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Anisotropic potential around charged absorbing small particle in a collisional electronegative plasma with ion drift

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A distribution of the electric potentials around a charged absorbing particle in a drifting weakly ionised collisional plasma with negative ions is calculated in the linear hydrodynamic approach. Coulomb-like asymptote of the electric potential around the absorbing particle deforms under the action of the negative ions' flow and exhibits a valley profile along the flow behind the particle. The presence of the flowing negative ions can be conducive to string formation in the dust structures at relatively large pressures.

Key words: dusty plasmas, plasma flows, complex plasmas

1. Introduction

The interactions between charged macroscopic dust particles immersed in a plasma defines the properties of a strongly coupled complex plasma. Often, the complex plasma is subjected to an external electric field (for example, in the plasma sheath, in the positive column etc.) and becomes anisotropic. The anisotropic interaction between the solid particles in a plasma is exhibited in the formation of inhomogeneous dust structures such as string-like structures (Ivlev *et al.* 2008, 2011; Mitic *et al.* 2013). The interaction caused by ion flow perturbations is non-reciprocal and can be responsible for heating and instabilities in the dusty plasma (see Melzer 2001; Ivlev *et al.* 2015). When a negatively charged particle is placed in a flow of positive ions, an excess of positive charge is formed downstream of the particle due to the ion focusing (see Melandsø & Goree 1995; Lampe *et al.* 2000; Lapenta 2000).

In the collisionless case, the anisotropic potential decays at large distances as $1/r^3$, where r is the distance from the particle (see Montgomery, Joyce & Sugihara 1968; Kompaneets, Morfill & Ivlev 2016). Collisions of the ions with neutrals change the asymptote and give a slower decaying component $\sim 1/r^2$ at distances larger than an ion mean free path length (Stenflo, Yu & Shukla 1973; Kompaneets *et al.* 2007, 2016). In a weakly ionised collision-dominated flowing plasma containing electrons and positive ions as mobile charges, the long-range potential around an absorbing charged particle consists of a dipole-like component and an isotropic Coulomb-like one caused by absorption, which is usually dominant (see Chaudhuri,

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Khrapak & Morfill 2007; Filippov *et al.* 2007; Chaudhuri, Khrapak & Morfill 2010). The Coulomb-like potential is a field of the ambipolar diffusion on the dust grain. It is isotropic in a uniform plasma with constant ion mobility (the case of a field-dependent mobility has been considered by Zobnin (2018)).

The situation drastically changes in the presence of negative ions in the plasma. The anisotropic part of the potential around the negatively charged absorbing particle decreases with distance as $1/r$ in the presence of the negative ion flow, as will be derived below.

2. Theory

Let us consider a small negatively charged absorbing particle in a weakly ionised quasi-neutral collisional plasma containing electrons, single charged positive ions and single charged negative ions, under the action of a uniform external electric field $\boldsymbol{E}$. We assume that both kinds of ions have the same temperature T_i (drifting velocities are sub-thermal), while the temperature of the electrons T_e may be differ. Assuming that the drift–diffusion approach is valid at the distances of interest and neglecting ionisation and space recombination processes we can write the flow continuity equations for unperturbed densities of the electrons, negative and positive ions as

$$\nabla(z_\alpha n_\alpha \boldsymbol{E} - (k_B T_\alpha/e)\nabla n_\alpha) = 0, \tag{2.1}$$

where the index α denotes 'e', '$-$' or '$+$' for the electrons, negative ions and positive ions, respectively, z_α is the charge number, $z_{\mathrm{e}} = z_- = -1$ for the electrons and the negative ions and $z_+ = 1$ for the positive ions, k_B is the Boltzmann constant, e is the elementary charge. The total electric field can be split into the uniform external field and an additional potential distribution $\varphi(r)$ staying finite at infinite distances.

