Macroeconomic Dynamics, **5**, 2001, 621–646. Printed in the United States of America.
DOI: 10.1017.S1365100500000481

NONLINEAR RISK

MARCELLE CHAUVET
University of California, Riverside

SIMON POTTER
Federal Reserve Bank of New York

This paper analyzes the joint time-series properties of the level and volatility of expected excess stock returns. An unobservable dynamic factor is constructed as a nonlinear proxy for the market risk premia with its first moment and conditional volatility driven by a latent Markov variable. The model allows for the possibility that the risk–return relationship may not be constant across the Markov states or over time. We find an overall negative contemporaneous relationship between the conditional expectation and variance of the monthly value-weighted excess return. However, the sign of the correlation is not stable, but instead varies according to the stage of the business cycle. In particular, around the beginning of recessions, volatility rises substantially, reflecting great uncertainty associated with these periods, while expected return falls, anticipating a decline in earnings. Thus, around economic peaks there is a negative relationship between conditional expectation and variance. However, toward the end of a recession expected return is at its highest value as an anticipation of the economic recovery, and volatility is still very high in anticipation of the end of the contraction. That is, the risk–return relation is positive around business-cycle troughs. This time-varying behavior also holds for noncontemporaneous correlations of these two conditional moments.

Keywords: Expected Excess Return, Risk Premia, Conditional Variance, Dynamic Factor, Markov Process

1. INTRODUCTION

In the past 20 years, great progress has been made in modeling the relation between risk and expected return. Most of this research has focused on the single-period risk–return trade-off among different securities. There is general agreement that riskier securities are rewarded by larger expected returns, within a given time period. However, there are less obvious conclusions about the joint dynamics of risk and return over time. On a marketwide level, there is no general consensus in most related empirical work concerning the temporal behavior of both stock market

The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of New York or the Federal Reserve System. We thank participants in the 1998 Meetings of the Society of Economic Dynamics, 1999 North American Summer Meetings, and the Latin American Meetings of the Econometric Society, Conference on Nonlinear Modeling of Multivariate Macroeconomic Relations (Rotterdam), and seminar at the State University of Maringa (Brazil), and two anonymous referees for helpful comments. Address correspondence to: Marcelle Chauvet, Department of Economics, University of California, Riverside, CA 92521-0247, USA; e-mail: chauvet@mail.ucr.edu.

 1365-1005/01 $9.50

returns and their volatility, although there is substantial evidence of nonlinearity in their dynamics.[1] In particular, recent findings show that a distinct pattern is revealed in expected stock returns and their conditional variances when they are grouped according to the state of the business cycle.[2] This implies that stocks may bear more risk at some times than others, but it is not indisputable whether, on average, investors require larger risk premia during times when stocks are riskier.

Theory also does not yield unambiguous insights about the relationship between risk and excess return. Backus and Gregory (1993), for example, find that theoretical models are consistent with virtually any sort of relationship between excess return and its conditional variance proxying for risk, depending on model preferences and the probability structure across states. Further, using equilibrium asset-pricing models, one would expect the relationship between excess return and variables proxying for corporate cash flows and investors' discount rates to be nonlinear.

Related empirical research has focused on modeling the dynamics of time-varying conditional second moments of stock returns as proxies for risk premia. From a theoretical point of view, the predictability of the level and volatility of returns should be connected.[3] Thus, rather than modeling them separately, considerable effort has gone into modeling their joint dynamic behavior. New models such as ARCH, GARCH, and stochastic volatility (SV) have been developed to capture the persistence in the volatility of returns. The main empirical framework of the joint determination of the conditional mean and variance of stock returns is the ARCH-M, in which time-varying conditional second moments account for changes in risk premia. The underlying assumption of these models is that risk premia on assets can be represented as linear increasing functions of their conditional covariance with the market.[4]

In this paper, we are particularly interested in using an empirical framework that does not impose a priori structure between the conditional mean and volatility of stock returns. We estimate an unobservable dynamic factor as a nonlinear proxy for the market risk premia with first and second conditional moments driven by a latent two-state Markov variable. That is, we consider the possibility that market return and its volatility are not necessarily related directly but are a function of a third variable—the Markov process, which represents the state of financial market conditions.

The approach captures potential asymmetric responses by investors to changes in risk, depending on their perception of the state of business conditions. The two Markov states can be interpreted as bull and bear markets.[5] Since expected returns account for changes in the level of the market value and risk due to discrete changes in the state of financial conditions, bad news may cause a switch to a bear market (low-return state), whereas positive news can lead to a bull phase (high-return state). Bear and bull markets could be associated with an increasing relation between mean and variance for the market returns. However, they could be associated with an inverse relation as well. In particular, in our framework, expected stock returns can be higher or lower during periods when the market is more volatile. For example, it could be the case that, in those times, investors desiring to hedge against risk might move back and forth from stocks, driving changes in expected stock returns and the direction of the risk–return relation according to the stage of the economy. Thus, as the market value and the level of risk switch regimes, the model can account for the leverage effect—where negative shocks to returns increase volatility; and to the volatility feedback—where high expected future volatility is associated with low current stock prices and high expected future returns.[6]

The proposed framework allows the use of multivariate information with a parsimonious variance–covariance structure to produce the sort of predictions obtained from regression models. In contrast, most ARCH, GARCH, and SV models use only the information contained in returns. The multivariate information is introduced by constructing a stock market index, subject to switches between bull and bear markets, from a range of financial variables, as in Chauvet and Potter (2000). We also examine the risk–return relationship for stocks from different firm sizes, which captures potential asymmetric behavior across financial states, depending on different market capitalization. Ultimately, forecasts of excess returns can be obtained from forecasts of the mean and volatility of the stock market index. Our analysis focuses on the risk–return dynamics across the Markov states and the business-cycle phases. In particular, we study their contemporaneous as well as offset correlations around business cycle turning points as dated by the NBER.

In terms of results, we find a significant asymmetric behavior of conditional excess returns according to firm size. In particular, excess returns on stocks of small firms, as proxied by the CRSP equal-weighted index, are more reactive to changes in the state of financial markets than large firms. In addition, a business cycle pattern is present in the conditional expectation and variance of the value-weighted excess return. Typically, the conditional mean decreases a couple of months before or at the peak of expansions, and increases before the end of recessions. On the other hand, the conditional volatility rises considerably during economic recessions.[7]

With respect to the risk–return relation, we find that, during bear markets, expected excess returns are low whereas the conditional volatility is high. In bull markets, the conditional mean is high whereas the volatility is low. However, the contemporaneous correlation is not stable, but changes signs according to the state of the business cycle, as measured by the NBER. In particular, around the beginning of recessions, expected returns decrease, anticipating a decline in earnings, and volatility increases substantially, reflecting great uncertainty associated with these periods. Thus, around economic peaks, there is a negative relationship between conditional expectation and variance, as in the leverage effect. Toward the middle of a recession, volatility is still very high and expected returns are at their highest value, anticipating the imminent economic recovery. That is, the risk–return relation is positive around business-cycle troughs, as in the volatility feedback effect. This time-varying behavior also holds for noncontemporaneous correlations of these two conditional moments.

The paper is organized as follows: The second section describes the model and interprets nonlinear risk premia within the Markov-switching dynamic factor framework. The third section discusses estimation and derives analytical expressions of the conditional moments of the excess returns. In the fourth section, the empirical results are presented and compared to extant literature. The fifth section concludes.

2. MODEL DESCRIPTION

We propose modeling excess returns on stocks, $\boldsymbol{Y}_{kt}$, as a function of a common unobserved dynamic factor, F_t, and individual idiosyncratic noises, $\boldsymbol{\varepsilon}_{kt}$. The factor captures marketwide comovements underlying these stocks, and it is a parsimonious proxy for the market risk premium:

$$\boldsymbol{Y}_{kt} = \boldsymbol{\lambda}_k^{st} F_t + \boldsymbol{\varepsilon}_{kt}, \quad k = 1, \ldots, 4; \quad S_t = 0, 1; \quad \boldsymbol{\varepsilon}_{kt} \sim \text{i.i.d. } N(0, \boldsymbol{\Sigma}). \qquad \textbf{(1)}$$

In a first specification, $\boldsymbol{Y}_{kt}$ is a 4×1 vector of monthly excess stock return (defined as the difference between continuously compounded stock returns and the 3-month T-bill rate) on the valued-weighted index, the equal-weighted index, IBM stock, and GM stock. In a second specification, $\boldsymbol{Y}_{kt}$ includes other financial variables such as price/earnings ratio, dividend yield, 3-month T-bill rate, in addition to the excess return on the valued-weighted index. The state-dependent factor loadings, $\boldsymbol{\lambda}_k^{st}$, measure the sensitivity of the kth series to the market risk premia, F_t, in Markov state S_t. The factor loading for the value-weighted excess return is set equal to 1 in both states to provide a scale for the latent dynamic factor.[8]

