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ASSET PRICING IN DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM MODELS WITH INDETERMINACY

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We explore asset pricing in the context of the one-sector Benhabib-Farmer-Guo (BFG) model with increasing returns to scale in production and compare our results with financial implications of the standard dynamic stochastic general equilibrium (DSGE) model. Our main goal is to determine the effects of local indeterminacy and the presence of sunspot shocks on asset pricing. We find that the BFG model does not adequately represent key stylized facts of U.S. capital markets and does not improve on the asset-pricing results obtained in the standard DSGE model.

Keywords: Equity Premium, Volatility Puzzle, Indeterminacy

1. INTRODUCTION

Over more than two decades, financial economists have been striving to explain time- variation in interest rates and cross-sectional variation in returns on average stocks and bonds in the dynamic stochastic general equilibrium (DSGE) framework. DSGE models, in which macroeconomic factors affect both output and asset prices, seem to be a natural context for asset-pricing explorations. Typically the analysis is undertaken in an exchange context, although production style real business cycle (RBC) formulations have been studied as well.[1] The common feature of neoclassical RBC models is that they rely on technology shocks as the main source of fluctuations. Although remarkably successful in matching business cycle statistics, most neoclassical RBC models are unable to replicate one or several stylized financial facts: the high equity premium, the low risk-free rate, and the high volatility of equity returns with corresponding low volatility of bond returns.[2]

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More recently, starting with the pioneering work of Benhabib and Farmer (1994) and Farmer and Guo (1994), a large body of literature has developed in which DSGE models, modified to include increasing returns to scale in production, can result in a continuum of equilibria indexed by agents' expectations.[3] In these models, economic agents' self-fulfilling beliefs, also referred to as sunspots or animal spirits, can generate business cycle fluctuations that are difficult to distinguish from the dynamics of neoclassical RBC models driven by technology shocks. To our knowledge, the financial implications of these models have not been investigated.

In this research we attempt to study financial properties of DSGE models, in which economic agents' (investors') beliefs, alone or in combination with technology shocks, generate fluctuations. Because financial markets are theorized to be driven, at least in part, by agents' expectations, one might expect that models with indeterminacy would reflect well the behavior of such markets. Our main objective therefore is to explore whether inclusion of nonfundamental belief shocks and a different (endogenous) shock propagation mechanism, which arises in indeterminate models, enhances asset pricing performance.

As a framework for examining the behavior of financial assets in an indeterminate one-sector RBC economy, we adopt the Benhabib-Farmer-Guo (BFG) model. The one-sector BFG model requires large increasing returns to scale to support sunspot equilibria. To the extent that one objects to the high returns-to-scale calibration, the quantitative experiments that we present below should be viewed more from a methodological perspective as evaluating the value added of indeterminacy and sunspots in accounting for the stylized financial facts.[4] We also realize that it is not possible to represent stylized financial and macroeconomic facts adequately in the context of the BFG model at the same time because the volatility of the pricing kernel in the BFG model depends solely on the volatility of consumption growth, just as in the standard consumption-based asset-pricing models. In the data, consumption growth does not vary much and therefore the standard deviation of the pricing kernel is bound to be low if one wishes to replicate this feature of the data. In the determinate DSGE models, the standard deviation of consumption growth is typically lower than its empirical counterpart. From another perspective, the work of Hansen and Jagannathan (1991) implies that accounting for the equity premium in the U.S. data requires the volatility of the stochastic discount factor to be at least 50% annually. Our goal in this research is therefore not to resolve the financial puzzles but to understand if sunspots and indeterminacy help to alleviate them to some degree.

We compare the asset-pricing results obtained in the indeterminate BFG model with the results from the indivisible-labor business cycle model of Hansen (1985). These two models are canonical in their respective classes, indeterminate and neoclassical. Apart from the nature of shocks, they differ only in the level of returns to scale in production, facilitating a controlled comparison of financial implications of two competing paradigms.

In this comparison, we see the second contribution of our research. The empirical productivity analysis literature [e.g., Caballero and Lyons (1992), Basu and

Fernald (1997), Laitner and Stolyarov (2004)] has not achieved a broad consensus on the degree of aggregate increasing returns in the data, and therefore on the relevance of sunspots to many models with indeterminacy. This problem is well understood in light of Kamihigashi's (1996) observational equivalence argument: if the shock behind economic fluctuations is left unrestricted, the observed time series of consumption, investment, output, capital, and labor input can be generated by an economy with any value of returns to scale. Cole and Ohanian (1999) show that even with a restricted shock process the measurement of increasing returns is imprecise, making it difficult to discriminate between models. The two formulations, however, have different implications for economic policy and for that reason any additional information in favor of choosing one setting over the other is of value. Several studies [for instance, Farmer and Guo (1994) and Thomas (1999)] have shown the two types of models to be comparably successful in replicating essential macroeconomic features of the business cycle. Success along asset-pricing dimensions would certainly present strong support for sunspot formulations.

Unfortunately, our results indicate that the BFG model does not substantially improve upon the performance of the neoclassical RBC model of Hansen (1985) in representing stylized financial facts of the data. The 4.0176% log gross return on equity is significantly below the 7.58% average log gross return on equity in the U.S. data. The risk-free rate is high: 4.0163% versus 1.54% in the data.[5] The reported numbers clearly illustrate the presence of the equity premium puzzle of Mehra and Prescott (1985) and the risk-free rate puzzle of Weil (1989) in the BFG model. In addition, the volatility of the log return on equity in the model economy is only 0.37% in contrast to the 15.47% in the U.S. economy, indicating the volatility puzzle. The corresponding financial stylized facts from Hansen's model are almost identical to those of the BFG model when similarly parameterized.

Our conclusion is that the presence of the sunspot shock and the endogenous shock propagation mechanism does not have any significant impact on financial performance. For example, the equity premium depends on the covariance between the return on equity and consumption growth. The covariance, in turn, is the product of the standard deviations of the return on equity and consumption growth and their correlation coefficient. Indeterminacy and the sunspot do not increase these statistics and therefore do not alleviate the severity of the equity premium and other financial puzzles. Overall, the indeterminate BFG model fails to resolve financial puzzles for the same reason as does Hansen's model: in both settings risk-averse agents can adjust consumption, labor supply, and the rate of capital accumulation in response to shocks, and they smooth consumption "too much," even relative to empirically observed consumption growth. Because in both formulations the stochastic discount factor is equal to the rate of consumption growth, the pricing kernel is insufficiently volatile. The natural conclusion is that improvement along the asset-pricing dimension requires some technology that breaks the link between the pricing kernel and consumption growth. One way to disconnect the two variables is to introduce habit formation into investors' preferences. However, previous studies in asset pricing [Jermann (1998), Avalos (2001), Boldrin, Christiano, and Fisher (2001), and others] have shown that such modification by itself does not help to resolve asset-pricing puzzles in production economies. Habit formation increases agents' local risk aversion. Very risk-averse investors are eager to smooth their consumption, which is easy to accomplish by adjusting labor and capital inputs. Therefore, an additional mechanism, which prevents easy consumption smoothing, must be put in place. This mechanism, however, would also prevent indeterminacy, since for swings in optimism and pessimism to translate into corresponding movements in economic activity there must be enough flexibility in the model economy to allow agents to act on their expectations. For example, Kim (2003) shows that indeterminacy disappears when capital adjustment costs are incorporated into the BFG model.[6]

The rest of this paper proceeds as follows: In Section 2 we describe the model and its equilibrium; in Section 3 we discuss the solution method and its application to asset pricing, in Section 4 we choose parameter values and present our results; in Section 5 we conclude.

