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ARTICLES

ENDOGENOUS INEQUALITY OF NATIONS THROUGH FINANCIAL ASSET MARKET INTEGRATION

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The paper analyzes an endogenous mechanism leading perfectly symmetric economies to diverge in the long run after unifying their financial asset markets. The standard growth model with overlapping generations of consumers (OLG) is extended to include uncertainty and a financial asset. In the absence of an international asset market, the two autarkic economies converge to the same globally attracting steady state under rational expectations dynamics. When the two asset markets are unified internationally, additional asymmetric steady states appear, implying that the steady state with equal levels of capital becomes unstable, causing symmetry breaking. The paper derives general sufficient conditions for a saddle node bifurcation of the symmetric steady state. A numerical example shows that these effects occur, in particular when the production function and the function of absolute risk aversion are isoelastic.

Keywords: Asset Market Integration, Capital Accumulation, Nonconvergence, Symmetry Breaking

1. INTRODUCTION

The convergence hypothesis in development economics suggests that countries with similar structural characteristics should exhibit similar levels of per capita income in the long run, regardless of their initial capital stock. Several empirical studies have documented evidence against such a general convergence hypothesis. Bianchi (1997), Jones (1997), and Quah (1997) show that the shape of intercountry income distribution has transformed from a unimodal one in the early 1960s to a bimodal one in the 1990s. Durlauf and Johnson (1995) confirm a positive relationship between the starting level of per capita output and subsequent growth rates, implying divergence of income levels over time. Although these findings provide empirical evidence for nonconvergence, it is less clear from a theoretical point of view which mechanisms cause or could explain divergence. Because the theory of pure trade seems to offer very little in this direction, the role and structure

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of international financial markets is often mentioned, implying that the forces in such markets cause funds to flow from poor to rich countries and thus may induce intertemporal as well as interregional distortions.

Along these general lines, Matsuyama (2004) develops a model of the world economy without foreign direct investment and without uncertainty but with an international market for deposits. He argues that the integration of intercountry deposit markets can cause a divergence of otherwise symmetric economies. He shows that imperfections in credit markets *coupled* with minimum requirements for capital investment can promote deposits to flow from capital scarce to capital abundant countries after the deposit markets are integrated. This occurs because the distortion created by the credit market imperfection is smaller in rich countries than in poor countries. As a result, agents in rich countries are not credit constrained and can finance all profitable projects, whereas agents in poor countries are credit constrained and can finance only part of profitable projects. In autarky, deposit rates would adjust independently in each country implying that the world economy converges to a symmetric steady state. When capital endowments differ before deposit markets are integrated, the deposit rate can be lower in a country with scarce capital, due to the imperfection in the credit market and the indivisibility in investment. The integration implies equalization of the deposit rates. This causes funds to flow from capital-scarce to capital-abundant countries, setting off a mechanism so that initially rich countries increase their income gradually and lower the credit market imperfections, whereas initially poor countries suffer more and more from low income, low investment, and high credit market imperfections. Thus, the integration of deposit markets may cause symmetry breaking in the sense of Matsuyama (1996), i.e., the symmetric steady state of the world economy might become unstable. As a result, depending on the initial distribution of capital, the world economy can converge to an asymmetric steady state.

The result in Matsuyama (2004) relies on three main assumptions: (a) investment in physical capital is nondivisible, and there is a minimum investment requirement; (b) consumers are credit constrained and need to borrow funds in order to become entrepreneurs; and (c) there is no foreign direct investment. It is clear that the result of symmetry breaking no longer holds if one of these assumptions is removed. In particular, dropping the assumption of the minimum capital investment requirement, one obtains that funds will not flow from capital-scarce to capital-abundant countries, because of higher capital returns in capital-scarce countries. Thus, credit market imperfections alone do not guarantee the result of symmetry breaking; rich countries will not benefit from deposit market integration because deposits will not flow from poor to rich.

In the present paper the symmetry breaking result is caused by a different mechanism demonstrated by a different model. Although we maintain the assumption that foreign direct investment is not allowed, we introduce a market for a financial asset.[1] Each country is represented by a standard growth model with overlapping generations of consumers (OLG) to which a market for a financial asset has been added. The asset is traded between generations and serves as an intertemporal device to distribute the random dividends from an exogenous production process, a so-called "Lucas tree."

