Robotica (2013) volume 31, pp. 589–598. © Cambridge University Press 2012
doi:10.1017/S0263574712000604

A sliding-mode controller for dual-user teleoperation with unknown constant time delays

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(Accepted September 10, 2012. First published online: October 19, 2012)

SUMMARY
In this paper, a control methodology is proposed for dual-user teleoperation system in the presence of unknown constant communication time delay. To satisfy dual-user system-desired objectives, three impedance characteristics are defined as the desired closed-loop system. In order to satisfy the desired impedance characteristics, a sliding-mode-based impedance controller is applied. The proposed controller affords unknown communication delay, an issue that is disregarded in the previous studies performed on dual-user systems. The nonlinear gain of the controller is achieved independent of time delay caused by communication channels. Therefore, the necessity of measurement or estimation of the time delay is relaxed. In addition, the stability analysis is presented for the closed-loop system using the passivity theory. The validity of the proposed controller scheme is demonstrated via experimental results performed on a dual-user system in the presence of unknown communication delay. In addition, due to lack of availability of forces corresponding to the operators' hand that are required in the proposed controller, a Kalman Filter-based Force Observer (KFFO) is proposed.

KEYWORDS: Dual-user system; Teleoperation; Sliding-mode control; Multi-master/single-slave system; Unknown time delay.

Nomenclature

M	number of degree of freedom
x_s	(M × 1) position vector of the slave robot
x_{m_1}	(M × 1) position vector of the master 1 robot
x_{m_2}	(M × 1) position vector of the master 2 robot
α	dominance factor
d_1	delay between master 1 and slave
d_2	delay between master 2 and slave
d_3	delay between master 1 and master 2
$M_{i,d}$, $B_{i,d}$, $K_{i,d}$ $(i = 1, 2, s)$	desired scalar impedance parameters for master 1, master 2 and the slave robots
F_{h_1}	(M × 1) hand force vector of operator 1
F_{h_2}	(M × 1) hand force vector of operator 2
F_e	(M × 1) vector of the environment force

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1. Introduction

Teleoperation systems as robotics applications have achieved considerable attentions. Referring to their ability in performing a remote operation, they have found substantial applications in hazardous as well as out of reach areas.[1] For example, under water exploration, space exploration, mining, nuclear as well as toxic material handling, telesurgery, and minimally invasive surgery are some of the teleoperation systems applications.[2,3] Stability and transparency, as the main teleoperation objectives, usually contradict each other and as the consequence, usually a trade-off is required in controller design. The contradiction rises when the communication channels suffer from considerable values of time delay, which leads a more complicated design procedure.[4–6]

As the conventional category of teleoperation systems, single-master/single-slave (SMSS) has received great attention in the past few decades. In this category, one operator applies one master robot in order to manipulate a slave robot in a remote environment.[7] Several control approaches have been proposed for SMSS teleoperation systems. In ref. [8], some of the control approaches are summarized.

In addition to conventional SMSS, another teleoperation category is introduced recently. In this category, multiusers manipulate a common operation cooperatively. The desire for collaborative operation in order to increase the accuracy or even utilizing two hands of an operator in case of no collaboration has led to expand such systems in a wide range. As its two main divisions, multi-master/multi-slave systems and dual-user systems can be denoted.[9] The field has found vast applications in rehabilitation and surgical training.[3,10]

In multi-master/multi-slave teleoperation, multiusers manoeuvre multi-slaves through multi-master robots in order to control a common task in a remote environment. A few control architectures have been proposed for mutli-master/multi-slave systems.[11–13] Sirouspour[12] has suggested a multilateral μ -synthesis-based methodology for a system with multiple slave robots manipulating a common environment

https://doi.org/10.1017/S0263574712000604 Published online by Cambridge University Press

through an intervening tool. In ref. [13], a two-channel adaptive nonlinear control architecture has been presented.

In dual-user system as the second division, two operators accomplish a common task through one slave robot cooperatively. In this system, despite of multi-master/multi-slave teleoperation system, each operator has a specified authority over the slave. The authority is adjusted through a factor called "dominance factor."[9] In ref. [11], an H_∞-based controller is proposed to accomplish collaboration between two operators performing surgical training. In this scheme, the slave robot is controlled unilaterally, although no kinesthetic feedback is provided to the users from the environment side. In ref. [10], a cooperative architecture is presented in which both master robots interact separately and the operators' dominance has not been considered.

In ref. [9], a multilateral-shared control architecture is introduced for dual-user system. The control law has been developed in such a way that the impedance of the environment is reflected to the user, once the user has the full authority via adjusting the dominance factor. However, the most challenging issue of communication delay has been disregarded in this study.

An H_∞-based shared controller is proposed in ref. [14]. In this scheme, equality of the users' hand impedance is a necessity in order to satisfy the desired objective, since the masters' controller has a simple linear structure, which is a restrictive assumption.

Addition to the limitations mentioned for the previous works, the issue of time-delay in dual-user teleoperation has been disregarded in most of the previous studies despite of its importance and undesired outcomes on the stability and performance.[15]

In ref. [16], a robust controller based on μ -synthesis is proposed for a delayed dual-user system. In this architecture, a constant and known delay is considered between the slave side and the masters' side. In other words, the master robots are assumed to communicate with each other via a delay free channel that is a restricting assumption. On the other hand, although this paper presents a strategy to handle delay, however considering the eight-order Padé approximation of delay leads the controller to be so sophisticated while it is just able to handle known as well as constant delays.

Therefore, in this paper, a multilateral-shared sliding-mode controller is proposed for dual-user systems in the presence of unknown communication delay. In the previous studies performed on the SMSS teleoperation systems, effectiveness of the sliding-mode controller has been shown.[17] However, in dual-user systems, in addition to interaction of each master robot and the slave robot, there is an interaction between the master robots. This interaction turns the case of dual-user systems into a different and more complicated challenge in comparison with SMSS systems, where the derived formulas for the SMSS cannot be used.

In the proposed sliding-mode controller, the controller gain is determined independent of the delay. Therefore, there is no necessity to estimate time delay or have any *a priori* knowledge about value of the delay. In fact, the proposed controller employs delayed signals sent through communication channels and does not require any information upon the time delay value and consequently, it is able to overcome the destructive effects of unknown communication delay in dual-user systems. Stability analysis is presented for the closed-loop system through passivity theory and it has been shown that by proper selection of the controller parameters, it is able to guarantee the system stability in presence of unknown constant communication time delays. In order to show the controller ability in handling unknown time delay, some experiments have been performed. The experimental setup includes two Phantom-Omni as the master robots and one virtual slave robot. Each robot is connected to a separate computer and the computers communicate with each other through a delayed network. In addition, due to lack of availability of the operators' hand forces, which are required in the proposed controller, a Kalman Filter-based Force Observer (KFFO) is proposed. The KFFO estimates the operators' hand forces in order to use in the experiments.

