

**$L^p$  EXTENSION OF HOLOMORPHIC FUNCTIONS  
FROM SUBMANIFOLDS TO STRICTLY  
PSEUDOCONVEX DOMAINS WITH NON-SMOOTH  
BOUNDARY**

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**Abstract.** Let  $D$  be a bounded strictly pseudoconvex domain in  $\mathbb{C}^n$  (with not necessarily smooth boundary) and let  $X$  be a submanifold in a neighborhood of  $\overline{D}$ . Then any  $L^p$  ( $1 \leq p < \infty$ ) holomorphic function in  $X \cap D$  can be extended to an  $L^p$  holomorphic function in  $D$ .

**§1. Introduction**

Let  $D$  be a bounded strictly pseudoconvex domain in  $\mathbb{C}^n$  with smooth boundary and let  $X$  be a submanifold in a neighborhood of  $\overline{D}$  which intersects  $\partial D$  transversally. Then Henkin [4] proved that any bounded holomorphic function  $f$  in  $X \cap D$  can be extended to a bounded holomorphic function  $F$  in  $D$ . Moreover, he proved that if  $f$  is holomorphic in  $X \cap D$  and continuous on  $\overline{X \cap D}$ , then  $F$  is holomorphic in  $D$  and continuous on  $\overline{D}$ . Henkin-Leiterer [5] obtained the above results in the case when  $D$  is a bounded strictly pseudoconvex domain in  $\mathbb{C}^n$  with non-smooth boundary, without assuming that the submanifold  $X$  and  $\partial D$  intersect transversally. On the other hand, Beatrous [1] and Cumenge [3] obtained  $L^p$  extensions of holomorphic functions from a submanifold  $X \cap D$  of a bounded strictly pseudoconvex domain  $D$  in  $\mathbb{C}^n$  with smooth boundary under the hypothesis that the submanifold  $X$  and  $\partial D$  intersect transversally. Using a quite different method, Ohsawa-Takegoshi [6] have done the remarkable discovery concerning  $L^2$  extensions. They obtained the  $L^2$  extension of holomorphic functions from the intersection of a complex hyperplane and a bounded pseudoconvex domain which involves weight functions. In their theorem

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the transversality is not assumed. When  $p > 2$ , Cho [2] gave a counter-example in some pseudoconvex domain such that the  $L^p$  extension does not hold. In this paper, we show that any  $L^p$  ( $1 \leq p < \infty$ ) holomorphic function in  $X \cap D$  can be extended to an  $L^p$  holomorphic function in  $D$  in the case when  $D$  is a bounded strictly pseudoconvex domain in  $\mathbb{C}^n$  with non-smooth boundary, without assuming that the submanifold  $X$  and  $\partial D$  intersect transversally. The proof is based on the estimates of the integral formula for holomorphic functions in  $X \cap D$  which was used to prove the bounded and continuous extension of holomorphic functions by Henkin-Leiterer [5]. We also use the estimate of the volume form by means of local coordinates in a neighborhood of a singular points of  $X \cap \partial D$  obtained by Schmalz [7].

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**§2. Preliminaries**

Let  $D \Subset \mathbb{C}^n$  be a strictly pseudoconvex open set and let  $\theta \Subset \mathbb{C}^n$  be a neighborhood of  $\partial D$ , and let  $\rho$  be a strictly plurisubharmonic  $C^2$  function in a neighborhood of  $\bar{\theta}$  such that

$$D \cap \theta = \{z \in \theta : \rho(z) < 0\}.$$

Let  $N(\rho) = \{z \in \bar{\theta} : \rho(z) = 0\}$ , and assume that  $N(\rho) \Subset \theta$ . By Henkin-Leiterer [4], we can choose numbers  $\varepsilon, \beta > 0$  and  $C^1$  functions  $a_{jk}$  on  $\bar{\theta}$  such that the following estimates hold:

