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PROGRESSIVE TAXATION AND MACROECONOMICS (IN)STABILITY UNDER HOUSEHOLD HETEROGENEITY

SHU-HUA CHEN
National Taipei University

It has been shown that progressive income taxation may stabilize an otherwise standard representative-agent real business cycle model with an indeterminate steady state against aggregate fluctuations caused by agents' animal spirits. By contrast, within an identical model that allows for sustained economic growth, progressive taxation could lead to equilibrium indeterminacy and sunspot-driven fluctuations. In the context of household heterogeneity that gives rise to income and asset inequality, the fiscal authority has (at least) two options of setting the baseline level of taxable income: (i) the economy-wide average level of income and (ii) the economy's steady-state level of per capita income. I show that the adoption of a fiscal rule (i) invalidates the effects that a progressive tax can exert on the model's local stability properties. Progressive income taxation thus no longer operates as an automatic stabilizer that mitigates belief-driven cyclical fluctuations in a no-growth economy, nor as an automatic destabilizer that leads to local indeterminacy in a sustained-growth economy. If a tax policy rule (ii) is instead adopted, then the existing literature's findings of the (de)stabilizing roles of progressive taxation are robust to the inclusion of household heterogeneity.

Keywords: Progressive Income Taxation, Equilibrium Indeterminacy, Income Inequality

1. INTRODUCTION

Ever since the influential works of Benhabib and Farmer (1994) and Farmer and Guo (1994), it is now widely known that a one-sector real business cycle (RBC) model with sufficiently strong aggregate increasing returns-to-scale is prone to self-fulfilling expectations and business cycle fluctuations driven by the "animal spirits" of agents. This has motivated not only the investigation of mechanisms that give rise to the existence of multiple equilibria in dynamic general equilibrium models[1] but also the exploration of alternative stabilizing policies that

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mitigate business cycle fluctuations. In the latter topic, there has been an extensive research on the interrelations between fiscal policy rules and macroeconomic (in)stability. One significant contribution is from Guo and Lansing (1998), who show that a progressive income tax policy by "taxing away" the higher returns from belief-driven labor or investment spurts prevents agents' expectations from becoming self-fulfilling, thereby *ceteris paribus* it reduces the cyclical volatilities of output and employment in the Benhabib–Farmer–Guo model.

The policy implication of Guo and Lansing (1998) runs obviously in line with the conventional Keynesian view toward progressive taxation. However, the recent finding of Chen and Guo (2019) overturns this traditional viewpoint about Keynesian-type stabilization policies when the model exhibits sustained economic growth, that is, progressive income taxation may operate like an automatic *destabilizer* that generates equilibrium indeterminacy and belief-driven fluctuations in Benhabib and Farmer's (1994, Section 5) endogenously growing macroeconomy. No matter what, Guo and Lansing (1998) and Chen and Guo (2019) both demonstrate that progressive income taxation does exert (de)stabilization effects on the macroeconomy.[2,3]

In this paper, I extend the Benhabib–Farmer–Guo analysis by incorporating household time-preference heterogeneity that gives rise to income and asset inequality. In particular, the majority of existing works on the interrelation between progressive taxation and macroeconomic (in)stability—including the above-mentioned Guo and Lansing (1998) and Chen and Guo (2019)—conduct their analysis under the assumption of homogeneous agents, which obviously is not consistent with the observed household heterogeneity in both developed and developing countries. Another model specification commonly adopted is that the benchmark level of taxable income in the progressive income tax schedule is set at the economy's steady-state level of per capita income. Specifically, under homogeneous agents, the representative household's level of income is *identical* to the economy-wide average level. It follows that, when the benchmark level of taxable income is set at the economy-wide average level of income, each (identical) household will be levied a common and constant income tax rate, and that this constant tax rate is applied to *all* levels of income. This means that only a flat income tax rate is feasible in this context. By setting the baseline level of taxable income at the economy's steady-state level of per capita income, the existing literature thus can capture the observed progressivity feature—that is, both the average and marginal income tax rates increase with the representative household's taxable income, for example, the US tax code illustrated in Chen and Guo (2013a, Figures 4 and 5).

When households are heterogeneous, individual households' levels of income can be different from the economy-wide average level. The fiscal authority thus has the options of choosing either the economy-wide average level of income or the economy's steady-state level of per capita income as the baseline level of taxable income.[4,5] In the context of heterogeneous households, this paper shows that the interrelations between (local) stability of competitive equilibria and progressive income tax policy crucially depend on the selection of the baseline level of taxable income.

The analysis of this paper comprises two parts. The first part investigates a heterogeneous-household version of the no-growth Benhabib–Farmer–Guo model under progressive income taxation. To facilitate comparison with Benhabib–Farmer–Guo's result, government spending is postulated to be wasteful and does not contribute to utility or production.[6] I analytically derive that, when the economy-wide average level of income is taken as the benchmark level of taxable income, the necessary and sufficient condition for equilibrium indeterminacy is identical to that in the laissez-faire economy of Benhabib–Farmer–Guo, that is, when the degree of productive externalities is sufficiently high such that the aggregate after-tax equilibrium wage-hours locus is upward sloping and steeper than the aggregate labor supply curve. Under this progressive income tax rule, the slopes of both the aggregate after-tax labor demand schedule and the aggregate supply curve are independent of the tax-slope and level parameters. The progressive tax structure thus exerts no effect on local dynamics of the economy's unique steady state. This result runs in stark contrast to that found in Guo and Lansing (1998), who postulate that the benchmark level of taxable income is set at the economy's steady-state level of per capita income and finds that a sufficiently high progressivity of the tax schedule overturns the relative steepness criteria derived by Benhabib–Farmer–Guo, thereby functioning as an automatic stabilizer that reduces the scope of belief-driven business fluctuations.

In the latter part of this paper, I allow for sufficiently strong productive externalities such that, as in Benhabib and Farmer (1994, Section 5), the social technology is linear in physical capital, and therefore the economy exhibits sustained economic growth. I focus on the local stability properties of the economy's interior balanced growth path (BGP) along which hours worked are stationary, and output, consumption, and physical capital all grow at a common constant rate. It turns out that the no-growth economy's result, in which the adoption of the economy-wide average level of income as the benchmark level of taxable income breaks the linkage between progressive tax and the model's local stability properties, is robust to the inclusion of sustained economic growth. The Benhabib–Farmer–Guo condition for equilibrium indeterminacy thus prevails, implying that a progressive income tax does not expand the scope of equilibrium indeterminacy as found in Chen and Guo (2019), whereby the BGP's level of per capita income is used as the baseline level of taxable income.

The remainder of this paper is organized as follows. Section 2 describes the heterogeneous-household extension of the no-growth Benhabib–Farmer–Guo model and analyzes its equilibrium conditions under a progressive fiscal policy rule that sets the baseline level of taxable income at the economy-wide average level of output. Section 3 investigates the local stability properties associated with the economy's steady state. Section 4 examines an alternative progressive income tax rule that sets the benchmark level of taxable income at the economy's

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steady-state level of per capita income. Section 5 analytically examines the interrelations between progressive taxation and equilibrium (in)determinacy within the heterogeneous-household version of Benhabib and Farmer's (1994, Section 5) endogenously growing macroeconomy. Section 6 concludes.

2. THE BENCHMARK MODEL

Within Benhabib and Farmer's (1994) laissez-faire one-sector indeterminate RBC model, I incorporate household time-preference heterogeneity *à la* Li and Sarte (2004) and Koyuncu and Turnovsky (2016), as well as Guo and Lansing's (1998) and Li and Sarte's (2004) non-linear progressive income tax schedule that displays continuously increasing average and marginal tax rates. To simplify the analysis and without loss of generality, I follow Li and Sarte (2004, Section II.A.) and Koyuncu and Turnovsky (2016, Section 5) by considering a two-class economy comprised of infinitely lived "Poor" and "Rich" households, each of which derives utility from consumption and leisure. In particular, class/group-1 (2) contains the impatient (patient) households who possess a higher (lower) rate of time preference. Group-1 (the impatient) households thus earn income below the economy-wide average level, and hence become Poor (Rich) households. The production side consists of a social technology that displays increasing returns-to-scale due to positive productive externalities from aggregate capital and labor inputs. The government balances the budget each period by spending its tax revenue on goods and services that do not contribute to the households' utility or the firms' production. I assume that there are no fundamental uncertainties present in the economy.

2.1. Firms

There is a continuum of identical competitive firms indexed by j, with the total number normalized to one. The representative firm produces output Y_{jt} according to a Cobb–Douglas production function:

$$Y_{jt} = X_t K_{jt}^{1-\beta} L_{jt}^{\beta}, \quad 0 < \beta < 1, \tag{1}$$

where K_{jt} and L_{jt} are capital and labor inputs, respectively, and X_t represents positive productive externalities that are taken as given by each individual firm.

As in Benhabib and Farmer (1994), I postulate that externalities take the form:

$$X_t = A\left(K_t^{1-\beta} L_t^{\beta}\right)^{\chi}, \quad A > 0, \quad \chi \geq 0, \tag{2}$$

where K_t and L_t, respectively, denote the economy-wide average levels of capital and labor services. In a symmetric equilibrium, all firms make the same decisions such that $K_{jt} = K_t$ and $L_{jt} = L_t$, for all j and t. As a result, by substituting (2) into (1), I obtain the following aggregate increasing returns-to-scale production function for total output Y_t:

$$Y_t = AK_t^{\Omega_K} L_t^{\Omega_L}, \tag{3}$$

where $\Omega_K \equiv (1-\beta)(1+\chi) < 1$, such that externalities are not strong enough to generate sustained economic growth; in addition, $\Omega_L \equiv \beta(1+\chi) \gtreqless 1$, depending on the strength of productive externalities, χ.

Under the assumption that factor markets are perfectly competitive, the firm's profit maximization conditions are given by:

$$r_t = (1-\beta)\frac{Y_t}{K_t} \quad \text{and} \quad w_t = \beta\frac{Y_t}{L_t}, \tag{4}$$

where r_t is the capital rental rate, and w_t is the real wage. In addition, $1-\beta$ and β represent the capital and labor share of national income, respectively.

2.2. Households

The economy is populated by two classes/groups of infinitely lived households, indexed by $i = 1, 2$. Households are identical in all aspects except for their rates of time preference, $\rho_i > 0$, and their initial capital endowments, K_{i0}. I consider that $\rho_1 > \rho_2 > 0$. Group-1 households are thus less patient than group-2 households. Each representative household in class/group-i is endowed with one unit of time and maximizes a discounted stream of utilities over its lifetime:

$$\int_0^\infty (\log C_{it} - BL_{it})\, e^{-\rho_i t} dt, \quad B > 0, \tag{5}$$

where C_{it} and L_{it} are the group-i representative household's consumption and hours worked, respectively. The linearity of (5) in hours worked draws on the formulation of indivisible labor (Hansen (1985) and Rogerson (1988)) that is commonly adopted in the RBC-based indeterminacy literature.

The budget constraint faced by household i is given by:

$$\dot{K}_{it} = (1-\tau_{it})(r_t K_{it} + w_t L_{it}) - \delta K_{it} - C_{it}, \quad K_{i0} > 0 \text{ given}, \tag{6}$$

where K_{it} is household i's capital stock, and $\delta \in (0,1)$ is the capital depreciation rate. Households derive income by providing capital and labor services to firms, taking factor prices r_t and w_t as given. The income tax rate τ_{it} is postulated to take the form:

$$\tau_{it} = 1 - \eta \left(\frac{Y_t}{Y_{it}}\right)^{\phi}, \quad \eta \in (0,1) \quad \text{and} \quad \phi \in (0,1), \tag{7}$$

where $Y_{it} = r_t K_{it} + w_t L_{it}$ represents household i's taxable income, $Y_t = \frac{Y_{1t}+Y_{2t}}{2} = r_t K_t + w_t L_t$ is the economy-wide average level of income, with $K_t = \frac{K_{1t}+K_{2t}}{2}$ and $L_t = \frac{L_{1t}+L_{2t}}{2}$, respectively, denoting the economy-wide average levels of capital stock and labor hours, and the parameters η and ϕ govern the level and slope of the

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tax schedule, respectively. As in Li and Sarte (2004) and Koyuncu and Turnovsky (2016), the income tax rule (7) specifies that the baseline level of taxable income is set at the economy-wide average level of income Y_t.