The flow continuity equations in the presence of a small absorbing particle are

$$\nabla(z_\alpha(n_\alpha + \hat{n}_\alpha)(\boldsymbol{E} - \nabla\varphi)) - (k_B T_\alpha/e)\Delta(n_\alpha + \hat{n}_\alpha) = -(F_\alpha/b_\alpha)\delta(\boldsymbol{r}), \tag{2.2}$$

where hats over n_α denote small perturbations of the corresponding number densities, $F_e = F_+$ are the fluxes of electrons and positive ions on the particle (flux of the negative ions is absent, $F_- = 0$), b_α is the mobility of the corresponding species and $\delta(\boldsymbol{r})$ is the three-dimensional delta function, $\boldsymbol{r}$ is the radius vector from the particle centre. The point-like sink ascribed to the delta function has been used early in a number of papers (Chaudhuri *et al.* 2007; Filippov *et al.* 2007; Khrapak, Klumov & Mofill 2008) for the description of the electric field around a small absorbing particle. Neglecting the terms $\nabla(\hat{n}_\alpha \nabla\varphi)$ and taking into account (2.1) we derive the linearised equations

$$z_\alpha((\nabla\hat{n}_\alpha \cdot \boldsymbol{E}) - n_\alpha \Delta\varphi) - (k_B T_\alpha/e)\Delta\hat{n}_\alpha = -(F_\alpha/b_\alpha)\delta(\boldsymbol{r}). \tag{2.3}$$

Poisson's equation is

$$-\varepsilon_0 \Delta\varphi = e(\hat{n}_+ - \hat{n}_- - \hat{n}_{\mathrm{e}} - z_d \delta(\boldsymbol{r})), \tag{2.4}$$

where ε_0 is the electric constant and z_d is the charge number of the particle. After the Fourier transformation of (2.3) and (2.4) in a standard way we have

$$\varphi(\boldsymbol{k}) = \frac{-ez_d - eJ(\chi_+ - \chi_{\mathrm{e}} n_+ b_+/(b_{\mathrm{e}} n_{\mathrm{e}}))}{(2\pi)^{3/2}\varepsilon_0 \boldsymbol{k}^2 (1 + \chi_+ + \chi_- + \chi_{\mathrm{e}})}, \tag{2.5}$$

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where $\chi_+ = k_{D+}^2/(\boldsymbol{k}^2 + \mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{k}_E))$, $\chi_- = k_{D-}^2/(\boldsymbol{k}^2 - \mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{k}_E))$, $\chi_\mathrm{e} = k_{De}^2/(\boldsymbol{k}^2 - \mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{k}_E)/\tau)$ are the susceptibilities, $J = F_+\varepsilon_0/(en_+b_+)$ is the dimensionless sink, k_{D+}, k_{D-} and k_{De} are the inverse Debye radii for the positive ions, negative ions and electrons, respectively, $\boldsymbol{k}_E$ is the normalised external electric field or the gas-sound Mach number divided by the ion mean free pass length, $\boldsymbol{k}_E = e\boldsymbol{E}/(k_BT) = \boldsymbol{u}\nu/(2\mathrm{v}_t^2)$, where u is the drift velocity vector of the positive ions, ν is the ion–neutral collision frequency, v_t is the thermal velocity and τ is the ratio of electron and ion temperatures. Dependence of the electric potentials on the spatial coordinates is given by the inverse Fourier transformation

$$\varphi(\boldsymbol{r}) = \frac{1}{(2\pi)^{3/2}}\int \varphi(\boldsymbol{k})\exp(\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}))\,\mathrm{d}^3\boldsymbol{k}. \tag{2.6}$$

The term $\chi_\mathrm{e} n_+ b_+/(b_\mathrm{e} n_\mathrm{e})$ in (2.5) is small with respect to the term χ_+ due to the small ratio of the ion and electron mobilities (with the exception of the case of a strongly electronegative plasma with $n_-/n_\mathrm{e} \sim b_\mathrm{e}/b_+$) and will be omitted below.