$$\text{dist}(N(\rho), \partial\theta) > 2\varepsilon,$$

$$\inf_{\zeta \in \bar{\theta}} \sum_{j,k=1}^n \frac{\partial^2 \rho(\zeta)}{\partial \zeta_j \partial \bar{\zeta}_k} \xi_j \bar{\xi}_k > 3\beta |\xi|^2 \quad \text{for all } 0 \neq \xi \in \mathbb{C}^n,$$

$$\sup_{\zeta \in \bar{\theta}} \left| \frac{\partial^2 \rho(\zeta)}{\partial \zeta_j \partial \bar{\zeta}_k} - a_{jk}(\zeta) \right| < \frac{\beta}{n^2},$$

$$\left| \frac{\partial^2 \rho(\zeta)}{\partial x_j \partial x_k} - \frac{\partial^2 \rho(z)}{\partial x_j \partial x_k} \right| < \frac{\beta}{2n^2} \quad \text{for } \zeta, z \in \bar{\theta} \text{ with } |\zeta - z| \leq 2\varepsilon,$$

where  $\zeta_j = x_j + ix_{j+n}$ . We define

$$F(z, \zeta) = 2 \sum_{j=1}^n \frac{\partial \rho(\zeta)}{\partial \zeta_j} (\zeta_j - z_j) - \sum_{j,k=1}^n a_{jk}(\zeta) (\zeta_j - z_j) (\zeta_k - z_k).$$

Then, by Henkin-Leiterer [5] there exist  $\varepsilon > 0$  and  $c > 0$  such that

$$\operatorname{Re} F(z, \zeta) \geq \rho(\zeta) - \rho(z) + c|\zeta - z|^2 \quad (\zeta, z \in \bar{\theta}, |\zeta - z| \leq 2\varepsilon).$$

Moreover, Henkin-Leiterer [5] proved the following:

**THEOREM 1.** *There exist a neighborhood  $U \Subset \theta$  of  $N(\rho)$  and  $C^1$  functions  $\Phi(z, \zeta)$ ,  $\tilde{\Phi}(z, \zeta)$ ,  $M(z, \zeta)$  and  $\tilde{M}(z, \zeta)$  for  $\zeta \in U$  and  $z \in U \cup D$  such that the following conditions are fulfilled:*

- (i)  $\Phi(z, \zeta)$  and  $\tilde{\Phi}(z, \zeta)$  depends holomorphically on  $z \in U \cup D$ .
- (ii)  $\Phi(z, \zeta) \neq 0$  and  $\tilde{\Phi}(z, \zeta) \neq 0$  for  $\zeta \in U$ ,  $z \in U \cup D$  with  $|\zeta - z| \geq \varepsilon$ .
- (iii)  $M(z, \zeta) \neq 0$  and  $\tilde{M}(z, \zeta) \neq 0$  for  $\zeta \in U$ ,  $z \in U \cup D$ .
- (iv)  $\Phi(z, \zeta) = F(z, \zeta)M(z, \zeta)$  and  $\tilde{\Phi}(z, \zeta) = (F(z, \zeta) - 2\rho(\zeta))\tilde{M}(z, \zeta)$  for  $\zeta \in U$ ,  $z \in U \cup D$  with  $|\zeta - z| \leq \varepsilon$ .
- (v) Let  $V_1, V_0$  be neighborhoods of  $N(\rho)$  such that  $V_0 \cup D$  is a strictly pseudoconvex open set and  $V_1 \Subset V_0 \Subset U$ . Then there exist the  $C^1$  map  $w(z, \zeta) = (w_1(z, \zeta), \dots, w_n(z, \zeta))$  for  $\zeta \in V_0$ ,  $z \in V_0 \cup D$  with the following properties:

(a)

$$\langle w(z, \zeta), \zeta - z \rangle = \Phi(z, \zeta) \quad (\zeta \in V_0, z \in V_0 \cup D).$$

(b) We choose a neighborhood  $V_2$  of  $N(\rho)$  such that  $V_2 \Subset V_1$  and a  $C^\infty$  function  $\chi$  on  $\mathbb{C}^n$  such that

$$\chi = 0 \text{ on } \mathbb{C}^n \setminus V_1 \text{ and } \chi = 1 \text{ on } V_2.$$