2. THE MODEL

The economy is populated by a continuum of identical households indexed by [0,1] and by a representative "stand-in" firm. There exists a legitimate financial market in which equity claims to the representative firm's net income stream and possibly other assets are traded.

2.1. Households

Households maximize their expected lifetime utility defined over consumption and leisure by deciding on the time they wish to work and by choosing their financial asset holdings,

$$\text{Max}_{\{Z_{t+1},N_{\text{h}t}\}} E_0 \left(\sum_{t=0}^{\infty} \beta^t \frac{(C_t)^{1-\xi} - 1}{1-\xi} \right) - \Lambda N_{\text{h}t} \tag{1}$$

subject to

$$C_t + Z'_{t+1} P_t^z \leq Z'_t \left(P_t^z + D_t^z \right) + W_t N_{ht}, \tag{2}$$

where $\beta (0 < \beta < 1)$ is the subjective time discount factor and parameter $\xi (0 < \xi < \infty)$ is the coefficient of the relative risk aversion; C_t and $N_{\text{h}t}$ are per capita consumption and labor services, respectively, each in period t. Each household is endowed with one unit of time and the parameter $\Lambda (\Lambda > 0)$ in the utility function is chosen to match the fraction of that time devoted to work in the data.[7] W_t is period t's wage rate. Z_t is a vector of financial assets held at period t and chosen at t-1. P_t^z and D_t^z are vectors of asset prices and current period payouts (dividends).[8] Vector Z_t includes an equity security, whose price and dividend are denoted by P_t^{e} and D_t^{e}. We normalize number of equity shares to 1. Another asset included in Z_t is the one-period risk-free bond, whose price is P_t^{b}. The bond is in zero net supply.

The period preference ordering of the representative household is assumed to be separable in consumption and leisure and has its origins in Hansen (1985). The representative household's marginal utility of consumption is given by $U_{\mathrm{c}}(C_t, N_{\mathrm{h}t}) = (C_t)^{-\xi}$ and its intertemporal marginal rate of substitution in consumption, also known as the stochastic discount factor or pricing kernel, by

$$M_{t+1} = \beta \frac{U_{\mathrm{c}}(C_{t+1}, N_{\mathrm{h}t+1})}{U_{\mathrm{c}}(C_t, N_{\mathrm{h}t})} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\xi}. \tag{3}$$

The first-order conditions for optimization program (1)–(2) with respect to financial asset holdings produce asset pricing equations. For instance, the equation for the price of the equity security, P_t^{e}, which is a claim to the infinite sequence of dividends, paid by the firm $\{D_{t+j}^{\mathrm{e}}\}_{j=1}^{\infty}$, is given by

$$P_t^{\mathrm{e}} = E_t \left[M_{t+1} \left(P_{t+1}^{\mathrm{e}} + D_{t+1}^{\mathrm{e}} \right) \right]. \tag{4}$$

Equation (4) means that in his intertemporal choice problem, a typical investor equates the loss in utility associated with buying an additional unit of the financial asset (equity) at time t—$P_t^{\mathrm{e}} U_{\mathrm{c}}(C_t, N_{\mathrm{h}t})$—to the discounted expected utility of the resulting additional consumption in the next period ($\beta E_t[(P_{t+1}^{\mathrm{e}} + D_{t+1}^{\mathrm{e}}) U_{\mathrm{c}}(C_{t+1}, N_{\mathrm{h}t+1})]$). Substituting forward for $P_{t+j}^{\mathrm{e}} (j = 1, \ldots, \infty)$ and using the law of iterated expectations, we obtain a unique nonexplosive solution for (4):

$$P_t^{\mathrm{e}} = E_t \left[\sum_{j=1}^{\infty} M_{t+j} D_{t+j}^{\mathrm{e}} \right]. \tag{5}$$

The price of a one-period risk-free bond—an asset that pays one unit of consumption in every state next period—is just the expectation of the stochastic discount factor:

$$P_t^{\mathrm{b}} = E_t[M_{t+1}]. \tag{6}$$

The first-order condition for the household's labor decision equates the utility of extra consumption obtained by working longer to the disutility of the additional work effort:

$$(C_t)^{-\xi} W_t = \Lambda. \tag{7}$$

The conditional expectations in (4) and (6) are taken over two exogenous sources of fluctuations: technology shock and nonfundamental belief or sunspot shock. The latter is an extra return on the equity security (in terms of a utility increment), which the household believes to materialize over the period. Under certain parameterizations of the model, beliefs become self-fulfilling.

2.2. Firms

A representative firm begins period t with the stock of capital, K_t, carried over from the previous period. The evolution of the capital stock is given by

$$\begin{aligned} &K_{t+1} = (1-\Omega)K_t + I_t \\ &K_0 \text{ given,} \end{aligned} \tag{8}$$

where I_t is period t investment and $\Omega (0 < \Omega < 1)$ is the depreciation rate. The firm produces output via a standard Cobb-Douglas function,

$$Y_t = A_t X_t (K_t)^{\alpha} (N_{\mathrm{f}t})^{-\alpha}, \tag{9}$$

with two inputs—capital, K_t, and labor, $N_{\mathrm{f}t}$—and the current level of technology A_t, the log of which is assumed to follow an AR(1) process with the persistence coefficient $\rho \in (0, 1)$:

$$\begin{aligned} &a_t = \ln A_t = (1-\rho)\ln \bar{A} + \rho \ln A_{t-1} + \varepsilon_t \\ &A_0 \text{ given.} \end{aligned} \tag{10}$$

There is an external effect, X_t, which depends on the economywide quantities of capital and labor, denoted by variables with bars:

$$X_t = \bar{K}_t^{\alpha\eta} \bar{N}_t^{(1-\alpha)\eta}. \tag{11}$$

The parameter $\eta (\eta \geq 0)$ captures the size of the aggregate production externality.