3. Large distances asymptote

To investigate the potential profile at large distances, we assume that $\chi = \chi_+ + \chi_- + \chi_\mathrm{e} \gg 1$, expand (2.5) in the set on the χ negative powers and conserve terms up to χ^{-1}. The result is

$$\begin{aligned}\varphi(\boldsymbol{k}) \approx{}& \frac{-(\boldsymbol{k}^4 + (\boldsymbol{k}\cdot\boldsymbol{k}_E)^2)(\boldsymbol{k}^2 - \mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{k}_E)/\tau)}{(2\pi)^{3/2}\varepsilon_0 k_D^2\boldsymbol{k}^4(\boldsymbol{k}^2 - \mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{k}_E)\eta)}\\ &\times\left(ez_d + \frac{eJk_{D+}^2}{(\boldsymbol{k}^2 + \mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{k}_E))}\left(1 - \frac{(\boldsymbol{k}^4 + (\boldsymbol{k}\cdot\boldsymbol{k}_E)^2)(\boldsymbol{k}^2 - \mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{k}_E)/\tau)}{k_D^2\boldsymbol{k}^2(\boldsymbol{k}^2 - \mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{k}_E)\eta)}\right)\right),\end{aligned} \tag{3.1}$$

where $k_D^2 = k_{D+}^2 + k_{D-}^2 + k_{De}^2$ is the square of the inverse Debye length λ_D, and the parameter η is defined by the equation $\eta = (k_{D+}^2(1 + 1/\tau) - k_{D-}^2(1 - 1/\tau))/k_D^2$. The expression (3.1) allows for analytical Fourier transformation, but the resulting equation appears to be too complex for analysis. So, we restrict the analysis via the condition $\tau \to \infty$. Thus, $\eta = (k_{D+}^2 - k_{D-}^2)/k_D^2 = (1 + 2\zeta)^{-1}$, where $\zeta = n_-/n_\mathrm{e}$ is the electronegativity parameter, and the spatial potential distribution can be expressed in the form

$$\varphi(r) = \frac{e}{4\pi\varepsilon_0}\left(-\frac{A}{r} + B\frac{k_E\cos\theta}{r^2k_D^2} + \exp\left(-\eta k_E r\cos^2\frac{\theta}{2}\right)\frac{(C - H(r,\theta))}{r}\right), \tag{3.2}$$

where k_E and r are the absolute values of $\boldsymbol{k}_E$ and $\boldsymbol{r}$, respectively, θ is the angle between $\boldsymbol{E}$ and $\boldsymbol{r}$ and A, B, C, D, E and F are the parameters

$$A = J(1+\zeta),\quad B = z_d(1+2\zeta) - J(1+\zeta)(1+2\zeta),\quad C = \frac{2J\zeta(1+\zeta)}{1+2\zeta},$$

$$\begin{aligned}H(r,\theta) ={}& \frac{D}{k_D^2}\left(\frac{k_E}{r}\cos\theta + \eta k_E^2\cos^2\frac{\theta}{2}\right)\\ &- 8J\zeta^2\frac{(1+\zeta)^2}{(1+2\zeta)^3}\frac{k_E^2}{k_D^2}\left(\sin^2\frac{\theta}{2} - \eta k_E r\cos^4\frac{\theta}{2}\right),\end{aligned}$$

$$D = 4z_d\zeta\frac{1+\zeta}{1+2\zeta} - 4J\zeta\frac{(1+\zeta)(2+3\zeta+2\zeta^2)}{(1+2\zeta)^2}.$$

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Equation (3.2) is valid when both conditions

$$r \gg \lambda_D \quad \text{and} \quad r \gg k_E \lambda_D^2/\eta \tag{3.3a,b}$$

are satisfied.

The ion flux on the grain can be easily calculated only for a small enough spherical grain in a collision-dominant plasma, when

$$l_i < a(-e\varphi_s/k_B T)^{1/2} \ll \lambda_D, \tag{3.4}$$

where a is the grain radius, φ_s is the surface potential and l_i is the ion free path length. In this case

$$J \approx z_d, \tag{3.5}$$

which is well known for the two-component plasma (see Su & Lam 1963; Khrapak *et al.* 2006) and remains true in the electronegative plasma, because (3.5) is derived neglecting space charge in Poisson's equation. The conditions (3.4) are stronger than necessary for validity of (2.3) because the distances of interest for dust–dust interaction are typically greater than the ion capturing radius and the Debye length. Calculation of the ion flux on the grain in a general case is a rather complicated task even in a two-component plasma. However, equation (3.5) gives the upper limit of the ion flux on the attractive particle, so $J \leqslant z_d$.