To further understand the progressivity features of the above taxation scheme, let us first note that the marginal tax rate τ_{imt}, defined as the change in taxes paid by household i divided by the change in its taxable income, is given by:

$$\tau_{imt} \equiv \frac{\partial\left(\tau_{it} Y_{it}\right)}{\partial Y_{it}} = 1 - \eta\left(1-\phi\right)\left(\frac{Y_t}{Y_{it}}\right)^{\phi} = \tau_{it} + \phi\left(1-\tau_{it}\right). \quad \mathbf{(8)}$$

I restrict the analysis to an environment wherein both the average and marginal tax rates lie between 0 and 1, and consequently (i) the government does not have access to lump-sum taxes or transfers, (ii) the government cannot confiscate all productive resources, and (iii) households have an incentive to supply factor services to the firm's production process. Under $\eta \in (0, 1)$ and $\phi \in (0, 1)$, these requirements impose a lower bound for the equilibrium relative income, $\frac{Y_{it}}{Y_t} > \eta^{\frac{1}{\phi}}$, for all t and $i = 1, 2$. It is clear from (8) that, under $\phi > 0$, the marginal tax rate is higher than the average tax rate ($\tau_{imt} > \tau_{it}$), which is a "progressivity" feature. Note that both the average income tax rate τ_{it} and the marginal tax rate τ_{imt} rise with household i's taxable income Y_{it}. Given that $Y_{1t} < Y_t < Y_{2t}$, the government levies higher (lower) average and marginal tax rates on group-1 (2) households, that is, $\tau_{1t} < \tau_{2t}$ and $\tau_{1mt} < \tau_{2mt}$. Because households face a progressive tax schedule, the patient group will not end up owning all capital in equilibrium.

I postulate that agents take into account the way in which the tax schedule affects their earnings when they decide how much to consume, invest, and work over their lifetimes. Therefore, it is the marginal tax rate of income τ_{imt} that governs the household's economic decisions. The first-order conditions for household i with respect to the indicated variables and the associated transversality condition (TVC) are

$$C_{it}: \quad C_{it}^{-1} = \lambda_{it}, \quad \mathbf{(9)}$$

$$L_{it}: \quad \frac{B}{\lambda_{it}} = \underbrace{\eta(1-\phi)\left(\frac{Y_t}{Y_{it}}\right)^{\phi}}_{(1-\tau_{imt})} \underbrace{\beta\frac{Y_t}{L_t}}_{w_t}, \quad \mathbf{(10)}$$

$$K_{it}: \quad -\frac{\dot{\lambda}_{it}}{\lambda_{it}} = \underbrace{\eta(1-\phi)\left(\frac{Y_t}{Y_{it}}\right)^{\phi}}_{(1-\tau_{imt})} \underbrace{(1-\beta)\frac{Y_t}{K_t}}_{r_t} - \delta - \rho_i, \quad \mathbf{(11)}$$

$$TVC: \quad \lim_{t\to\infty} e^{-\rho_i t}\lambda_{it}K_{it} = 0, \quad \mathbf{(12)}$$

where $\lambda_{it} > 0$ is the Lagrange multiplier on the budget constraint (6), (10) equates the slope of household i's indifference curve to the after-tax real wage, (11) is the

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modified consumption Euler equation, and (12) is the TVC. Notice that under the restrictions $\eta \in (0, 1)$ and $\phi \in (0, 1)$, household i's budget constraint (6) is jointly concave in the state and control variables, that is, K_{it}, C_{it}, and L_{it}. Thus, equations (9)–(11) are not only necessary but also sufficient conditions for the unique global maximum of the household's dynamic optimization problem.

2.3. Government

The government sets the tax rate τ_{it} according to (7) and balances its budget at each point in time. Hence, its instantaneous budget constraint is given by:

$$G_t = \tau_{1t} Y_{1t} + \tau_{2t} Y_{2t}, \tag{13}$$

where G_t is public spending on goods and services. By putting together households' and the government's budget constraints, the aggregate resource constraint for the economy is derived as:

$$C_t + \dot{K}_t + \delta K_t + \frac{G_t}{2} = Y_t, \tag{14}$$

where the economy-wide average level of consumption is $C_t = \frac{C_{1t}+C_{2t}}{2}$, with $C_{it} = \frac{\eta(1-\phi)}{B}(\frac{Y_t}{Y_{it}})^{\phi} \beta \frac{Y_t}{L_t}$ derived by putting together (4), (9), and (10).

3. MACROECONOMIC (IN)STABILITY

To facilitate the analysis of the model's local stability properties, I follow García-Peñalosa and Turnovsky (2006, 2007, 2011) in denoting $y_{it} \equiv \frac{Y_{it}}{Y_t}$ as household i's relative income. It follows that the relative income has mean 1: $\frac{y_{1t}+y_{2t}}{2} = 1$. By using the expressions of the average and marginal income tax rates, $\tau_{it} = 1 - \eta(y_{it})^{-\phi}$ and $\tau_{imt} = 1 - \eta(1-\phi)(y_{it})^{-\phi}$, I then define the arithmetic mean of the average and marginal tax rates as $\bar{\tau}_t \equiv \frac{\tau_{1t}+\tau_{2t}}{2}$ and $\bar{\tau}_{mt} \equiv \frac{\tau_{1mt}+\tau_{2mt}}{2}$, respectively. In addition, as in Li and Sarte (2004), the effective average tax rate is defined as $\tilde{\tau}_t \equiv \frac{\tau_{1t}y_{1t}+\tau_{2t}y_{2t}}{2}$. The model's equilibrium conditions can then be collapsed into the following autonomous dynamical system:

$$\dot{y}_{1t} = \left[\rho_1 - \rho_2 + \frac{(\tau_{1mt} - \tau_{2mt})(1-\beta) Y_t}{K_t}\right] \frac{y_{1t} y_{2t}}{2\phi}, \tag{15}$$

$$\dot{L}_t = \left[\frac{(1-\tau_{1mt})(1-\beta) Y_t}{K_t} - \delta - \rho_1 + \phi \frac{\dot{y}_{1t}}{y_{1t}} - \Omega_K \frac{\dot{K}_t}{K_t}\right] \frac{L_t}{\Omega_L - 1}, \tag{16}$$

$$\dot{K}_t = (1-\tilde{\tau}_t) Y_t - \frac{(1-\bar{\tau}_{mt}) \beta Y_t}{BL_t} - \delta K_t, \tag{17}$$

$$\dot{K}_{1t} = (1-\tau_{1t}) y_{1t} Y_t - \frac{(1-\tau_{1mt}) \beta Y_t}{BL_t} - \delta K_{1t}, \tag{18}$$

where total output Y_t is given by (3).

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An interior steady state is characterized by positive real numbers (y_1^*, L^*, K^*, K_1^*) that satisfy $\dot{y}_{1t} = \dot{L}_t = \dot{K}_t = \dot{K}_{1t} = 0$. It is straightforward to derive from (15)–(18) that the model exhibits a unique interior steady state given by:

$$y_1^* = 2\left[1 + \left(\frac{\delta + \rho_1}{\delta + \rho_2}\right)^{1/\phi}\right]^{-1}, \tag{19}$$

$$L^* = \frac{\beta\left(1 - \bar{\tau}_m^*\right)}{B\left[1 - \tilde{\tau}^* - \delta\left(1 - \beta\right)\left(1 - \tau_{1m}^*\right) / \left(\delta + \rho_1\right)\right]}, \tag{20}$$

$$K^* = \left[\frac{\delta + \rho_1}{\left(1 - \beta\right)\left(1 - \tau_{1m}^*\right) A \left(L^*\right)^{\Omega_L}}\right]^{\frac{1}{\Omega_K - 1}}, \tag{21}$$

$$K_1^* = \frac{\left(1 - \tau_1\right) Y^*}{\delta}\left[y_1^* - \frac{\left(1 - \phi\right)\beta}{BL^*}\right]. \tag{22}$$

The remaining endogenous variables at the economy's steady state can then be derived accordingly.[7] Note that the requirement whereby $y_1^*(< y_2^*)$ exceeds the lower bound for the equilibrium relative income, denoted as $\underline{y} \equiv \eta^{\frac{1}{\phi}}$, leads to a restriction on the values of the parameters: $1 + (\frac{\delta+\rho_1}{\delta+\rho_2})^{1/\phi} < \frac{2}{\eta^{1/\phi}}$. I thus have the following proposition.

PROPOSITION 1. *Under household heterogeneity and the fiscal rule of (7), the Benhabib–Farmer economy without ongoing growth exhibits a unique steady state.*

In terms of the local stability properties of the unique steady state, it is clear that (15)–(17) are independent of K_{1t} and constitute a 3-dimensional subsystem of (y_{1t}, L_t, K_t); afterward, the evolution through time of the stock of capital owned by household 1, K_{1t}, is determined by (18). It is straightforward to derive that (18) possesses a negative eigenvalue equaling $-\delta$. This, together with the fact that K_{1t} is a predetermined variable, implies the existence of a unique rational expectations equilibrium converging to K_1^*.

To examine the local dynamics of the subsystem (15)-(17), I compute the associated Jacobian matrix $\mathbf{J}$ evaluated at (y_1^*, L^*, K^*) and derive the determinant and trace of $\mathbf{J}$ as follows:

$$\begin{aligned} Det &= \frac{\left(1 - \Omega_K\right)\left(\delta + \rho_1\right)\left(1 - \beta\right)\beta\left(1 - \tau_{2m}^*\right)\left(1 - \bar{\tau}_m^*\right)\left(Y^*/K^*\right)^2}{\left(\Omega_L - 1\right)BL^*} \\ &\gtrless 0 \text{ when } \Omega_L - 1 \gtrless 0, \end{aligned} \tag{23}$$

and

$$Tr = \frac{\Phi^*\left(1 - \beta\right)Y^*/K^* - \delta\chi}{\Omega_L - 1} \lessgtr 0 \text{ when } \Omega_L - 1 \gtrless 0, \tag{24}$$

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where $\Phi^* \equiv \Omega_L(1-\bar{\tau}_m^*)(2-\frac{1}{BL^*}) - \frac{(1-\tau_{1m}^*)y_2^*+(1-\tau_{2m}^*)y_1^*}{2} < 0$.[8] The subsystem (15)-(17)'s local stability property is determined by comparing the eigenvalues of **J** that have negative real parts with the number of initial conditions in the subsystem, which is one, because y_{1t} and L_t are both non-predetermined jump variables. As a result, the economy displays saddle-path stability and equilibrium uniqueness if and only if one eigenvalue of **J** has a negative real part. This is true when $\Omega_L - 1 < 0$, under which the Jacobian **J** displays $Det < 0$ and $Tr > 0$. When $\Omega_L - 1 > 0$, the fact that $Det > 0$ and $Tr < 0$ indicates the existence of two negative eigenvalues. The steady state is thus a locally indeterminate sink that can be exploited to generate endogenous cyclical fluctuations driven by agents' self-fulfilling expectations or sunspots.

To understand the above indeterminacy condition, I present the log-linear forms of the aggregate labor supply schedule, L^s, the after-tax labor demand schedule, L^d, and the after-tax equilibrium wage-hours locus, L^D, under the income tax rule of (7) as follows:[9]

$$\hat{\omega}_t^s = \log B + \hat{C}_t, \tag{25}$$

$$\hat{\omega}_t^d = \log\left(1-\bar{\tau}_{mt}\right) + \log\left(\beta\right) + \hat{X}_t + \left(1-\beta\right)\hat{K}_t + \left(\beta-1\right)\hat{L}_t, \tag{26}$$

$$\hat{\omega}_t^D = \log\left(1-\bar{\tau}_{mt}\right) + \log\left(\beta A\right) + \Omega_K\hat{K}_t + \left(\Omega_L-1\right)\hat{L}_t, \tag{27}$$

where $\hat{X}_t = \log A + \chi(1-\beta)\hat{K}_t + \chi\beta\hat{L}_t$ represents productive externalities, and $1-\bar{\tau}_{mt} = \frac{\eta(1-\phi)[(y_{1t})^{-\phi}+(y_{2t})^{-\phi}]}{2}$. The above equations state that: (i) the slope of the aggregate labor supply curve is 0, because of the specification of indivisible labor; (ii) since each individual firm takes productive externalities $\hat{X}_t$ as given, the after-tax labor demand schedule exhibits a negative slope: $\frac{\partial\hat{\omega}_t^d}{\partial\hat{L}_t} = \beta - 1 < 0$; and (iii) after incorporating productive externalities, the aggregate after-tax equilibrium wage-hours locus is positively/negatively sloped when χ is above/below a critical level: $\frac{\partial\hat{\omega}_t^D}{\partial\hat{L}_t} = \Omega_L - 1 \gtrless 0$, when $\chi \gtrless \chi^{BF} \equiv \frac{1-\beta}{\beta}$, where χ^{BF} is the minimum level of productive externalities above which the laissez-faire economy of Benhabib and Farmer (1994) possesses an indeterminate steady state.[10] It follows that the necessary and sufficient condition needed to generate belief-driven fluctuations in this paper's heterogeneous-agent model under the progressive tax rule of (7) (i.e. $\Omega_L - 1 > 0$) states that the aggregate after-tax equilibrium wage-hours locus L^D is upward sloping and steeper than the aggregate labor supply curve L^s. Interestingly, this turns out to be exactly the same (necessary and sufficient) condition for equilibrium indeterminacy in Benhabib and Farmer's (1994) one-sector homogeneous-agent RBC model under laissez-faire.

Note that the slopes of L^s and L^D are both independent of the level and slope parameters in the income tax schedule, η and ϕ. The income tax structure thus exerts no effect on the model's local stability properties, indicating that progressive income taxation no longer serves as an automatic stabilizer as found in Guo and Lansing (1998), where sufficiently strong tax progressivity is able to stabilize the Benhabib–Farmer–Guo economy against business cycles driven by agents' animal spirits or sunspots. I thus have the following proposition.