The firm takes the external effect as given and views its production function as having constant returns to scale. As a result, the firm behaves competitively. However, there are increasing returns to scale in production at the aggregate level because of the externality. If $\eta = 0$, private and social returns to scale are both constant.

After period t output is produced, the firm sells it and uses the proceeds of the sale to pay the wage bill, $W_t N_{\mathrm{f}t}$, and to finance investments, I_t, under the knowledge of the equation of motion of the capital stock (8). The remaining output is distributed as dividends to the shareholders (households):

$$D_t = Y_t - W_t N_{\mathrm{f}t} - I_t. \tag{12}$$

In this complete market setting the representative firm's objective is to maximize its predividend stock market value, period by period, by choosing its investment and labor input. The competitive firm realizes that shareholders' intertemporal marginal rates of substitution are crucial for asset pricing and uses investors' valuation for the price of equity[9] provided by equation (5). Expression (5) simply equates the share price to the expected present discounted value of the infinite

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dividend stream paid by the firm. The representative firm's dynamic optimization program is

$$\mathrm{Max}_{\{I_t, N_{ft}\}}\left(D_t^{e} + P_t^{e}\right) \tag{13}$$

subject to (5), (8), (9), and (12).

The program (13) is a decentralized version of the stochastic growth model proposed by Danthine and Donaldson (2002a). This interpretation requires shareholders to convey to the firm a complete listing of their future intertemporal marginal rates of substitution [equation (5)]. Danthine and Donaldson (2002a) show that in the complete market setting with homogenous agents there is perfect unanimity about the provided information. Alternatively, shareholders appoint one of their cohort to manage the firm, realizing that his or her preferences over future consumption are identical to their own.

The first-order conditions for the firm's problem (13) with respect to its labor hiring and investment decisions are

$$(1-\alpha)\frac{Y_t}{N_{ft}} = W_t \tag{14}$$

and

$$-1 + E_t\left[M_{t+1}\left\{\alpha\frac{Y_{t+1}}{K_{t+1}} + 1 - \Omega\right\}\right] = 0. \tag{15}$$

Because the private technology of a representative firm is convex, an interior solution to the model exists and the equilibrium is well defined.

2.3. Equilibrium

An equilibrium in this economy is a vector of price sequences, $\{W_t\}_{t=0}^{\infty}$, $\{P_t^z\}_{t=0}^{\infty}$, and the set of policy functions $\{Y_t\}_{t=0}^{\infty}$, $\{C_t\}_{t=0}^{\infty}$, $\{N_t\}_{t=0}^{\infty}$, $\{K_t\}_{t=0}^{\infty}$, $\{I_t\}_{t=0}^{\infty}$, and $\{D_t\}_{t=0}^{\infty}$ such that

(1) The first-order conditions of the representative household, (4), (6), and (7), and of the representative firm, (14) and (15), are satisfied together with the usual transversality condition $\lim_{t\to\infty}\beta^t U_c(C_t, N_t)K_{t+1} = 0$.

(2) The labor, goods, and capital markets clear: $N_{ht} = N_{ft} = \bar{N}_t$, $Y_t = C_t + I_t$, and $K_t = \bar{K}_t$. Equilibrium in the financial market requires that investors hold all outstanding equity shares and all other assets are in zero net supply: $Z_t^{e} = 1$ and $Z_t^{b} = 0$.

If the externality exists (η is positive) the decentralized equilibrium is not Pareto optimal because the representative firm fails to take the external effect into account when choosing optimal labor and investment.

3. MODEL SOLUTION AND ASSET PRICING

First, we solve the model for the approximate dynamics of the macroeconomic variables by log-linearizing the equations around the unique steady state implied by the above equilibrium conditions as in King et al. (1988). The solution of the approximated model can be represented by a log-linear state space system with the vector of state variables s_t following a first-order autoregressive process with multivariate normal i.i.d. impulses:

$$\begin{aligned} s_{t+1} &= Ps_t + Q\omega_t \\ k_0 &\text{ given.} \end{aligned} \tag{16}$$

The lower case letters denote the log deviations of variables from their steady-state values. For the economies considered in this research, s_t contains capital stock k_t, level of technology a_t, and possibly an additional endogenous state variable.[10] The square matrix P governs the dynamics of the system. When the equilibrium is unique the steady state is a saddle point, and the third variable in s_t has to be chosen so that the system is always on the stable branch. Alternatively, the transversality condition is violated. If two eigenvalues of the matrix P, corresponding to capital stock and the third variable in s_t, are both within the unit circle, the equilibrium is indeterminate. In this case the transversality condition is satisfied for any value of the third variable in s_t, which then becomes an additional endogenous state variable.

The vector of exogenous shocks ω_t consists of two variables: the technology shock, ε_t, and the sunspot, ν_t. Consistency with rational expectations requires that the sunspot be i.i.d. with $E_t[\nu_t] = 0$.

For asset pricing, we obtain the log of dividends d and the log of the stochastic discount factor m as linear combinations of the state vector.

3.1. Rates of Return

The next step in our solution is to apply the lognormal pricing method developed by Jermann (1998), which combines the linearization approach detailed above with nonlinear asset-pricing formulae. The main advantage of this technique over nonlinear value-function iteration, used for example in Danthine et al. (1992) and Rouwenhorst (1995), is its ability to handle a model with multiple endogenous state variables with ease. This would be a hurdle in purely nonlinear discrete state space methods. On the other hand, our return computations do not impose equal ex ante returns across securities, as is generally true under pure linearization of the utility, which results in risk neutrality.

The basic pricing equation requires that the time t price of a claim to a single uncertain future payout (dividend), D_{t+j}, is equal to its expected present value discounted using the stochastic discount factor $M_{t+j} = \beta^j \left(\frac{C_{t+j}}{C_t}\right)^{-\xi}$:

$$P_t(D_{t+j}) = E_t[M_{t+j} D_{t+j}] = \beta^j E_t[\exp\{\xi(c_t - c_{t+j}) + d_{t+j}\}]. \tag{17}$$

Since the log deviations of the stochastic discount factor and dividends with respect to the steady state values are conditionally normal, M and D are conditionally

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lognormal. Using the well-known theorem about the expectation of lognormal variables, the Euler equation (17) can be written as

$$P_t(D_{t+j}) = \beta^j \exp\left\{E_t[\xi(c_t - c_{t+j}) + d_{t+j}] + \frac{1}{2}\mathrm{Var}_t[\xi(c_t - c_{t+j}) + d_{t+j}]\right\}. \tag{18}$$

It is possible to obtain closed-form solutions for first and second moments of returns on assets with a single-period payout as in equation (17). For example, the return on a one-period bond, which pays one unit of consumption good in every state, i.e., per-period risk-free rate, is given by

$$R^{\mathrm{b}}_{t,t+1}(1_{t+1}) = \beta^{-1}\exp\left\{-E_t[\xi(c_t - c_{t+1})] - \frac{1}{2}\mathrm{Var}_t[\xi(c_t - c_{t+1})]\right\}. \tag{19}$$