The rest of the paper is organized as follows: The dynamics of the dual-user teleoperation system as well as the system-desired objectives are introduced in Section 2. The controller design procedure for both masters and the slave is introduced in Section 3. Section 4 presents the stability analysis for the closed-loop system. Experimental results on a dual-user system in the presence of unknown communication delay are presented in Section 5, and Section 6 concludes the paper.

2. Daul-User System Dynamics and Desired Objectives

2.1. System dynamics

In dual-user teleoperation system, the masters ($\gamma = m_1, m_2$) and slave ($\gamma = s$) robots have a nonlinear dynamics in the following form:[18]

$$D_\gamma(x_\gamma)\ddot{x}_\gamma + C_\gamma(x_\gamma, \dot{x}_\gamma)\dot{x}_\gamma + G_\gamma(x_\gamma) = F_{c\gamma} - F_{ext\gamma}, \quad (1)$$

where $\gamma = m_1, m_2$ refers to masters, $\gamma = s$ refers to the slave, and x_γ refers to the position of the robots end-effector. $D_\gamma(x_\gamma)_{M\times M}$ is the mass matrix, $C_\gamma(x_\gamma, \dot{x}_\gamma)_{M\times M}$ corresponds to the velocity-dependent elements, and $G_\gamma(x_\gamma)_{M\times 1}$ represents forces depend on position such as the gravity. In addition, $F_{c\gamma_{M\times 1}}$ stands for the control signal and $F_{ext\gamma_{M\times l}}$ goes with the external force acting at the robots end-effector.

The external forces acting on each masters and slave correspond to the operators' hand and environment forces, respectively. The operator dynamics and environment can be modeled by means of second-order linear time invariant systems.[3] Consequently, the operators' hand forces as well as the environment forces are given by:

$$F_{ext\,s} = F_e = M_e\ddot{x}_e + B_e\dot{x}_e + K_e\left(x_e - x_{e0}\right), \quad (2)$$

$$\begin{aligned} F_{ext\,m_i} &= -F_{h_i} \\ &= -(F^*_{h_i} - M_{h_i}\ddot{x}_{h_i} - B_{h_i}\dot{x}_{h_i} - K_{h_i}[x_{h_i} - x_{h_i0}]), \end{aligned} \quad (3)$$

where ($i = 1, 2$), M_Λ, B_Λ, and $K_\Lambda(\Lambda = h_i, e)$ correspond to mass, damping, and stiffness of the operators' hand and the environment, respectively, and $F^*_{h_i}$ represents the users exogenous force. Moreover, x_e and x_{h_i} refer to the positions

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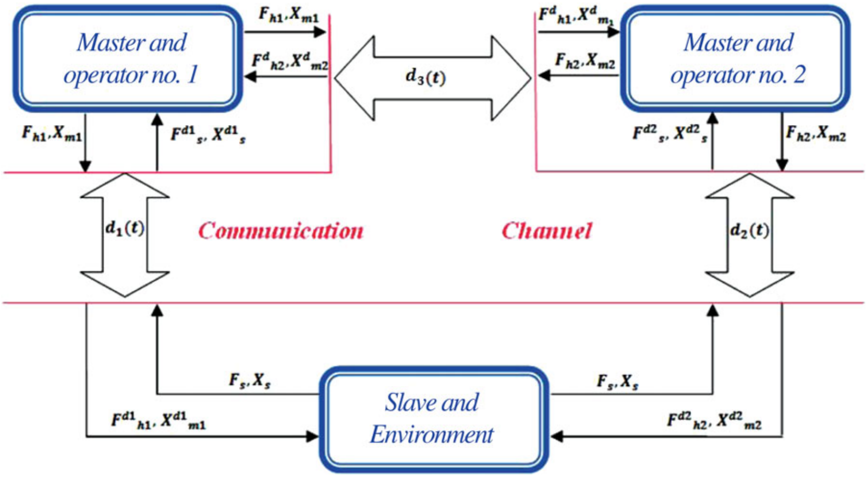

Fig. 1. (Colour online) The block diagram of the delayed dual-user system.

of the hands and the environment that are equivalent to the positions of the masters and the slave, respectively.

2.2. *Desired objective in dual-user systems*

In dual-user system, each operator has a determined authority over the task. This authority is adjusted through a dominance factor "α." In an ideal situation, it is desired that each robot position be an α-based combination of the two other robots position. Similarly, it is desired that each of the operators' hand force as well as the environment force be an α-based combination of the other forces. Therefore, in the presence of communication time delay, dual-user system objectives are in the following form:[9]

$$x_{sd} = \alpha x^{d_1}{}_{m_1} + (1-\alpha)x^{d_2}{}_{m_2}, \tag{4}$$

$$x_{m_1 d} = \alpha x^{d_1}{}_{s} + (1-\alpha)x^{d_3}{}_{m_2}, \tag{5}$$

$$x_{m_2 d} = \alpha x^{d_3}{}_{m_1} + (1-\alpha)x^{d_2}{}_{s}, \tag{6}$$

and

$$F_{h_1 d} = \alpha F^{d_1}{}_{e} + (1-\alpha)F^{d_3}{}_{h_2}, \tag{7}$$

$$F_{h_2 d} = \alpha F^{d_3}{}_{h_1} + (1-\alpha)F^{d_2}{}_{e}, \tag{8}$$

$$F_{ed} = \alpha F^{d_1}{}_{h_1} + (1-\alpha)F^{d_2}{}_{h_2}, \tag{9}$$

where scalar $\alpha \in [0, 1]$ stands for the dominance factor; x_{m1}, x_{m2}, and x_s represent the positions of masters and slave; F_{h_1}, F_{h_2}, and F_e represent the forces exerted on masters and slave by operators' hand and the environment, respectively. The subscript "d" corresponds to the desired value and the signals having the superscripts "d_1," "d_2," and "d_3" refer to the delayed signals passed through communication channels between the two masters, master 1 and slave as well as master 2 and slave correspondingly. The superscript d_1 corresponds to the delay caused by communication channel between the slave robot and master number 1. Similarly, d_2 corresponds to the delay caused by channel between slave and master number 2. In addition, superscript "d_3" refers to the delay caused by channel between the two masters. The block diagram of a dual-user system including its communication channels in the presence of latency is shown in Fig. 1.

In order to satisfy the desired objectives given in Eqs. (4)–(9) in this paper, a sliding-mode-based impedance control methodology is proposed, which is discussed in the next section.