The expression (3.2) contains the components proportional to the parameters A and C, which decrease as $1/r$, while other components decrease as $1/r^2$. The first ones arise from the absorption and are proportional to J. In the absence of the external field they give a Coulomb-like asymptote

$$\varphi \approx \frac{A - C}{r} = \frac{eJ}{4\pi\varepsilon_0 r}\frac{k_{D+}^2}{k_D^2} \tag{3.6}$$

in accordance with Chaudhuri *et al.* (2007) and Khrapak *et al.* (2008). In the presence of the external field, the same asymptote remains only in the direction opposite to the field ($\theta = \pi$), while in other directions the asymptote is

$$\varphi \approx \frac{eJ}{4\pi\varepsilon_0 r}\frac{k_{D+}^2}{k_D^2}(1 + 2\zeta). \tag{3.7}$$

The anisotropic term decaying as $1/r$ appears only in the electronegative plasma and is connected with rarefaction of the negative ion flow behind the negatively charged particle.

Figure 1 illustrates the potential distributions $\varphi(r)$ around the charged absorbing particle depending on the electronegativity parameter for the ratios of $J/z_d = 0.1$ and 1.0. The calculations were performed for a fixed normalised external field and a finite temperature ratio $\tau = 100$ by the numerical Fourier transformation of (2.5). The additional potential is normalised by the value $-ez_d k_D/(4\pi\varepsilon_0)$, and the levels 0.3, 0.25, 0.2, 0.15, 0.1, 0.05, 0.02, ± 0.01, ± 0.005, 0 are shown in the figure by the solid lines. The levels calculated by the approximation (3.2) are shown by the dashed lines. Note, that, according to the conditions (3.3), the approximation (3.2) for $\zeta = 5$ and $k_E = k_D$ is valid only for $r \gg 11\lambda_D$.

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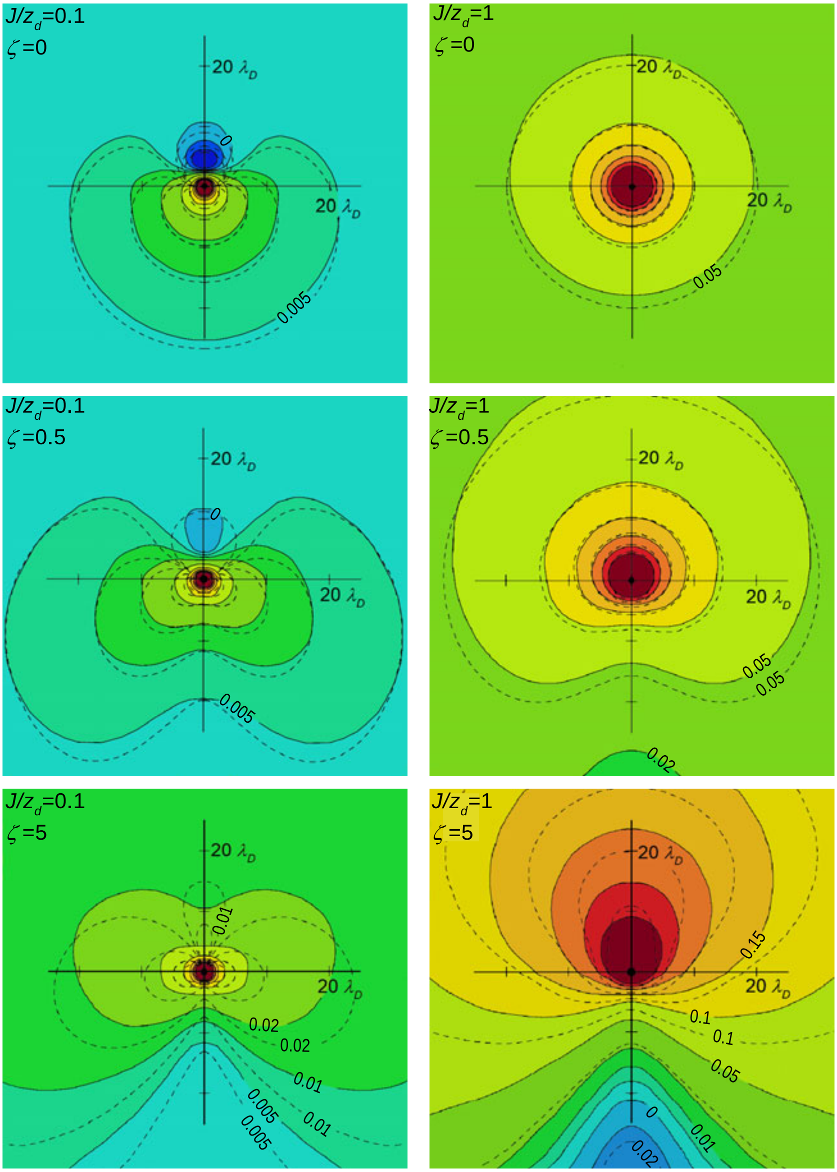