The unconditional mean risk-free rate and its variance can be shown to be equal to

$$E\left[R^{\mathrm{b}}_{t,t+1}\right] = \beta^{-1}\exp\left\{\frac{1}{2}\mathrm{Var}[E_t[\xi(c_t - c_{t+1})]] - \frac{1}{2}\mathrm{Var}[\xi(c_t - c_{t+1}) - E_t[\xi(c_t - c_{t+1})]]\right\}, \tag{20}$$

$$\mathrm{Var}\left[R^{\mathrm{b}}_{t,t+1}\right] = \beta^{-2}\exp\{\mathrm{Var}[E_t[\xi(c_t - c_{t+1})]] - \mathrm{Var}[\xi(c_t - c_{t+1}) - E_t[\xi(c_t - c_{t+1})]]\}(\exp\{\mathrm{Var}[E_t\xi(c_t - c_{t+1})]\} - 1). \tag{21}$$

An equity security is the claim to the infinite sequence of dividends $\{D_{t+j}\}_{j=1}^{\infty}$ and can be regarded as an infinite composite of single-strip securities, which are priced according to equation (18). The period gross return to the firm's equity is given by

$$R^{\mathrm{e}}_{t,t+1}\left(\{D_{t+j}\}_{j=1}^{\infty}\right) = \frac{P^{\mathrm{e}}_{t+1}\left(\{D_{t+j}\}_{j=1}^{\infty}\right) + D_{t+1}}{P^{\mathrm{e}}_t\left(\{D_{t+j}\}_{j=1}^{\infty}\right)} = \frac{\sum_{j=1}^{\infty}\beta^j E_{t+1}[\exp\{\xi(c_{t+1} - c_{t+1+j}) + d_{t+1+j}\}] + \exp d_{t+1}}{\sum_{j=1}^{\infty}\beta^j E_t[\exp\{\xi(c_t - c_{t+j}) + d_{t+j}\}]} \tag{22}$$

We can compute stock returns from the model's linear solution, but to calculate the unconditional expectation and variance of the return on equity, it is necessary to use simulations.[11]

3.2. Financial Puzzles in DSGE Models

The average excess stock return $E[R^{\mathrm{e}} - R^{\mathrm{b}}]$ in the U.S. data is almost 8%. The average excess stock return justified in the context of the standard DSGE model as

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a reward for bearing risk is close to zero. Herein lies the equity premium puzzle. To understand the determinants of the equity premium using the standard DSGE framework, we rewrite the asset-pricing Euler equations (5) and (6) in terms of returns[12]

$$1 = E_t\left[\left(1 + R^{\mathrm{e}}_{t,t+1}\right) M_{t+1}\right], \tag{23}$$

$$1 + R^{\mathrm{b}}_{t,t+1} = \frac{1}{E_t[M_{t+1}]}. \tag{24}$$

Denote the log gross returns on stocks and the risk-free asset by $r^{\mathrm{e}}_{t,t+1} = \ln(1 + R^{\mathrm{e}}_{t,t+1})$ and $r^{\mathrm{b}}_{t,t+1} = \ln(1 + R^{\mathrm{b}}_{t,t+1})$ and the log of the stochastic discount factor by $m_t = \ln(M_{t+1}) = \ln\beta - \xi(c_{t+1} - c_t) = \ln\beta - \xi\,\Delta c_{t+1}$. Because the marginal rates of substitution and asset returns are jointly lognormally distributed and homoscedastic, we can rewrite the equations (23) and (24) in terms of logs,[13]

$$r^{\mathrm{b}}_{t,t+1} = -E_t[m_{t+1}] - \frac{\mathrm{Var}(m)}{2}, \tag{25}$$

$$E_t\left[r^{\mathrm{e}}_{t,t+1}\right] = -E_t[m_{t+1}] - \frac{1}{2}(\mathrm{Var}(r^{\mathrm{e}}) + \mathrm{Var}(m) + \mathrm{Cov}(r^{\mathrm{e}}, m)), \tag{26}$$

$$\begin{aligned} E_t\left[r^{\mathrm{e}}_{t,t+1} - r^{\mathrm{b}}_{t,t+1}\right] + \frac{\mathrm{Var}(r^{\mathrm{e}})}{2} &= -\mathrm{Cov}(r^{\mathrm{e}}, m) = \xi\mathrm{Cov}(r^{\mathrm{e}}, \Delta c) \\ &= \xi\rho(r^{\mathrm{e}}, \Delta c)\sigma_{r^{\mathrm{e}}}\sigma_{\Delta c}, \end{aligned} \tag{27}$$

where Δc denotes log consumption growth. Equation (27) states that the log of the expected risk premium, adjusted for Jensen's inequality, is equal to the covariance between the log return on equity and log consumption growth. Intuitively, the asset whose return positively covaries with consumption growth pays in "good" times when the consumption level is high and the marginal utility of additional consumption is low. Such assets require high returns to induce investors to hold them. Alternatively, the assets that pay in the "bad" states are very desirable and command low returns because they allow risk-averse agents to smooth their consumption patterns. In the data, the standard deviation of the log consumption growth is 1.08% and the standard deviation of the log return on equity is around 16%. Even if one is willing to assume the perfect positive correlation between the two variables, a relative risk aversion coefficient (ξ) of about 50 is required to resolve the puzzle in the context of the DSGE models, in which the pricing kernel is equal to the consumption growth. In the literature, $\xi = 10$ is considered to be the maximum plausible value for this parameter.

The equity premium results can be also examined in the framework provided by the work of Hansen and Jagannathan (1991). They show that the unconditional version of the first-order condition for excess return on equity, $E[M_{t+1}(R^{\mathrm{e}}_{t,t+1} - R^{\mathrm{b}}_{t,t+1})] = 0$, implies the following restriction for the Sharpe

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ratio of any asset's excess return,

$$\frac{E\left[R^{\mathrm{e}}_{t,t+1} - R^{\mathrm{b}}_{t,t+1}\right]}{\sigma\left[R^{\mathrm{e}}_{t,t+1}\right]} = -\rho\left(M_{t+1}, R^{\mathrm{e}}_{t,t+1} - R^{\mathrm{b}}_{t,t+1}\right)\frac{\sigma[M_{t+1}]}{E[M_{t+1}]} \leq \frac{\sigma[M_{t+1}]}{E[M_{t+1}]}, \quad \mathbf{(28)}$$

where ρ denotes the unconditional correlation between two variables, which cannot be higher than 1 in absolute value. The inequality in (28) is the Hansen–Jagannathan lower bound (HJB) on the pricing kernel. In equation (28), $E[M_{t+1}]$ is the expected value of the price of a one-period risk-free discount bond and should be close to 1, which means that $\sigma[M_{t+1}]$ should be around 0.5 in annual terms.