Suppose that any two developing countries from some point in time on have the same technological facilities and are identical (or similar due to the free availability of technical know-how, etc.) in their characteristics including their market structure. Under autarky they would have necessarily converged to the same (or similar) levels of capital, incomes, and welfare in the long run. Would they also converge starting from historically *different* capital levels if they were to combine their asset markets before the long run is reached? In contrast to traditional trade theory, which argues that after combining markets the convergence hypothesis holds, we show that the answer of the above question can be negative. This shows that there must exist endogenous features in such economies that make the symmetric outcome not impossible but unstable.

Our result reveals and identifies those situations in which a more advanced country in terms of its capital level has an advantage over a less developed one, if they combine their asset markets prior to convergence. The result is primarily of theoretical relevance, showing that asset market integration may hurt a relatively poor country and benefit a relatively rich one. In reality, no two countries will be identical, so it may be difficult to allude directly to any empirical counterpart or regions of existing economies. However, formally, we would argue that countries with similar structural characteristics can diverge in the long run due to the integration of their asset markets.

The paper is organized as follows. Section 2 introduces the model and discusses its main properties for general production functions and general risk preferences. Section 3 demonstrates the existence, uniqueness, and stability of an interior rational expectations equilibrium. Section 4 presents the two-country model with an international financial asset market, discussing existence of rational expectations equilibria (Section 4.1) and deriving general sufficient conditions under which the symmetric equilibrium looses stability (Section 4.2). Section 5 presents global properties of a parameterized explicit example. Section 6 concludes.

2. THE MODEL

Consider an infinite-horizon economy in discrete time $t = 0, 1, \dots$. The economy is composed of a consumption sector and a production sector. The production sector consists of an infinitely lived neoclassical firm *plus* an exogenous production process generating a *random* stream of a consumption commodity in each period. Such an exogenous production process is often referred to as a "Lucas tree," underlining the fact that the stochastic process generating the proceeds is exogenous and completely independent of the production technology described in the model. The stochastic process yields ε_t units of the consumption commodity in each period payed to the owners as a dividend. We assume that $\{\varepsilon_t\}_{t=1,2,\dots}$ is a sequence of i.i.d. random variables taking values $d > 0$ and 0 with probabilities $q \in (0, 1)$ and $1 - q$, respectively. Intertemporal ownership of the tree and the right to the proceeds is traded in the form of a *financial asset* (a financial contract) between successive generations of consumers. The asset is infinitely lived. It yields a random dividend and is sold in a competitive market. The total number of tradable contracts (the supply of assets) is constant over time.

In addition to the exogenous production process, there is a neoclassical firm producing the consumption commodity using capital and labor as inputs. The technology of the firm is described by a standard production function $F : \mathbf{R}_+^2 \to \mathbf{R}_+$ with constant returns to scale. At any time t, let $Y_t = F(K_t, L_t)$ denote total output produced, where $K_t \geq 0$ and $L_t \geq 0$ are aggregate supplies of physical capital and labor, respectively. Output per worker is defined as $y_t = Y_t/L_t = F(K_t/L_t, 1) =: f(k_t)$, where $k_t = K_t/L_t$ denotes capital per worker and $f : \mathbf{R}_+ \to \mathbf{R}_+$ is the production function in intensive form. We assume that $f(0) = 0$, i.e., capital is essential in production. In addition, f is assumed to be twice continuously differentiable, strictly increasing, and strictly concave and satisfies the Inada conditions. Both factor markets in the economy are assumed to be competitive. Therefore, under full employment, factor rewards for capital and labor are determined by their respective marginal products. Let $r_t = r(k_t) := f'(k_t)$ denote the rental rate of capital and $w_t = w(k_t) := f(k_t) - k_t f'(k_t)$ denote the wage rate in any given period. The produced commodity can be either consumed or invested in physical capital, which becomes available in the next period. Old capital depreciates fully within a period.

