ON THE FORMULA FOR QUADRATURES.

To the Editor.

DEAR SIR,—The June number of the *Educational Times* contains a mathematical article "On approximation to a curvilinear area," by Professor De Morgan, in which he gives the results of a determination of the numerical coefficients appertaining to the formula usually employed in calculating quadratures. The coefficients, according to a method founded on the "Calculus of operations," are those of the symbolic expansion of $\{\log(1+\Delta)\}^{-1}$. They may, therefore, be found to any number of terms by working out, by long division, the reciprocal of the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\&c.$ Or, if C_n denote the nth coefficient, it may be computed, from those which precede it, by the formula

$$C_n = \frac{C_{n-1}}{2} - \frac{C_{n-2}}{3} + \frac{C_{n-3}}{4} - \ldots \pm \frac{C_1(=1)}{n}.$$

In allusion to these arithmetical values, Professor De Morgan's interesting note concludes as follows:—

"The coefficients of $\{\log(1+\Delta)\}^{-1}$, so far as usually given, are

$$1, \frac{1}{2}, -\frac{1}{12}, \frac{1}{24}, -\frac{19}{720}, \frac{3}{160}, -\frac{863}{60480}.$$

The three next, whether ever before printed I know not, are

$$\frac{275}{24192}, -\frac{33953}{3628800}, \frac{8183}{1036800}."$$

The object of the present communication is merely to point out that, by an investigation conducted on the principles of ordinary algebra, the whole of these coefficients had already been determined in my paper on "Summation," printed in the *Journal.* See vol. xi., page 309, where, with respect to a series of equidistant ordinates V_0, V_1, V_2, &c., it is found that "the curvilinear area bounded by V_0 and V_n=

$$(V_0+V_1 \ldots +V_n)-\tfrac{1}{2}(V_n+V_0)-\frac{1}{12}(a'-a)-\frac{1}{24}(b'+b)$$
$$-\frac{19}{720}(c'-c)-\frac{3}{160}(d'+d)-\frac{863}{60480}(e'-e)-\frac{275}{24192}(f'+f)$$
$$-\frac{33953}{3628800}(g'-g)-\frac{8183}{1036800}(h'+h) \ldots\ldots (B)."$$

It will be perceived that this formula includes all the coefficients stated by Professor De Morgan. Moreover, the general formula (A) expressing the result of the summation of an interpolated finite series of values is carried out to the same order of differences—that is, to the eighth order. It is, however, to the integration formula (B) only that reference is needed on this occasion; and the practical inference to be drawn is, that the identity of Professor De Morgan's numerical coefficients with those previously determined by a process so entirely different may be accepted as a satisfactory proof of their accuracy.

I am, dear Sir,

Yours most truly,

W. S. B. WOOLHOUSE.

Alwyne Lodge,
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