

Since an event with probability 1 of occurring must (to draw a very weak consequence) be a possible event we have the following corollary:

Corollary B. *If $c_0, c_1, \dots \in \mathbb{C}, \sum_{j=0}^{\infty} |c_j|^2 = \infty$ and $\limsup |c_n|^{1/n} = 1$ then there exist $\lambda_0, \lambda_1, \dots \in \{-1, 1\}$ such that the circle of convergence $|z| = 1$ is a natural boundary for the Taylor series $\sum_{j=0}^{\infty} c_j z^j$.*

At this point I would advise readers to try and prove the corollary in the special case $c_j = 1$ for all j by non-probabilistic means. Since this is not too difficult they could also try the following problem:

Problem. For which α does there exist a $C(\alpha)$ with

$$\sup_{t \in \mathbb{R}} \left| \sum_{j=1}^N \sin m_{jN} t \right| \leq C(\alpha) N^\alpha$$

for suitably chosen integers $0 < m_{1N} < m_{2N} < \dots < m_{NN}$?

In Section 6 of Chapter 6 Kahane describes Bourgain’s proof that we can take $\alpha = 2/3$. At a much more modest level the reader is invited to anticipate the typically elegant and typically probabilistic argument by which in Section 3 of Chapter 4, Kahane improves on our Theorem A.

Theorem A’. *If $\limsup |c_n|^{1/n} = 1$ then, with probability 1, the circle of convergence $|z| = 1$ is a natural boundary for the Taylor series $\sum_{j=0}^{\infty} \pm c_j z^j$.*

Probabilistic methods in analysis have a fairly long history starting with Borel’s 1896 study of the questions discussed above. Important results were obtained by Paley and Zygmund, Littlewood and Offord, Wiener and others. However even in the 1960s few analysts kept probabilistic methods in their mathematical tool box and one of the purposes of the first edition was to make them think again. Since then many important new results and proofs have been obtained by probabilistic methods by mathematicians both within and without Kahane’s mathematical sphere of influence.

Wisely, however, Kahane has not chosen to rewrite from scratch so we still get the zeal of a “missionary preaching to cannibals” which, according to Littlewood, distinguished Hardy’s *Pure Mathematics*. Instead he has rewritten a few sections to take account of new developments (often inspired by the first edition) and added a selection of new results. (Incidentally, anyone teaching a first course on Brownian motion will find Section 4 of Chapter 16 repays thoughtful attention.) Altogether about a third of the material is new. One or two results present in the first edition have gone but, so far as I can see, only the reviewer and a couple of other specialists will miss them.

It is, I think, true that the very greatest mathematics requires the same kind of effort of the reader as a mountaineering expedition requires of its participants. Hours or days of hard and often tedious toil are required to attain a magnificent vista. Kahane’s book is more like a ramble through fine countryside. At every mile one is rewarded with a singing waterfall or an old farmhouse and at virtually every step with a new wayside flower. Cambridge University Press is to be congratulated on obtaining this second edition for us.

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TOMKINSON, M. J. *FC-groups* (Research Notes in Mathematics 96, Pitman, 1984), 188 pp. £8.95.

In the study of infinite groups, it is necessary to impose conditions on the groups in order to obtain satisfactory structural results. These conditions are normally either that the group has some sort of commutativity, or that it has some sort of finiteness, or a combination of both. Such a class of groups which has received a good deal of attention in the last thirty years or so is the subject of this book, and the author has played a substantial part in the development of the theory during the latter half of this period.

This is the class of FC groups, so called because it consists precisely of those groups all of whose conjugacy classes are finite. Alternatively the centralizers of all one-element subsets are of finite index. These conditions lead to groups which are sufficiently far removed from finite groups to provide interesting differences, but tractable enough so that a rich theory can be developed. Preliminary work on FC groups was done by B. H. Neumann and P. Hall. A number of others have done some work in the area, but the major contributor of recent years outside Russia has been M. J. Tomkinson. Here he has given us a survey of most of what is now known about FC groups, often improving and streamlining the proofs.

After dealing with the basic results in the first chapter, we are shown results due to Hall, Gorchakov and the author. These concern those FC groups which are subgroups of direct products of finite groups. So the main concern here is the class of residually finite periodic FC groups. The next chapter extends this to sections of direct products of finite groups, a section meaning a homomorphic image of a subgroup. This therefore deals mainly with periodic FC groups.

Chapter 4 is the beginning of a part needing different techniques. The first theme considered is that of inverse limits of finite groups and locally inner automorphisms, that is automorphisms which act like inner automorphisms on finite subsets of the group. Information on local conjugacy classes and conjugacy classes of subgroups is also obtained. The next topic is that of Sylow theory. By using local systems of finite normal subgroups, a very satisfactory theory of Sylow subgroups is developed for FC groups by analogy with that for finite groups. This is taken further in the next chapter where a theory of formations and Fitting classes is developed. A different approach is needed when dealing with those FC groups which are centre-by-finite or finite-by-abelian. Here combinatorial set theory is a key part of the proofs. The last chapter deals with a variety of topics: FL groups, that is groups which have only finitely many elements of a given order, infinite abelian and nilpotent subgroups of FC groups and minimal non-FC groups. A number of open questions are stated at appropriate points, and the book ends with exercises for the serious reader, and a very good bibliography.

This book covers an area of group theory which has had to develop a number of techniques of its own and which has had considerable success in obtaining satisfying and interesting results. A thorough, well developed exposition is given and it will be a great help to any student who wishes to learn about FC groups. Any worker in the field will find it invaluable, and should undoubtedly buy it as it is not expensive by present day standards. It will also find its way into most good algebra libraries.

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