3. Controller Design

In this section, the proposed controller design approach is presented, which leads the system to reach to the desired objectives. The proposed decentralized controller utilizes the impedance control approach in order to satisfy the objectives. In fact, in this paper, for each of the masters and the slave robots, one impedance equation is defined, which leads the objectives to be satisfied. For each robot, a control law is presented in order to satisfy the desired impedance equation. The masters' controllers include feedback linearization and an impedance controller. In addition, a sliding-mode controller is applied in the slave side in order to satisfy the desired-slave-impedance in the presence of communication delay.

3.1. *Masters controller*

Considering Eqs. (7) and (8) in dual-user systems, each master robot must reflect an α-based combination of the environment and the other master's operator forces to the human operating on it. Evidently, if there is latency in the system, the reflected force should be a combination of delayed forces of the environment and the other operator. The impact forces imposed on each human operator can be decreased effectively, applying impedance control scheme. Furthermore, the desired characteristics between the operating and external forces can be determined.[3] Considering this, the desired impedance characteristics for both master robots can have the following general form:

$$M_{1,d}\ddot{x}_{m_1} + B_{1,d}\dot{x}_{m_1} + K_{1,d}x_{m_1} = F_{h_1 d} - \alpha F^{d_1}{}_{e} - (1-\alpha)F^{d_3}{}_{h_2}, \tag{10}$$

$$M_{2,d}\ddot{x}_{m_2} + B_{2,d}\dot{x}_{m_2} + K_{2,d}x_{m_2} = F_{h_2 d} - \alpha F^{d_2}{}_{e} - (1-\alpha)F^{d_3}{}_{h_1}, \tag{11}$$

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where $M_{i,d}$, $B_{i,d}$, and $K_{i,d}$ $(i = 1, 2)$ correspond to the desired inertia, damping, and stiffness for each master robot, respectively. Each of these desired parameters is considered the same for all of degrees of freedom and consequently is scalar.

In order to satisfy the desired impedance characteristics presented in Eqs. (10) and (11) for the master robots, the control law is obtained for them as following:

$$F_{cm_1} = -F_{h_1} + C_{m_1}\dot{x}_{m_1} + G_{m_1} + D_{m_1} M^{-1}{}_{1,d} \begin{pmatrix} F_{h_1} - \alpha F^{d_{1e}} - (1-\alpha)F^{d_{3h_2}} \\ -B_{1,d}\dot{x}_{m_1} - K_{1,d}x_{m_1} \end{pmatrix}, \tag{12}$$

$$F_{cm_2} = -F_{h_2} + C_{m_2}\dot{x}_{m_2} + G_{m_2} + D_{m_2} M^{-1}{}_{2,d} \begin{pmatrix} F_{h_2} - \alpha F^{d_{2e}} - (1-\alpha)F^{d_{3h_1}} \\ -B_{2,d}\dot{x}_{m_2} - K_{2,d}x_{m_2} \end{pmatrix}. \tag{13}$$

3.2. Slave controller

The desired impedance characteristic for the slave robot is defined as follows:

$$M_{s,d}\tilde{x}_s'' + B_{s,d}\tilde{x}_s' + K_{s,d}\tilde{x}_s = -F_e, \tag{14}$$

where scalars $M_{s,d}$, $B_{s,d}$, and $K_{s,d}$ stand for the desired inertia, damping, and stiffness of the slave robot, respectively. Furthermore:

$$\tilde{x}_s'' = \ddot{x}_s - \alpha\ddot{x}^{d_1}{}_{m_1} - (1-\alpha)\ddot{x}^{d_2}{}_{m_2}, \tag{15}$$

$$\tilde{x}_s' = \dot{x}_s - \alpha\dot{x}^{d_1}{}_{m_1} - (1-\alpha)\dot{x}^{d_2}{}_{m_2}, \tag{16}$$

$$\tilde{x}_s = x_s - \alpha x^{d_1}{}_{m_1} - (1-\alpha)x^{d_2}{}_{m_2}. \tag{17}$$

In order to satisfy the desired impedance equation given in Eq. (14) in this paper, a sliding-mode impedance controller is designed and applied to the system. Sliding-mode impedance control, which brings the advantages of nonlinear and robust control together,[19] can overcome the effects of imperfection caused by the uncertainties as well as the latency in the system. Considering this powerful capability, sliding-mode control can offer appropriate scheme to satisfy the desired objectives in teleoperation systems in the presence of delay. Toward this end and in order to satisfy Eq. (14) as the slave desired impedance equation, the sliding surface is defined in a way to minimize the impedance error given in Eq. (18):

$$I_e := M_{s,d}\tilde{x}_s'' + B_{s,d}\tilde{x}_s' + K_{s,d}\tilde{x}_s + F_e. \tag{18}$$

An appropriate alternative for the sliding surface can be defined as the scaled integration of impedance error,[3] which, for the dual-user systems, is accomplished in the form of:

$$\begin{aligned} S(t) = \frac{1}{M_{s,d}}\int_0^t I_e(\tau)d\tau = \dot{x}_s + M_{s,d}^{-1}B_{s,d}x_s \\ + M_{s,d}^{-1}K_{s,d}\int_0^t x_s(\tau)d\tau + \alpha\big(M_{1,d}^{-1}K_{1,d} - M_{s,d}^{-1}K_{s,d}\big) \\ \times \int_0^t x^{d_1}{}_{m_1}(\tau)d\tau + (1-\alpha)\big(M_{2,d}^{-1}K_{2,d} - M_{s,d}^{-1}K_{s,d}\big) \\ \times \int_0^t x^{d_2}{}_{m_2}(\tau)d\tau + \alpha\big(M_{1,d}^{-1}B_{1,d} - M_{s,d}^{-1}B_{s,d}\big) \\ \times \int_0^t \dot{x}^{d_1}{}_{m_1}(\tau)d\tau + (1-\alpha)\big(M_{2,d}^{-1}B_{2,d} - M_{s,d}^{-1}B_{s,d}\big) \\ \times \int_0^t \dot{x}^{d_2}{}_{m_2}(\tau)d\tau - \int_0^t \alpha M_{1,d}^{-1}\big(F_{h_1}^{d_1}(\tau) - \alpha F_e^{d_1d_1}(\tau) \\ - (1-\alpha)F_{h_2}^{d_1d_3}(\tau)\big)d\tau - \int_0^t (1-\alpha)M_{2,d}^{-1} \\ \times \begin{pmatrix} F_{h_2}^{d_2}(\tau) - (1-\alpha)F_e^{d_2d_2}(\tau) \\ -\alpha F_{h_1}^{d_2d_3}(\tau) \end{pmatrix} d\tau \int_0^t M_{s,d}^{-1}F_e(\tau)d\tau, \end{aligned} \tag{19}$$