To examine potential changes in conditional excess return and in its volatility across different states of the financial markets, we allow the first and second moments of the factor to switch regimes according to a Markov variable, S_t, representing the state of financial conditions[9]:

$$F_t = \alpha_1 + \alpha_0 S_t + \phi F_{t-1} + \eta_{S_t}, \quad S_t = 0, 1; \quad \eta_{S_t} \sim N\left(0, \sigma^2_{\eta_{S_t}}\right); \qquad \textbf{(2)}$$

that is, financial markets can be either in an expansion period (bull market), $S_t = 1$, or in a contraction state (bear market), $S_t = 0$, with the switching ruled by the transition probabilities of the first-order two-state Markov process, $p^{ij} = \text{Prob}[S_t = j \mid S_{t-1} = i]$, $\sum_{j=0}^{1} p^{ij} = 1$, $i, j = 0, 1$.[10] The dynamic factor is, therefore, a representation of nonlinear market risk across Markov states. Cyclical variation in the nonlinear risk is generated from the common shock (η_{S_t}) to each of the observable variables ($\boldsymbol{Y}_{kt}$), and all idiosyncratic movements arise from the term $\boldsymbol{\varepsilon}_{kt}$. That is, we assume that η_{S_t} and $\boldsymbol{\varepsilon}_{kt}$ are mutually independent at all leads and lags, for all $k = 1, \ldots, 4$, for each model specification. The dynamic factor is the common element among the financial variables and is produced as a nonlinear combination of the observable variables $\boldsymbol{Y}_{kt}$. This factor has a time-varying conditional mean and variance and, therefore, should play a role in determining the time-series behavior of market risk premia.

3. ESTIMATION AND ANALYSIS OF CONDITIONAL MOMENTS

The parameters of the model are estimated using a nonlinear discrete version of the Kalman filter combined with Hamilton's (1989) filter in one algorithm, as suggested by Kim (1994) based on the work of Harrison and Stevens (1976). The model is cast in state-space form, where equations (1) and (2) are, respectively, the measurement and transition equations. The goal of the nonlinear filter is to form forecasts of the factor and the associated mean squared error matrices, based not only on information available up to time $t-1$, $I_{t-1} \equiv [\boldsymbol{Y}'_{t-1}, \boldsymbol{Y}'_{t-2}, \ldots, \boldsymbol{Y}'_1]'$, but also on the Markov state S_t taking on the value j, and on S_{t-1} taking on the value i. That is,

$$F^{(i,j)}_{t|t-1} = E[F_t \mid I_{t-1}, S_t = j, S_{t-1} = i], \tag{3}$$

$$\theta^{(i,j)}_{t|t-1} = E[(F_t - F_{t|t-1})(F_t - F_{t|t-1})' \mid I_{t-1}, S_t = j, S_{t-1} = i], \tag{4}$$

where $F_{t|t-1} = E(F_t \mid I_{t-1})$. The nonlinear Kalman filter is

$$F^{(i,j)}_{t|t-1} = \mu^j + \phi F^i_{t-1|t-1}, \tag{5}$$

$$\theta^{(i,j)}_{t|t-1} = \phi^2 \theta^i_{t-1|t-1} + \sigma^2_{s_t}, \qquad \text{(prediction equations)} \tag{6}$$

$$F^{(i,j)}_{t|t} = F^{(i,j)}_{t|t-1} + \boldsymbol{K}^{(i,j)}_t \boldsymbol{N}^{(i,j)}_{t|t-1}, \tag{7}$$

$$\theta^{(i,j)}_{t|t} = \left(1 - \boldsymbol{K}^{(i,j)}_t \boldsymbol{\lambda}^j\right)\theta^{(i,j)}_{t|t-1}, \qquad \text{(updating equations)} \tag{8}$$

where $\mu^j = \alpha_1 + \alpha_0 S_t$, $\boldsymbol{K}^{(i,j)}_t = \theta^{(i,j)}_{t|t-1}\boldsymbol{\lambda}^{j\prime}[\boldsymbol{Q}^{(i,j)}_t]^{-1}$, $\boldsymbol{N}^{(i,j)}_{t|t-1} = \boldsymbol{Y}_t - \boldsymbol{\lambda}^j F^{(i,j)}_{t|t-1}$ is the conditional forecast error of $\boldsymbol{Y}_t$, and $\boldsymbol{Q}^{(i,j)}_t = \boldsymbol{\lambda}^j \theta^{(i,j)}_{t|t-1} \boldsymbol{\lambda}^{j\prime} + \boldsymbol{\Sigma}$ is its conditional variance. Hamilton's nonlinear filter is

$$\text{Prob}(S_{t-1} = i, S_t = j \mid I_{t-1}) = p^{ij} \sum_{h=0}^{1} \text{Prob}(S_{t-2} = h, S_{t-1} = i \mid I_{t-1}). \tag{9}$$

From these joint conditional probabilities, the density of $\boldsymbol{Y}_t$ conditional on S_{t-1}, S_t, and I_{t-1} is

$$f(\boldsymbol{Y}_t \mid S_{t-1} = i, S_t = j, I_{t-1}) = \left[(2\pi)^{-k/2}\left|\boldsymbol{Q}^{(i,j)}_t\right|^{-1/2} \times \exp\left(-\frac{1}{2}\boldsymbol{N}^{(i,j)\prime}_{t|t-1}\boldsymbol{Q}^{(i,j)^{-1}}_t \boldsymbol{N}^{(i,j)}_{t|t-1}\right)\right]. \tag{10}$$

The joint probability density of states and observations is then calculated by multiplying each element of (9) by the corresponding element of (10):

$$f(\boldsymbol{Y}_t, S_{t-1} = i, S_t = j \mid I_{t-1}) = f(\boldsymbol{Y}_t \mid S_{t-1} = i, S_t = j, I_{t-1}) \times \text{Prob}(S_{t-1} = i, S_t = j \mid I_{t-1}). \tag{11}$$

https://doi.org/10.1017/S1365100501023082 Published online by Cambridge University Press

The probability density of $\boldsymbol{Y}_t$ given I_{t-1} is

$$f(\boldsymbol{Y}_t \mid I_{t-1}) = \sum_{j=0}^{1}\sum_{i=0}^{1} f(\boldsymbol{Y}_t, S_{t-1} = i, S_t = j \mid I_{t-1}). \tag{12}$$

The joint probability density of states is calculated by dividing each element of (11) by the corresponding element of (12):

$$\text{Prob}(S_{t-1} = i, S_t = j \mid I_t) = f(\boldsymbol{Y}_t, S_{t-1} = i, S_t = j \mid I_{t-1}) / f(\boldsymbol{Y}_t \mid I_{t-1}). \tag{13}$$

Finally, summing over the states in (13), we obtain the filtered probabilities of bull or bear markets:

$$\text{Prob}(S_t = j \mid I_t) = \sum_{i=0}^{1} \text{Prob}(S_{t-1} = i, S_t = j \mid I_t). \tag{14}$$

The link between the two filters arises as an approximation introduced through $F^j_{t|t}$ and $\theta^j_{t|t}$, which truncates the forecasts at each iteration.[11] The approximation is required to make the filter computationally tractable, since at each date t the nonlinear filter computes four forecasts, and at each iteration the number of possible cases is multiplied by the number of states. The approximation consists of a weighted average of the updating procedures by the probabilities of the Markov state:

$$F^j_{t|t} = \frac{\sum_{i=0}^{1} \text{Prob}[S_{t-1} = i, S_t = j \mid I_t] F^{(i,j)}_{t|t}}{\text{Prob}[S_t = j \mid I_t]}, \tag{15}$$

$$\theta^j_{t|t} = \frac{\sum_{i=0}^{1} \text{Prob}[S_{t-1} = i, S_t = j \mid I_t]\left\{\theta^{(i,j)}_{t|t} + \left(F^j_{t|t} - F^{(i,j)}_{t|t}\right)\left(F^j_{t|t} - F^{(i,j)}_{t|t}\right)'\right\}}{\text{Prob}[S_t = j \mid I_t]}. \tag{16}$$

The nonlinear filter allows recursive calculation of the predicted equations using only observations on $\boldsymbol{Y}_{kt}$, $k = 1, \ldots, 4$, given values for the parameters in ϕ, $\boldsymbol{\lambda}^j$, μ^j, p^{ij}, $\boldsymbol{\Sigma}$, and $\sigma^2_{\eta_{S_t}}$, and initial inferences for the factor, $F^j_{t|t}$, the mean squared error, $\theta^j_{t|t}$, and the joint probability of the Markov-switching states. The outputs are their one-step updated values. This permits estimation of the unobserved state vector as well as the probabilities associated with the latent Markov state. A byproduct of this algorithm is the conditional likelihood of the observable variable, which can be evaluated at each t. The log likelihood function is

$$\log f(\boldsymbol{Y}_T, \boldsymbol{Y}_{T-1,\ldots} \mid I_0)) = \sum_{t=1}^{T} \log \sum_{j=0}^{1}\sum_{i=0}^{1} \Bigg[2\pi^{-k/2} \left|\boldsymbol{Q}^{(i,j)}_t\right|^{-1/2} \times \exp\left(-\frac{1}{2}\boldsymbol{N}^{(i,j)'}_{t|t-1}\boldsymbol{Q}^{(i,j)^{-1}}_t \boldsymbol{N}^{(i,j)}_{t|t-1}\right)\Bigg] \text{Prob}(S_{t-1} = i, S_t = j \mid I_{t-1}). \tag{17}$$

https://doi.org/10.1017/S1365100501023082 Published online by Cambridge University Press