PROPOSITION 2. *In the heterogeneous-household version of the Benhabib–Farmer economy without ongoing growth, progressive income taxation no longer influences the model's local stability properties and no longer serves as an automatic stabilizer when the fiscal rule of* (7)*, where the baseline level of taxable income is set at the time-varying economy-wide average level of income, is implemented. The original Benhabib–Farmer condition for equilibrium (in)determinacy under laissez faire prevails.*

In terms of the intuition behind the above indeterminacy result, let us start the economy from its steady-state equilibrium E^0 illustrated in Figure 1, wherein the degree of productive externalities is high enough ($\chi > \chi^{BF}$), such that L^D shown in the bottom panel is positively sloped and steeper than L^s. I then suppose that agents anticipate an increase in future economic activities. Acting upon this belief, both types of households will decrease today's consumption and invest more for future returns (the intertemporal substitution effect). By raising the time $t+1$ aggregate stock of capital, this shifts the labor demand curve L^d shown in the top panel of Figure 1 up and to the right, since (26) implies that $\frac{\partial \hat{\omega}_t^d}{\partial \hat{K}_t} = 1-\beta > 0$. Labor hours are thereby enhanced. In the presence of productive externalities, the higher economy-wide levels of capital stock and hours worked by raising $\hat{X}_{t+1}$ cause a further shift in L^d to the right; note from the expression $\hat{X}_{t+1} = \log A + \chi(1-\beta)\hat{K}_{t+1} + \chi\beta\hat{L}_{t+1}$ that a higher value of χ will magnify the increase in $\hat{X}_{t+1}$ and the consequential shift in L^d.

As the top panel of Figure 1 indicates, L^d shifts up also because of a decline in household 1's relative income y_{1t+1}, since (26) implies that $\frac{\partial \hat{\omega}_t^d}{\partial y_{1t}} = -\frac{\partial \bar{\tau}_{mt}/\partial y_{1t}}{1-\bar{\tau}_{mt}} < 0$, where $\frac{\partial \bar{\tau}_{mt}}{\partial y_{1t}} = \frac{\phi\eta(1-\phi)[(y_{1t})^{-\phi-1}-(y_{2t})^{-\phi-1}]}{2} > 0$. In particular, the intertemporal substitution effect, which leads the patient household 2 to reduce consumption and accumulate capital more than the impatient household 1, causes household 2's capital income to rise more than household 1's capital income. Consequently, Y_{2t+1} increases more than Y_{1t+1}, resulting in a decrease in $y_{1t+1} = \frac{2}{1+Y_{2t+1}/Y_{1t+1}}$, thus implying a deterioration in (pre-tax) income inequality.

The top panel of Figure 1 illustrates that the increase in labor demand that expands real output enhances aggregate consumption $\hat{C}_{t+1}$, which by shifting up the aggregate labor supply curve L^s ($\frac{\partial \hat{\omega}_t^s}{\partial \hat{C}_t} = 1$) reduces labor supply (the income effect). Thus, the top panel of Figure 1 shows that the economy moves from E^0 to E^1, leading labor hours to increase from $\hat{L}_{t+1}^0$ to $\hat{L}_{t+1}^1$.

In terms of the bottom panel of Figure 1, which presents a positively sloped aggregate after-tax equilibrium wage-hour locus L^D that intersects the aggregate labor supply curve L^s from below, the increase in K_{t+1} and the decrease in y_{1t+1} that shift up L^D tend to reduce labor hours. On the other hand, the higher level of $\hat{C}_{t+1}$ that shifts up L^s enhances hours worked. Because of the presence of the income effect, aggregate hours worked rise from $\hat{L}_{t+1}^0$ to $\hat{L}_{t+1}^1$ as the economy moves from E^0 to E^1.[11] For agents' initial rosy expectations about the economy's future to be validated as a self-fulfilling equilibrium, the economy-wide average

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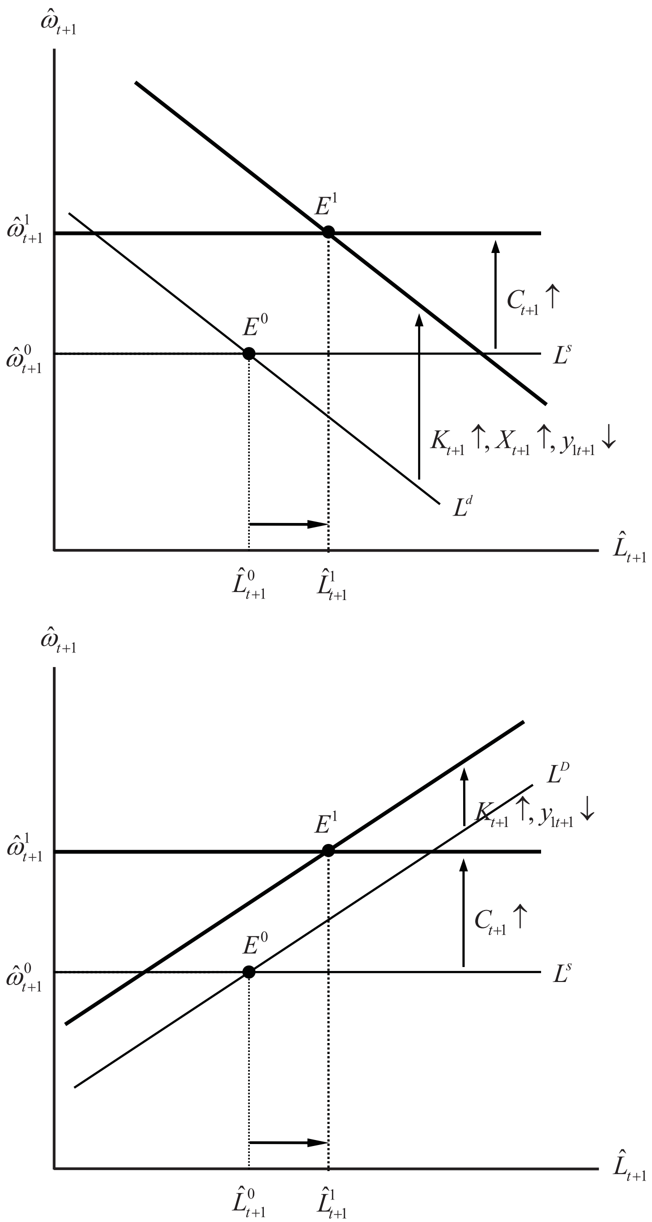

FIGURE 1. Labor market adjustment to a sunspot shock: L^D steeper than L^s

after-marginal tax rate of return on capital investment, $(1-\bar{\tau}_{mt+1})r_{t+1}$, should also be higher than its original steady-state level in transition. As it turns out, this requires a degree of productive externality χ that exceeds the threshold level of $\chi^{BF}=\frac{1-\beta}{\beta}$.

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I present in Figure 2 the case of a weaker degree of productive externalities such that $\chi < \chi^{BF}$ and $\Omega_L - 1 < 0$. The aggregate after-tax equilibrium wage-hours locus L^D thus is negatively sloped and is flatter than the labor demand curve, because of the presence of productive externalities: $\frac{\partial \hat{\omega}_t^d}{\partial \hat{L}_t} = \beta - 1 < \Omega_L - 1 = \frac{\partial \hat{\omega}_t^D}{\partial \hat{L}_t} < 0$. In this specification, when households become optimistic about the economy's future real activities and decide to raise their investment expenditures today, Figure 2 shows that the mechanism described above that creates multiple equilibria will lead to a decline in aggregate hours worked. This will in turn decrease the marginal product of capital and shrink real activities. As a result, belief-driven business fluctuations will not occur.

Based on the preceding analytical result, in what follows I quantitatively examine a calibrated version of the model under indeterminacy ($\Omega_L - 1 > 0$) in response to a 1% sunspot innovation.[12] As is common in the RBC literature, in the benchmark specification I set the labor share of national income $\beta = 0.6$, and the quarterly depreciation rate of physical capital $\delta = 0.1/4$. Household 1's time discount rate, ρ_1, is set equal to 0.04/4, and the scale parameter in the production function A is normalized to 1. In terms of the level and slope parameters of the tax schedule, they are respectively set equal to the average values of Chen and Guo's (2013a) year-by-year point estimates of ϕ and η from the 1966–2005 US federal individual income tax schedule. Hence, $\eta = 0.8$ and $\phi = 0.12$. The preference parameter B is chosen to be 0.088 such that the aggregate hours worked are one-fourth [Prescott (2006)]. Finally, household 2's rate of time preference, ρ_2, takes on the value of 0.031/4 such that household 2's relative income equals 1.25 (implying that $y_1^* = 0.75$, which is above the lower bound $\underline{y} = \eta^{\frac{1}{\phi}} = 0.1557$).

Under the baseline calibration of β, δ, χ, η, ϕ, ρ_1, ρ_2, A, and B, the Jacobian matrix $\mathbf{J}$, given by (A.1), associated with the subsystem (15)-(17) displays two negative roots and one positive root. The steady state thus displays indeterminacy of dimension 1. The selected value of $\beta = 0.6$ implies that the threshold level of χ that satisfies the necessary and sufficient condition for equilibrium indeterminacy is $\chi^{BF} = 0.067$. I thus set χ equal to $0.075 (> \chi^{BF})$ when plotting Figure 3, which describes the impulse response functions of the model economy to the above one-time sunspot innovation.

As can been seen from Figure 3, the related variables exhibit non-monotonic adjustment paths that are in line with the previously described mechanism. When the economy is hit by the animal spirits shock, the impact is a decline in household 1's relative income (y_{1t}), and increases in aggregate output (Y_t), aggregate hours worked (L_t), aggregate consumption (C_t), aggregate investment (I_t), as well as the after-marginal tax rate of return on capital ($(1 - \bar{\tau}_{mt})r_t$).[13] During the transitional period, y_{1t} remains below its original steady-state level, and C_t and Y_t remain higher. All other variables, L_t, I_t, and $(1 - \bar{\tau}_{mt})r_t$, are above their original steady-state levels while continuously falling, thereby slightly overshooting their steady-state levels.

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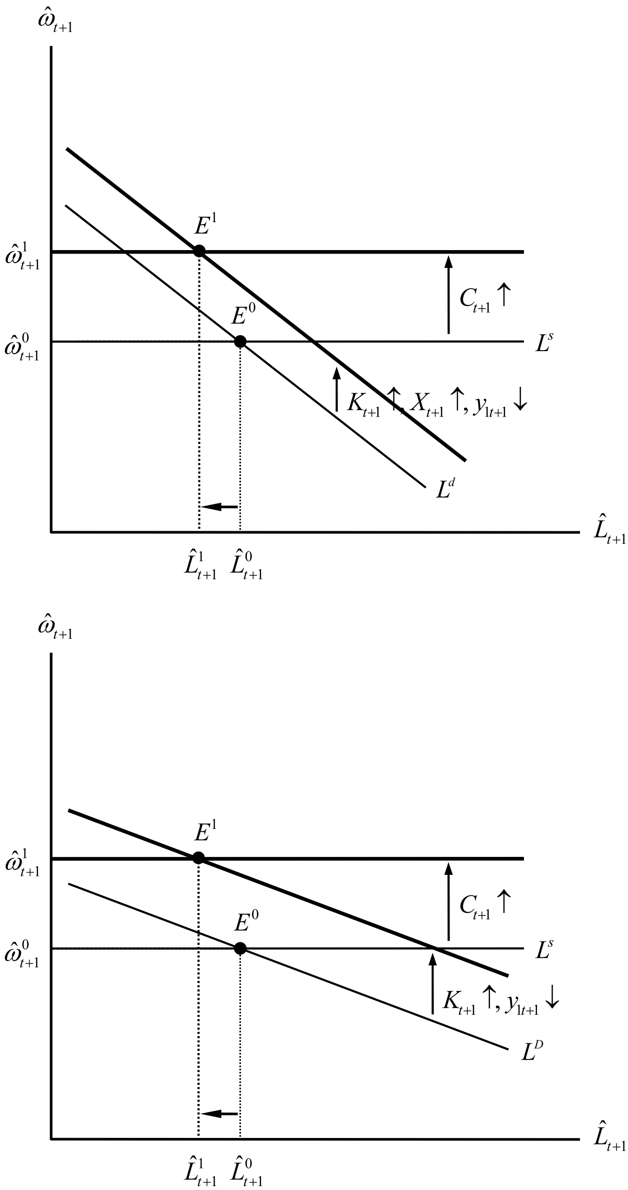

FIGURE 2. Labor market adjustment to a sunspot shock: L^D Flatter than L^s.