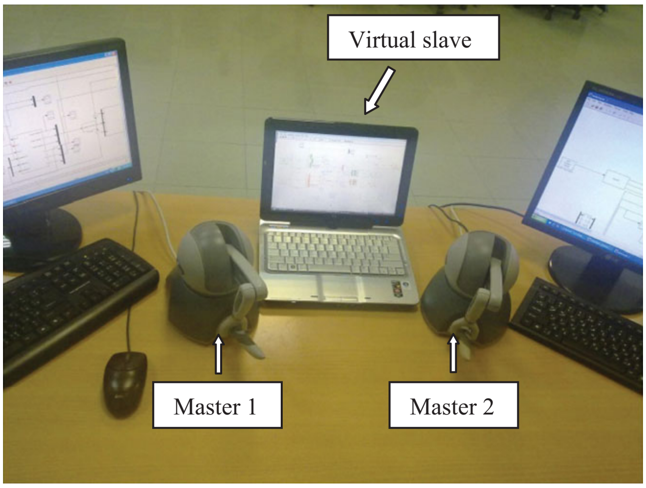

Fig. 2. (Colour online) Experimental setup.

where $F_e^{d_1d_1}(\tau) = F_e(\tau - d_1(\tau) - d_1(\tau))$ refers to delayed signal F_e, which has been sent to the master number 1 side and sent back to the slave side. Similarly, $F_{h_1}^{d_2d_3}(\tau) = F_{h_1}(\tau - d_2(\tau) - d_3(\tau))$ corresponds to F_{h_1} sent to master number 2 side and then to the slave side. As another example, $x_{m_1}^{d_1}(\tau) = x_{m_1}(\tau - d_1(\tau))$ refers to x_{m_1} sent from master number 1 side to the slave side through the communication channel between the two sides, which has caused the signal to be delayed with the amount of x_{m_1}. The same procedure is applied to the other delayed signals in order to calculate $S(t)$ at the slave side. Consequently, by applying the delayed signals sent through communication channels and received at the slave side, there is no necessity to know the amount of time delay. Figure 2 shows how the delayed signals are created to be used in the controllers.

Letting $\dot{S}(t) = S(t) = 0$, the equivalent control signal is obtained.[20] Substituting the linearized slave and masters dynamics into $\dot{S}(t)$, considering the parameters uncertainties, the sliding-based slave control law is calculated as following:

$$U_{cs} = -M_{s,d}^{-1}\big(B_{s,d}\tilde{x}_s'(t) + K_{s,d}\tilde{x}_s(t)\big) + \big(1 - M_{s,d}^{-1}\big)F_e(t) + \alpha M_{1,d}^{-1}\begin{pmatrix} F_{h_1}^{d_1}(t) - \alpha F_e^{d_1d_1}(t) - (1-\alpha)F_{h_2}^{d_1d_3}(t) \\ -B_{1,d}\dot{x}^{d_1}{}_{m_1}(t) - K_{1,d}x^{d_1}{}_{m_1}(t) \end{pmatrix}$$

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$$+(1-\alpha)M_{2,d}^{-1}\begin{pmatrix}F_{h_2}^{d_2}(t)-(1-\alpha)F_e^{d_2d_2}(t)-\alpha F_{h_1}^{d_2d_3}(t)\\-B_{2,d}\dot{x}^{d_2}{}_{m_2}(t)-K_{2,d}x^{d_2}{}_{m_2}(t)\end{pmatrix}$$
$$-K_{slid}Sat\left(\frac{S}{\varphi}\right), \quad (20)$$

where K_{slid} stands for nonlinear gain, and $Sat(.)$ refers to saturation function. Moreover, φ corresponds to thickness of the boundary layer used to reduce chattering phenomena in the control input. Satisfying $S(t)\dot{S}(t)\leq-\eta|S(t)|$ by means of control input given by Eq. (20), lets the states reach to the sliding manifold in a finite time.

Considering all control terms, which lead to have a linearized slave dynamics, the entire slave control law is specified as follows:

$$F_{cs}=D_s(U_{cs}-F_e)+F_e+C_s\dot{x}_s+G_s. \quad (21)$$

Applying the control signal given in Eq. (21) and expressing the slave dynamics in terms of $S(t)$ leads Eq. (22) to be satisfied. The intermediate steps to reach to the following equality are presented in Appendix A.

$$\dot{S}(t)+K_{slid}Sat\left(\frac{S(t)}{\varphi}\right)=0. \quad (22)$$

Considering Eq. (22), the boundary condition for nonlinear gain, K_{slid}, in order to fulfill the sliding manifold condition is:[20]

$$K_{slid}\geq\eta>0. \quad (23)$$

Regarding to the nonlinear gain boundary, since it simply refers to a positive constant, independency of gain, K_{slid}, on the delay is obvious. Therefore, the gain applies no constraint on delay existed in communication channel. With the appropriate selection of gain, K_{slid}, state trajectory is kept around the sliding manifold. Consequently, the slave robot reaches to the desired impedance characteristics, while there is no necessity to have any *a priori* knowledge upon the communication delay and the controller satisfy the desired impedance equation independent of time delay.

4. Stability Analysis

The proposed decentralized controller introduced in the previous section leads the desired impedance equations to be held as the closed-loop system. In order to investigate the stability of the closed-loop system, passivity theory is utilized, which is discussed in this section.

As mentioned before, the dual-user teleoperation system includes two operators, an environment as well as three robots and the communication channels. By the assumption of passivity of the both operators as well as the environment, the only source of the instability of the system is communication channels. According to the system structure, the communication channels can be analyzed as a 3-port network with the inputs and outputs as follows:

$$Y=HU, \quad (24)$$

where the network output is $Y=[F_{h_1}F_{h_2}\dot{x}_s]'$, the network input is $U=[\dot{x}_{m_1}\dot{x}_{m_2}-F_e]'$, and the matrix H stands for the hybrid matrix of the system as follows:[3]

$$H=\begin{bmatrix}\left.\frac{F_{h_1}}{\dot{x}_{m_1}}\right|_{\dot{x}_{m_2}=F_e=0} & \left.\frac{F_{h_1}}{\dot{x}_{m_2}}\right|_{\dot{x}_{m_1}=F_e=0} & \left.\frac{F_{h_1}}{-F_e}\right|_{\dot{x}_{m_2}=\dot{x}_{m_1}=0}\\ \left.\frac{F_{h_2}}{\dot{x}_{m_1}}\right|_{\dot{x}_{m_2}=F_e=0} & \left.\frac{F_{h_2}}{\dot{x}_{m_2}}\right|_{\dot{x}_{m_1}=F_e=0} & \left.\frac{F_{h_2}}{-F_e}\right|_{\dot{x}_{m_2}=\dot{x}_{m_1}=0}\\ \left.\frac{\dot{x}_s}{\dot{x}_{m_1}}\right|_{\dot{x}_{m_2}=F_e=0} & \left.\frac{\dot{x}_s}{\dot{x}_{m_2}}\right|_{\dot{x}_{m_1}=F_e=0} & \left.\frac{\dot{x}_s}{-F_e}\right|_{\dot{x}_{m_2}=\dot{x}_{m_1}=0}\end{bmatrix}. \quad (25)$$