The filter evaluates this likelihood function at each t, which can be maximized with respect to the model parameters using a nonlinear optimization algorithm. Thus, the factor is constructed as a nonlinear combination of the observable variables weighted by the probabilities of the Markov state, using information available through time t:

$$F_{t|t} = E(F_t \mid I_t) = \sum_{j=0}^{1} \text{Prob}(S_t = j \mid I_t) F_{t|t}^{j}. \tag{18}$$

Finally, the conditional moments of the excess returns are obtained from forecasts of the mean and volatility of the dynamic factor. The estimation yields first and second conditional moments for each of the components of the dynamic factor. From equations (1) and (2) and from the nonlinear algorithm, the conditional expectation of excess returns is

$$E(\boldsymbol{Y}_t \mid I_{t-1}) = \sum_j \sum_i \boldsymbol{\lambda}^j \left[\mu^j + \phi F_{t-1|t-1}^{i}\right] \text{Prob}(S_t = j, S_{t-1} = i \mid I_{t-1}) \tag{19}$$

The conditional variances of excess returns are the conditional variance of the forecast error of $\boldsymbol{Y}_t$ obtained from the Kalman iterations:

$$\text{Var}(\boldsymbol{Y}_t \mid I_{t-1}) = \sum_j \sum_i \boldsymbol{\lambda}^j \left[\theta_{t|t-1}^{(i,j)} \text{Prob}(S_t = j, S_{t-1} = i \mid I_{t-1})\right] \boldsymbol{\lambda}^{j\prime} + \boldsymbol{\Sigma} \tag{20}$$

Hence, the Sharpe ratio is

$$\text{SR} = E(\boldsymbol{Y}_t \mid I_{t-1}) / \sqrt{\text{Var}(\boldsymbol{Y}_t \mid I_{t-1})}, \tag{21}$$

which in this framework corresponds to the price of the marketwide risk.

As seen in equations (19) and (20), both expected excess returns and conditional volatility are functions of the latent states of the stock market, as represented by the Markov process. However, the model does not impose a direct relation between the level and volatility of excess returns. In fact, expected excess return and its conditional volatility may not be related directly but may be a nonlinear function of the phase of the stock market, whether bull or bear. Thus, expected excess returns and conditional volatility could be positively or negatively associated or they could exhibit no relationship at all.[12]

4. EMPIRICAL RESULTS

4.1. Specification Tests and Results

Two specifications of the nonlinear dynamic factor model are reported for monthly data from 1954:02 to 1997:12, in an application to the postwar U.S. financial market.[13] In Model 1, $\boldsymbol{Y}_{kt}$ is composed of the excess return on the CRSP value-weighted (VW) index, on the CRSP equal-weighted (EW) index, on the IBM stock, and on the GM stock. The excess return is defined as the difference between continuously compounded stock returns and the 3-month T-bill rate in annual terms.[14] That is, in Model 1, excess returns are conditioned on the Markov process and on a latent factor that captures comovements on past values of different measures of excess returns. We use the excess returns on IBM and GM stocks to represent large firms, and the excess returns on the EW index to proxy for the dynamic behavior of small firms.[15] The excess return on the value-weighted index represents the market premium. Modeling the factor loadings as state dependent in this setting allows analysis of potential asymmetric behavior of the risk–return relationship for stocks with different market capitalization, across financial states.

The variables that compose the factor are highly correlated with each other. The dynamic factor structure captures commonalities underlying the observable variables. The resulting dynamic factor is also highly correlated with all the excess return series used to construct the factor, indicating that the structure was not simply imposed on the data by assuming large idiosyncratic errors. In addition, tests for the number of states support the single-factor specification.[16]

In Model 2, $\boldsymbol{Y}_{kt}$ includes the growth rate of the S&P 500 dividend yield (Dyield), changes in the 3-month T-bill rate (TB3) and in the S&P 500 price-earnings (P/E) ratio, in addition to the excess return on the VW index. In this case, excess return on the VW index is conditioned on a switching latent factor constructed from comovements underlying past values of other financial variables. There is a large literature on the predictability of excess stock returns that use lagged values of the dividend yield, P/E ratios, interest rates, and measures of the term premium or default risk as proxies for unobserved time-varying risk premia. The findings systematically link variations in unobserved risk premia to the business cycle.[17] Our framework allows the use of multivariate information with a parsimonious variance–covariance structure to produce the sort of predictions obtained from regression models—but in a nonlinear setting.

The dynamic factor obtained from Model 2 is highly correlated with the financial variables underlying the factor. Here again, the factor structure captures the commonalities underlying the fast movements of *changes* in P/E ratio, dividend yield, interest rates, and excess stock returns. Tests for the number of factors also support the single-factor structure for Model 2.

The Markov process captures the potential switches between bear and bull markets underlying financial variables. To verify this, we fit an AR(1) univariate Markov-switching model to each of the components of the factors in Models 1 and 2, allowing both the mean and the volatility of the variables to switch regime. Tests for the number of states favor the two-state specification.[18] Figure 1 plots the estimated filtered probabilities of bear markets against NBER-dated recessions. The results show a dichotomous pattern in the series associated with the phases of the stock market cycle, which, in turn, are related to future recessions and expansions.[19]

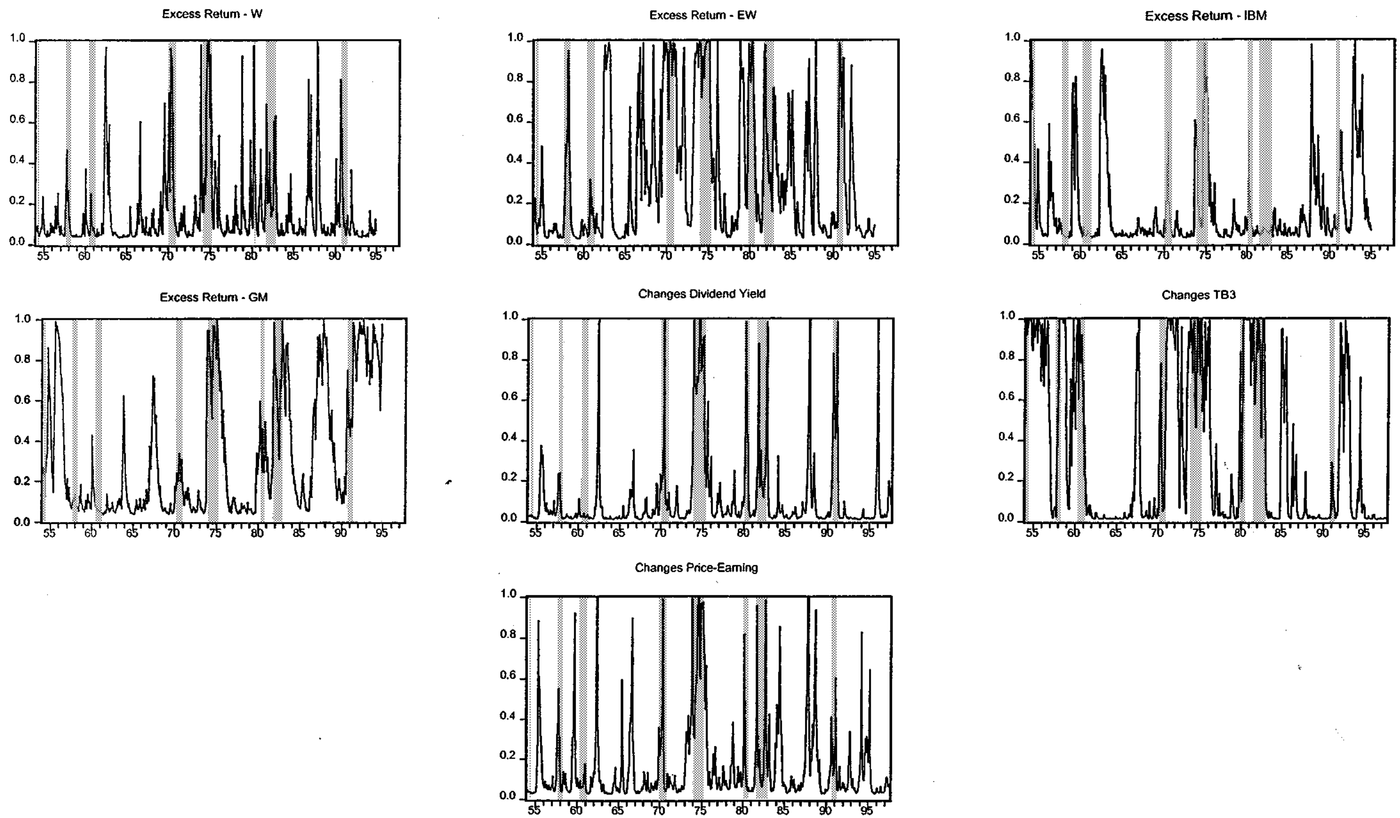

FIGURE 1. Filtered probabilities of bear market obtained from fitting a univariate AR(1) Markov-switching model to the factor components, and NBER dated recessions (shaded area).