Then there exist constants  $\alpha > 0$  and  $c < \infty$  such that

$$|\tilde{\Phi}(z, \zeta)| \geq \alpha(|\rho(\zeta)| + |\rho(z)| + |\operatorname{Im} F(z, \zeta)| + |\zeta - z|^2) \quad \text{for } z, \zeta \in V_2 \cap D.$$

$$|w(z, \zeta)| \leq c(\|d\rho(\zeta)\| + |\zeta - z|) \quad \text{for } \zeta, z \in V_2.$$

$$\left| \frac{\partial \tilde{\Phi}(z, \zeta)}{\partial \bar{\zeta}_j} \right| \leq c \left( \left| \frac{\partial \rho(\zeta)}{\partial \bar{\zeta}_j} \right| + |\zeta - z| + |\rho(\zeta)| \right) \quad \text{for } \zeta, z \in V_2, j = 1, \dots, n.$$

### §3. $L^p$ extension

We define

$$\zeta' = (\zeta_1, \dots, \zeta_{n-1}), \quad (w(z, \zeta))' = (w_1(z, \zeta), \dots, w_{n-1}(z, \zeta)),$$

$$\bar{\partial}_{\zeta'} = \sum_{j=1}^{n-1} \frac{\partial}{\partial \bar{\zeta}_j} d\bar{\zeta}_j, \quad \omega_{\zeta'}(\zeta) = d\zeta_1 \wedge \cdots \wedge d\zeta_{n-1},$$

$$\bar{\omega}_{\zeta'} \left( \frac{\chi(\zeta)(w(z, \zeta))'}{\tilde{\Phi}(z, \zeta)} \right) = \bigwedge_{j=1}^{n-1} \bar{\partial}_{\zeta'} \left( \frac{\chi(\zeta)w_j(z, \zeta)}{\tilde{\Phi}(z, \zeta)} \right).$$

Let  $X = \{z \in \mathbb{C}^n : z_n = 0\}$ . We denote by  $dV$  and  $dV'$  the volume form on  $\mathbb{C}^n$  and  $\mathbb{C}^{n-1}$ , respectively. For an  $L^p$  holomorphic function  $f$  in  $D \cap X$  ( $p \geq 1$ ) and  $z \in D$ , we define

$$(3.1) \quad Ef(z) = \frac{(n-1)!}{(2\pi i)^{n-1}} \int_{D \cap X} f(\zeta) \bar{\omega}_{\zeta'} \left( \frac{\chi(\zeta)(w(z, \zeta))'}{\tilde{\Phi}(z, \zeta)} \right) \wedge \omega_{\zeta'}(\zeta).$$

Then  $Ef$  is holomorphic in  $D$  and satisfies  $Ef|_{D \cap X} = f$ .

Using Schmalz [7], we have the following lemma:

LEMMA 1. *Let  $t(z, \zeta) = \text{Im}\langle w(z, \zeta), \zeta - z \rangle$ . We set  $\zeta_j = \xi_j + i\xi_{j+n}$ ,  $z_j = \eta_j + i\eta_{j+n}$  and  $E_\delta(z) = \{\zeta \in D : |\zeta - z| < \delta \|d\rho(z)\|\}$  for all  $\delta > 0$ . Then there are constants  $c < \infty$ ,  $\gamma > 0$ , and numbers  $\mu, \nu \in \{1, \dots, 2n\}$  such that,  $\{\rho, t(z, \zeta), \xi_1, \dots, \hat{\mu}, \hat{\nu}, \dots, \xi_{2n}\}$  ( $\xi_\mu$  and  $\xi_\nu$  have to be omitted) forms coordinates system in  $E_\gamma(z)$  ( $\{\rho, t(z, \zeta), \eta_1, \dots, \hat{\mu}, \hat{\nu}, \dots, \eta_{2n}\}$  forms coordinates system in  $E_\gamma(\zeta)$ , respectively) and we have the estimates*

$$dV \leq \frac{c}{\|d\rho(z)\|^2} |d\rho(\zeta) \wedge d_\zeta t(z, \zeta) \wedge \dots, \hat{\mu}, \hat{\nu}, \dots \wedge d\xi_{2n}| \quad \text{on } E_\gamma(z)$$

$$dV \leq \frac{c}{\|d\rho(\zeta)\|^2} |d\rho(z) \wedge d_z t(z, \zeta) \wedge \dots, \hat{\mu}, \hat{\nu}, \dots \wedge d\eta_{2n}| \quad \text{on } E_\gamma(\zeta),$$

where  $dV$  is the Euclidean volume form on  $\mathbb{C}^n$ .