Regarding the desired objectives of a dual-user system, in an ideal situation, the hybrid matrix will be as given by Eq. (26). In order to have the ideal situation, it is required to have complete transparency to let the operators completely feel the environment. Therefore, the operators' hand forces should be equal to the environment force. In order to have this ideal situation, the hybrid matrix should be 0 at $H(i,j)(i,j=1,2)$ and -1 at $H(i,3)(i=1,2)$. On the other hand, in the ideal situation, it is required for the slave robot to follow the α-based combination of the master robots positions. Therefore, considering structure of the hybrid matrix given in Eq. (25), in the ideal format, H should be α, $1-\alpha$, and 0 at $H(3,1)$, $H(3,2)$, and $H(3,3)$, respectively, which makes the slave position equivalent to the α-based combination of the master robots position. Consequently, the ideal hybrid matrix will be as follows:

$$H_{ideal}=\begin{bmatrix}0&0&-1\\0&0&-1\\\alpha&1-\alpha&0\end{bmatrix}. \quad (26)$$

Applying the sliding-mode-based controller proposed in this paper to a dual-user system, the hybrid matrix elements' are determined as Eqs. (27)–(35). The detail calculation has been given in Appendix B.

$$H_{11}=\frac{M_{1,d}s^2+B_{1,d}s+K_{1,d}}{s(1-(\alpha-\alpha^2)e^{-2d_3s})}, \quad (27)$$

$$H_{12}=\frac{(M_{2,d}s^2+B_{2,d}s+K_{2,d})(1-\alpha)e^{-d_3s}}{s(1-(\alpha-\alpha^2)e^{-2d_3s})}, \quad (28)$$

$$H_{13}=-\frac{\alpha e^{-d_1s}+(1-\alpha)^2e^{-(d_3+d_2)s}}{1-(\alpha-\alpha^2)e^{-2ds}}, \quad (29)$$

$$H_{21}=\frac{(M_{1,d}s^2+B_{1,d}s+K_{1,d})\alpha e^{-d_3s}}{s(1-(\alpha-\alpha^2)e^{-2d_3s})}, \quad (30)$$

$$H_{22}=\frac{M_{2,d}s^2+B_{2,d}s+K_{2,d}}{s(1-(\alpha-\alpha^2)e^{-2d_3s})}, \quad (31)$$

$$H_{23}=-\frac{(1-\alpha)e^{-d_2s}+\alpha^2e^{-(d_3+d_1)s}}{1-(\alpha-\alpha^2)e^{-2d_3s}}, \quad (32)$$

$$H_{31}=\alpha e^{-d_1s}, \quad (33)$$

$$H_{32}=(1-\alpha)e^{-d_2s}, \quad (34)$$

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$$H_{33} = \frac{s}{M_{s,d}s^2 + B_{s,d}s + K_{s,d}}. \tag{35}$$

Although Passivity theory is a more conservative tool than absolute stability, however it can be applied in order to analyze the system stability and it presents a sufficient condition for stability.[21]

Theorem 1: A linear time-invariant n-port network possessing a general hybrid matrix, which is analytic in the open RHS is passive if and only if the general hybrid matrix is positive real.[22]

Theorem 2: Matrix H is positive real if the following conditions are held:[22]

1. *H is analytic in the open right half plane (RHP).*
2. *$\bar{H}(s) = H(\bar{s})$ for all s in the open RHP.*
3. *The hermittian part of H is nonnegative definite for all s in open RHP.*

Considering structure of the hybrid matrix, referring to the fact that $0 \le \alpha \le 1$, the first two conditions of positive realness are held immediately. On the other hand, in order to guarantee the third condition, proper tuning of the desired impedances and the dominance factor are required. Therefore, tuning of these parameters should be performed in a way to achieve a positive real hybrid matrix and consequently passivity of the entire system. Making the system passive means that the system is absolutely stable, since passivity is a stronger condition than system stability. Additionally, the effect of the dominance factor upon the system stability illustrates the importance of each operator's authority on the operation. It should be noted that referring to Theorem 1, which is presented for time-invariant n-port networks, the given stability analysis covers *constant* time-delays.

5. Experimental Results

In this section, performance of the proposed control methodology is evaluated by means of experimental results performed on a dual-user teleoperation system in the presence of unknown communication delay.

5.1. Experimental setup

In these experiments, two Phantom-Omni robots are used as the master robots and each master is controlled by one separate computer. To interact with the Phantom-Omni robots, we prepared an interface in MATLAB 2008, which uses Open Haptic Libraries. In addition, the virtual slave robot is considered as an RR robot (a serial manipulator with two rotational joints) and has been simulated on a third computer in MATLAB as well. The computers communicate with each other utilizing a LAN network. For each robot, the local controller is implemented in MATLAB 2008 on its corresponding computer. Figure 2 illustrates the experimental setup utilized in the paper.

To show the superior ability of the proposed controller, it is required to have a considerable communication delay. For this reason, addition to the network latency, virtual delay is applied to the communicated-signals. This virtual delay is applied using MATLAB facilities.

As can be seen, the proposed control structure requires force of the operators' hand and environment in Cartesian space. However, due to lack of availability of the forces, their estimations are used in the experiments. In order to estimate the operators' hand forces, a force-observer is designed, which is discussed in the next section.

5.2. Force observer

In order to establish the proposed teleoperation scheme, interaction forces between the hands and the master robots are required. The utilized setup (Phantom-Omni) does not support this kind of measurement (there is no attached force sensor). Consequently, a force observer is required in the experiments. Toward this end, a KFFO has been proposed in this paper to estimate the applied forces on the master robots in the experiments. Utilizing the filtering characteristics of KFFO, the challenge of the measurement noise is alleviated. The observation algorithm is introduced below.