https://doi.org/10.1017/S1365100501023082 Published online by Cambridge University Press

Thus, both the underlying correlation between the variables and the evidence of a Markov-switching structure indicate that the switching dynamic factor structure may well depict features of the data. The maximum likelihood estimates are shown in Table 1. For both models the coefficients of the two Markov states are statistically significant. In particular, we find that state 0 exhibits negative mean, high volatility, and a shorter average duration, which is associated with the short-lived and nervous bear markets. State 1 has a positive mean, low volatility, and a longer average duration, capturing the features of bull markets. Here again, tests for the number of states support the two-state specification.[20]

Specification tests are also implemented regarding the assumptions on the residuals. We implement Brock et al.'s (1996) diagnostic test and it fails to reject the hypothesis of i.i.d. disturbances.[21] In addition, the one-step-ahead forecast errors obtained from the Kalman filter are not predictable by lags of the observable variables.

In both models, we set the factor loading of the value-weighted index to one ($\lambda_{VW} = 1$).[22] Thus, we can compare the sensitivity of the other components to the factor in the same units as the excess returns on the VW index. The state-dependent factor loadings can capture the asymmetric behavior of returns, depending on the size of the firm across financial market states. Table 2 summarizes these findings. In bull markets excess stock returns of large and small firms exhibit a similar behavior ($\boldsymbol{\lambda}$ values around 1), but in bear markets, firm size makes a difference. That is, during periods of low market excess return, small firms are the most reactive to market risk ($\lambda_{EW} = 1.32$), whereas large firms are less sensitive to the market ($\lambda_{IBM} = 0.89$, $\lambda_{GM} = 0.97$). That is, stock returns of large firms decrease less than those of small firms during bear markets. These results also can be seen from the estimated filtered probabilities of bear markets plotted in Figure 1. The probabilities of bear markets from excess returns on small firms, as proxied by the EW index, are the most volatile and strongly react to most of the economic recessions in the sample data. On the other hand, the probabilities of bear markets for IBM and GM excess stock returns are less volatile and are less associated with the NBER-dated economic recessions.

Table 3 reports dating of the U.S. stock market cycle phases. The framework adopted in this paper provides probabilities that can be used as filtering rules for dating turning points. We use information from the frequency distribution of the smoothing probabilities from Model 1 to define turning points: a peak (trough) occurs if the smoothing probabilities of bear markets are greater (smaller) than their mean plus one-half their standard deviation.[23] The results for our sample data confirm the empirical observation that there have been more bear markets (10) than recessions (7), as measured by the NBER. With the exception of the 1960–1961 recession, all others in the sample data were associated with a bear market. Generally, bear markets begin a couple of months before a recession and end in the middle of it, anticipating economic recovery.[24] These findings are illustrated in Figure 2, which shows the smoothing probabilities of bear markets and the NBER recessions.

TABLE 1. Maximum likelihood estimates, 1954:02–1997:12

Parameters	Model 1[a]	Parameters	Model 2[b]
α_1	0.132	α_1	0.038
	(0.022)[c]		(0.015)
α_0	−0.355	α_0	−0.113
	(0.136)		(0.079)
ϕ	−0.078	ϕ	0.295
	(0.041)		(0.049)
$\sigma^2\varepsilon_{\mathrm{VW}}$	0.013	$\sigma^2\varepsilon_{\mathrm{VW}}$	0.135
	(0.005)		(0.009)
$\sigma^2\varepsilon_{\mathrm{EW}}$	0.094	$\sigma^2\varepsilon_{\mathrm{DY}}$	1.359
	(0.008)		(0.350)
$\sigma^2\varepsilon_{\mathrm{GM}}$	0.364	$\sigma^2\varepsilon_{\mathrm{TB3}}$	61.218
	(0.023)		(3.794)
$\sigma^2\varepsilon_{\mathrm{IBM}}$	0.392	$\sigma^2\varepsilon_{\mathrm{P/E}}$	5.621
	(0.024)		(0.514)
λ^0_{VW}	1	λ^0_{VW}	1
	—		—
λ^0_{EW}	1.317	λ^0_{DY}	−9.134
	(0.066)		(0.737)
λ^0_{GM}	0.968	λ^0_{TB3}	−3.589
	(0.106)		(1.704)
λ^0_{IBM}	0.888	$\lambda^0_{\mathrm{P/E}}$	10.070
	(0.098)		(0.892)
λ^1_{VW}	1	λ^1_{VW}	1
	—		—
λ^1_{EW}	1.002	λ^1_{DY}	−11.204
	(0.054)		(1.001)
λ^1_{GM}	0.978	λ^1_{TB3}	−2.683
	(0.096)		(1.922)
λ^1_{IBM}	1.022	$\lambda^1_{\mathrm{P/E}}$	10.859
	(0.090)		(1.036)
$\boldsymbol{p}_{11}$	0.961	$\boldsymbol{p}_{11}$	0.948
	(0.018)		(0.023)
$\boldsymbol{p}_{00}$	0.813	$\boldsymbol{p}_{00}$	0.703
	(0.094)		(0.133)
$\sigma^2_{\eta 1}$	0.149	$\sigma^2_{\eta 1}$	0.066
	(0.015)		(0.012)
$\sigma^2_{\eta 0}$	0.511	$\sigma^2_{\eta 0}$	0.306
	(0.100)		(0.090)
LogL(θ)	−1,517.79	LogL(θ)	−4,671.76

[a] $\boldsymbol{Y}_{kt} = \boldsymbol{\lambda}_k^{st} F_t + \boldsymbol{\varepsilon}_{kt}$; $F_t = \alpha_1 + \alpha_0 S_t + \phi F_{t-1} + \eta_{S_t}$, $S_t = 0, 1$; k = excess returns on VW, EW, GM, IBM.
[b] $\boldsymbol{Y}_{kt} = \boldsymbol{\lambda}_k^{st} F_t + \boldsymbol{\varepsilon}_{kt}$; $F_t = \alpha_1 + \alpha_0 S_t + \phi F_{t-1} + \eta_{S_t}$, $S_t = 0, 1$; k = excess returns on VW, changes in dividend yield, TB-3, P/E.
[c] Asymptotic standard errors are in parentheses.

Table 2. Firm size asymmetries across states (Model 1)

Asymmetries	Bear market	Bull market
Market: λ_{VW}	1	1
Large firms: λ_{IBM}	0.887	1.021
Large firms: λ_{GM}	0.969	0.978
Small firms: λ_{EW}	1.317	1.003

Table 3. Dating of the U.S. bear markets,[a] smoothed probabilities, Model 1

NBER recessions		Bear markets	
Peak	Trough	Peak	Trough
1957:08	1958:04	1957:08	1957:12
1960:04	1961:02	—	—
—	—	1962:03	1962:10
—	—	1966:05	1966:09
1969:12	1970:11	1969:02	1970:09
1973:11	1975:03	1973:01	1975:02
—	—	1978:08	1978:11
1980:01	1980:07	1979:09	1980:04
1981:07	1982:11	1981:06	1982:02
—	—	1987:09	1987:11
1990:07	1991:03	1990:07	1990:10

[a] The stock market is assumed to be in a bear market if the smoothed probabilities of bear markets, $P(S_t = 0 \mid I_T)$, is greater than their mean plus half their standard deviation.

4.2. Contemporaneous Relationship of the Conditional Moments

To investigate the empirical relationship between conditional expected excess return and its volatility, we derive these moments as described in equations (19)–(21) of Section 3.[25] Figures 3 and 4 plot the Sharpe ratio obtained from Models 1 and 2, the dating of bear and bull markets from Table 3, and NBER-dated recessions. We find that the price of the marketwide risk varies across stock market phases and business-cycle states.