On the other hand, for the investor whose preferences are given by the standard power utility, high relative risk aversion implies low elasticity of intertemporal substitution (reciprocal of the relative risk aversion coefficient). That is, a very risk-averse person is also extremely unwilling to substitute consumption across time. Such investor would hold risk-free bonds only if they offered high return, which is the essence of the risk-free rate puzzle of Weil (1989). More specifically, let us take the unconditional expectations of (25):

$$E[r^{\mathrm{b}}] = -\ln\beta + \xi E[\Delta c] - \frac{\xi^2\sigma^2_{\Delta c}}{2}. \quad \mathbf{(29)}$$

From equation (29), it is clear that higher relative risk aversion parameter ξ has two opposite effects on the average risk-free rate: it increases the risk-free rate through the intertemporal consumption smoothing effect ($\xi E[\Delta c]$) and reduces the risk-free rate through the precautionary savings effect ($\xi^2\sigma^2_{\Delta c}/2$). Intuitively, given the positive average rate of consumption growth, agents would like to borrow from the future to reduce the growth in their consumption, but also save to protect themselves against future uncertainty. For the precautionary savings effect to dominate the intertemporal consumption smoothing effect the coefficient of the relative risk aversion needs to be implausibly high. But even with high ξ the low risk-free rate and the reasonable rate of time preferences (β) can be simultaneously obtained only for a very narrow range of relative risk aversion coefficient values. It follows that the increase in the relative risk aversion parameter would not satisfactory resolve financial puzzles.

The equity premium puzzle is related to the volatility puzzle because the standard deviation of stock returns also depends on the volatility of consumption growth in addition to the volatility of dividend growth. Campbell (1999) shows that the standard deviation of dividends needs to be counterfactually high to achieve the 16% standard deviation of stock returns found in the U.S. data.

In the models investigated in this research, the stochastic discount factor is equal to the consumption growth rate. Given the arguments outlined above, one would not expect to solve asset-pricing puzzles without altering the pricing kernel, and that is not our objective here. Rather, our focus is on understanding the impact that local indeterminacy and sunspot have on asset pricing. We are interested in knowing whether these two features help to alleviate financial puzzles and to what degree.

4. QUANTITATIVE RESULTS

The main purpose of the quantitative evaluation presented in this section is to examine the potential of the indeterminate BFG model to explain the historic equity premium, the average risk-free rate, and the volatility of financial asset returns found in the U.S. data, while maintaining its ability to replicate the stylized business cycle facts. We start the discussion of asset-pricing implications of one-sector DSGE models with a review of the financial implications of the indivisible-labor model of Hansen (1985). As noted earlier, the BFG model and Hansen's model differ in the degree of aggregate returns in production and in the nature of the driving processes. We would like to disentangle the impact of increasing returns on asset pricing from the impact of the sunspot shock. To achieve this goal, we examine three types of model economies. First we consider economies with different values of the increasing returns parameter, η, with fluctuations driven solely by technology shocks. Then we turn our attention to BFG economies—those with increasing returns to scale high enough to be driven by belief shocks—and consider these shocks alone. Finally, we explore one version of the BFG economy with both sources of uncertainty: the nonfundamental belief shock and the productivity (technology) shock.

4.1. Calibration

Each of these model economies shares commonly calibrated parameter values. All are in line with empirical estimates and the values commonly used in the literature. [See for instance Hansen (1985), Mehra and Prescott (1985), Juster and Stafford (1991), Poterba (1998), Jermann (1998), Boldrin, et al. (2001), and King and Rebelo (1999).] Capital's share of output, α, is 0.3 and the quarterly depreciation rate, Ω, is 0.025. The subjective discount factor, β, is 0.99, corresponding to a steady-state risk-free rate of return of 4% per year. We choose Λ to yield steady-state work time of the representative household equal to 1/3 of its time endowment. Estimations of the Solow residual typically yield a highly persistent AR(1) process [see Prescott (1986) for details]. For the AR(1) process describing A_t, we choose the value of the persistence parameter, ρ, equal to 0.95.

Our benchmark value of ξ, the relative risk aversion (RRA) coefficient, is equal to 1. Because the RRA coefficient is a critical parameter for asset pricing, we present cases in which the RRA coefficient is equal to 5 in our second group of simulations.

The degree of increasing returns to scale (IRS) in the model economy is given by $1 + \eta$. Despite numerous attempts to estimate the level of returns to scale in the data, there is no broad agreement in the literature on its value. Basu and Fernald (1997) demonstrate that although the average U.S. industry exhibits approximately constant returns to scale, the aggregate private business economy appears to exhibit large increasing returns. The largest aggregate estimate they obtain is 1.72 (standard error equal to 0.36). They suggest that aggregate estimates are appropriate for calibration of one-sector models. This argument is very helpful for the proposed research because the one-sector BFG model requires a minimum externality of about 0.55 to support sunspot equilibria. Our benchmark value is $\eta = 0.6$, but we present results using higher and lower values in our simulations in the first and second groups of results.

We have freedom in setting the variances of our shocks. In each case we set them to match the standard deviation of output in the U.S. data (1.82%). When we use both shocks, they are uncorrelated.[14] Because there is no obvious way to estimate the variances of the shocks individually, we choose these parameters to maximize the log equity premium in the BFG model with two shocks.[15]

4.2. Effect of Increasing Returns on Asset Prices

Our objective in this section is to investigate the effect of the change in the level of returns to scale in production on asset pricing implications of the indivisible labor model. In Table 1, we collect statistics from the model economies in our first group. For easy comparison, the first column of Table 1 replicates the point estimates of moments of the U.S. data (standard errors are in parentheses).