Using Lemma 1, we prove the following theorem:

THEOREM 2. *Let  $X$  be a closed complex submanifold of some neighborhood of  $\bar{D}$ . Let  $f$  be an  $L^p$  holomorphic function in  $D \cap X$  ( $p \geq 1$ ). Then there exists an  $L^p$  holomorphic function  $F$  in  $D$  such that  $F|_{D \cap X} = f$ .*

*Proof.* We may assume  $X = \{z_n = 0\}$ . We set  $\tilde{U} = D \cap U$ . The integral of the right hand side of (3.1) consists of the following two types

integrals:

$$I_1(z) = \int_{X \cap \tilde{U}} f(\zeta) \frac{G(z, \zeta)}{\tilde{\Phi}(z, \zeta)^{n-1}} dV'(\zeta),$$

$$I_2(z) = \int_{X \cap \tilde{U}} f(\zeta) G(z, \zeta) \frac{w_j(z, \zeta) \frac{\partial}{\partial \zeta_\nu} \tilde{\Phi}(z, \zeta)}{\tilde{\Phi}(z, \zeta)^n} dV'(\zeta),$$

where  $G(z, \zeta)$  is a smooth function in  $\overline{D} \times \overline{D}$ . At first we prove the theorem in the case when  $p = 1$ . Using Fubini's theorem, we have

$$\begin{aligned} \int_D |I_1(z)| dV(z) &\lesssim \int_{X \cap \tilde{U}} |f(\zeta)| \left\{ \int_D \frac{1}{|\tilde{\Phi}(z, \zeta)|^{n-1}} dV(z) \right\} dV'(\zeta) \\ &\lesssim \int_{X \cap \tilde{U}} |f(\zeta)| \left\{ \int_{|\zeta - z| \leq M} \frac{1}{(|\zeta - z|^2)^{n-1}} dV(z) \right\} dV'(\zeta) \\ &\lesssim \int_{X \cap \tilde{U}} |f(\zeta)| dV'(\zeta). \end{aligned}$$

Using the inequality

$$|w_j(z, \zeta)| \left| \frac{\partial \tilde{\Phi}(z, \zeta)}{\partial \zeta_\nu} \right| \lesssim (\|d\rho(\zeta)\|^2 + |\zeta - z| + |\rho(\zeta)|),$$

we have

$$\begin{aligned} &\int_D |I_2(z)| dV(z) \\ &\lesssim \int_{X \cap \tilde{U}} |f(\zeta)| \left( \int_D \frac{\|d\rho(\zeta)\|^2 + |\zeta - z| + |\rho(\zeta)|}{|\tilde{\Phi}(z, \zeta)|^n} dV(z) \right) dV'(\zeta). \end{aligned}$$

In view of Lemma 1, if we set  $t' = (t_3, \dots, t_{2n})$ , we obtain

$$\begin{aligned} &\int_D \frac{\|d\rho(\zeta)\|^2}{|\tilde{\Phi}(z, \zeta)|^n} dV(z) \\ &= \int_{z \in E_\gamma(\zeta)} \frac{\|d\rho(\zeta)\|^2}{|\tilde{\Phi}(z, \zeta)|^n} dV(z) + \int_{z \notin E_\gamma(\zeta)} \frac{\|d\rho(\zeta)\|^2}{|\tilde{\Phi}(z, \zeta)|^n} dV(z) \\ &\lesssim \int_{|t| \leq M} \frac{dt_1 dt_2 dt'}{(|t_1| + |t_2| + |t'|)^n} + \int_{z \notin E_\gamma(\zeta)} \frac{|\zeta - z|^2}{|\tilde{\Phi}(z, \zeta)|^n} dV(z) \\ &\lesssim \int_0^M \frac{r^{2n-3}}{(r^2)^{n-2}} dr \lesssim 1. \end{aligned}$$