FIGURE 1. Potential distribution profiles for $k_E\lambda_D = 1$ (the external field is upwards directed), $\tau = 100$, $J/z_d = 0.1$ and 1, the negativity parameters $\zeta = 0$, 0.5 and 5, numerically calculated by (2.5), (2.6) (solid lines) and according to (3.2) (dashed lines).

4. Conclusions

The obtained results have a direct relation to the dusty gas discharge plasma. Negative ions can appear in chemically active or contaminated plasmas (Amemiya 1990; Klumov, Ivlev & Morfill 2003). The presence of negative ions essentially affects the dust component. It not only decreases the dust particle charge (Klumov *et al.* 2003; Merlino & Kim 2006), but drastically changes the interaction between grains in the drifting plasma.

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The negative ion flow produces the anisotropic interaction even in the hydrodynamic charging mode, when the positive ion flow cannot give significant anisotropy. The anisotropic interaction produced by a negative ion flow fosters string formation. A slow decrease in the potential with distance allows one to expect enhanced capture of the particle in a string with an increasing number of particles. The effect of string formation under the action of the negative ion flow is an analogy to the lane formation in the driven binary complex plasmas, which has been experimentally observed by Sütterlin *et al.* (2009) and Du *et al.* (2012), because a flow of small dust particles penetrating a cloud of big particles can be treated as the flow of multi-charged negative ions. Of cause, a large charge number limits application of the small perturbation approach used here.

REFERENCES

AMEMIYA, H. 1990 Plasmas with negative ions – probe measurements and charge equilibrium. *J. Phys.* D*: Appl. Phys.* **23**, 999–1014.

CHAUDHURI, M., KHRAPAK, S. A. & MORFILL, G. E. 2007 Effective charge of a small absorbing body in highly collisional plasma subject to an external electric field. *Phys. Plasmas* **14**, 022102.

CHAUDHURI, M., KHRAPAK, S. A. & MORFILL, G. E. 2010 A note on the electrical potential distribution around a test charge in anisotropic collisional plasmas. *J. Plasma Phys.* **76**, 603–606.

DU, C. R., SÜTTERLIN, K. R., JIANG, K., RÄTH, C., IVLEV, A. V., KHRAPAK, S., SCHWABE, M., THOMAS, H. M., FORTOV, V. E. & LIPAEV, A. M. 2012 Experimental investigation on lane formation in complex plasmas under microgravity conditions. *New J. Phys.* **14**, 073058.

FILIPPOV, A. V., ZAGORODNY, A. G., PAL', A. F., STAROSTIN, A. N. & MOMOT, A. I. 2007 Kinetic description of the screening of the charge of macroparticles in a nonequilibrium plasma. *J. Expl Theor. Phys. Lett.* **86** (12), 761–766.

IVLEV, A. V., BARTNICK, J., HEINEN, M., DU, C. R., NOSENKO, V. & LÖWEN, H. 2015 Statistical mechanics where Newton's third law is broken. *Phys. Rev.* X **5**, 011035.