For both models the expected excess return and volatility—and hence the Sharpe ratio—display distinct business-cycle dynamics. Conditional excess return falls during expansions, reaching a minimum in the middle of a recession, and rises in the second half of a recession, reaching a maximum at its trough. On the other hand, the conditional volatility is generally higher during economic recessions and lower during expansions. This result can also be seen in Table 4, which reports a series of regressions of the price of risk on measures of business cycles, such

https://doi.org/10.1017/S1365100501023082 Published online by Cambridge University Press

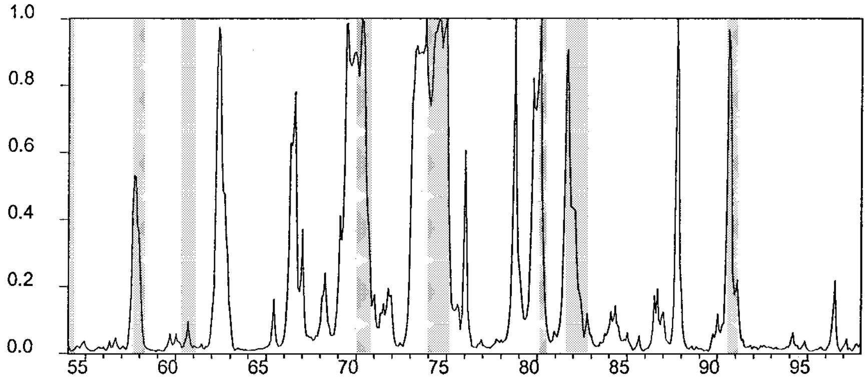

Figure 2. Smoothing probabilities of bear market from Model 1 and NBER-dated recessions.

as a 0/1 dummy variable representing recessions as dated by the NBER, changes in industrial production, and changes in the business-cycle indicator generated by Chauvet (1998).[26] We find that the regression coefficients are statistically significant in all regressions and the Sharpe ratio displays a strong countercyclical business pattern due to its behavior around economic turning points.

Table 5 shows the contemporaneous correlation between conditional expectation and variance of excess returns across financial-cycle phases. For Model 1, where the level and volatility of expected return are conditioned only to a Markov process and no a priori association is imposed on them, there is a significant contemporaneous negative risk–return relationship at the monthly frequency.[27] This result reflects the relationship between expected excess returns and volatility across bull and bear markets. In bear markets the conditional mean is low and the volatility is high, implying a low Sharpe ratio. In bull markets, with a high conditional mean and low volatility, the Sharpe ratio is much higher (Figure 3).

A negative but weak relationship is also found, for example, by Fama and Schwert (1977), Campbell (1987), Nelson (1991), and Glosten et al. (1993). As discussed by Backus and Gregory (1993), negative, nonmonotonic, or positive relationships between the first and second conditional moments of stock returns can arise from equilibrium models. The empirical literature reports mixed findings, depending on the way the moments are modeled and on the conditional variables used.[28] The analysis of conditional moments from Model 1 can add to the discussion in that it reflects expectations based only on past information of different measures of excess returns and on the state of the economy, as represented by the Markov process.

Using other financial variables in addition to the Markov state, as in Model 2, allows us to study the role of conditioning variables and to compare our results to existing literature. Figure 5 plots scatter diagrams for the mean and volatility of the excess return on the VW index for Models 1 and 2. Notice that although the overall

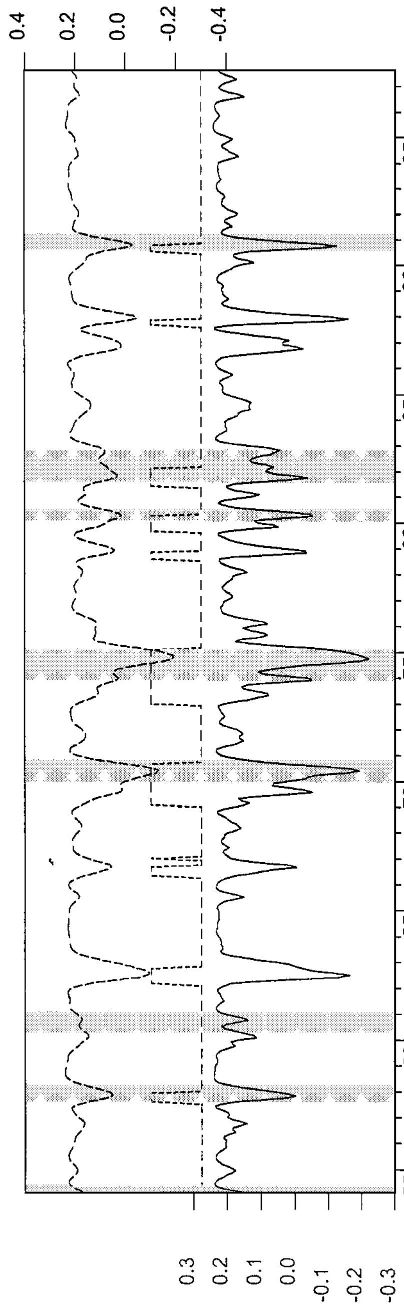

Figure 3. Model 1: Sharpe ratio of the VW excess return filtered (—) and smoothed (- - -) series, bear markets (- - -), and NBER-dated recessions (shaded area).

https://doi.org/10.1017/S1365100501023082 Published online by Cambridge University Press

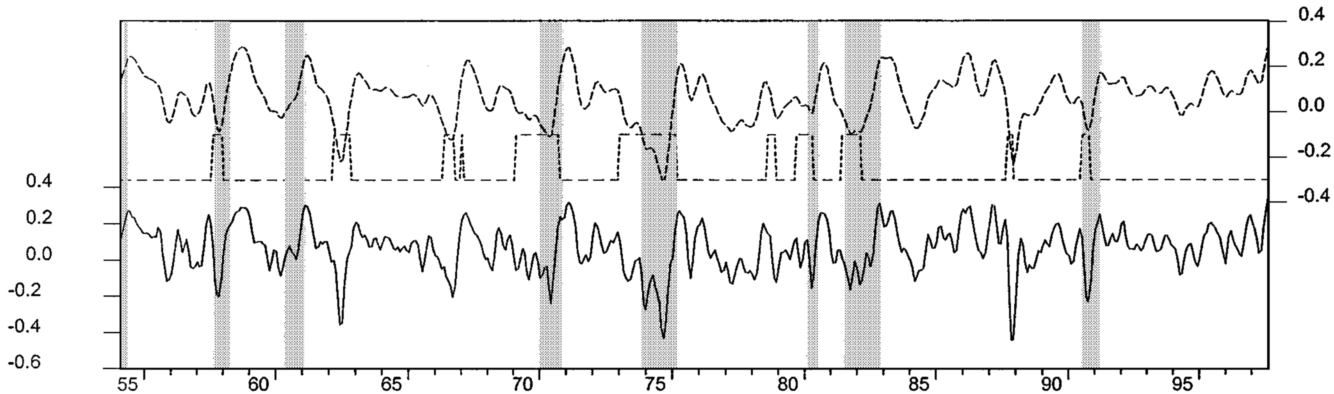

Figure 4. Model 2: Sharpe ratio of the VW excess return filtered (—) and smoothed (−−−) series, bear markets (- - -), and NBER-dated recessions (shaded area).

https://doi.org/10.1017/S1365100501023082 Published online by Cambridge University Press

Table 4. Individual regressions of the Sharpe ratio on economic variables[a]

Independent variable	Model 1			Model 2		
	NBER[b]	ΔlnIP[c]	SFC[d]	NBER[b]	ΔlnIP[c]	SFC[d]
Coefficient	−1.214	0.294	0.020	−1.199	0.311	0.011
	(−12.115)	(6.873)	(8.416)	(−3.743)	(2.324)	(2.805)
Adj. R^2	0.217	0.086	0.130	0.021	0.009	0.015

[a] t-statistic inside parentheses.
[b] NBER is a 0/1 dummy variable taking the value of 1 at NBER-dated recessions.
[c] Log first difference of Industrial Production.
[d] Business-cycle index built from a switching dynamic factor, which is highly correlated with GDP growth [see Chauvet (1998)]. We ran three simple regressions of the Sharpe ratio on a constant and each of the independent variables, for Models 1 and 2. The constant term is positive and significant in all equations.

Table 5. Contemporaneous correlation between conditional expectation (CE) and conditional variance (CV) of VW excess returns across business and financial cycles

CE and CV During	Correlation	
	Model 1	Model 2
Bear market	−0.505	−0.443
Bull market	−0.978	−0.152
CE < 0	−0.985	−0.906
CE > 0	−0.971	0.438
Full sample	−0.995	−0.399

Bear and Bull markets refer to the smoothed probabilities of bear and bull markets, respectively, obtained from each model. P-values are approximately zero for each entry of columns 1 and 2.

contemporaneous relation is still negative for Model 2, the relation between these two moments is weaker than in Model 1. In particular, for Model 2 the volatility is high for low values of the conditional expectation during bear markets. In bull markets the reverse occurs for low values of the conditional expectation (Figure 6).