4. ALTERNATIVE INCOME TAX RULE

As explained in Section 1, the majority of the related works base the analysis on homogeneous-agent settings. In such a context, the representative household's level of income will be identical to the economy-wide average level, that is,

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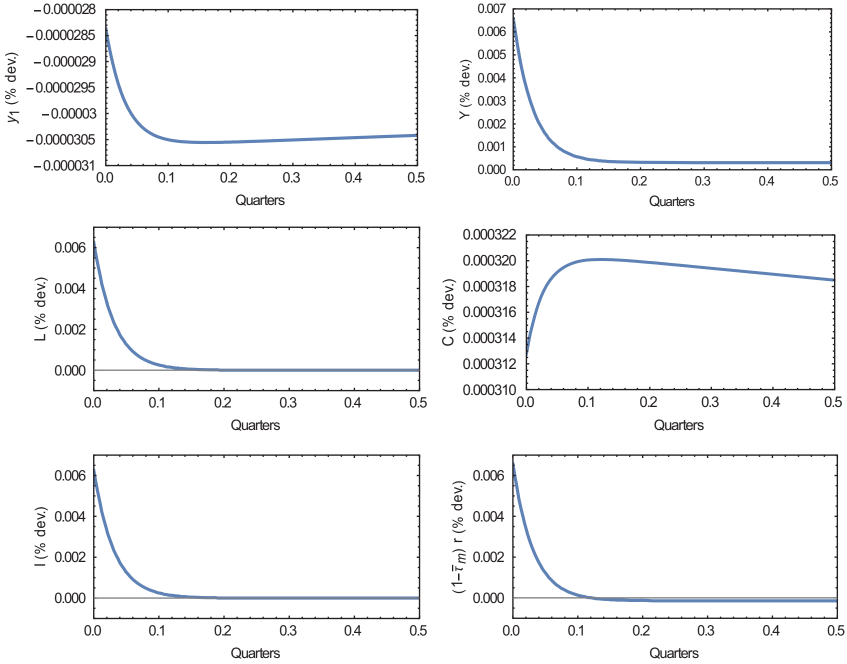

FIGURE 3. Impulse response functions to a sunspot shock: No-growth economy with $\Omega_L - 1 > 0$ and Y_t as the baseline level of taxable income.

$Y_{it} = Y_t$, for all i. If the fiscal authority adopts the fiscal rule of (7), then it will levy each household the same constant rate that equals $\tau_{it} = 1 - \eta$, for all i. Note that this constant income tax rate applies to all levels of taxable income Y_{it}, meaning that only a flat income tax rate is feasible. As a result, the related literature, including Guo and Lansing (1998), sets the baseline level of taxable income at the economy's steady-state level of per capita income. Under such a fiscal rule, both the average and marginal income tax rates can rise when the representative household earns a higher level of income, which is consistent with the US tax code illustrated in Chen and Guo (2013a, Figures 4 and 5).

The time-invariant steady-state level of per capita income as the benchmark level of taxable income is also an option for the fiscal authority under heterogeneous-agent settings. To investigate the resultant effect on the model's local dynamics, I postulate the following income tax rate τ_{it} faced by household i:

$$\tau_{it}^{s} = 1 - \eta \left(\frac{Y^*}{Y_{it}} \right)^{\phi}, \quad \eta \in (0, 1) \quad \text{and} \quad \phi \in (0, 1), \tag{28}$$

where $Y^*(= r^*K^* + w^*L^*)$ is the steady-state level of per capita income that is taken as given by each household. The first part of Appendix C presents the dynamical system that governs the dynamics of the model as well as the determinant and trace of the associated Jacobian matrix under the fiscal rule of (28). Based on that, I establish the following proposition.

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PROPOSITION 3. *In the heterogeneous-household version of the Benhabib–Farmer economy without ongoing growth, progressive income taxation serves as an automatic stabilizer when the fiscal rule of (28), where the baseline level of taxable income is set at the time-invariant steady-state level of per capita income, is implemented. The Guo–Lansing condition for equilibrium (in)determinacy prevails.*

Proposition 3 proves the robustness of Guo and Lansing's (1998) result to the inclusion of household heterogeneity when the fiscal rule of (28) is adopted. Specifically, a sufficiently strong tax progressivity operates like an automatic stabilizer that eliminates aggregate fluctuations caused by agents' animal spirits when the after-tax equilibrium wage-hours locus is upward sloping and steeper than the labor supply curve, that is, $(1-\phi)\Omega_L - 1 < 0$. The cyclical volatility of output can thus be reduced.

To understand the economic intuition, I find that, when the progressive income tax scheme of (28) is adopted, the aggregate labor supply schedule is still described by (25). However, the aggregate after-tax equilibrium wage-hours locus is now expressed as:

$$\begin{aligned}\hat{\omega}_t^{D,s} &= \log\left(1-\bar{\tau}_{mt}^s\right) + \log(\beta A) + \Omega_K \hat{K}_t + (\Omega_L - 1)\hat{L}_t \\ &= \log\left\{\Delta\left(Y^*\right)^{\phi}\left[(y_{1t})^{-\phi} + (y_{2t})^{-\phi}\right]\right\} + (1-\phi)\,\Omega_K \hat{K}_t + \left[(1-\phi)\,\Omega_L - 1\right]\hat{L}_t,\end{aligned} \tag{29}$$

where $1-\bar{\tau}_{mt}^s = \frac{\eta(1-\phi)[(y_{1t})^{-\phi}+(y_{2t})^{-\phi}](\frac{Y^*}{Y_t})^{\phi}}{2}$, and $\Delta \equiv \frac{\eta(1-\phi)\beta A^{1-\phi}}{2} > 0$.

It follows from (29) that the slope of the aggregate after-tax equilibrium wage-hours schedule under the tax policy rule of (28) is given by: $\frac{\partial \hat{\omega}_t^{D,s}}{\partial \hat{L}_t} = (1-\phi)\Omega_L - 1 \gtreqless 0$. When productive externalities are strong enough such that $\Omega_L - 1 > 0$ and hence the laissez-faire economy of Benhabib and Farmer (1994) displays belief-driven cyclical fluctuations, then by adopting the income tax scheme of (28) with the tax progressivity parameter being set above a critical level, $\phi > \tilde{\phi} \equiv \frac{\Omega_L - 1}{\Omega_L} \in (0,1)$, the fiscal authority is able to turn the slope of L^D from positive to negative. As a result, Figure 2 illustrates that agents' initial optimistic anticipation about the economy's future cannot be validated in equilibrium. Hence, under the income tax rule of (28), the fiscal authority is able to tax away the higher returns from belief-driven labor and investment spurts, thereby stabilizing the economy against sunspot fluctuations.[14]

5. THE DYNAMICS OF ENDOGENOUS GROWTH

This section turns to the heterogeneous-household version of Benhabib and Farmer's (1994, Section 5) model, where sufficiently strong productive externalities give rise to sustained economic growth. Households' and the government's behavior are exactly the same as those described in Section 2's macroeconomy. The only modification to the model is on the production side. Specifically, while

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firm j's production function is still of the form (1), productive externalities X_t are now given by:

$$X_t = AK_t^{\beta} L_t^{\beta\chi}, \quad A > 0, \quad \chi \geq 0, \tag{30}$$

In a symmetric equilibrium, all firms make the same decisions such that $K_{jt} = K_t$ and $L_{jt} = L_t$, for all j and t. As a result, (30) can be substituted into (1) to obtain the following social technology, which displays linearity in physical capital:

$$Y_t = AK_t L_t^{\Omega_L}. \tag{31}$$

Under the assumption that factor markets are perfectly competitive, the first-order conditions for the representative firm's profit maximization problem are exactly the same as those given by (4).

I focus on the economy's BGP along which labor hours are stationary, whereas output, consumption, and physical capital all grow at a common constant rate ψ. It is straightforward to show that, under the income tax rule (7), which sets the baseline level of taxable income at the economy-wide average level of output Y_t, the model's equilibrium conditions can be collapsed into the following autonomous dynamical system:

$$\dot{y}_{1t} = \left[\rho_1 - \rho_2 + (\tau_{1mt} - \tau_{2mt})(1-\beta) AL_t^{\Omega_L}\right] \frac{y_{1t} y_{2t}}{2\phi}, \tag{32}$$

$$\dot{L}_t = \frac{\left\{\left[(1-\beta)(1-\tau_{1mt}) - (1-\tilde{\tau}_t) + \frac{\beta(1-\bar{\tau}_{mt})}{BL_t}\right] AL_t^{\Omega_L} - \rho_1 + \phi \frac{\dot{y}_{1t}}{y_{1t}}\right\} L_t}{\Omega_L - 1}, \tag{33}$$

$$\dot{k}_{1t} = \left[(1-\tau_{1t}) y_{1t} - \beta(1-\tau_{1mt}) - (1-\tilde{\tau}_t) k_{1t} + \frac{\beta(1-\bar{\tau}_{mt}) k_{1t}}{BL_t}\right] AL_t^{\Omega_L}. \tag{34}$$

To examine the existence and number of the economy's BGPs, notice first that, under a given pair of the economy's BGP values of (y_1^*, L^*), equation (34) with $\dot{k}_{1t} = 0$ uniquely determines the BGP value of k_1^*. Thus, it comes down to examining the pair(s) of positive real numbers (y_1^*, L^*) such that $\dot{y}_{1t} = \dot{L}_t = 0$. To this end, I respectively derive from (32) and (33) that:

$$\left.\frac{dL^*}{dy_1^*}\right|_{\dot{y}_{1t}=0} = \frac{\phi \Pi_1^* L^*}{\Omega_L\left(\tau_{2m}^* - \tau_{1m}^*\right)} > 0, \tag{35}$$

$$\left.\frac{dL^*}{dy_1^*}\right|_{\dot{L}_t=0} = \frac{(\rho_2 - \rho_1)\left\{\frac{[\phi(1-\beta)+1]L^*}{\beta} + \frac{\phi \Pi_1^*}{B\left(\tau_{2m}^* - \tau_{1m}^*\right)}\right\}}{(1-\beta)\left[(1+\chi)(\rho_1 - \rho_2) y_2^* + 2\Psi^*\right]} < 0, \tag{36}$$

where $\Pi_1^* \equiv \eta(1-\phi)[(y_1^*)^{-\phi-1} + (y_2^*)^{-\phi-1}] > 0$ and $\Psi^* \equiv \frac{(1-\bar{\tau}_m^*)Y_t^*}{BL^* K_t^*} - \rho_1(1+\chi) > 0$.[15]

Equation (35) indicates that, regardless of the parametric configurations, the equilibrium locus of $\dot{y}_{1t} = 0$ is positively sloped in the $y_1^* - L^*$ space. Equation (36) then shows that $\dot{L}_t = 0$ is negatively sloped in the $y_1^* - L^*$ space. Hence, as Figure 4 illustrates, whatever the BGP's local stability properties are, there exists an intersection of the loci of $\dot{y}_{1t} = 0$ and $\dot{L}_t = 0$ that uniquely determines the model's BGP equilibrium. This result is distinct from the homogeneous-agent model of Benhabib and Farmer (1994, Section 5) under laissez faire, where two BGP equilibria emerge under sufficiently strong productive externalities from hours worked. I thus present the following proposition.

PROPOSITION 4. *Under household heterogeneity and the fiscal rule of (7), the endogenous-growth version of the Benhabib–Farmer model exhibits a unique balanced growth equilibrium.*

In terms of the local stability properties of the unique balanced growth equilibrium, it is clear that, similar to the preceding section's no-growth economy, the dynamical system (32)–(34) can be separated into two subsystems. Specifically, since (32) and (33) are independent of household 1's relative capital stock, $k_{1t} \equiv \frac{K_1^*}{K_t^*}$, these two equations constitute a 2-dimensional subsystem of (y_{1t}, L_t); afterward, the evolution of k_{1t} is determined by (34). Because k_{1t} is a predetermined variable and (34) possesses a negative eigenvalue that equals $-(\delta + \psi) < 0$, there exists a unique rational expectations equilibrium converging to k_1^*.

In terms of the local stability properties of the subsystem (32) and (33), I analytically compute the associated Jacobian matrix $\mathbf{J}^\psi$ evaluated at (y_1^*, L^*) and derive the corresponding determinant and trace as follows:

$$Det^\psi = -\frac{\left\{\Pi_1^* \left[(1-\beta)\,\Psi^* + \frac{\Omega_L(\rho_1-\rho_2)}{2BL^*}\right] + \Gamma^*\right\} \beta y_1^* y_2^* Y^*}{2\,(\Omega_L - 1)\,K^*} \gtrless 0 \text{ when } \Omega_L - 1 \lessgtr 0, \quad (37)$$

$$Tr^\psi = \frac{\Pi_1^*\,(1-\beta)\,y_1^* y_2^* Y^*}{2K^*} - \left[\frac{(1+\chi)\,(\rho_1-\rho_2)}{2} + \Psi^*\right] \frac{\beta}{\Omega_L - 1}, \quad (38)$$

where $\Gamma^* \equiv [\frac{(1-\beta)(1-\tau_{1m}^*)}{y_1^*} + \frac{\tau_{2m}^* - \tau_{1m}^*}{2\phi}](1+\chi)(\rho_1 - \rho_2) > 0$. Note that there is no initial condition for the dynamical system (32) and (33). As a result, the BGP displays equilibrium uniqueness if and only if both eigenvalues of $\mathbf{J}^\psi$ have positive real parts ($Det^\psi > 0$ and $Tr^\psi > 0$). Equations (37) and (38) indicate that this is true when $\Omega_L - 1 < 0$. If, by contrast, $\Omega_L - 1 > 0$, then the two eigenvalues of $\mathbf{J}^\psi$ are of opposite signs ($Det^\psi < 0$), and hence the BGP is a locally indeterminate sink that may lead to macroeconomic instability.