TABLE 1. Effect of increasing returns, models with technology shocks only

		Models with technology shocks only		
	U.S. data	$\eta = 0$ (Hansen)	$\eta = 0.15$	$\eta = 0.6$
	A. Select business cycle moments			
σ_y	1.82	1.82	1.82	1.82
σ_c	0.87	0.57	0.56	0.66
$\sigma_{\Delta c}$	1.08	0.76	0.75	0.9
$\sigma_{\Delta d}$	28	16.87	17.09	7.78
	B. Financial moments			
$E[r^{\mathrm{e}}]$	7.58 (2.33)	4.0188	4.0186	4.0141
$\sigma_{r^{\mathrm{e}}}$	15.47 (0.13)	0.25	0.25	0.37
$E[r^{\mathrm{b}}]$	1.54 (0.19)	4.0178	4.0175	4.0135
$\sigma_{r^{\mathrm{b}}}$	2.5 (0.002)	0.2	0.2	0.29
$E[r^{\mathrm{e}} - r^{\mathrm{b}}]$	6.04 (2.27)	0.001	0.0011	0.0006
$\sigma_M/E[M]$	$\geq$0.53	0.0038	0.0038	0.0045
Corr(r^{e}, Δc)	0.2	0.69	0.65	0.386
Cov(r^{e}, Δc)	3.41	0.13	0.12	0.128

Notation: variables y, c, and d denote log-deviations of the Hodrick-Prescott filtered series of output, consumption and dividends respectively. σ_y and σ_c are quarterly standard deviations of output and consumption. Δc and Δd are the logs of consumption growth and dividend growth and $\sigma_{\Delta c}$ and $\sigma_{\Delta d}$ are their annualized quarterly standard deviations. $r^{\mathrm{e}} = \ln(1 + R^{\mathrm{e}})$ and $r^{\mathrm{b}} = \ln(1 + R^{\mathrm{b}})$ denote the log gross return on equity and on the one-quarter risk-free bond. All financial statistics are reported in annualized percentage points.

Column two of Table 1 shows statistics obtained from simulations of the Hansen indivisible labor model with constant returns to scale in production ($\eta = 0$). In columns three and four we increase the level of returns to scale first to 1.15 and then to 1.6. In each case, we adjust the standard deviation of the technology shock (σ_ε) to match the standard deviation of output in the U.S. data. In each case the coefficient of the relative risk aversion (ξ) is equal to one. The information in this and subsequent tables is provided in two panels: Panel A presents selected macroeconomic statistics, which determine prices of financial assets in our framework, and panel B contains financial results. In panel B we choose to report statistics in the form consistent with equation (27).

Comparing the statistics in panel A, we note that in the indivisible labor economy the standard deviation of log consumption growth is 0.76% and the standard deviation of log dividend growth is 16.87%, whereas both of these statistics are higher for the U.S. data—1.08% and 28% respectively.

The log gross return on equity in the Hansen model is 4.0188%—fairly low in comparison with the 7.58% average log gross return on U.S. equities. The log gross risk-free rate in the indivisible labor model is 4.0178%, almost the same as the return on the risky equity security, whereas the risk-free rate in the U.S. data is 1.54%. The log equity premium in the model is close to zero, in stark contrast to the 6% log risk premium observed in the U.S. data. The model's financial asset return volatilities, especially the volatility of the return on equity, are an order of magnitude lower than their empirical counterparts: 0.25% versus 15.47% for the stock and 0.2% versus 2.5% for the risk-free bond. Presented numbers clearly demonstrate the equity premium, risk-free rate, and volatility puzzles in the Hansen model.

In column three, we report results from the model economy with mildly increasing returns (parameter $\eta = 0.15$). Comparison of results in columns two and three indicates that the increase in the level of returns to scale from 0 (Hansen's model) to 0.15 has a negligible effect on the results. The standard deviation of the log consumption growth remains at around 0.76%. The standard deviation of the log dividend growth increases slightly to 17.09%. The covariances between the log consumption growth and the return on equity in columns two and three are 0.13% and 0.12%, respectively, versus the empirical covariance of 3.41% (column one). The HJB in both model economies is 0.0038, whereas the lower bound on the volatility of the pricing kernel inferred from the U.S. data is greater than 0.5. The returns on financial assets and their volatilities in the mildly increasing-returns economy are almost identical to those in the Hansen model. We note that with the level of IRS equal to 0.15 indeterminacy cannot arise in this one-sector production model.

In column four the level of IRS is 1.6 ($\eta = 0.6$) and the model economy is capable of supporting sunspot equilibria. The volatility of log consumption growth increases to 0.9% but this improvement is offset by the decrease in the volatility of the dividend growth to 7.78%. As a result, the return on equity in this economy actually drops slightly to 4.0141%. The risk-free rate is a little lower and is equal to 4.0135%. This is due to the small increase in the stochastic discount factor volatility. The covariance between the log consumption growth and return on equity is 0.128% and the equity premium is 0.0006%. The volatilities of financial asset returns in the model economy with $\eta = 0.6$ are 0.37% for equity and 0.29% for the riskless bond, a slight improvement on the results from the constant-returns-to-scale economy, but still an order of magnitude lower than the empirical return volatilities.

Our conclusion is that the increase in the level of returns to scale in production has a negligible effect on the financial implications of DSGE models.

4.3. Effect of the Sunspot Shock on Asset Prices

Our next step is to explore asset pricing in the increasing-returns-to-scale economies, where η is sufficiently high to allow for indeterminacy. This is the BFG framework. BFG economies can generate business cycle fluctuations when driven by belief (sunspot) shocks alone. Our goal now is to investigate the impact of the nonfundamental belief shock on the financial performance of DSGE models. Table 2 displays results from several parameterizations of the BFG model with only one extrinsic shock: the sunspot. Similarly to Table 1, we replicate the corresponding statistics for the U.S. economy in column one and for the Hansen model in column two. In column three we present statistics from the economy with the benchmark parameter values, i.e., the RRA coefficient $\xi = 1$ and $\eta = 0.6$. In

TABLE 2. Quantitative results for the BFG model with sunspot shocks only

	U.S. data	Hansen model $\xi = 1, \eta = 0$	BFG with sunspot shocks only $\xi = 1, \eta = 0.6$	$\xi = 1, \eta = 0.72$	$\xi = 5, \eta = 0.6$
		A. Select business cycle moments			
σ_y	1.82	1.82	1.82	1.82	1.82
σ_c	0.87	0.57	0.31	0.42	0.06
$\sigma_{\Delta c}$	1.08	0.76	0.34	0.5	0.07
$\sigma_{\Delta d}$	28	16.87	22.86	18.96	24.14
		B. Financial moments			
$E[r^e]$	7.58	4.0188	4.0196	4.0204	4.0192
σ_{r^e}	15.47	0.25	0.21	0.225	0.18
$E[r^b]$	1.54	4.0178	4.0195	4.0201	4.0192
σ_{r^b}	2.5	0.2	0.13	0.173	0.13
$E[r^e - r^b]$	6.04	0.001	0.0001	0.0003	0
$\sigma_M/E[M]$	$\geq$0.53	0.0038	0.0017	0.0025	0.0016
Corr(r^e, Δc)	0.2	0.69	0.45	0.49	0.28
Cov(r^e, Δc)	3.41	0.13	0.03	0.06	0.03

Notation: variables y, c, and d denote log-deviations of the Hodrick-Prescott filtered series of output, consumption and dividends, respectively. σ_y and σ_c are quarterly standard deviations of output and consumption. Δc and Δd are the logs of consumption growth and dividend growth and $\sigma_{\Delta c}$ and $\sigma_{\Delta d}$ are their annualized quarterly standard deviations. $r^e = \ln(1 + R^e)$ and $r^b = \ln(1 + R^b)$ denote the log gross return on equity and on the one-quarter risk-free bond. All financial statistics are reported in annualized percentage points.