A closer examination suggests that there is a nonlinear relationship between these moments, depending on whether conditional expectations are positive or negative. In spite of being counterintuitive in theory, empirical evidence reveals that, in certain periods, excess return is predicted to be negative.[29] We find that negative expected excess returns occur mostly right before the beginning of economic recessions. Dividing the sample into periods when the conditional expectation is positive or negative shows a remarkable result—the risk–return relationship is weakly positive if we exclude periods of negative conditional excess returns, and significantly negative otherwise (Figure 6). This nonlinear behavior may be behind

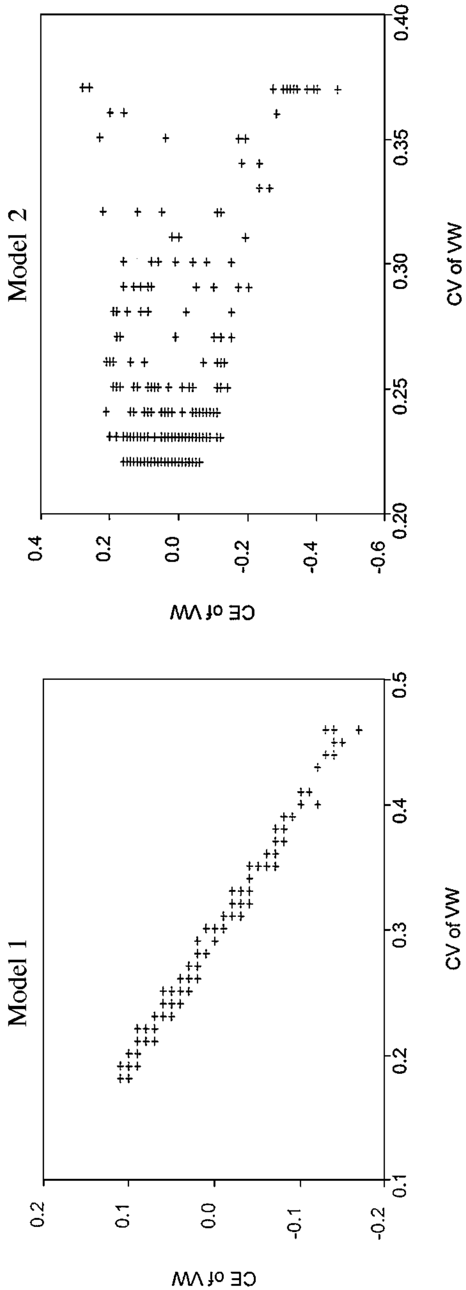

FIGURE 5. Scatter diagrams of conditional expectation (CE) and conditional variance (CV) of the VW excess return, full sample.

https://doi.org/10.1017/S1365100501023082 Published online by Cambridge University Press

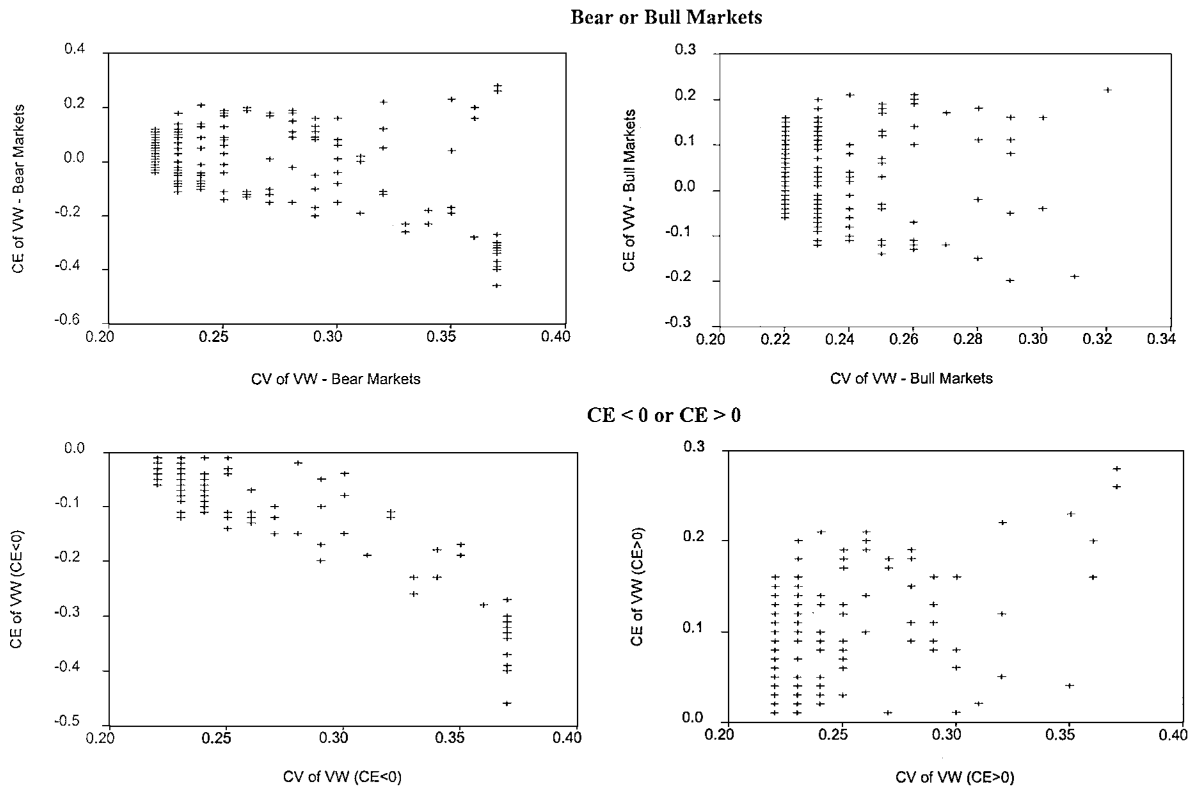

Figure 6. Model 2: Scatter diagrams of conditional expectation (CE) and conditional variance (CV) of the VW excess return, restricted sample, bear or bull markets, and CE $<$ 0 or CE $>$ 0.

https://doi.org/10.1017/S1365100501023082 Published online by Cambridge University Press

the diversity of empirical results found in the literature regarding the risk–return relationship.

This finding arises from the dynamics of conditional expected return near the beginning and end of business-cycle contractions. The conditional volatility is at its highest values near peaks and troughs of business cycles. In fact, conditional variance moves up and down during economic recessions, reflecting the great uncertainty of these periods. On the other hand, expected excess return reaches its minimum and its maximum values immediately before and during economic recessions. Since the decrease in the expected excess returns is substantial (reaching negative values) at the beginning of economic contractions, a net negative contemporaneous relationship between risk–return prevails for the whole sample.

Table 6. Model 1 correlogram: Conditional expectation (CE) and conditional variance (CV) of the VW

i	CE of VW, CV of VW($-i$)	CE of VW, CV of VW($+i$)
0	−0.9950[b]	−0.9950[b]
1	−0.7631[b]	−0.7336[b]
2	−0.5475[b]	−0.5195[b]
3	−0.4402[b]	−0.4213[b]
4	−0.4191[b]	−0.4018[b]
5	−0.3931[b]	−0.3708[b]
6	−0.3136[a]	−0.2912[a]
7	−0.2319	−0.2206
8	−0.1886	−0.1799
9	−0.1678	−0.1636
10	−0.1437	−0.1350
11	−0.1127	−0.1061
12	−0.1296	−0.1292
13	−0.1332	−0.1294
14	−0.1364	−0.1322
15	−0.1049	−0.1126
16	−0.0967	−0.1022
17	−0.0922	−0.0894
18	−0.0720	−0.0621
19	−0.0497	−0.0405
20	−0.0139	−0.0142
21	0.0195	0.0186
22	0.0481	0.0465
23	0.0502	0.0391
24	0.0640	0.0586

[a] Statistically significant at the 5% level.
[b] Statistically significant at the 1% level.

https://doi.org/10.1017/S1365100501023082 Published online by Cambridge University Press

However, toward the end of the recession, expected returns increase, foreseeing rises in earnings. Thus, the risk–return relationship is strongly negative in the first half of recessions and positive in the second half. This suggests that the risk–return dynamic relationship can be better understood if studied within and as a function of the different stages of the economy.

Table 5 summarizes these results. For Model 1, the correlation is negative across the financial cycle. However, when excess returns are conditioned on other financial variables in addition to the Markov state, as in Model 2, the correlation is −0.91 for times when the conditional expectation is negative and 0.44 for periods when it is positive. These results are statistically significant for any standard level.