It is clear that I again derive the original Benhabib–Farmer–Guo condition for equilibrium (in)determinacy under the fiscal rule of (7), which sets the baseline level of taxable income at the economy-wide average level of output Y_t. The income tax structures η and ϕ thus no longer influence the model's local stability properties, and progressive income taxation no longer serves as an automatic

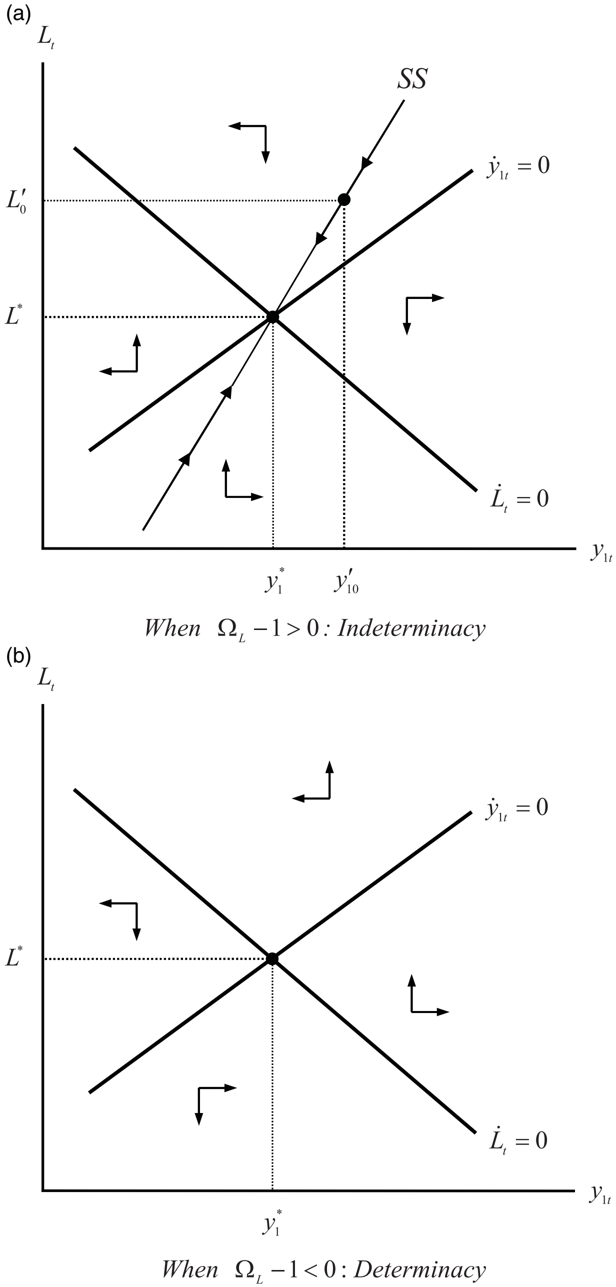

FIGURE 4. Phase diagram: Growth economy and Y_t as the baseline level of taxable income.

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destabilizer that generates local indeterminacy and belief-driven fluctuations in the endogenous-growth version of Benhabib and Farmer's (1994, Section 5) economy as found in Chen and Guo (2019). I thus establish the following proposition.

PROPOSITION 5. *In the heterogeneous-household version of the Benhabib–Farmer economy that displays sustained economic growth, progressive income taxation no longer influences the model's local stability properties when the fiscal rule of (7), where the baseline level of taxable income is set at the economy-wide average level of income, is implemented. The original Benhabib–Farmer condition for equilibrium (in)determinacy under laissez faire prevails.*

The intuition behind Proposition 5's indeterminacy result for the configuration where $\Omega_L - 1 > 0$ can be understood through the model's phase diagram illustrated in Figure 4(a). In this specification, there exists a positively sloped stable arm (denoted as SS) corresponding to the negative eigenvalue $\upsilon_1 < 0$ that is steeper than the equilibrium locus of $\dot{y}_{1t} = 0$.

Let us start from the economy's unique BGP equilibrium characterized by (y_1^*, L^*), and then consider a slight deviation caused by households' optimistic anticipation about an expansion of future economic activities. Acting upon this anticipation, households will increase current consumption through a positive wealth effect. On the other hand, a higher expected after-tax rate of return on capital investment induces households to reduce their consumption and invest more today through an intertemporal substitution effect. With a stronger wealth effect, households' current consumption rises in response to the animal spirits shock. This causes an immediate rise in household's hours worked, and hence an increase in aggregate labor hours, $L_t = [\frac{BC_t}{(1-\bar{\tau}_{mt})\beta AK_t}]^{\frac{1}{\Omega_L - 1}}$. Another dynamic trajectory $\{y'_{1t}, L'_t\}$ that begins at (y'_{10}, L'_0) with $y'_{10} > y_1^*$ and $L'_0 > L^*$ is thus initiated. Figure 4(a) shows that, for this alternative path to become a self-fulfilling equilibrium, the after-tax rate of return on investment for both types of households must be higher than their original BGP levels along the transitional path SS where $y_{1t} > y_1^*$. This is true since:

$$\left.\frac{d\,[(1-\tau_{1mt})\,MPK_t]}{dy_{1t}}\right|_{SS} = \frac{\Lambda^*\phi(1-\tau_{1mt})MPK_t}{y_{1t}} > 0, \quad (39)$$

and

$$\left.\frac{d\,[(1-\tau_{2mt})\,MPK_t]}{dy_{1t}}\right|_{SS} = \frac{(2+\Lambda^* y_{2t})\,\phi(1-\tau_{2mt})MPK_t}{y_{1t}y_{2t}} > 0, \quad (40)$$

where $\Lambda^* \equiv \frac{2(\mathbf{J}_{11}^{\psi} - \upsilon_1)}{(\rho_1 - \rho_2)y_2^*} - 1 > 0.$[16] As a consequence, agents' initial rosy expectations are validated under the indeterminate configuration where $\Omega_L - 1 > 0$. It follows from (39) and (40) that the economy-wide average after-marginal tax rate of return on capital investment, $(1-\bar{\tau}_{mt})MPK_t$, is also above its original BGP level during the transitional period.

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If the degree of productive externality χ is not high enough to meet the condition for equilibrium indeterminacy such that $\Omega_L - 1 < 0$, then Figure 4(b) shows that the model's BGP will be a completely unstable source. When households decide to raise their investment expenditures today, the preceding mechanism that makes for multiple equilibria will generate divergent trajectories away from the original BGP (y_1^*, L^*). This implies that, given household 1's initial capital endowment K_{10}, the period-0 levels of household 1's relative income y_{10} and the economy's aggregate labor hours L_0 are uniquely determined, such that the economy immediately jumps onto its original balanced growth equilibrium (y_1^*, L^*) and always stays there without any possibility of deviating transitional dynamics. It follows that equilibrium indeterminacy and endogenous growth fluctuations can never occur in this setting.

I next turn to examining the macroeconomic stabilizing properties of the fiscal rule that sets the baseline level of taxable income at the economy's BGP level of per capita output Y_t^*, whereby $\frac{\dot{Y}_t^*}{Y_t^*} = \psi$ for all t.[17] The second part of Appendix C presents the differential equations that govern the model's dynamics under the income tax rule where $\tau_{it}^s = 1 - \eta(\frac{Y_t^*}{Y_{it}})^\phi$, with $\eta \in (0, 1)$ and $\phi \in [0, 1)$. I also derive there the determinant and trace of the associated Jacobian matrix, and based on that I establish the following proposition, which demonstrates the robustness of Chen and Guo's (2019) result to the inclusion of household heterogeneity. Specifically, when tax progressivity falls below a critical level such that the aggregate after-tax equilibrium wage-hours locus is *flatter* than the labor supply curve (i.e. when $(1 - \phi)\Omega_L - 1 < 0)$), the economy's unique BGP must be locally indeterminate, indicating that progressive income taxation may work as an automatic *destabilizer*.

PROPOSITION 6. *In the heterogeneous-household version of the Benhabib–Farmer economy that displays sustained economic growth, progressive income taxation serves as an automatic destabilizer that exacerbates belief-driven growth fluctuations, when a tax policy rule that sets the baseline level of taxable income at the economy's BGP level of per capita output is implemented. The Chen–Guo sufficient condition for equilibrium indeterminacy prevails.*

Figure 5 describes the impulse responses of the growth economy under indeterminacy associated with a one-time animal spirits innovation, where panels (*a*) and (*b*) respectively, illustrate the cases where the baseline level of taxable income is set equal to Y_t and Y_t^*. The time unit is taken to be 1 year. I first adopt the proceeding section's baseline parameterization of $\beta = 0.6$, $\delta = 0.1$, $\chi = 0.75$, $\eta = 0.8$, $\phi = 0.12$, and $\rho_1 = 0.04$. I then set the preference parameter $B = 2.93$, the scale parameter in the production function $A = 2.35$, and household 2's rate of time preference $\rho_2 = 0.03$, such that the aggregate hours worked $L^* = 0.25$, the output growth rate $\psi = \frac{(1-\tau_{1m}^*)(1-\beta)Y^*}{K^*} - \delta - \rho_1 = 2\%$, and household 2's relative income $y_2^* = 1.25$.

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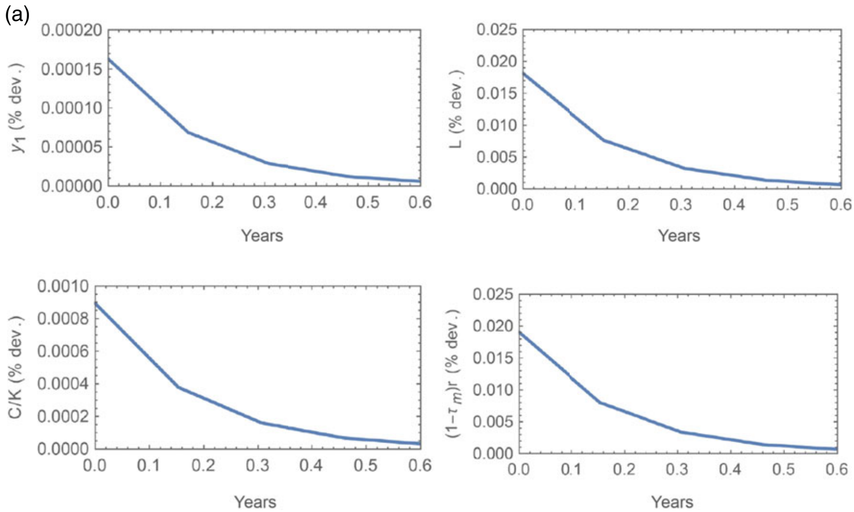
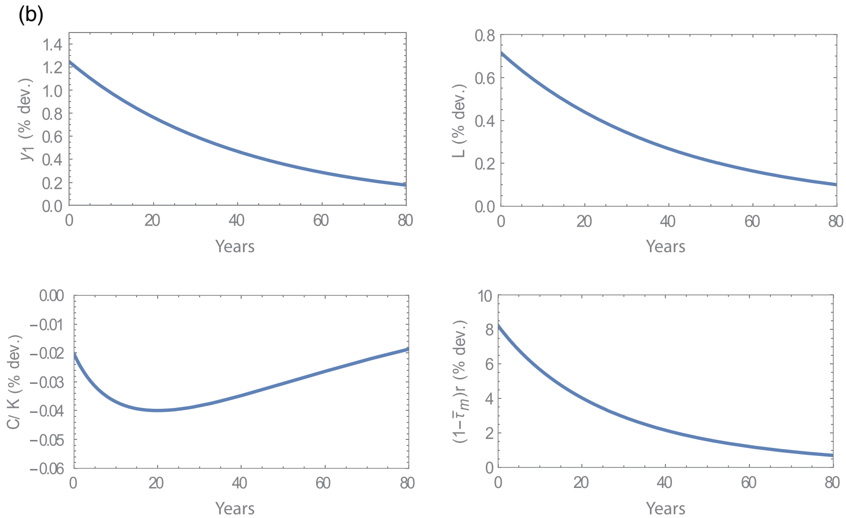


When $\Omega_L - 1 > 0$ *and* Y_t *as the baseline level of taxable income*



When $(1-\phi)\Omega_L - 1 < 0$ *and* Y_t^* *as the baseline level of taxable*

FIGURE 5. Impulse response functions to a sunspot shock: Growth economy.

Given the calibrated values of β, δ, χ, η, ϕ, ρ_1, ρ_2, A, and B, the economy under the progressive income tax rule, which sets the baseline level of taxable income at Y_t (Y_t^*), exhibits an aggregate after-tax equilibrium wage-hours locus that is steeper (flatter) than the aggregate labor supply curve, since $\Omega_L - 1 = 0.05 > 0$ and $(1-\phi)\Omega_L - 1 = -0.076 < 0$. Under either of the income tax rules, the model's unique BGP exhibits indeterminacy of dimension 1.