The consumption sector consists of overlapping generations of consumers who live for two successive periods. Thus, in any period, there are two generations alive, referred to as young and old. Each generation, which consists of a continuum of homogeneous agents with unit mass, is identified by its date of birth. For simplicity we assume no population growth. A typical young consumer of any generation $t = 0, 1, \ldots$ supplies one unit of labor endowment inelastically in the first period of his life for which he receives labor income w_t. He does not consume in the first period but invests all income in a portfolio consisting of physical capital and the financial asset. The budget constraint of a young consumer is given by $i_t + x_t p_t = w_t$, where $i_t \geq 0$ denotes the amount of investment in physical capital and $x_t \geq 0$ is the number of assets purchased at the price p_t (measured in units of the consumption good).

The initial old generation, which lives for only one period, is endowed with $x > 0$ units of the financial asset and k_0 units of capital. They consume the total receipts to both of them, which consist of the return on their capital, the proceeds from the tree, and the value at which they sell the asset in the market. Old consumers of succeeding generations acquire their endowment of the asset and of physical capital from saving their wage income when young. Old consumers do not leave bequests to future generations and consume their entire wealth. Therefore, their random second period consumption is

$$c_{t+1} = i_t r_{t+1} + x_t \left(p_{t+1} + \varepsilon_{t+1} \right), \tag{1}$$

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where $i_t r_{t+1}$, $x_t \varepsilon_{t+1}$, and $x_t p_{t+1}$ are the returns (in units of consumption good) received from capital investment, from asset holding as dividends, and from selling the financial asset. Depending on the realization of ε_{t+1}, consumption in old age can take values $\overline{c}_{t+1} = i_t r_{t+1} + x_t(p_{t+1} + d)$ or $\underline{c}_{t+1} = i_t r_{t+1} + x_t p_{t+1}$ with probabilities q and $1 - q$, respectively. In the sequel of the paper, $\overline{c}_{t+1}$ and $\underline{c}_{t+1}$ will be referred to as realizations of consumption in good and bad states. Since $i_t = w_t - x_t p_t$, old age consumption can be rewritten as

$$\overline{c}_{t+1}=w_t r_{t+1}+x_t d+x_t(p_{t+1}-p_t r_{t+1}) \quad \text{and} \quad \underline{c}_{t+1} = w_t r_{t+1} + x_t(p_{t+1} - p_t r_{t+1}). \tag{2}$$

Preferences over old age consumption is described by a utility function $u : \mathbf{R}_+ \to \mathbf{R}$. We assume that u is twice continuously differentiable, strictly increasing, and strictly concave. For given values of wage income w_t, next period's rate of return on capital r_{t+1}, and next period's asset price p_{t+1}, the consumer's demand for the asset is defined as

$$\varphi(w_t, r_{t+1}, p_{t+1}, p_t) := \arg \max_{x \in B(w_t, p_t)} \left\{ q u(\overline{c}_{t+1}) + (1 - q) u(\underline{c}_{t+1}) \right\}, \tag{3}$$

where $\overline{c}_{t+1}$ and $\underline{c}_{t+1}$ are consumptions in good and bad states and $B(w_t, p_t) = \{x | x \geq 0,\ x p_t \leq w_t\}$ is the budget set. Given the assumptions made, asset demand of consumers takes a particularly simple form. All proofs are provided in the appendix.

PROPOSITION 1. *For any given nonnegative vector* $(w_t, r_{t+1}, p_{t+1}) \geq 0$*, asset demand is given by*

$$\varphi(w_t, r_{t+1}, p_{t+1}, p_t) = \begin{cases} 0 & \text{if} \quad p_t \geq p_3^* \\ \varphi_m(p_{t+1} - p_t r_{t+1}, w_t r_{t+1}) & \text{if} \quad p_2^* < p_t < p_3^* \\ w_t / p_t & \text{if} \quad p_t \leq p_2^* \end{cases}. \tag{4}$$

The function $\varphi_m : \mathbf{R}_+^2 \to \mathbf{R}_+$ *is increasing in both arguments, and the critical levels* p_2^* *and* p_3^* *are defined by the unique solutions of* $p_2^* \varphi_m(p_{t+1} - p_2^* r_{t+1}, w_t r_{t+1}) = w_t$ *and* $p_3^* r_{t+1} = p_{t+1} + dq$.