TABLE 7. Model 2 correlogram: Conditional expectation (CE) and conditional variance (CV) of the VW

i	CE of VW, CV of VW($-i$)	CE of VW, CV of VW($+i$)
0	−0.3993[b]	−0.3993[b]
1	−0.3330[b]	−0.1192[b]
2	−0.2066	−0.0164[b]
3	−0.1358	−0.0011[b]
4	−0.1500	0.0058[b]
5	−0.2220	0.0083[b]
6	−0.1948	0.0549[b]
7	−0.1561	0.0261[b]
8	−0.1550	0.0188[b]
9	−0.1077	0.0126[a]
10	−0.0524	0.0611
11	−0.0266[a]	0.1109
12	−0.0549[a]	0.0988
13	−0.0892	0.0514[a]
14	−0.0330	0.0644
15	0.0139	0.0403
16	0.0235	0.0149
17	0.0021	0.0263
18	−0.0313	0.0414
19	0.0279	0.0424
20	0.0071	0.0091
21	−0.0479	0.0046
22	0.0148	0.0091
23	0.0472	−0.0194
24	0.0305	−0.0643

[a] Statistically significant at the 5% level.
[b] Statistically significant at the 1% level.

https://doi.org/10.1017/S1365100501023082 Published online by Cambridge University Press

4.3. Noncontemporaneous Correlations

We find that the contemporaneous relationship between expected excess returns and the conditional variance is not only time-varying, but also changes signs within economic recessions. We further examine their correlation at leads and lags. Tables 6 and 7 show the cross-correlogram between conditional excess return and variance for 24-month leads and lags of the conditional variance. For Model 1, their cross-correlation is negative and significant up to six leads and lags. The offset correlations are symmetric, implying that cyclical variations in risk and return are negatively related but coincident. For Model 2, the cross-correlation between the two moments is weaker. It is significantly negative up to two lags of the conditional variance and statistically insignificant for higher lags. For leads of the conditional variance, the relation is negative and significant up to 9 months. Using Granger causality and spectral analysis, we find that expected excess return slightly leads volatility. No strong conclusion can be drawn from this, since the relationship between these two moments may be driven by the phase of the business cycle, as examined here. However, using the whole sample, it seems that when excess returns are expected to be low, an immediate increase in market volatility follows as investors seek to move their position to hedge against noise.

Based on the findings of the previous session, an interesting question is whether the noncontemporaneous relationship between expected excess returns and volatility is also nonstable around business-cycle turning points. Table 8 shows the cross-correlation for the subsample of the data corresponding to periods of negative or positive conditional expectation for Model 2. In fact, when the conditional expectation is positive, as during expansions and the second half of economic contractions,

Table 8. Model 2 correlogram: Conditional expectation (CE) and conditional variance (CV) of the VW excess return

i	CE of VW, CV of VW($-i$) CE $<$ 0	CE of VW, CV of VW($+i$) CE $<$ 0	CE of VW, CV of VW($-i$) CE $>$ 0	CE of VW, CV of VW($+i$) CE $>$ 0
0	−0.9064[b]	−0.9064[b]	0.4383[b]	0.4383[b]
1	−0.3151[b]	−0.2111[b]	0.1267[b]	0.1756[b]
2	−0.1193	−0.0702[a]	0.0046	0.0820[a]
3	−0.0837	−0.0542	−0.0109	0.0347
4	−0.0975	−0.0614	0.1021[a]	−0.0572
5	−0.1117	−0.0566	0.0304	−0.0502
6	0.0156	0.0063	0.0744	−0.0376
7	−0.0592	−0.0710	−0.0274	−0.0248
8	−0.0914	−0.0764	−0.0333	0.0002
9	−0.0837	−0.0473	0.0173	0.0148
10	−0.0610	−0.0421	0.0461	−0.0156

[a] Statistically significant at the 5% level.
[b] Statistically significant at the 1% level.

https://doi.org/10.1017/S1365100501023082 Published online by Cambridge University Press

we find that offset correlations are positive and significant for up to two leads and lags (volatility feedback). However, restricting the sample for times when the conditional expectation is negative—during the first half of contractions—the offset correlations are significant and negative up to two leads and lags (leverage effect).[30]

5. CONCLUSIONS

This paper proposes an empirical framework that offers a flexible description of the joint time-series properties of the level and volatility of expected stock returns. An unobservable dynamic factor is built as a nonlinear proxy for the market risk premia with conditional first and second moments driven by a latent Markov variable. That is, we consider the possibility that the market expected return and its conditional volatility are not necessarily directly related but are a function of a third variable—the two states of the Markov process, which can be interpreted as bull and bear markets.

We find a significant asymmetric behavior of conditional excess returns according to firm size. In particular, excess returns on small firm stocks are more reactive to changes in the state of financial markets than large firms. With respect to the risk–return relationship, during bear markets, expected excess return is low whereas the conditional volatility is high. In bull markets, the conditional mean increases whereas the volatility decreases. That is, we find an overall contemporaneous negative risk–return relationship at the monthly frequency. This negative relation is less significant when other conditional financial variables are considered.

Most important, the contemporaneous correlation is not stable, but instead it changes signs according to the state of the business cycle. Around peaks and during the first half of economic recessions as measured by the NBER, the trade-off between risk and return is negative, whereas during the second half of economic recessions, the relationship is positive. This result arises from the dynamics of conditional expected returns near business-cycle peaks and troughs. In particular, around the beginning of recessions, volatility rises considerably, whereas expected return falls, anticipating a decrease in earnings. Thus, there is a negative relationship between conditional expectation and variance. Toward the end of a recession, expected returns are at their highest value as an anticipation of the economic recovery, and volatility is still very high in anticipation of the turning point. In fact, we find that the conditional volatility is at its highest values near peaks and troughs of business cycles, reflecting the uncertainty about the timing of these turns. Thus, during times of high volatility, investors move back and forth from stocks, driving changes in expected returns and the direction of the relationship, depending on the stage of the economy.

This time-varying behavior also holds for noncontemporaneous correlations. When the conditional expectation is positive, we find that offset correlations between conditional mean and variance are positive and significant for shorter leads and lags (volatility feedback). However, restricting the sample for times when the conditional expectation is negative, the offset correlation is significant and negative (leverage effect). The results suggest that the contemporaneous and offset risk–return relationship change over time, as a result of the dynamics of conditional expected returns around business-cycle peaks and troughs.

NOTES

1. Fama and Schwert (1977), Campbell (1987), Nelson (1991), Glosten et al. (1993), among others, find a negative relation between conditional expected stock return and variance. On the other hand, Bollerslev et al. (1988), Harvey (1989), and Chan et al. (1992) find a weak or no statistically significant relationship between expected return and conditional variance in the stock market. Others, such as French et al. (1987) and Campbell and Hentschel (1992) find a positive relation between expected returns and conditional second moments.

2. For example, Fama and French (1989), Whitelaw (1994), Perez-Quiros and Timmermann (1998, 2001), Chauvet (1998/1999), and Chauvet and Potter (2000) find evidence of a significant state dependence in the conditional distribution of stock returns, where financial variables proxying for risk forecast business-cycle phases.

3. That is, predictability of the level implies predictability of volatility. However, if the level of returns is difficult to predict, it does not imply that the volatility should be.

4. Further, when conditional variance is used to proxy for risk, the ARCH-M restricts the conditional mean of excess returns to be positive.

5. In stock market jargon, bear markets are periods of persistent decrease in stock prices. Thus, bear markets are also associated with periods when the excess return is negative.

6. For further discussion of the leverage effect and volatility feedback see, for example, Black (1976), Christie (1982), Pindyck (1984), Poterba and Summers (1986), French et al. (1987), Turner et al. (1989), Campbell and Hentschel (1992), Glosten et al. (1993), or Chauvet (2000b).

7. These findings are also corroborated by Perez-Quiros and Timmermann (1998, 2001).

8. Generally, to assign a scale to the factor, either its variance or one of the factor loadings is set to 1. In our models, normalization of the factor is attained through the factor loadings because the variances are state dependent.

9. In an initial stage of this project, different specifications were estimated in which the factor mean, variance, and factor loadings do not switch across states. The likelihood ratio test rejects these models at the 0.5% significance level.

10. The transition probabilities are assumed to be constant in this setting. Allowing the transition probabilities to be time-varying in this multivariate framework does not change the results qualitatively, while it amounts to additional complication in interpreting the model.

11. Smith and Makov (1980) simulate the filter and find that the approximation yields the smallest mean squared error compared to any other estimator based on a linear function. Also it performs well compared with other nonlinear approximation methods. The method yields the smallest sum of the squared errors, tracks most closely the true observations, and is the best in estimating the jumps.

12. Notice that the sign of μ^j only determines the level of the stock market—bull or bear phases. Whether bull or bear markets are associated with low or high conditional variances depends on the properties of the data and not on any model restrictions.

13. The data are obtained from the 1997 release of the DRI Basic Economic Database, and from the CRSP files.

14. We also estimate the model using only the excess returns on the value-weighted and on the equal-weighted ($k = 2$), or using only one large firm ($k = 3$). The results are very similar to the ones obtained for the four-variable model.