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The numerical simulation in Figure 5 illustrates that, under either of the progressive tax rules, the response of the economy to the animal spirits shock is immediate rises in household 1's relative income, aggregate labor hours, and the economy-wide average after-tax rate of return on capital investment. All these variables remain above their original BGP levels during the entire transitional period. The aggregate consumption-to-capital ratio C_t/K_t exhibits distinct adjustment patterns under the two income tax rules: in Figure 5(a) (Figure 5(b)) where Y_t (Y_t^*) is taken to be the baseline level of taxable income, C_t/K_t rises (drops) upon the animal spirits shock, and remains above (below) its original BGP level in transition.[18]

To understand the above indeterminacy result by examining labor market adjustment, I first define transformed variables $W_t \equiv \frac{\omega_t}{C_t}$ and the aggregate consumption-to-capital ratio $Z_t \equiv \frac{C_t}{K_t}$, such that I am able to graphically illustrate labor market equilibrium in an ongoing growth economy. I then express the log-linear forms of the aggregate labor supply schedule, L^s, and the after-tax equilibrium wage-hours locus, L^D, in transformed variables W_t and Z_t under the fiscal rule that sets the baseline level of taxable income at Y_t^* as follows:

$$\hat{W}_t^s = \log B, \tag{41}$$

$$\hat{W}_t^D = \log\left\{\Delta A^{\phi}\left[(y_{1t})^{-\phi} + (y_{2t})^{-\phi}\right]\right\} + (\Omega_L - 1)\,\hat{L}_t - Z_t. \tag{42}$$

When the baseline level of taxable income is instead set at the economy's BGP level of per capita output Y_t^*, (41) still describes the aggregate labor supply schedule. However, the aggregate after-tax equilibrium wage-hours locus is given by:

$$\hat{W}_t^{D,s} = \log\left\{\Delta\left(Y_t^*\right)^{\phi}\left[(y_{1t})^{-\phi} + (y_{2t})^{-\phi}\right]\right\} - \phi\hat{K}_t + \left[(1-\phi)\,\Omega_L - 1\right]\hat{L}_t - Z_t. \tag{43}$$

Based on (41), Figure 6 presents a horizontal aggregate labor supply curve, since $\frac{\partial \hat{W}_t^s}{\partial \hat{L}_t} = 0$. In addition, panel (*a*) illustrates the indeterminate configuration when Y_t is taken to be the baseline level of taxable income: according to (42), L^D is positively sloped: $\frac{\partial \hat{W}_t^D}{\partial \hat{L}_t} = \Omega_L - 1 > 0$. Panel (*b*) of Figure 6, on the other hand, corresponds to Figure 5(b) where Y_t^* is chosen to be the baseline level of taxable income and the Chen–Guo sufficient condition for indeterminacy is met: equation (43) implies that L^D is negatively sloped, because of a sufficiently progressive income tax: $\frac{\partial \hat{W}_t^{D,s}}{\partial \hat{L}_t} = (1-\phi)\Omega_L - 1 < 0$.

I start the economy as before from its steady-state equilibrium E^0, and then suppose that agents anticipate an increase in future economic activities. The resultant intertemporal substitution effect induces households to decrease consumption and invest more today, while the anticipation of higher levels of future income leads households to increase today's consumption. The simulation result shown

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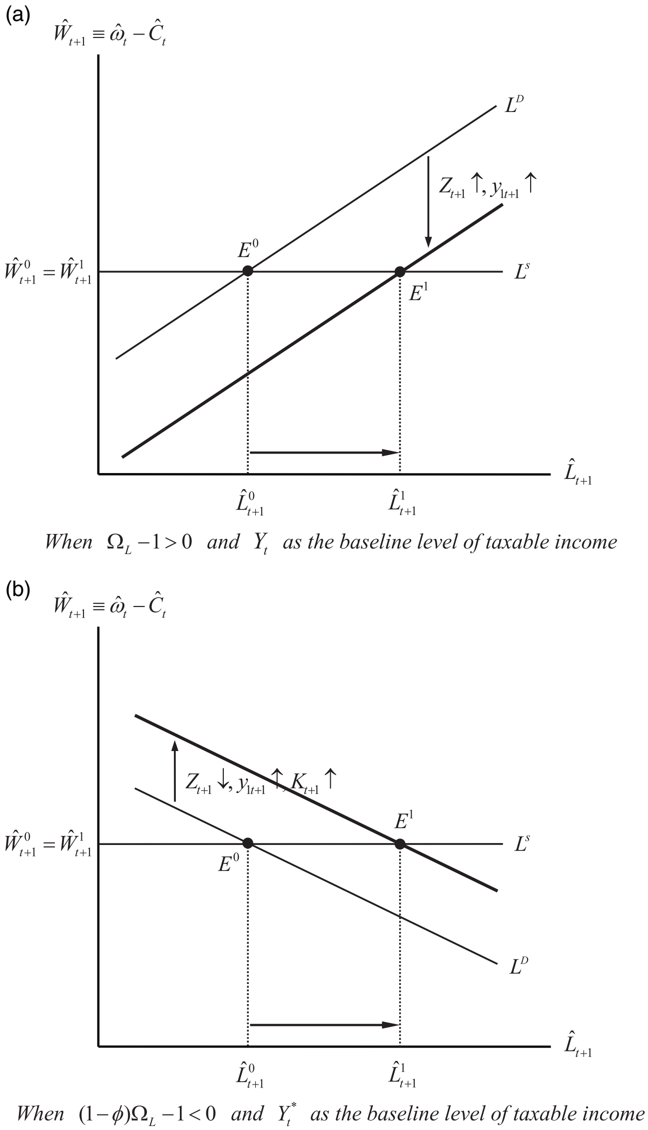

FIGURE 6. Labor market adjustment to a sunspot shock: Growth economy.

in Figure 5(a) depicts that, because of a stronger wealth effect, current consumption rises. In transition, the aggregate consumption-to-capital ratio remains above its original BGP level, and household 1's relative income is also at a higher level. Both will shift L^D down and to the right (see equation (42)), leading to an increase in aggregate hours worked. Real activities of the economy are thereby

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enhanced, and agents' initial optimistic anticipations about the economy's future are validated in equilibrium.

In terms of Figure 6(b), a sufficiently progressive income tax that reduces households' disposable income weakens the wealth effect. Hence, as shown in Figure 5(b), households' current consumption drops on impact, and the aggregate consumption-to-capital ratio remains below its original BGP level during the transitional period. Equation (43) indicates that, while the increases in both household 1's relative income and the capital stock tend to shift down L^D, the drop in the aggregate consumption-to-capital ratio is enough to lead to an upward shift in L^D. Hence, as Figure 6(b) illustrates, aggregate labor hours increase, and hence agents' expectations become self-fulfilling.

6. CONCLUSION

By incorporating household preference heterogeneity into Benhabib and Farmer's (1994) one-sector indeterminate model with aggregate increasing returns to scale, this paper systematically examines the interrelations between progressive income taxation and macroeconomic (in)stability under two specifications of the baseline level of taxable income: (i) the economy-wide average level of output and (ii) the economy's steady-state level of per capita output. When tax policy rule (i) is adopted, it turns out that the progressive tax structure—including both the tax-slope and level parameters—exerts no effect on the model's local stability properties, no matter whether or not the economy exhibits sustained economic growth. When tax policy rule (ii) is instead implemented, a sufficiently progressive income tax is an automatic stabilizer that reduces cyclical volatility of output in a no-growth economy, and progressive taxation may operate like an automatic destabilizer that exacerbates growth and employment volatility in an ongoing-growth economy.

This paper's study of the model's local dynamics utilizes an application of the Hartman–Grobman theorem.[19] The indeterminacy literature has cautioned that local determinacy of the steady state/BGP may co-exist with various forms of global indeterminacy and chaotic equilibrium paths.[20] Indeed, this paper's results imply that, under tax policy rule (i), the no-growth (sustained-growth) model economy undergoes a flip (Hopf) bifurcation, as the degree of productive externalities passes through the bifurcation value. Such a co-existence of local determinacy and global indeterminacy also appears when the no-growth economy is subject to tax policy rule (ii), and consequently the tax-slope parameter also serves as a flip bifurcation parameter. In this case, the fiscal authority's design of a progressive income tax rule aiming at stabilizing the economy against sunspot fluctuations near the steady state may give rise to the form of endogenous fluctuations arising from global indeterminacy.

This paper can be extended in several directions. First of all, it would be worthwhile to incorporate productive or utility-generating government spending *à la* Barro (1990), or to consider more general forms of the utility and/or production functions.[21] In addition, I can explore alternative mechanisms for generating endogenous growth (e.g. human capital accumulation) or examine an economy with multiple production sectors. These possible extensions will allow me to examine the robustness of this paper's theoretical results and policy implications, as well as further enhance the understanding of the relationship between progressive taxation and macroeconomic (in)stability in the presence of household heterogeneity and income inequality. I plan to pursue these research projects in the near future.

NOTES

1. See Benhabib and Farmer (1999) for an excellent survey of the RBC-based indeterminacy literature.
2. Other works that find the potential (de)stabilization role of progressive income taxation include Bosi and Seegmuller (2010) who incorporate heterogeneous agents and borrowing constraints, Chen and Guo (2013a, 2013b, 2014) who consider useful government spending contributing to utility or production, and the analyses within two-sector models by Guo and Harrison (2001) and Chen et al. (2018), among many others.
3. In a similar vein, Schmitt-Grohé and Uribe (1997) show that equilibrium indeterminacy can arise within standard one-sector RBC models under constant returns-to-scale in production and a balanced-budget rule with fixed government spending, where the latter implies a regressive income tax. Guo and Harrison (2001) find that regressive income taxation may stabilize a two-sector RBC economy against sunspot-driven business cycles.
4. Dromel and Pintus (2008), Lloyd-Braga et al. (2008), Carroll and Young (2009), Bosi and Seegmuller (2010), and Mino and Nakamoto (2012) also consider heterogeneous-agent settings. The first two papers base their analyses on the heterogeneous-consumer model of Woodford (1986), where one type of consumers are capitalists, and the other are workers who are subject to liquidity constraints. Carroll and Young (2008) examine how the minimum degree of increasing returns needed for generating indeterminacy depends on exogenous changes in wealth and wage inequalities under a progressive income tax. Bosi and Seegmuller (2010) focused on the case where only the most patient households hold capital and are subject to borrowing constraints. Mino and Nakamoto (2012) emphasize the role of intragroup and intergroup consumption externalities when both types of households supply labor and hold capital in equilibrium. Among the above-cited papers, Lloyd-Braga et al. (2008) is the only one that incorporates a baseline level of taxable income, and they set the baseline level of income at the economy's steady-state level of per capita GDP.
5. Within two-sector models with production externalities under laissez-faire, Ghiglino and Olszak-Duquenne (2005) and Ghiglino and Venditti (2011) also explore the interrelations between inequality and indeterminacy.
6. Useless or wasteful government spending is a simplifying assumption that is commonly adopted in the literature as the benchmark theoretical specification. Subsequent research may consider useful public expenditures that are productive (e.g. infrastructure) and/or utility-generating (e.g. national health care and public education).
7. In particular, it can be shown that total output and the economy-wide average level of consumption are expressed as $Y^* = A(K^*)^{\Omega_K}(L^*)^{\Omega_L}$ and $C^* = \frac{(1-\bar{\tau}_m^*)\beta Y^*}{BL^*}$, respectively. In addition, for group-1 households, the relative labor hours and consumption are $l_1^* \equiv \frac{L_1^*}{L^*} = \frac{y_1^*-(1-\beta)k_1^*}{\beta}$ and $c_1^* \equiv \frac{C_1^*}{C^*} = \frac{1-\tau_1^*}{2(1-\bar{\tau}^*)}$, respectively. For group-2 households, $y_2^* = 2 - y_1^*$, $k_2^* = 2 - k_1^*$, $l_2^* = 2 - l_1^*$, and $c_2^* = 2 - c_1^*$.
8. The empirically realistic value of the economy-wide average level of hours worked L^* is smaller than 0.5; for example, U.S. households allocate about one quarter of their time to production activities [Prescott (2006)]. When setting the scaling parameter B at 1, one can immediately tell that $\Phi^* < 0$.

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9. The term "equilibrium wage-hours locus" is dubbed as "equilibrium labor demand schedule" in Schmitt-Grohé and Uribe (1997) and "aggregate labor demand curve" in Wen (1998).

10. While it is widely known that this required degree of increasing returns-to-scale for local indeterminacy is too high to be empirically plausible when judged by most recent empirical estimates (e.g. Burnside (1996); Basu and Fernald (1997)), it is also known that the requisite degree of productive externalities can be reduced to an empirically plausible level when incorporating into the model features like variable capital utilization (Wen (1998)) and multiple production sectors with sector-specific externalities (Benhabib and Farmer (1996); Weder (2000); and Harrison (2001)).

11. See Wen (2001), Jaimovich (2008), Meng and Yip (2008), and Nourry et al. (2013), among others, for the presence of the income effect on the demand for leisure as a necessary condition for indeterminacy to occur in one-sector RBC models.