column four we increase the degree of increasing returns to 0.72 [the value used in Farmer and Guo (1994)] while leaving the risk aversion parameter unchanged. And finally, in column five, we make investors more risk-averse ($\xi = 5$), but leave $\eta = 0.6$.[16]

In panel A, we note that in all considered "sunspot-only" cases, the standard deviation of the log consumption growth is lower than in the Hansen model and in the U.S. data. In column four, the change in the level of returns to scale (η) from 0.6 to 0.72 increases the volatility of consumption growth from 0.34% to 0.51%. Among the sunspot-driven economies, the economy with high relative risk aversion in column five has the least volatile consumption growth, with standard deviation of only 0.07%.[17] This is not surprising because more risk averse agents strive to achieve smoother consumption patterns. In all sunspot-only parameterizations HJBs and covariances between the log consumption growth and the return on equity are an order of magnitude lower than the corresponding values in the U.S. economy and are slightly lower than values obtained in the indivisible labor model. It comes as no surprise that the financial statistics from sunspot-only economies are marginally worse than those produced by the Hansen model. The highest equity premium we are able to obtain is 0.0003% in the economy with level of IRS equal to 0.72 and RRA coefficient equal to 1 (column four in Table 2). The economy with high relative risk aversion (column five Table 2) has a zero excess return on stocks because the negative effect of the reduction in the standard deviation of the stochastic discount factor and the volatility of the equity return on premium outweighs the positive effect of the increase in RRA coefficient [see equation (27)]. Therefore increasing the RRA coefficient within empirically plausible values does not help to solve the equity premium puzzle in the BFG model. Of all considered sunspot-only cases the economy with $\eta = 0.72$ (column four) has the highest volatilities of asset returns (0.225% for equity and 0.173% for the risk-free bond), but these numbers are slightly lower than in the Hansen model.

We summarize the results of the exercise presented in this section as follows: The equity risk premium in the BFG model, driven by sunspot shocks only, is close to zero and the standard deviations of the returns on financial assets are an order of magnitude lower than their empirical estimates obtained from the U.S. data. The BFG model describes the stylized financial facts of the U.S. economy marginally worse than the Hansen model. We suspect that without the persistent technology shock, successive i.i.d. sunspot shocks cancel each other out, resulting in smaller standard deviations of the stochastic discount factor and returns on equity and lower covariance between them than the corresponding statistics in the Hansen model.

4.4. Financial Implications of the BFG Model with Simultaneous Sunspot and Technology Shocks

We now turn to Table 3, which presents results from the BFG model with two exogenous shocks: sunspot and technology (column three). Note that we use the

TABLE 3. Quantitative results for the BFG model with two schocks

	U.S. data	Hansen model	BFG model with two schocks
	A. Select business cycle moments		
σ_y	1.82	1.82	1.82
σ_c	0.87	0.57	0.447
$\sigma_{\Delta c}$	1.08	0.76	0.92
$\sigma_{\Delta d}$	28	16.87	8
	B. Financial moments		
$E[r^{\mathrm{e}}]$	7.58	4.0188	4.0176
$\sigma_{r^{\mathrm{e}}}$	15.47	0.25	0.37
$E[r^{\mathrm{b}}]$	1.54	4.0178	4.0163
$\sigma_{r^{\mathrm{b}}}$	2.5	0.2	0.3
$E[r^{\mathrm{e}} - r^{b}]$	6.04	0.001	0.0013
$\sigma_M/E[M]$	$\geq$0.53	0.0038	0.0046
$\mathrm{Corr}(r^{\mathrm{e}}, \Delta c)$	0.2	0.69	0.58
$\mathrm{Cov}(r^{\mathrm{e}}, \Delta c)$	3.41	0.13	0.2

Notation: variables y, c, and d denote log-deviations of the Hodrick-Prescott filtered series of output, consumption, and dividends, respectively. σ_y and σ_c are quarterly standard deviations of output and consumption. Δc and Δd are the logs of consumption growth and dividend growth and $\sigma_{\Delta c}$ and $\sigma_{\Delta d}$ are their annualized quarterly standard deviations. $r^{\mathrm{e}} = \ln(1 + R^{\mathrm{e}})$ and $r^{\mathrm{b}} = \ln(1 + R^{\mathrm{b}})$ denote the log gross return on equity and on the one-quarter risk-free bond. All financial statistics are reported in the annualized percentage points.

benchmark values of both $\xi = 1$ and $\eta = 0.6$. To facilitate comparison, the first column of Table 3 displays the point estimates of moments of the U.S. data and the second column presents the corresponding statistics obtained from Hansen's indivisible labor model.

In panel A, we note that with the addition of the technology shock, the standard deviation of the log consumption growth in the BFG model increases to 0.92%, which is quite close to 1.08% observed empirically and higher than 0.76% in the Hansen model. But the log dividend growth is not sufficiently volatile: its standard deviation is only 8%. These statistics are almost identical to the corresponding quantities in the similarly parameterized economy driven by technology shocks only (Table 1, column four), which confirms our results from the previous section, namely that the addition of the sunspot shock does not improve the financial performance of DSGE models.

In panel B, the log gross return on equity in the BFG model with two shocks is 4.0176% and the log gross risk-free rate is 4.0163%. The log equity premium is 0.0013%. It is the highest value of all cases considered thus far, including the Hansen model (0.001%), but still a negligible amount when compared to the data. The standard deviation of the return on equity is 0.37%, which presents a marginal improvement over the 0.25% standard deviation of the stock returns in the Hansen model.

The covariance between the log consumption growth and log return on equity is 0.2% compared to 0.13% covariance in the indivisible labor economy. The covariance value is slightly higher than in all previously considered cases due to the higher standard deviations of the log consumption growth, return on stocks, and slightly higher correlation coefficient between them.

In summary, the BFG model with two exogenous shocks presents a marginal improvement over the Hansen model (and all previously considered cases) in replicating stylized financial facts, but this improvement is clearly insufficient to judge it a success.

The reason for the poor financial performance of the BFG model[18] is evident from equation (27), which states that the log risk premium is determined by the product of the RRA coefficient (ξ) and the covariance of the log consumption growth and log return on equity [$\mathrm{Cov}(r^{\mathrm{e}}, \Delta c) = \mathrm{Corr}(r^{\mathrm{e}}, \Delta c)\sigma_{\Delta \mathrm{c}}\sigma_{r^{\mathrm{e}}}$]. In the data, covariance between two variables is 3.41% and even with this value, very high RRA coefficients are necessary to match the equity premium. In the BFG model with two shocks, the covariance between the logs of consumption growth and stock returns is only 0.2%, which is more than 15 times lower than that in the data. The BFG model comes close to matching the empirical volatility of consumption growth. However, the volatility of the equity return in the BFG model, although higher than that in the determinate model, is 42 times lower than in the data, resulting in low $\mathrm{Cov}(r^{\mathrm{e}}, \Delta c)$.