15. Small firms that were participating in the stock market in the beginning of the sample tended to disappear over our 40 years of data whereas those that survived generally became medium or large firms. Thus, we use the EW return index to proxy for small-firm dynamics instead of selecting individual firms.

16. The number of factors underlying the variables was tested by examining the eigenvalues from the correlation matrix of the common factors. The magnitude of the eigenvalues for each factor reflects how much of the correlation among the observable variables is explained by a particular factor. The procedure indicates strong evidence for the single-factor specification for both models.

17. Some examples are Fama and Schwert (1977), Keim and Stambaugh (1986), Campbell and Shiller (1988), Fama and French (1989), Whitelaw (1994), Perez-Quiros and Timmermann (1998, 2001), Chauvet and Potter (2000), or Chauvet (1998/1999).

18. We test for the number of states using the approach proposed by Garcia (1998), based on Hansen (1993). The test provides strong evidence for the two-state model.

19. This evidence is also found by Perez-Quiros and Timmermann (1998, 2001), Chauvet (1998/1999), and Chauvet and Potter (2000), among others.

20. Although Garcia's critical values are designed for a univariate AR(1) regime-switching model and the test is parameter dependent, the value of the likelihood ratios are about 3 times larger than the highest value in Garcia's table for the 1% significance level.

21. For a vector $\boldsymbol{\varepsilon}_t^m = \varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+m-1}$, we use $m = 2, 3, 4$ and we set the distance d between any two vectors $\boldsymbol{\varepsilon}_t^m$ and $\boldsymbol{\varepsilon}_S^m$, equal to the standard deviation of ε_t. The test estimates the probability that these vectors are within the distance d.

22. The normalization affects only the scale of the factor. None of the time-series properties of the dynamic factor or the correlation with its components is affected by the choice of the parameter scale.

23. These probabilities are obtained recursively from the Kalman filter based on full sample information, $\text{Prob}(S_t = 0 \mid I_T)$. Similar dating of bull and bear markets is also obtained using different threshold values for defining a turning point [see Chauvet (1998/1999)].

24. Chauvet (1998/1999) also finds that bear and bull market cycles, although more frequent, are closely associated with future business-cycle phases.

25. We find that the dynamics of the risk–return relationship for the excess return series are all qualitatively very similar. Thus, we focus mainly on the results for the VW excess return.

26. This monthly coincident indicator is constructed from a Markov-switching dynamic factor using economic variables that move contemporaneously with business cycles, such as sales, personal income, industrial production, and employment.

27. For Model 1, the autoregressive parameter is small and not significant, reflecting the low persistence underlying the stock excess returns. However, the variables are strongly related to the factor through the state-dependent first and second moments.

28. The role of conditioning and misspecification in determining the direction of the relationship is discussed by Harvey (1991), Pagan and Hong (1991), and Glosten et al. (1993), among others, particularly when a symmetric relation between risk and return is imposed.

29. See, for example, Whitelaw (1994), Harrison and Zhang (1999), Perez-Quiros and Timmermann (1998, 2001), Chauvet and Potter (2000), or Chauvet (2000a).

30. These results are further examined by Chauvet (2000b).

REFERENCES

Backus, D.K. & A.W. Gregory (1993) Theoretical relationships between risk premiums and conditional variances. *Journal of Business and Economic Statistics* 11, 177–185.

Black, F. (1976) Studies of stock price volatility changes. In *Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economics Statistics Section*, pp. 177–181. Alexandria, VA: American Statistical Association.

Bollerslev, T.R., R. Engle & J. Wooldridge (1988) A capital asset pricing model with time varying covariances. *Journal of Political Economy* 96, 116–131.

Brock, W., D. Dechert, J. Scheinkman & B. LeBaron (1996) A test for independence based on the correlation dimension. *Econometric Reviews* 15, 197–235.

Campbell, J. (1987) Stock returns and the term structure. *Journal of Financial Economics* 18, 373–399.

Campbell, J. & L. Hentschel (1992) No news is good news: An asymmetric model of changing volatility in stock returns. *Journal of Financial Economics* 31, 281–318.

Campbell, J. & R. Shiller (1988) The dividend-price ratio and expectations of future dividends and discount factors. *Review of Financial Studies* 1, 195–228.

Chan, K.C., G.A. Karolyi & R. Stulz (1992) Global financial markets and the risk premium on U.S. equity. *Journal of Financial Economics* 31, 281–318.

Chauvet, M. (1998) An econometric characterization of business cycle dynamics with factor structure and regime switches. *International Economic Review* 39, 969–996.

Chauvet, M. (1998/1999) Stock market fluctuations and the business cycle. *Journal of Economic and Social Measurement* 25, 235–258.

Chauvet, M. (2000a) An Empirical Analysis of the Equity Premium Puzzle at the Business Cycle Frequency. Mimeo, University of California, Riverside.

Chauvet, M. (2000b) Volatility Feedback or Leverage Effect? An Event Timing Study. Mimeo, University of California, Riverside.

Chauvet, M. & S. Potter (2000) Coincident and leading indicators of the stock market. *Journal of Empirical Finance* 7, 87–111.

Christie, A.A. (1982) The stochastic behavior of common stock variances: Value, leverage, and interest rate effects. *Journal of Financial Economics* 10, 407–432.

Fama, E. & K.R. French (1989) Business conditions and expected returns on stock and bonds. *Journal of Financial Economics* 25, 23–49.

Fama, E. & G.W. Schwert (1977) Asset returns and inflation. *Journal of Financial Economics* 5, 115–146.

French, K.R., G.S. Schwert, & R. Stambaugh (1987) Expected stock returns and volatility. *Journal of Financial Economics* 19, 3–30.

Garcia, R. (1998) Asymptotic null distribution of the likelihood ratio test in Markov switching models. *International Economic Review* 38, 763–788.

Glosten, C., R. Jagannathan & D. Runkle (1993) On the relation between the expected value and volatility of the nominal excess returns on stocks. *Journal of Finance* 48, 1779–1802.

Hamilton, J. (1989) A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica* 57, 357–384.

Hansen, B.E. (1993) The likelihood ratio test under non-standard conditions: Testing the Markov trend model of GNP. In M.H. Pesaran & S. Potter (eds.), *Nonlinear Dynamics Chaos and Econometrics*, pp. 53–73. New York: John Wiley & Sons.

Harrison, P.J. & C.F. Stevens (1976) Bayesian forecasting. *Journal of the Royal Statistic Society Series B* 38, 205–247.

Harrison, P. & H. Zhang (1999) An investigation of the risk and return relation at long horizons. *Review of Economics and Statistics* 81, 399–408.

Harvey, C.R. (1989) Time-varying conditional covariances in tests of asset pricing models. *Journal of Financial Economics* 24, 289–317.

Harvey, C.R. (1991) The Specification of Conditional Expectations. Working paper, Duke University.

Keim, D. & R.F. Stambaugh (1986) Predicting returns in the stock and bond markets. *Journal of Financial Economics* 17, 357–390.

Kim, C.J. (1994) Dynamic linear models with Markov-switching. *Journal of Econometrics* 60, 1–22.

Nelson, D. (1991) Conditional heterokesdasticity in asset returns: A new approach. *Econometrica* 59, 347–370.

Pagan, A. & Y.S. Hong (1991) Nonparametric estimation and the risk premium. In W. Barnett, J. Powell, & G. Tauchen (eds.), *Nonparametric and Semiparametric Methods in Econometrics and Statistic*, pp. 51–75. Cambridge, UK: Cambridge University Press.

Perez-Quiros, G. & A. Timmermann (1998) Variations in the mean and volatility of stock returns around turning points of the business cycle. In J. Knight & S. Satchell (eds.), *Forecasting Volatility in the Financial Markets*, pp. 287–306. Oxford: Butterworth-Heinemann.

Perez-Quiros, G. & A. Timmermann (2001) Business cycle asymmetries in stock returns: Evidence from higher order moments and conditional densities. *Journal of Econometrics* 103, 259–306.

Pindyck, R.S. (1984) Risk, inflation, and the stock market. *American Economic Review* 74, 335–351.

Poterba, J.M. & L.H. Summers (1986) The persistence of volatility and stock market fluctuations. *American Economic Review* 76, 1142–1151.

Smith, A.F.M. & U.E. Makov (1980) Bayesian detection and estimation of jumps in linear systems. In O.L.R. Jacobs, M.H. Davis, M.A. Dempster, C.J. Harris, & P.C. Parks (eds.), *Analysis and Optimization of Stochastic Systems*, pp. 333–345. New York: Academic Press.

Turner, C.M., R. Staptz, & C.R. Nelson (1989) A Markov model of heteroskedasticity, risk, and learning in the stock market. *Journal of Financial Economics* 25, 3–22.

Whitelaw, R. (1994) Time variations and covariations in the expectation and volatility of stock market return. *Journal of Finance* 2, 515–541.