpha}}q^{k(k,j)\frac{\alpha}{1-\alpha}}$$

$$rs\,(k,j,t) = \left\{\left(\frac{q-1}{q}\right)\left[(\alpha P_m A)^{\frac{1}{1-\alpha}}\bar{m}^{\frac{\beta}{1-\alpha}}\right] - r(t)\frac{\zeta}{\varrho.f\,(j)}\right\}m^{\frac{\beta}{1-\alpha}}q^{k(k,j)\frac{\alpha}{1-\alpha}}.$$

Equilibrium aggregate R&D spending, $R(t)$, can be computed as

$$R\,(t) = \int_0^1 rs\,(k,j,t)\,dj = \int_0^J rs\,(k,j,t)\,dj + \int_J^1 rs\,(k,j,t)\,dj$$

$$R\,(t) =$$
$$\left\{\left(\frac{q-1}{q}\right)\left[(\alpha P_L A)^{\frac{1}{1-\alpha}}\bar{l}^{\frac{\beta}{1-\alpha}}\right] - r(t)\frac{\zeta}{\varrho.f\,(j)}\right\}L^{\frac{\beta}{1-\alpha}}\underbrace{\int_0^J q^{k(k,j)\frac{\alpha}{1-\alpha}}}_{Q_L}$$
$$+\left\{\left(\frac{q-1}{q}\right)\left[(\alpha P_H A)^{\frac{1}{1-\alpha}}\bar{h}^{\frac{\beta}{1-\alpha}}\right] - r(t)\frac{\zeta}{\varrho.f\,(j)}\right\}H^{\frac{\beta}{1-\alpha}}\underbrace{\int_J^1 q^{k(k,j)\frac{\alpha}{1-\alpha}}}_{Q_H.}$$

APPENDIX F: LAW OF MOTION OF Q_m

Suppose a new quality of intermediate good j is introduced, all else remaining equal, the change in the corresponding aggregate quality indexes is given by

$$\triangle Q_m = \left(q^{k_j+1}\right)^{\frac{\alpha}{1-\alpha}} - \left(q^{k_j}\right)^{\frac{\alpha}{1-\alpha}} = \left(q^{k_j}\right)^{\frac{\alpha}{1-\alpha}}\left(q^{\frac{\alpha}{1-\alpha}} - 1\right).$$

Nevertheless, in order to understand the law of motion, we do also need to take into consideration the probability of a new quality arrives. When the free entry condition holds, we can prove that

$$pb\,(k, j, t)\; V\,(k+1, j, t) = rs\,(k, j, t)\,.$$

Notice that

$$pb\,(k-1, j, t)\; V\,(k, j, t) = rs\,(k-1, j, t)$$

$$V\,(k, j, t) = \frac{\Pi\,(k, j, t)}{r(t) + bp\,(k, j, t)} = \frac{(q-1)\left\{\left[\frac{P_m(t)A\alpha}{q}\right]^{\frac{1}{1-\alpha}} (\bar{m}m)^{\frac{\beta}{1-\alpha}}\, q^{k(k,j)\,\frac{\alpha}{1-\alpha}}\right\}}{r(t) + bp\,(k, j, t)}$$

$$pb\,(k-1, j, t) = rs\,(k-1, j, t)\,.\varrho q^{k-1(j,t)}\zeta^{-1} q^{-(1-\alpha)^{-1}(k-1)(j,t)}.m^{-\frac{\beta}{1-\alpha}}.f\,(j)\,.$$

Thus,

$$\varrho q^{k-1(j,t)}\zeta^{-1} q^{-(1-\alpha)^{-1}(k-1)(j,t)}.m^{-\frac{\beta}{1-\alpha}}.f\,(j)\,\Pi\,(k, j, t) = r(t) + pb\,(k, j, t)$$

$$\frac{\varrho}{\zeta} q^{-\frac{\alpha}{1-\alpha}(k-1)(j,t)} f\,(j)\, m^{-\frac{\beta}{1-\alpha}} \left\{(q-1)\left[\frac{P_m(t)A\alpha}{q}\right]^{\frac{1}{1-\alpha}} (\bar{m}m)^{\frac{\beta}{1-\alpha}}\, q^{k(k,j)\,\frac{\alpha}{1-\alpha}}\right\}$$

$$= r(t) + pb\,(k, j, t)$$

$$\frac{\varrho}{\zeta} f\,(j)\, \bar{m}^{\frac{\beta}{1-\alpha}} \left\{(q-1)\left[\frac{P_m(t)A\alpha}{q}\right]^{\frac{1}{1-\alpha}}\right\} q^{\frac{\alpha}{1-\alpha}} = r(t) + pb\,(k, j, t)$$

$$pb\,(k, j, t) = \frac{\varrho}{\zeta} f\,(j)\, \bar{m}^{\frac{\beta}{1-\alpha}} \left(\frac{q-1}{q}\right) [P_m(t)A\alpha]^{\frac{1}{1-\alpha}} - r(t).$$

Substituting $\bar{m}$ for h or l and m for L or H, we get

$$pb_L = \frac{\varrho}{\zeta} f\,(j)\, l^{\frac{\beta}{1-\alpha}} \left(\frac{q-1}{q}\right) [P_L(t)A\alpha]^{\frac{1}{1-\alpha}} - r(t),$$

$$pb_H = \frac{\varrho}{\zeta} f\,(j)\, h^{\frac{\beta}{1-\alpha}} \left(\frac{q-1}{q}\right) [P_H(t)A\alpha]^{\frac{1}{1-\alpha}} - r(t).$$

Finally, notice that

$$\hat{Q}_m\,(t) = \frac{\dot{Q}_m\,(t)}{Q_m\,(t)} = \frac{\int_0^1 b_p\,(k, j, t)\,\triangle Q_m\,(t)}{Q_m\,(t)} = \frac{\int_0^1 b_p\,(k, j, t)\,.\,\overbrace{\left(q^{k_j}\right)^{\frac{\alpha}{1-\alpha}}}^{Q_m\,(t)}\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)}{Q_m\,(t)}$$

$$\hat{Q}_m\,(t) = \left\{\frac{\varrho}{\zeta} f\,(j)\, \bar{m}^{\frac{\beta}{1-\alpha}} \left(\frac{q-1}{q}\right) [P_m(t)A\alpha]^{\frac{1}{1-\alpha}} - r(t)\right\}\left(q^{\frac{\alpha}{1-\alpha}} - 1\right).$$

Combining the $rs(k, t) = \int_0^1 rs\,(k, t)\,dj$ already derived with pb_L and pb_H, we obtain

$$R = \int_0^1 rs\,(k, j)\,dj = \frac{\zeta}{\varrho}\left(Q_L L^{\frac{\beta}{1-\alpha}}\, pb_L + Q_H H^{\frac{\beta}{1-\alpha}}\, pb_H\right).$$