12. Following Farmer and Guo (1994) and Farmer (1999), among others, to study the economy's impulse response dynamics, I add into the model sunspot innovations, denoted as ε_{1t} and ε_{2t}, that have zero unconditional means. Equations (15) and (16) are thus respectively modified as: $\dot{y}_{1t} = [\rho_1 - \rho_2 + \frac{(\tau_{1mt}-\tau_{2mt})(1-\beta)Y_t}{K_t} + \varepsilon_{2t} - \varepsilon_{1t}]\frac{y_{1t}y_{2t}}{2\phi}$ and $\dot{L}_t = [\frac{(1-\tau_{1mt})(1-\beta)Y_t}{K_t} - \delta - \rho_1 + \phi\frac{\dot{y}_{1t}}{y_{1t}} - \Omega_K\frac{\dot{K}_t}{K_t} + \varepsilon_{1t}]\frac{L_t}{\Omega_L - 1}$; while both (17) and (18) remain unchanged. Figure 3 shows the case of homogenous expectations among households, whereby $\varepsilon_{1t} = \varepsilon_{2t}$.

13. Specifically, in the current labor market, there are two opposing forces that determine the shift in the aggregate labor supply curve: (i) the intertemporal substitution effect that reduces aggregate consumption shifts L^s downward; and (ii) the positive income effect that increases aggregate consumption shifts L^s upward. The numerical simulation shows that, because of a stronger income effect, aggregate labor hours rises on impact. This in turn expands real output and raises the after-marginal tax return on capital, where the former fulfills the increase in aggregate consumption.

14. Under the baseline parameterization of $\beta = 0.6$, $\delta = 0.1/4$, $\chi = 0.75$, $\eta = .0.8$, $\phi = 0.12$, $\rho_1 = 0.04/4$, $\rho_2 = 0.031/4$, $A = 1$, and $B = 0.088$, the slope of the aggregate after-tax equilibrium wage-hours locus is $(1-\phi)\Omega_L - 1 = -0.076$. In addition, the Jacobian matrix $\mathbf{J}^s$, given by (C.5), of the dynamical system (C.1)-(C.3) with one initial condition displays one negative root and two positive roots. This guarantees saddle-path stability of the steady state.

15. See the first part of Appendix B, where I numerically demonstrate that Ψ^* is positive under a wide range of empirically plausible parameter values.

16. See the second part of Appendix B for the proof of $\Lambda^* > 0$.

17. In order for a balanced-growth equilibrium to exist in the model economy, the household's taxable income Y_{it} needs to grow at the same rate as the baseline level of output Y_t^*.

18. Also note that, when Y_t^* is adopted as the benchmark level of taxable income, the variables return to their original BGP levels very slowly, showing a high degree of persistence imparted by a transitory white noise animal spirits shock. See also Farmer and Guo (1994) and Weder (2000), among others, for the analysis of one- and two-sector models under laissez-faire.

19. See, for example, Benhabib and Farmer (1994), Benhabib and Perli (1994), and Farmer and Guo (1994) for earlier works in this vast literature.

20. The co-existence of global indeterminacy with local determinacy is common in models with multiple steady states or balanced growth paths [see, for example, Benhabib and Perli (1994), Cazzavillan et al. (1998), Evans et al. (1998), Christiano and Harrison (1999), Benhabib et al. (2001), and Stockman (2010)]. Examples in models with a single interior steady state include Grandmont et al. (1998), Venditti (1998), Pintus et al. (2000), Zhang (2000), and Guo and Lansing (2002), among others. In this paper's setting where the model displays a unique steady state/balanced-growth path, regime switching sunspot fluctuations like those in Christiano and Harrison (1999) and Stockman (2010), as well as transcritical bifurcation as in Cazzavillan et al. (1998) will not occur.

21. The specification of indivisible labor in (5) allows me to obtain analytical stability conditions in this paper. When considering divisible labor whereby household i's instantaneous utility function takes the form of $U_{it} = \log C_{it} - B\frac{L_{it}^{1+\gamma}}{1+\gamma}$, where $\gamma > 0$ denotes the inverse of the intertemporal elasticity of substitution in labor supply, the subsystem (15)-(17) of the no-growth model will no longer be independent of K_{1t}; likewise, the subsystem (32) and (33) of the sustained-growth model will not be

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independent of k_{1t}. Because of the complicated structure of the subsequent dynamical systems of the models, the requisite conditions that govern the steady state's (the BGP's) local stability properties cannot analytically be obtained. One thus has to resort to numerical methods for studying the models' local dynamics. This is also true under, for example, García-Peñalosa and Turnovsky's (2006, 2007) specification of a homogeneous utility function: $U_{it} = \frac{1}{1-\sigma}[C_{it}(1-L_{it})^{\theta}]^{1-\sigma}$, where σ, $\theta > 0$, as well as under more general forms of the utility and/or production functions as in Cazzavillan et al. (1998), Meng and Yip (2008), Dromel and Pintus (2008), Bosi and Seegmuller (2010), and Nourry et al. (2013), among others. The related preference and/or technology parameters may then exert substantial influences on the stability conditions.

22. Under the baseline parameterization of $\beta = 0.6$, $\delta = 0.1/4$, $\eta = 0.8$, $\phi = 0.12$, $\rho_1 = 0.04/4$, $\rho_2 = 0.031/4$, $A = 1$, and $B = 0.088$, the numerator of Tr^s is negative for all $\chi \geq 0$ and $\phi \in (0, 1)$. Hence, when $(1-\phi)\Omega_L - 1 < 0$, the Jacobian $\mathbf{J}^s$ has only one root with a negative real part since $Det^s < 0$ and $Tr^s > 0$. The steady state thus displays saddle-path stability and equilibrium determinacy.

23. When $(1-\phi)\Omega_L - 1 > 0$, equation (C.6) implies that $Det^s > 0$. In addition, $(1-\phi)\Omega_L - 1 > 0$ guarantees $\Delta < 0$, and hence $Tr^s < 0$. I thus analytically obtain that the Jacobian $\mathbf{J}^s$ has two roots with negative real parts, and hence the steady state displays indeterminacy of dimension 1.

24. When adopting the baseline calibration of $\beta = 0.6$, $\delta = 0.1$, $\eta = 0.8$, $\rho_1 = 0.04$, $\rho_2 = 0.03$, $A = 2.35$, and $B = 2.93$, the Jacobian $\mathbf{J}^{s,\psi}$ displays one (two) root(s) with negative real part(s) for all $\chi \geq 0$ and $\phi \in (0, 1)$, provided that $(1-\phi)\Omega_L - 1 < (>)0$. Hence, if $(1-\phi)\Omega_L - 1 < (>)0$, then the model's unique BGP exhibits indeterminacy of dimension 1 (2). This indeterminacy result is robust to a wide range of parameter values that are consistent with the data. Notably, Chen and Guo (2019) find that dual BGPs emerge when $(1-\phi)\Omega_L - 1 > 0$, wherein the high-growth BGP is a locally indeterminate sink, whereas the low-growth BGP displays equilibrium uniqueness.

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APPENDIX A

This appendix details the Jacobian matrices of the dynamical systems for the no-growth and sustained-growth models when the economy-wide average level of income is specified as the baseline level of taxable income. For the no-growth model, I derive in the neighborhood of the steady state (y_1^*, L^*, K^*) of the subsystem (15)–(17) the associated 3×3 Jacobian matrix J as follows:

$$\mathbf{J} = \begin{bmatrix} \frac{(\rho_1-\rho_2)\Pi_1^* y_1^* y_2^*}{2\left(\tau_{2m}^*-\tau_{1m}^*\right)} & -\frac{\Omega_L(\rho_1-\rho_2)y_1^* y_2^*}{2\phi L^*} & \frac{(1-\Omega_K)(\rho_1-\rho_2)y_1^* y_2^*}{2\phi K^*} \\ \frac{L^*\left[\frac{\phi(\mathbf{J}_{11}-\delta-\rho_1)}{y_1^*}-\frac{\Omega_K \mathbf{J}_{31}}{K^*}\right]}{\Omega_L-1} & \frac{L^*\left[\frac{\Omega_L(\delta+\rho_1)}{L^*}+\frac{\phi \mathbf{J}_{12}}{y_1^*}-\frac{\Omega_K \mathbf{J}_{32}}{K^*}\right]}{\Omega_L-1} & \frac{L^*\left[\frac{(\Omega_K-1)(\delta+\rho_1)}{K^*}+\frac{\phi \mathbf{J}_{13}}{y_1^*}-\frac{\Omega_K \mathbf{J}_{33}}{K^*}\right]}{\Omega_L-1} \\ \frac{(1-\phi)\eta Y^*\left(\tau_{2m}^*-\tau_{1m}^*+\frac{\Pi_2^*\phi\beta}{BL^*}\right)}{2} & \frac{\beta\left[(1+\chi)\delta K^*+\frac{(1-\bar{\tau}_m)Y^*}{BL^*}\right]}{L^*} & \delta\,(\Omega_K-1) \end{bmatrix}. \tag{A.1}$$

For the subsystem (32) and (33) of the ongoing-growth model, on the other hand, the 2×2 Jacobian matrix J^ψ evaluated at (y_1^*, L^*) is given by:

$$\mathbf{J}^\psi = \begin{bmatrix} \frac{(\rho_1-\rho_2)\Pi_1^* y_1^* y_2^*}{2\left(\tau_{2m}^*-\tau_{1m}^*\right)} & -\frac{\Omega_L(\rho_1-\rho_2)y_1^* y_2^*}{2\phi L^*} \\ \frac{(\rho_1-\rho_2)\left\{[\phi(1-\beta)+1]L^*+\frac{\beta\phi\Pi_2^*}{B\left(\tau_{2m}^*-\tau_{1m}^*\right)}\right\}}{2(1-\beta)(1-\Omega_L)} & \frac{\beta}{1-\Omega_L}\left[\frac{(1+\chi)(\rho_1-\rho_2)y_2^*}{2}+\Psi^*\right] \end{bmatrix}. \tag{A.2}$$

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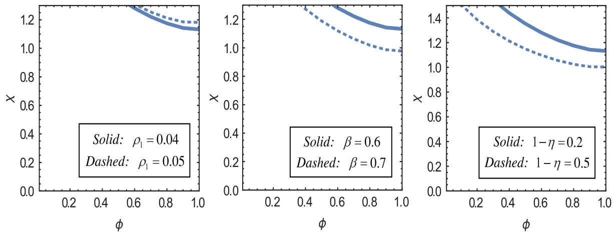

FIGURE B.1. Locus of $\Psi^* = 0$ (above: $\Psi^* < 0$; below: $\Psi^* > 0$).

APPENDIX B

Proof of $\Psi^* > 0$. Being unable to analytically pin down the sign of Ψ^*, I numerically show that $\Psi^* > 0$ is empirically realistic. In addition to the baseline parameterization of $\beta = 0.6$, $\delta = 0.1$, $\eta = 0.8$, $\phi = 0.12$, $\rho_1 = 0.04$, and $\chi = 0.03$, I set the preference parameter $B = 2.93$, the scale parameter in the production function $A = 2.35$, and household 2's rate of time preference $\rho_2 = 0.03$, such that the aggregate hours worked L^* are one quarter, the output growth rate ψ equals 2%, and household 2's relative income y_2^* is 1.25.

Given the above calibrated parameter values, the downward-sloped loci in the left panel of Figure B.1 illustrates the combinations of (ϕ, χ) such that $\Psi^* = 0$, where the solid (dashed) locus is plotted under $\rho_1 = 0.04$ (0.05). For both loci, the points located above (below) $\Psi^* = 0$ are associated with higher (lower) values of χ such that $\Psi^* < (>)0$. It is clear under any given value of the tax-slope parameter ϕ that the requisite degree of productive externalities from hours worked, χ, is too high to square with the data. Likewise, the middle (right) panel of Figure B.1 plots the loci of $\Psi^* = 0$ when β $(1-\eta)$ takes on different values. All panels in Figure B.1 demonstrate that $\Psi^* > 0$ when χ lies within an empirically reasonable range. ■

Proof of $\Lambda^* > 0$. By using $v_1 = \frac{-Tr^{\psi} - \sqrt{(Tr^{\psi})^2 - 4Det^{\psi}}}{2}$, I re-express Λ^* as: $\Lambda^* = \frac{\sqrt{\Lambda_1^*} - \sqrt{\Lambda_2^*}}{(\rho_1 - \rho_2) y_2^*}$, where $\Lambda_1^* \equiv (\mathbf{J}_{22}^{\psi} - \mathbf{J}_{11}^{\psi})^2 + 4\mathbf{J}_{12}^{\psi}\mathbf{J}_{21}^{\psi}$ and $\Lambda_2^* \equiv [\mathbf{J}_{22}^{\psi} - \mathbf{J}_{11}^{\psi} + (\rho_1 - \rho_2) y_2^*]^2$. I then define $\Pi_2^* \equiv \eta(1-\phi)[(y_1^*)^{-\phi-1} - (y_2^*)^{-\phi-1}] > 0$, and derive that:

$$\Lambda_1^* - \Lambda_2^* = \left(\frac{1}{\Omega_L - 1} + \frac{\Pi_1^* y_1^*}{\tau_{2m}^* - \tau_{1m}^*} \right) \left[(\rho_1 - \rho_2) \, y_2^* \right]^2 + \frac{2 \, (\rho_1 - \rho_2) \, \beta \Psi^* y_2^*}{\Omega_L - 1}$$

$$+ \frac{\Omega_L \, (\rho_1 - \rho_2)^2 \, y_1^* y_2^*}{(\Omega_L - 1) \, \phi \, (1-\beta)} \left[\phi \, (1-\beta) + 1 + \frac{\beta \phi \Pi_2^*}{B \left(\tau_{2m}^* - \tau_{1m}^* \right) L^*} \right] > 0.$$

As a result, $\Lambda^* = \frac{\sqrt{\Lambda_1^*} - \sqrt{\Lambda_2^*}}{(\rho_1 - \rho_2) y_2^*} > 0$. ■

APPENDIX C

This appendix provides the dynamical systems for the no-growth as well as the sustained-growth models when the economy's steady state/BGP level of per capital output is adopted as the baseline level of taxable income. Under this fiscal formulation, the average and

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marginal income tax rates are $\tau_{it}^s = 1 - \eta(y_{it})^{-\phi}(\frac{Y^*}{Y_t})^{\phi}$ and $\tau_{imt}^s = 1 - \eta(1-\phi)(y_{it})^{-\phi}(\frac{Y^*}{Y_t})^{\phi}$, respectively. I then define the arithmetic mean of the average and marginal tax rates as well as the effective average tax rate as $\bar{\tau}_t^s \equiv \frac{\tau_{1t}^s + \tau_{2t}^s}{2}$, $\bar{\tau}_{mt}^s \equiv \frac{\tau_{1mt}^s + \tau_{2mt}^s}{2}$, and $\tilde{\tau}_t^s \equiv \frac{\tau_{1t}^s y_{1t} + \tau_{2t}^s y_{2t}}{2}$, respectively.