First, consider the applied desired dynamics on the master side, denoted in Eqs. (10)–(11). The discrete-desired dynamics of the masters could be achieved as Eqs. (36)–(38) when T_s denotes the sampling time and the measurements are position and velocity. Note that, since the utilized desired dynamics of each dimension are decoupled so the force observation is decoupled as well. As a result, only the formulation of one-dimensional force-observer is shown in Eqs. (36)–(49) and other dimensions have similar separate observers.

$$X_M = [x_{m_i} \dot{x}_{m_i}]^T, \tag{36}$$

$$X_M(k+1) = A_M X_M(k) + B_M F_{h_{i-x}}(k) - B_M(\alpha F_e(k) + (1-\alpha) F_{h_{j-x}}(k)), \tag{37}$$

$$y_M(k) = C_M X_M(k) + D_M F_{h_{i-x}}(k) - D_M(\alpha F_e(k) + (1-\alpha) F_{h_{j-x}}(k)), \tag{38}$$

where $F_{h_{i-x}}$ is the first element of F_{h_i}, X_M is the state vector of the master's desired dynamics in X direction, and A_M, B_M, C_M, D_M are defined in Eq. (39). In addition, for each master robot, F_{h_j}, $(j = 1, 2)$ refers to F_{h_i} of the other master and $F_{h_{j-x}}$ is the first element of F_{h_j}.

In this technique, a constant first time-derivation discrete dynamics is assumed for the applied hand forces. In this model, denoted in Eqs. (40)–(42), $\dot{F}_{h_i}$, $(i = 1, 2)$ assumes to be constant during each sample time "T_s." This assumption is reasonable when the sample time is relatively small enough. In these experiments, the sampling time is 10 ms.

$$A_M = \begin{bmatrix} 1 & Ts \\ -\left(\frac{K_{i,d}}{M_{i,d}}\right) T_s & 1 - \left(\frac{B_{i,d}}{M_{i,d}}\right) T_s \end{bmatrix},$$
$$B_M = \begin{bmatrix} 0 \\ \left(\frac{1}{M_{i,d}}\right) T_s \end{bmatrix}, C_M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D_M = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{39}$$

$$X_{F_{h_i-x}} = [\dot{F}_{h_{i-x}} F_{h_{i-x}}]^T, \tag{40}$$

$$X_{F_{h_{i-x}}}(K+1) = A_{Force} X_{F_{1-x}}(K), \tag{41}$$

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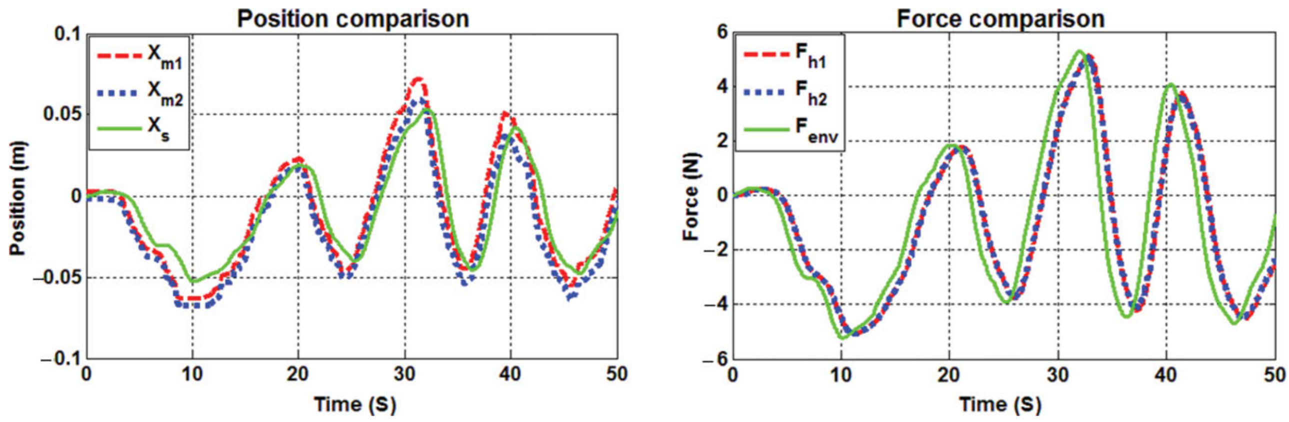

Fig. 3. (Colour online) Result of the first experiment, $d_3 = d_1 = d_2 = 400$ ms.

$$A_{Force} = \begin{bmatrix} 1 & 0 \\ T_s & 1 \end{bmatrix}. \tag{42}$$

In Eqs. (39)–(42), $X_{F_{h_{i-x}}}$ is the state vector of the assumed force dynamics. In the next step, an augmented discrete dynamics is established including the states of the master dynamics (X_M) and the states of the hands force ($X_{F_{h_{i-x}}}$). The augmented plant is denoted in Eqs. (43)–(49).

$$X_{aug}(K+1) = A_{aug}X_{aug} - B_{aug}(\alpha F_e(k) + (1-\alpha)F_{h_{j-x}}(k)), \tag{43}$$

$$Y_{aug} = C_{aug}X_{aug}, \tag{44}$$

$$X_{aug} = \begin{bmatrix} X_M & X_{F_{h_{i-x}}} \end{bmatrix}^T, \tag{45}$$

$$A_{aug} = \begin{bmatrix} A_M & [Zeros_{2,1}\ B_M] \\ Zeros_{2,2} & A_{Force} \end{bmatrix}, \tag{46}$$

$$B_{aug} = \begin{bmatrix} B_M \\ Zeros_{2,1} \end{bmatrix}, \tag{47}$$

$$C_{aug} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \tag{48}$$

$$Zeros_{2,1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, Zeros_{2,2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{49}$$

As mentioned before, the measurement data are the position and the velocity of the masters. Finally, the obtained discrete-augmented dynamics is incorporated into the *discrete recursive Kalman formulation*[23] in order to estimate the states of the augmented plant including the applied operators' hand forces. The Kalman formulation is not represented here, due to interest of space.