The other cases are similar. Thus we have

$$\int_D |I_2(z)| dV(z) \lesssim \int_{X \cap \tilde{U}} |f(\zeta)| dV'(\zeta),$$

which completes the proof when  $p = 1$ . Next we assume  $1 < p < \infty$ . Let  $q$  be a positive number such that  $p^{-1} + q^{-1} = 1$ . We choose  $\varepsilon > 0$  so small that  $2\varepsilon p < 1$  and  $2\varepsilon q < 1$ . Using Hölder’s inequality, we have

$$\begin{aligned} |I_1(z)|^p &\lesssim \left( \int_{X \cap \tilde{U}} \frac{|f(\zeta)|^p}{|\tilde{\Phi}(z, \zeta)|^{n-1+\varepsilon p}} dV'(\zeta) \right) \left( \int_{X \cap \tilde{U}} \frac{dV'(\zeta)}{|\tilde{\Phi}(z, \zeta)|^{n-1-\varepsilon q}} \right)^{p/q} \\ &\lesssim \int_{X \cap \tilde{U}} \frac{|f(\zeta)|^p}{|\tilde{\Phi}(z, \zeta)|^{n-1+\varepsilon p}} dV'(\zeta). \end{aligned}$$

Thus we have

$$\begin{aligned} \int_D |I_1(z)|^p dV(z) &\lesssim \int_{X \cap \tilde{U}} |f(\zeta)|^p \left( \int_D \frac{dV(z)}{|\tilde{\Phi}(z, \zeta)|^{n-1+\varepsilon p}} \right) dV'(\zeta) \\ &\lesssim \int_{X \cap \tilde{U}} |f(\zeta)|^p dV'(\zeta). \end{aligned}$$

Next we estimate  $I_2(z)$ . It is sufficient to prove that the following  $I_2^1(z)$ ,  $I_2^2(z)$  and  $I_2^3(z)$  belong to  $L^p(D)$ :

$$\begin{aligned} I_2^1(z) &= \int_{X \cap \tilde{U}} \frac{|f(\zeta)| \|d\rho(\zeta)\|^2}{|\tilde{\Phi}(z, \zeta)|^n} dV'(\zeta), \\ I_2^2(z) &= \int_{X \cap \tilde{U}} \frac{|f(\zeta)| \|d\rho(\zeta)\| |\zeta - z|}{|\tilde{\Phi}(z, \zeta)|^n} dV'(\zeta), \\ I_2^3(z) &= \int_{X \cap \tilde{U}} \frac{|f(\zeta)| \|d\rho(\zeta)\| |\rho(\zeta)|}{|\tilde{\Phi}(z, \zeta)|^n} dV'(\zeta). \end{aligned}$$

We prove that  $I_2^1(z)$  belongs to  $L^p(D)$ . The other cases are similar. Using Hölder’s inequality

$$\begin{aligned} I_2^1(z)^p &\leq \left( \int_{X \cap \tilde{U}} |f(\zeta)|^p \frac{\|d\rho(\zeta)\|^2}{|\tilde{\Phi}(z, \zeta)|^{n+\varepsilon p}} dV'(\zeta) \right) \\ &\quad \times \left( \int_{X \cap \tilde{U}} \frac{\|d\rho(\zeta)\|^2}{|\tilde{\Phi}(z, \zeta)|^{n-\varepsilon q}} dV'(\zeta) \right)^{p/q}. \end{aligned}$$