IVLEV, A. V., MORFILL, G. E., THOMAS, H. M., RÄTH, C., JOYCE, G., HUBER, P., KOMPANEETS, R., FORTOV, V. E., LIPAEV, A. M., MOLOTKOV, V. I. *et al.* 2008 First observation of electrorheological plasmas. *Phys. Rev. Lett.* **100**, 095003.

IVLEV, A. V., THOMAS, H. M., RÄTH, C., JOYCE, G. & MORFILL, G. E. 2011 Complex plasmas in external fields: the role of non-hamiltonian interactions. *Phys. Rev. Lett.* **106**, 155001.

KHRAPAK, S. A., KLUMOV, B. A. & MOFILL, G. 2008 Electric potential around an absorbing body in plasmas: effect of ion–neutral collisions. *Phys. Rev. Lett.* **100**, 225003.

KHRAPAK, S. A., MOFILL, G. E., KHRAPAK, A. G. & D'YACHKOV, L. G. 2006 Charging properties of a dust grain in collisional plasmas. *Phys. Plasmas* **13**, 052114.

KLUMOV, B. A., IVLEV, A. V. & MORFILL, G. 2003 The role of negative ions in experiments with complex plasma. *J. Expl Theor. Phys. Lett.* **78** (5), 300–304.

KOMPANEETS, R., KONOPKA, U., IVLEV, A. V., TSYTOVICH, V. N. & MORFILL, G. 2007 Potential around a charged dust particle in a collisional sheath. *Phys. Plasmas* **14** (5), 052108.

KOMPANEETS, R., MORFILL, G. E. & IVLEV, A. V. 2016 Wakes in complex plasmas: a self-consistent kinetic theory. *Phys. Rev.* E **93**, 063201.

LAMPE, M., JOYCE, G., GAUNGULI, G. & GAVRISHCHAKA, V. 2000 Interactions between dust grains in a dusty plasma. *Phys. Plasmas* **7**, 3851–3861.

LAPENTA, G. 2000 Linear theory of plasma wakes. *Phys. Rev.* E **62**, 1175–1181.

MELANDSØ, F. & GOREE, J. 1995 Polarized supersonic plasma flow simulation for charged bodies such as dust particles and spacecraft. *Phys. Rev.* E **52**, 5312–5326.

MELZER, A. 2001 Laser-experiments on particle interactions in strongly coupled dusty plasma crystals. *Phys. Scr.* T **89** (1), 33–36.

MERLINO, R. L. & KIM, S. H. 2006 Charge neutralization of dust particles in a plasma with negative ions. *Appl. Phys. Lett.* **89**, 091501.

MITIC, S., KLUMOV, B. A., KHRAPAK, S. A. & MORFILL, G. E. 2013 Three dimensional complex plasma structures in a combined radio frequency and direct current discharge. *Phys. Plasmas* **20**, 043701.

MONTGOMERY, D., JOYCE, G. & SUGIHARA, R. 1968 Inverse third power law for the shielding of test particles. *Plasma Phys.* **10** (7), 681–686.

STENFLO, L., YU, M. Y. & SHUKLA, P. K. 1973 Shielding of a slow test charge in a collisional plasma. *Phys. Fluids* **16** (3), 450–452.

SU, C. H. & LAM, S. H. 1963 Continuum theory of spherical electrostatic probes. *Phys. Fluids* **6**, 1479–1491.

SÜTTERLIN, K. R., WYSOCKI, A., IVLEV, A. V., RÄTH, C., THOMAS, H. M., RUBIN-ZUZIC, M., GOEDHEER, W. J., FORTOV, V. E., LIPAEV, A. M., MOLOTCOV, V. I. *et al.* 2009 Dynamic of lane formation in driven binary complex plasmas. *Phys. Rev. Lett.* **102**, 085003.

ZOBNIN, A. V. 2018 Potential distribution around the charged particle in the collisional weakly ionized plasma in an external electric field. *J. Phys.: Conf. Ser.* **946**, 012157.