The dynamics of the no-growth economy can be divided into two subsystems, where (y_{1t}, L_t, K_t) are first determined by the following 3-dimensional system:

$$\dot{y}_{1t} = \left[\rho_1 - \rho_2 + \frac{\left(\tau_{1mt}^s - \tau_{2mt}^s\right)(1-\beta)Y_t}{K_t}\right]\frac{y_{1t}y_{2t}}{2\phi}, \quad \textbf{(C.1)}$$

$$\dot{L}_t = \frac{\left[\frac{\left(1-\tau_{1mt}^s\right)(1-\beta)Y_t}{K_t} - \delta - \rho_1 + \phi\frac{\dot{y}_{1t}}{y_{1t}} - (1-\phi)\,\Omega_K\frac{\dot{K}_t}{K_t}\right]L_t}{(1-\phi)\,\Omega_L - 1}, \quad \textbf{(C.2)}$$

$$\dot{K}_t = \left(1 - \tilde{\tau}_t^s\right)Y_t - \frac{\left(1-\bar{\tau}_{mt}^s\right)\beta Y_t}{BL_t} - \delta K_t. \quad \textbf{(C.3)}$$

In turn, the evolution of K_{1t} is governed by:

$$\dot{K}_{1t} = \left(1-\tau_{1t}^s\right)y_{1t}Y_t - \frac{\left(1-\tau_{1mt}^s\right)\beta Y_t}{BL_t} - \delta K_{1t}, \quad \textbf{(C.4)}$$

which exhibits a negative eigenvalue equaling $-\delta$.

It is straightforward to show that the model displays the same unique interior steady state given by (19)–(22). In addition, the 3×3 Jacobian matrix $\mathbf{J}^s$ associated with (C.1)–(C.3) is given by:

$$\mathbf{J}^s = \begin{bmatrix} \frac{(\rho_1-\rho_2)\Pi_1^* y_1^* y_2^*}{2\left[\left(\tau_{2m}^s\right)^* - \left(\tau_{1m}^s\right)^*\right]} & -\frac{(1-\phi)\Omega_L(\rho_1-\rho_2)y_1^*y_2^*}{2\phi L^*} & \frac{[1-(1-\phi)\Omega_K](\rho_1-\rho_2)y_1^*y_2^*}{2\phi K^*} \\ \frac{L^*\left[\frac{\phi(\mathbf{J}_{11}-\delta-\rho_1)}{y_1^*} - \frac{(1-\phi)\Omega_K\mathbf{J}_{31}}{K^*}\right]}{(1-\phi)\Omega_L-1} & \frac{L^*\left[\frac{(1-\phi)\Omega_L(\delta+\rho_1)}{L^*} + \frac{\phi\mathbf{J}_{12}}{y_1^*} - \frac{(1-\phi)\Omega_K\mathbf{J}_{32}}{K^*}\right]}{(1-\phi)\Omega_L-1} & \mathbf{J}_{23}^s \\ \frac{(1-\phi)\eta Y^*\left[\left(\tau_{2m}^s\right)^* - \left(\tau_{1m}^s\right)^* + \frac{\Pi_2^*\phi\beta}{BL^*}\right]}{2} & \frac{\beta\left[(1-\phi)(1+\chi)\delta K^* + \frac{\left[1-(\bar{\tau}_m^s)^*\right]Y^*}{BL^*}\right]}{L^*} & \delta\left[(1-\phi)\,\Omega_K - 1\right] \end{bmatrix}, \quad \textbf{(C.5)}$$

where $\mathbf{J}_{23}^s = \frac{L^*\{\frac{[(1-\phi)\Omega_K-1](\delta+\rho_1)}{K^*} + \frac{\phi\mathbf{J}_{13}}{y_1^*} - \frac{(1-\phi)\Omega_K\mathbf{J}_{33}}{K^*}\}}{(1-\phi)\Omega_L-1}$. It follows that the trace and determinant of $\mathbf{J}^s$ are

$$Det^s = \frac{[1-(1-\phi)\,\Omega_K](\delta+\rho_1)(1-\beta)\beta\left[1-\left(\tau_{2m}^s\right)^*\right]\left[1-\left(\bar{\tau}_m^s\right)^*\right]\left(\frac{Y^*}{K^*}\right)^2}{[(1-\phi)\,\Omega_L - 1]\,BL^*}, \quad \textbf{(C.6)}$$

$$Tr^s = \frac{(\Phi^s)^*(1-\beta)\frac{Y^*}{K^*} + \delta\Delta}{(1-\phi)\,\Omega_L - 1}, \quad \textbf{(C.7)}$$

where $(\Phi^s)^* \equiv (1-\phi)\Omega_L[1-(\bar{\tau}_m^s)^*](2-\frac{1}{BL^*}) - \frac{[1-(\tau_{1m}^s)^*]y_2^* + [1-(\tau_{2m}^s)^*]y_1^*}{2} < 0$, and $\Delta \equiv \frac{\beta-(1-\phi)\Omega_L}{\beta} \gtrless 0$. Because the dynamical system (C.1)-(C.3) has one given initial condition, Det^s has to be negative for the steady state to display equilibrium uniqueness. Equation (C.6) implies that this necessary condition will be satisfied when $(1-\phi)\Omega_L - 1 < 0$.[22,23]

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The dynamics of the model with ongoing growth likewise can be divided into two subsystems, where $(y_{1t}, L_t, z_{1t} \equiv \frac{C_{1t}}{K_t})$ are first determined by the following 3-dimensional system:

$$\dot{y}_{1t} = \frac{\left[(\rho_1 - \rho_2)\,\beta\left(1 - \tau_{1t}^s\right) + \left(\tau_{1mt}^s - \tau_{2mt}^s\right)(1-\beta)\,Bz_{1t}L_t\right] y_{1t}y_{2t}}{2\phi\beta\left(1 - \tau_{1t}^s\right)}, \quad \textbf{(C.8)}$$

$$\dot{L}_t = \frac{\left[\frac{\phi(1-\beta)Bz_{1t}L_t}{\beta} + \phi\frac{\dot{y}_{1t}}{y_{1t}} + (1-\phi)\frac{\dot{z}_{1t}}{z_{1t}} - \phi\left(\psi^* + \delta + \rho_1\right)\right] L_t}{(1-\phi)\,\Omega_L - 1}, \quad \textbf{(C.9)}$$

$$\dot{z}_{1t} = \left\{\left[(1-\beta)\left(1 - \tau_{1t}^s\right) - \frac{1 - \tilde{\tau}_t^s}{1-\phi} + \frac{\beta\left(1 - \bar{\tau}_t^s\right)}{BL_t}\right]\frac{Bz_{1t}L_t}{\beta\left(1 - \tau_{1t}^s\right)} - \rho_1\right\} z_{1t}. \quad \textbf{(C.10)}$$

The evolution of k_{1t} is sequentially governed by:

$$\dot{k}_{1t} = \left\{\left[\frac{y_{1t}}{1-\phi} - \frac{\beta}{BL_t} - (1-\beta)\,k_{1t}\right]\frac{Bz_{1t}L_t}{\beta k_{1t}} + \frac{\dot{z}_{1t}}{z_{1t}} + \rho_1\right\} k_{1t}, \quad \textbf{(C.11)}$$

which exhibits a negative eigenvalue equaling $-\frac{(\psi+\delta)Bz_1^*K^*L^*}{\beta[1-(\tau_{1m}^s)^*]Y^*}$.

By imposing $\dot{y}_{1t} = \dot{L}_t = \dot{z}_{1t} = 0$, it can be shown that the model exhibits exactly the same unique interior BGP as presented in Figure 4. The 3×3 Jacobian matrix $\mathbf{J}^{s,\psi}$ associated with (C.8)-(C.10) is as follows:

$$\mathbf{J}^{s,\psi} = \begin{bmatrix} \frac{y_2^*\left[\rho_2 - \rho_1 + \frac{\Pi_1^*(1-\beta)Bz_1^*L^*y_1^*}{\beta\left[1-\left(\tau_{1m}^s\right)^*\right]}\right]}{2} & \frac{\left[\left(\tau_1^s\right)^* - \left(\tau_2^s\right)^*\right](1-\beta)Bz_1^*y_1^*y_2^*}{2\phi\left[1-\left(\tau_1^s\right)^*\right]\beta} & \frac{\left[\left(\tau_1^s\right)^* - \left(\tau_2^s\right)^*\right](1-\beta)BL^*y_1^*y_2^*}{2\eta\left[1-\left(\tau_1^s\right)^*\right]\beta} \\ \frac{L^*\left[\frac{\phi\mathbf{J}_{11}}{y_1^*} + \frac{(1-\phi)\mathbf{J}_{31}}{z_1^*}\right]}{(1-\phi)\Omega_L - 1} & \frac{L^*\left[\frac{\phi\mathbf{J}_{12}}{y_1^*} + \frac{(1-\phi)\mathbf{J}_{32}}{z_1^*} + \frac{\phi(1-\beta)Bz_1^*}{\beta}\right]}{(1-\phi)\Omega_L - 1} & \frac{L^*\left[\frac{\phi\mathbf{J}_{13}}{y_1^*} + \frac{(1-\phi)\mathbf{J}_{33}}{z_1^*} + \frac{\phi(1-\beta)BL^*}{\beta}\right]}{(1-\phi)\Omega_L - 1} \\ \mathbf{J}_{31}^{s,\psi} & \left[\rho_1 - \frac{\left[1-\left(\bar{\tau}^2\right)^*\right]z_1^*}{1-\left(\tau_1^s\right)^*}\right]\frac{z_1^*}{L^*} & \rho_1 \end{bmatrix}, \quad \textbf{(C.12)}$$

where $\mathbf{J}_{31}^{s,\psi} = \frac{\phi\rho_1 z_1^*}{y_1^*} - \frac{B(z_1^*)^2L^*\left[\frac{\phi(1-\beta)[1-(\tau_{1m}^s)^*]}{y_1^*} + \frac{(\tau_{2m}^s)^* - (\tau_{1m}^s)^*}{2} + \frac{\Pi_2^*\phi\beta}{2BL^*}\right]}{[1-(\tau_{1m}^s)^*]\beta}$. The trace and determinant of $\mathbf{J}^{s,\psi}$ are then derived as:

$$Det^{s,\psi} = \frac{\phi\left[1 - (\bar{\tau}^s)^*\right]\left[1 - \left(\tau_2^s\right)^*\right] z_1^*}{(1-\phi)\,\Omega_L - 1}\left[\frac{(1-\beta)\,Bz_1^*L^*}{\beta\left[1 - \left(\tau_1^s\right)^*\right]}\right]^2, \quad \textbf{(C.13)}$$

$$Tr^{s,\psi} = \frac{\left\{2\phi - y_2^* + \frac{\left[2(1-\phi)\Omega_L - y_1^*\right]\left[1-\left(\tau_2^s\right)^*\right]}{1-\left(\tau_1^s\right)^*}\right\}\frac{(1-\beta)Bz_1^*L^*}{2\beta} - \frac{\left[1-\left(\bar{\tau}_m^s\right)^*\right]z_1^*}{1-\left(\tau_1^s\right)^*} + \left[(1-\phi)\,\Omega_L - \phi\right]\rho_1}{(1-\phi)\,\Omega_L - 1}, \quad \textbf{(C.14)}$$

Since the dynamical system (C.8)–(C.10) has no initial conditions, $Det < 0$ guarantees the existence of infinitely many rational expectations equilibria, where there are either one or three dimensions of indeterminacy. Equation (C.6) implies that this sufficient condition for indeterminacy will be satisfied when $(1-\phi)\Omega_L - 1 < 0$.[24]

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