A FORMALISM FOR THE CLASSICAL SENTENCE-LOGIC

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I have introduced in my paper [1] a formalism for lower order predicate logics in which any sequence of variables and brackets ('[' and ']') can be regarded as a meaningful (well-formed) sentence. Also, in my paper [2], I have introduced another formalism of the same kind for a certain kind of predicate logics of higher order. Can we introduce a formalism of the same kind properly for sentence logics too? Since sentence logics can be regarded as a sub-system of predicate logics, we are apt to consider that our device introduced in [1] works well for sentence logics too.

However, to make every sequence of variables and brackets meaningful in sentence logics is a task other than to do it in predicate logics, because meaningful sentences in predicate logics are not necessarily meaningful in sentence logics. Indeed, my answer for the above-mentioned question is only PARTLY YES. I can really introduce a formalism of this kind for the classical sentence-logic. Namely, I can introduce a formalism of the same kind for sentence-logical part **LOS** of the primitive logic **LO**[1] having a special proposition symbol for CONTRADICTION, and I can interpret the **K**-series sentence-logics[2] faithfully in **LOS**. However, I can interpret only **J**-series logics (even the sentence-logical part of them) in **LO** as a predicate logic, so I can hardly give any formalism of this kind for **J**-series sentence logics.[3]

The device I am going to introduce in the present paper for expressing sentences of **LOS** is denoted by **FLOS** (FORMALIZED **LOS**). In **FLOS**, VARIABLES together with HEAD-BRACKET '[' and TAIL-BRACKET ']' are employed for expressing sentences. If we assume **FLO**, the formalism introduced in [1] for expressing sentences of the primitive logic **LO**, the formalism **FLOS**

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[1],[2] and [3] In my paper [3], I have studied on faithful interpretations of logics **LJ**, **LK**, **LM**, **LN**, **LP**, and **LQ** in **LO**. **LJ**, **LM**, and **LP** are called **J**-series logics, and **LK**, **LN**, and **LQ** are called **K**-series logics. As for detailed description of these logics, see [3].

can be described simply as follows:

Let $\mathfrak{a}$ be any sequence of variables and brackets, and let $\mathfrak{a}^*$ be the sequence obtained by replacing every variables p in $\mathfrak{a}$ by the sequence $[p]$. In **FLOS**, *the sequence $\mathfrak{a}$ means the proposition expressed by the sequence $\mathfrak{a}^*$ in* **FLO**.[4]

Accordingly, we can also express any formal deduction by a sequence of variables and brackets together with COMMAS. The device of expressing formal deductions by employing these symbols is just the same as that of [1] or [2], so I do not go into details concerning the matter.

Description of propositions of K-series sentence-logics

Any propositions of **K**-series sentence-logics can be expressed by a series of steps of logical operations such as negation or disjunction starting from some proposition variables. Proposition variables are the only variables in these systems. Negation can be expressed by implication $\rightarrow$ and the symbol $\curlywedge$ for a contradictory proposition, and operations other than implication and negation can be expressed by means of implication and negation in these logics. Hence, any proposition in these logics can be expressed by means of $\rightarrow$, $\curlywedge$, variables, and parentheses which show the order of operations. In **FLOS**, we can express $\curlywedge$ by $[]$, $\mathfrak{A} \rightarrow \mathfrak{B}$ by $[\mathfrak{a}][\mathfrak{b}]$, and $(\mathfrak{A})$ by $[\mathfrak{a}]$, where $\mathfrak{a}$ and $\mathfrak{b}$ stand for the sequences in **FLOS** denoting the propositions $\mathfrak{A}$ and $\mathfrak{B}$, respectively. It can be easily seen that every proposition of the sentence logics can be expressed univocally by a sequence of variables and brackets.

Interpretation of sequences of variables and brackets as sentences of FLOS

Let us now turn our attention to the problem of univocal interpretation of any sequence of variables and brackets as a sentence. Sequences $\mathfrak{a}$ which arise by above-mentioned ways are sequences satisfying the following conditions:

The number of tail-brackets in $\mathfrak{a}$ is equal to the number of head brackets in $\mathfrak{a}$, and, for any tail-bracket in $\mathfrak{a}$, the number of head-brackets standing before it exceeds

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[4] For example, Peirce's rule $((a \rightarrow b) \rightarrow a) \rightarrow a$ is expressed in **FLO** as $[[[a][b]][a]][a]$. The same formula can be regarded as expressing the same rule in **FLOS** too, but it can be reduced to a simpler form $[[ab]a]a$ in **FLOS**. Naturally, $[[ab]a]a$ has other meaning in **FLO**.

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the number of tail-brackets standing before it.

Any sequence $\mathfrak{a}$ is called NORMAL if and only if $\mathfrak{a}$ satisfies the above condition. Since most sequences $\mathfrak{a}$ of variables and brackets are not NORMAL in general, I will introduce a NORMALIZATION PROCESS of sequences by defining FORMAL EQUIVALENCE of sequences as follows:

Definition of FORMAL EQUIVALENCE: Any sequence $\mathfrak{a}$ *is FORMALLY EQUIVALENT to the sequence* $\mathfrak{a}]$ *as well as to the sequence* $[\mathfrak{a}$. *For any variable* p *and any sequences* $\mathfrak{a}$ *and* $\mathfrak{b}$, *the sequence* $\mathfrak{a}p\mathfrak{b}$ *is FORMALLY EQUIVALENT to the sequence* $\mathfrak{a}[p]\mathfrak{b}$. *For any NORMAL sequence* $\mathfrak{n}$ *and any sequences* $\mathfrak{a}$ *and* $\mathfrak{b}$, *the sequence* $\mathfrak{a}\mathfrak{n}]\mathfrak{b}$ *is FORMALLY EQUIVALENT to the sequence* $\mathfrak{a}[\mathfrak{n}]]\mathfrak{b}$.

It can be easily seen that any sequence is formally equivalent to a NORMAL sequence.

Logical meaning of sentences should be taken out from the following inference rule:

For any NORMAL sequence $\mathfrak{a}$ *and any sequence* $\mathfrak{b}$, *the sequence* $[\mathfrak{a}]\mathfrak{b}$ *is PROVABLE if the sequence* $\mathfrak{b}$ *can be deduced from the sequence* $[\mathfrak{a}]$, *and the sequence* $\mathfrak{b}$ *is PROVABLE if the sequences* $[\mathfrak{a}]$ *and* $[\mathfrak{a}]\mathfrak{b}$ *are both PROVABLE. Any sequence* $\mathfrak{a}$ *is PROVABLE if there is a PROVABLE SEQUENCE which is formally equivalent to* $\mathfrak{a}$.

References

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