5.3. *Experiments*

In order to investigate the controller performance, three sets of experiments are performed. In these experiments, sample time is 0.01 s and the dominance factor "α" is set to 0.5, which refers to equal authority for each user in the operation. In addition, K_{slid} and φ are set to 10 and 20, respectively. Additionally, the desired parameters for the introduced impedance equations are set as follows:

$$M_{1,d} = M_{2,d} = 0.1,\ B_{1,d} = B_{2,d} = 0.1,\ K_{1,d} = K_{2,d} = 0.1$$
$$M_{3,d} = 100,\ B_{3,d} = 200,\ K_{3,d} = 250.$$

In the first experiment, mass, damping, and stiffness of the environment are considered 5, 10, and 100, respectively. In addition, the communication delay in each channel is equal to 400 ms ($d_3 = d_1 = d_2 = 400$ ms), which is a considerable value. Results of this experiment are shown in Fig. 3. As can be seen, the proposed controller has satisfied the desired objectives in the presence of considerable value of

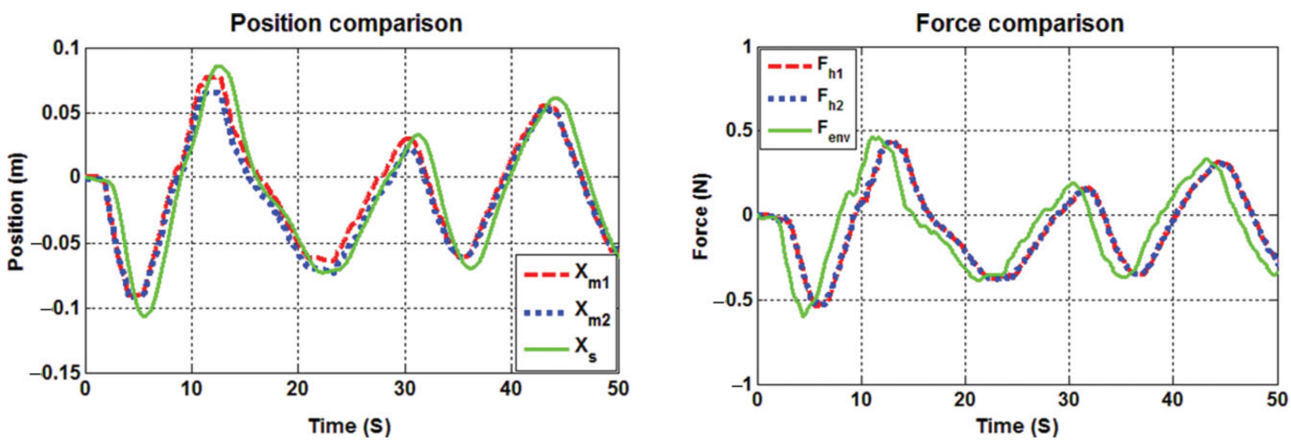

Fig. 4. (Colour online) Result of the second experiment performed in soft environment, $d_3 = 500$ ms, $d_1 = 400$ ms, $d_2 = 200$ ms.

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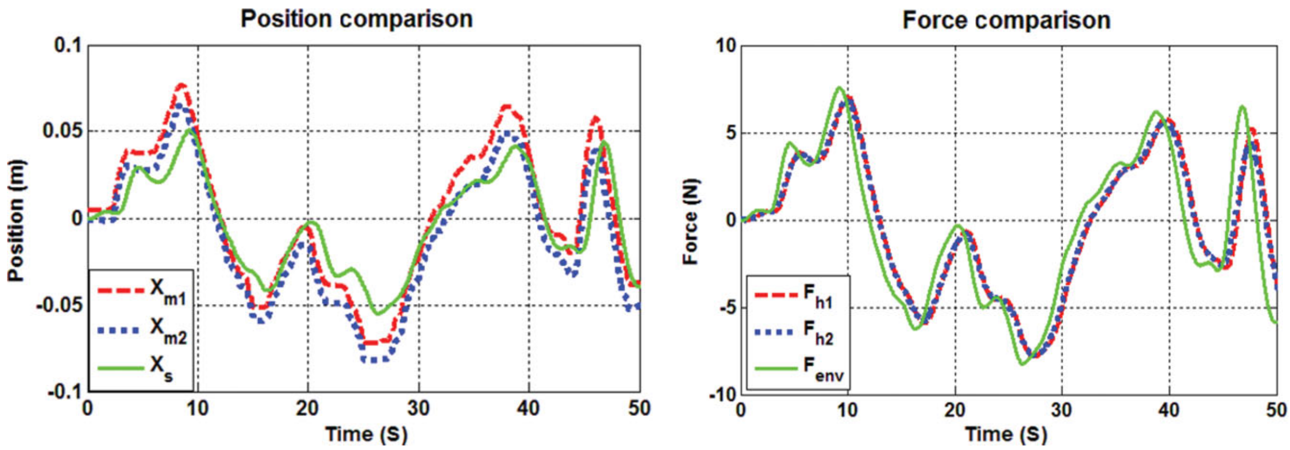

Fig. 5. (Colour online) Result of the second experiment performed in hard environment, $d_3 = 500$ ms, $d_1 = 400$ ms, $d_2 = 200$ ms.

communication time delay. It should be noted that the delay value is unknown to the controller and therefore, the proposed controller is able to handle unknown time delays.

In the next two experiments, delay values of the communication channels are different from each other. In these experiments, $d_3 = 500$ ms, $d_1 = 400$ ms, and $d_2 = 200$ ms. In addition, in these experiments, environment stiffness is set to two different values of 5 and 150 in order to evaluate the controller in both soft and hard environment. In both of these experiments, environment mass and damping are set to 1.5 and 5, respectively, while as mentioned, the stiffness of the environment is set differently to 5 and 150. Results of these experiments are shown in Figs. 4 and 5. Figure 4 illustrates the result of the experiment performed in the soft environment with the stiffness of 5, while Fig. 5 shows the experimental result correspond to the hard environment with the stiffness of 150. As can be seen in both figures, the proposed methodology is able to control the system in both soft and hard environment. In addition, it is obvious that the controller affords unknown communication delays and satisfies the system-desired objectives in the presence of different unknown time delays. Comparing Fig. 4 with Fig. 5 can be seen that in the soft environment, the operators' hand forces have smaller values in comparison with their value in the hard environment. In addition, comparing the results shows that the proposed methodology satisfies the desired objectives more satisfactorily in the soft environment.

6. Conclusions

A sliding-mode impedance controller is proposed for dual-user teleoperation systems in the presence of unknown communication time delay. The proposed controller satisfies three impedance characteristics defined in the paper, which lead the system-desired objectives to be satisfied. In the proposed methodology, the nonlinear gain of the controller is achieved independent of the communication time delay values. Therefore, unknown time delays can be afforded by employing the proposed controller and the necessity of having any *a-priory* information upon time delay value or its estimation is removed. The dominance factor, i.e., the parameter adjusting the authority of each user depending on the operator 2 skills,[14] has also been considered as an unknown parameter in the controller and can be set desirably. In addition, the stability analysis of the closed-loop system is presented through passivity theory. Finally, the efficiency of the proposed control scheme in overcoming the unknown communication time delays is demonstrated through some experiments. The experimental setup includes two Phantom-Omni robots as the masters and one virtual slave. Each robot is connected to a local computer and the computers communicate with each other through a delayed network. In addition, due to lack of availability of operators' hand forces that are required in the proposed controller, a KFFO is proposed. The KFFO estimates the operators' hand forces in order to utilize in the experiments. The results show that the proposed control methodology is able to satisfy the system-desired objectives satisfactorily in the presence of unknown communication time delays. Future work will concentrate on extension of the methodology to dual-user system with time-varying communication delays.