The typical features of asset demand are visualized in Figure 1, showing that the graph of the demand function consists of three sections. For a sufficiently high price, asset demand is zero, because the expected return from the financial asset is lower than the expected return from capital investment. In this case, consumers invest all of their wage income in physical capital. For a sufficiently low price, when the expected return from the financial asset investment exceeds the expected return from capital investment, the situation is the opposite. All wage income is invested in the asset market, and no new investment in physical capital occurs. For all intermediate prices, the optimal choice consists of an interior solution with a mixed portfolio containing the financial asset and physical capital. Moreover, as a consequence of portfolio theory, the function φ_m depends only on the expected risk premium $p_{t+1} - p_t r_{t+1}$ and on the discounted wage income $w_t r_{t+1}$.

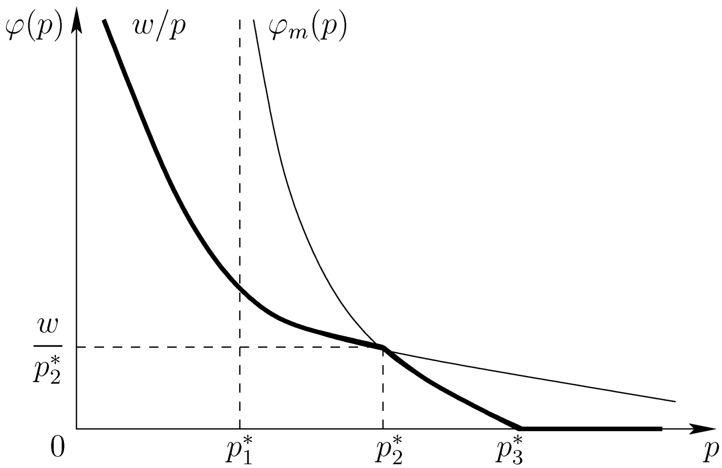

FIGURE 1. Asset demand function.

3. THE CLOSED ECONOMY

Consider first the case of autarky, when the asset market operates only domestically. The demographic structure of consumers implies that in autarky all assets sold by old consumers are purchased by young investors. Therefore, if no new assets are added in any period, the total number of assets will be constant through time. Then, for a given nonnegative vector $(w_t, r_{t+1}, p_{t+1}) \geq 0$ and for a given aggregate supply of assets $x > 0$, the asset market clearing price p_t solves the equation

$$\varphi(w_t, r_{t+1}, p_{t+1}, p_t) = x. \tag{5}$$

The strict monotonicity of the asset demand function implies that equation (5) has a unique solution. Let $p_t = \mathcal{S}(w_t, r_{t+1}, p_{t+1}, x)$ denote the unique market clearing asset price, where the function $\mathcal{S}$ is usually referred to as the temporary price law. Because next period's capital stock k_{t+1} is equal to new investment, one has $k_{t+1} = w_t - p_t x$, where $p_t x$ is total spending in the asset market.

3.1. Stationary Rational Expectations Equilibria

Consider next the situation when stationarity and perfect foresight prevail.

DEFINITION 1. *A stationary rational expectations equilibrium (SREE) is a pair* $(k, p) \in \mathbf{R}_+^2$ *such that*

- *given k, the price p clears the asset market under perfect foresight, i.e., p is a fixed point of the temporary price law*

$$p = \mathcal{S}[w(k), r(k), p, x], \tag{6}$$

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- *given p, the level of capital k is a fixed point of the capital accumulation equation*

$$k = A(k, p, x) := w(k) - px. \tag{7}$$

Even in very simple cases, the perfect foresight solutions of such economies cannot be determined explicitly, due to the interaction of the nonlinearities of the price law and of the law of capital accumulation. To show that there exist exactly two such solutions, a detailed implicit analysis is required involving features of the *inverse* asset demand function of consumers.