The inspection of the market price of risk—the ratio $\sigma_M/E[M]$—confirms the severity of the equity premium puzzle. The market price of risk in the data, implied by the HJB is greater than 0.53, meaning that the standard deviation of the stochastic discount factor (σ_M) should be at least 50% annually. The HJB in the BFG economy with two shocks is only 0.0046, the same order of magnitude as the HJB in the Hansen model (0.0038). In both models, low market price of risk ratios follow from the smoothness of the pricing kernel, which is equal to the consumption growth.

Our results clearly show that the one-sector DSGE model, modified to include increasing returns to scale in production sufficient for indeterminacy and the sunspot shock, does not explain the stylized financial facts of the U.S. data any better than the standard RBC model of Hansen (1985).

5. CONCLUSION

We have investigated the pricing of financial assets in the context of the one-sector indeterminate Benhabib-Farmer-Guo model with increasing returns to scale in production and sunspot shocks and compared the asset-pricing results from the models with indeterminacy with results obtained in Hansen's (1985) model, a standard in the RBC literature. The two formulations differ in the degree of increasing returns, the nature of shocks, and the shock propagation mechanism. The main purpose of this research has been to determine whether these differences affect the ability of consumption-based DSGE models to replicate key stylized financial facts observed in the U.S. data.

Our principal conclusion is that indeterminacy and the sunspot shock have almost no effect on the financial performance of the one-sector production model. We also find that the level of increasing returns does not influence financial statistics in any significant way. We show that neither the introduction of the sunspot shock nor a higher level of returns to scale in production increases the volatility of the stochastic discount factor and the standard deviation of the return on equity, both of which are necessary to account for the equity premium.

Moreover, it is not clear how to modify the model to increase the standard deviation of the pricing kernel and the return on equity and simultaneously preserve indeterminacy. In the previous asset pricing studies in the standard DSGE framework without indeterminacy [for example, Boldrin et al. (2001), Jermann (1998), and Avalos (2001)], improvement in the asset-pricing performance followed from the introduction of habit formation into the agent's preferences in combination with costs of adjustments in capital stock or other similar mechanisms, which prevented easy factor adjustments in response to shock. The combination of habit persistence and capital adjustment costs resolves the asset-pricing puzzles in production economies because the agents whose preferences display habit persistence are very risk-averse locally and are eager to avoid fluctuations in their consumption. With frictions such as adjustment costs, the equity security becomes an unattractive instrument for consumption smoothing relative to the risk-free asset. As a result, agents require a higher return for holding equity and accept a lower return on bonds. On the other hand, adjustment costs prevent the instantaneous response of the capital stock to exogenous shocks and therefore increase the volatility of the return on equity. Similar mechanisms are not consistent with indeterminacy because full mobility of factors of production is needed for agents to act on their beliefs.

We therefore conclude that the resolution of long-standing financial puzzles is an even more challenging task in the context of real models with self-fulfilling expectations than in the standard RBC framework because of two conflicting requirements. On the one hand, frictions that restrict the mobility of factors of production, especially of capital, are needed to generate a sufficient equity premium and volatility of asset returns. On the other hand, factors of production that respond flexibly to shocks are essential for the model economy to exhibit local indeterminacy with realistic increasing returns. Perhaps a model in which indeterminacy is introduced through channels other than aggregate increasing returns in production will fare better in this regard.

NOTES

1. Examples of such investigations include Brock (1979), Donaldson and Mehra (1984), Naik (1994), Rouwenhorst (1995), Boldrin et al. (2001), Jermann (1998), Tallarini (2000), Danthine and Donaldson (2002b), Lettau (2003), and Gomes et al. (2003), among others.

2. Excellent reviews of the literature on asset-pricing puzzles include Kocherlakota (1996), Mehra and Prescott (2003), and Campbell (1999).

3. Benhabib and Farmer (1999) present a comprehensive survey of the indeterminacy literature.

4. We discuss the issue of returns to scale calibration further in the Calibration section of this paper.

5. The cited financial statistics are obtained from the BFG model with simultaneous technology and sunspot shocks. The asset pricing results from other parameterizations of the BFG model, as presented in Sections 4.2 and 4.3, are similar.

6. Our results apply more generally to other models with indeterminacy, especially real models requiring a lower degree of increasing returns for indeterminacy. One example is Wen's (1998a) model with variable capacity utilization. The variable capacity functions as an additional factor of production, bringing down the level of IRS necessary for indeterminacy but providing risk-averse investors with one more channel for consumption smoothing in response to shocks. As a result, in Wen's model the standard deviation of consumption is close to zero, which leads to an extremely smooth pricing kernel. On the other hand, Wen(1998b) shows that in the presence of adjustment costs in the variable capacity utilization model, indeterminacy-disappears.

7. We choose Λ so that the steady-state value of N is 1/3.

8. We do not consider leverage effects because they have a negligible effect on asset prices in our economy.

9. The share price is equal to the ex-dividend value of the firm because the number of shares is normalized to one.

10. Consumption, investment, and shadow price of capital can serve as an additional variable in s_t.

11. The appendix with the detailed derivation of the financial asset returns is available and will be provided by the authors upon request.

12. A similar discussion is presented in Campbell (1999).

13. If variable X is conditionally lognormal, $\ln E_t[X] = E_t[\ln X] + \frac{1}{2}\mathrm{Var}_t[\ln X]$ with $\mathrm{Var}_t[\ln X] = \mathrm{Var}[\ln X]$ if X is homoscedastic.

14. We thank an anonymous referee for suggesting this. We considered a wide range of correlation coefficients between the two shocks and found that this parameter does not significantly affect the asset pricing implications of the studied economies.

15. With uncorrelated shocks, the highest value of the log equity premium that we are able to achieve is 0.0013%.

16. For detailed exploration of the macroeconomic performance of the BFG model see Farmer and Guo (1994) and Thomas (1999).

17. We do not present results from the cases with relative risk aversion coefficients smaller than one, in other words, when investors become almost risk-neutral. We simulated several parameterizations of the model with $\xi < 1$ and found results to be marginally worse than in the case with a logarithmic utility function. Generally, the logarithmic utility function is the lowest RRA coefficient considered in the asset-pricing literature.

18. The following discussion is relevant to all model economies considered in this research. We focus on the BFG model with two shocks because of its marginally better financial performance.

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