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APPENDIX A

Differentiating $S(t)$ given by the Eq. (19), $\dot{S}(t)$ is achieved as follows:

$$\begin{aligned}\dot{S}(t) = {} & \ddot{x}_s(t) + M_{s,d}^{-1} B_{s,d}\dot{x}_s(t) + M_{s,d}^{-1} K_{s,d} x_s(t) \\ & + \alpha\big(M_{1,d}^{-1}K_{1,d} - M_{s,d}^{-1}K_{s,d}\big)x^{d_1}{}_{m_1}(t) \\ & + (1-\alpha)\big(M_{2,d}^{-1}K_{2,d} - M_{s,d}^{-1}K_{s,d}\big)x^{d_2}{}_{m_2}(t) \\ & + \alpha\big(M_{1,d}^{-1}B_{1,d} - M_{s,d}^{-1}B_{s,d}\big)\dot{x}^{d_1}{}_{m_1}(t) \\ & + (1-\alpha)\big(M_{2,d}^{-1}B_{2,d} - M_{s,d}^{-1}B_{s,d}\big)\dot{x}^{d_2}{}_{m_2}(t) \\ & - \alpha M_{1,d}^{-1}\big(F_{h_1}^{d_1}(t) - \alpha F_e^{d_1 d_1}(t) - (1-\alpha)F_{h_2}^{d_1 d_3}(t)\big) \\ & - (1-\alpha)M_{2,d}^{-1}\big(F_{h_2}^{d_2}(t) - (1-\alpha)F_e^{d_2 d_2}(t) - \alpha F_{h_1}^{d_2 d_3}(t)\big) \\ & + M_{s,d}^{-1}F_e(t).\end{aligned} \tag{A1}$$

On the other hand, from the Eq. (1), $\ddot{x}_s$ is as follows:

$$\ddot{x}_s = D^{-1}{}_s F_{cs} - F_e - C_s\dot{x}_s - G_s. \tag{A2}$$

Substituting F_{cs} from Eq. (21) into Eq. (A2), we will have:

$$\ddot{x}_s = U_{cs} - F_e. \tag{A3}$$

Substituting U_{cs} from Eq. (20), Eq. (A3) will be as follows:

$$\begin{aligned}\ddot{x}_s = {} & -M_{s,d}^{-1}\big(B_{s,d}\tilde{x}_s'(t) + K_{s,d}\tilde{x}_s(t)\big) - M_{s,d}^{-1}F_e(t) \\ & + \alpha M_{1,d}^{-1}\begin{pmatrix} F_{h_1}^{d_1}(t) - \alpha F_e^{d_1 d_1}(t) - (1-\alpha)F_{h_2}^{d_1 d_3}(t) \\ -B_{1,d}\dot{x}_{m_1}^{d_1}(t) - K_{1,d}x_{m_1}^{d_1}(t)\end{pmatrix} \\ & + (1-\alpha)M_{2,d}^{-1}\begin{pmatrix} F_{h_2}^{d_2}(t) - (1-\alpha)F_e^{d_2 d_2}(t) - \alpha F_{h_1}^{d_2 d_3}(t) \\ -B_{2,d}\dot{x}_{m_2}^{d_2}(t) - K_{2,d}x_{m_2}^{d_2}(t)\end{pmatrix} \\ & - K_{slid}Sat\left(\frac{S}{\varphi}\right).\end{aligned} \tag{A4}$$

Now, by substituting $\ddot{x}_s$ from Eq. (A4) into $\dot{S}(t)$ given by Eq. (A1) and after simplifying the equation, we will have:

$$\dot{S}(t) = -K_{slid}Sat\left(\frac{S}{\varphi}\right), \tag{A5}$$

which implies that $\dot{S}(t) + K_{slid}Sat(\frac{S}{\emptyset}) = 0$. By satisfying this equality using the proposed sliding-mode controller given by Eqs. (20) and (21) and having the condition of$K_{slid} \geq \eta > 0$, the sliding manifold condition is satisfied.[20]

APPENDIX B

The proposed controller satisfies the desired impedance equations given by Eqs. (10), (11), and (14) as the closed-loop system. In order to calculate each element of the hybrid matrix, it is required to apply the given condition for each element and solve the closed-loop system equations for the required relation. For example, for H_{11}, we should calculate $\frac{F_{h_1}}{\dot{x}_{m_1}}|_{\dot{x}_{m_2}=F_e=0}$, which is performed as follows:

By applying the condition of $\dot{x}_{m2} = F_e = 0$, Eqs. (10) and (11) turn into Eqs. (B1) and (B2).

$$M_{1,d}\ddot{x}_{m_1} + B_{1,d}\dot{x}_{m_1} + K_{1,d}x_{m_1} = F_{h_1 d} - (1-\alpha)F_{h_2}^{d_3}, \tag{B1}$$

$$F_{h_2 d} - (1-\alpha)F^{d_3}{}_{h_1} = 0. \tag{B2}$$

Since time delay is constant and consequently the system is time invariant, we can use Laplace to write Eqs. (B1) and (B2) as follows:

$$(M_{1,d}s^2 + B_{1,d}s + K_{1,d})x_{m_1} = F_{h_1} - (1-\alpha)F_{h_2}e^{-d_3 s}, \tag{B3}$$

$$F_{h_2} - (1-\alpha)F_{h_1}e^{-d_3 s} = 0. \tag{B4}$$

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Now, by substituting F_{h_2} by $(1-\alpha)F_{h_1}e^{-d_3s}$ from Eq. (B4) into Eq. (B3), we have:

$$(M_{1,d}s^2 + B_{1,d}s + K_{1,d})x_{m_1} = F_{h_1} - (1-\alpha)^2 F_{h_1} e^{-2d_3s}, \tag{B5}$$

which can be simplified to:

$$(M_{1,d}s^2 + B_{1,d}s + K_{1,d})\dot{x}_{m_1} = F_{h_1}s(1-(1-\alpha)^2\, e^{-2d_3s}). \tag{B6}$$

Finally, it is required to find $\frac{F_{h_1}}{\dot{x}_{m_1}}$ as H_{11}, which is calculated as follows:

$$H_{11} = \frac{M_{1,d}s^2 + B_{1,d}s + K_{1,d}}{s(1-(\alpha-\alpha^2)e^{-2d_3s})}. \tag{B7}$$

Similarly, we can calculate all of the other elements of the hybrid matrix given by Eqs. (27)–(35).

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