We set  $\zeta' = (\zeta_1, \dots, \zeta_{n-1})$ ,  $z' = (z_1, \dots, z_{n-1})$ . Then we have

$$\begin{aligned} & \int_{X \cap \tilde{U}} \frac{\|d\rho(\zeta)\|^2}{|\tilde{\Phi}(z, \zeta)|^{n-\varepsilon q}} dV'(\zeta) \\ &= \int_{\zeta' \in E_\gamma(z')} \frac{\|d\rho(\zeta)\|^2}{|\tilde{\Phi}(z, \zeta)|^{n-\varepsilon q}} dV'(\zeta) + \int_{\zeta' \notin E_\gamma(z')} \frac{\|d\rho(\zeta)\|^2}{|\tilde{\Phi}(z, \zeta)|^{n-\varepsilon q}} dV'(\zeta). \end{aligned}$$

In view of Lemma 1, if we set  $t' = (t_3, \dots, t_{2n-2})$ , then there exists a positive constant  $M$  such that

$$\begin{aligned} \int_{\zeta' \in E_\gamma(z')} \frac{\|d\rho(\zeta)\|^2}{|\tilde{\Phi}(z, \zeta)|^{n-\varepsilon q}} dV'(\zeta) &\lesssim \int_{|t| \leq M} \frac{dt_1 dt_2 dt'}{(|t_1| + |t_2| + |t'|^2)^{n-\varepsilon q}} \\ &\lesssim \int_0^M \frac{dr}{r^{1-2\varepsilon q}} \lesssim 1. \\ \int_{\zeta' \notin E_\gamma(z')} \frac{\|d\rho(\zeta)\|^2}{|\tilde{\Phi}(z, \zeta)|^{n-\varepsilon q}} dV'(\zeta) &\lesssim \int_{X \cap \tilde{U}} \frac{|\zeta' - z'|^2}{|\tilde{\Phi}(z, \zeta)|^{n-\varepsilon q}} dV'(\zeta) \\ &\lesssim \int_0^M \frac{dr}{r^{1-2\varepsilon q}} \lesssim 1. \end{aligned}$$

By Fubini's theorem, we obtain

$$\int_D I_2^1(z)^p dV(z) \lesssim \int_{X \cap \tilde{U}} |f(\zeta)|^p \left( \int_D \frac{\|d\rho(\zeta)\|^2}{|\tilde{\Phi}(z, \zeta)|^{n+\varepsilon p}} dV(z) \right) dV'(\zeta).$$

Using the inequality

$$\|d\rho(\zeta)\| \lesssim \|d\rho(z)\| + |\zeta - z|,$$

it is sufficient to estimate the following two integrals  $J_1(\zeta)$  and  $J_2(\zeta)$ :

$$\begin{aligned} J_1(\zeta) &= \int_D \frac{\|d\rho(z)\|^2}{|\tilde{\Phi}(z, \zeta)|^{n+\varepsilon p}} dV(z), \\ J_2(\zeta) &= \int_D \frac{|\zeta - z|^2}{|\tilde{\Phi}(z, \zeta)|^{n+\varepsilon p}} dV(z). \end{aligned}$$

We estimate  $J_1(\zeta)$ . The other case is similar. In view of Lemma 1, we have

$$\begin{aligned} J_1(\zeta) &= \int_{z \in E_\gamma(\zeta)} \frac{\|d\rho(z)\|^2}{|\tilde{\Phi}(z, \zeta)|^{n+\varepsilon p}} dV(z) + \int_{z \notin E_\gamma(\zeta)} \frac{\|d\rho(z)\|^2}{|\tilde{\Phi}(z, \zeta)|^{n+\varepsilon p}} dV(z) \\ &\lesssim \int_{|t| \leq M} \frac{dt_1 dt_2 dt'}{(|t_1| + |t_2| + |t'|^2)^{n+\varepsilon p}} + \int_D \frac{dV(z)}{(|\zeta - z|^2)^{n-1+\varepsilon p}} \\ &\lesssim \int_0^M r^{1-2\varepsilon p} dr \lesssim 1. \end{aligned}$$

Thus we have proved that

$$\int_D I_2^1(z)^p dV(z) \lesssim \int_{X \cap \tilde{U}} |f(\zeta)|^p dV'(\zeta).$$

This completes the proof of Theorem 2.

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