First, we observe that young consumers must hold a mixed portfolio for the capital stock to be positive in an SREE. Let $\mathcal{L}(x) := qu(\overline{c}_{t+1}) + (1-q)u(\underline{c}_{t+1})$ denote the consumer's objective function. Then, from the first-order conditions for an interior optimum

$$\mathcal{L}'(x) = qu'(\overline{c}_{t+1})\,[d - (p_t r_{t+1} - p_{t+1})] - (1-q)u'(\underline{c}_{t+1})\,(p_t r_{t+1} - p_{t+1}) = 0, \tag{8}$$

one obtains the relation

$$\frac{d}{p_t r_{t+1} - p_{t+1}} = 1 + \frac{1-q}{q}\frac{u'(\underline{c}_{t+1})}{u'(\overline{c}_{t+1})}. \tag{9}$$

Second, we observe that stationary equilibrium consumptions in good and bad states are given by

$$\begin{aligned} \underline{c} &= w(k)r(k) - [(w(k)-k)(r(k)-1)] = f(k) - k, \\ \overline{c} &= w(k)r(k) + xd - [w(k)-k][r(k)-1] = f(k) - k + xd. \end{aligned} \tag{10}$$

Therefore, equations (9) and (10) imply that for a given SREE (k, x), the inverse demand function of the financial asset is given by

$$P(k, x) := \frac{d}{r(k)-1}h(k, x) \text{ with } h(k, x) := \frac{qu'(\overline{c})}{qu'(\overline{c}) + (1-q)u'(\underline{c})} \in [0, 1]. \tag{11}$$

The function h can be interpreted as the risk-neutral probability of the good state realization. Since h is always positive, equation (11) implies that in order to guarantee a positive asset price, the equilibrium k should belong to the interval $[0, \widehat{k}]$, where $\widehat{k}$ is the unique solution[2] of $r(k) = 1$. Equations (7) and (11) imply that capital at an interior SREE should satisfy the following equation:

$$\phi(k) = dxh(k, x), \tag{12}$$

where $\phi(k) := [w(k) - k][r(k) - 1]$. The following two assumptions will be used to prove existence of a unique positive SREE under autarky.

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Assumption 1. The elasticity of the production function with respect to capital (or the capital share in production) $\alpha : \mathbf{R}_+ \to [0, 1]$ defined as

$$\alpha(k) := \frac{kf'(k)}{f(k)}, \tag{13}$$

satisfies the inequality $\alpha(k) < 0.5$ for any $k \in [0, \widehat{k}]$.

Assumption 2. The consumer's absolute risk aversion $T : \mathbf{R}_+ \to \mathbf{R}_+$ defined as

$$T(c) := -\frac{u''(c)}{u'(c)}, \tag{14}$$

is a nonincreasing function.

Assumption 1, used in Lemma 1, implies some important properties of the function ϕ, and Assumption 2 will be used in Lemma 2 to establish a monotonicity property of the function h. The claims of Lemmas 1 and 2 provide the main arguments to prove the following proposition.

PROPOSITION 2. *If Assumptions 1 and 2 are satisfied, then in autarky there exists one corner and one interior SREE.*

Proof. Proposition 2 is a direct consequence of Lemmas 1 and 2, which are given in the appendix. Without loss of generality, let $x = 1$. Then, on the one hand, the function $\phi(k) := [w(k) - k][r(k) - 1]$ is strictly decreasing on the interval $k \in [0, \widehat{k}]$, $\phi(0) = \infty$, and $\phi(\widehat{k}) = 0$ (see Lemma 1). On the other hand, the function $k \mapsto h(k, 1)$ is positive and strictly increasing on the same interval $k \in [0, \widehat{k}]$ (see Lemma 2). This implies that equation (12) admits a unique interior solution $k^* \in (0, \widehat{k})$ for $x = 1$, with an associated interior asset price $p^* = P(k^*, 1) > 0$.

The expressions in (11) imply that $(k, p) = (0, 0)$ is a SREE on the boundary. If $k = 0$, wage income and the equilibrium asset price are both zero $(w, p) = (0, 0)$, and for a zero asset price, $k = 0$ is a fixed point of the capital accumulation equation. ■

Restricting the elasticity of production to be less than one half, as is done in Assumption 1, is crucial and important to obtain existence and uniqueness of an interior SREE. When it is violated, one finds that the function ϕ is not necessarily monotonic. Then, equation (12) can admit either no or multiple interior solutions [see Böhm and Vachadze (2008) for details]. On empirical grounds, assuming $\alpha(k) < 0.5$ can be justified immediately, because there is consensus that most empirical studies confirm such a value. Theoretically, however, the occurrence of multiple equilibria for $\alpha(k) > 0.5$ begs some explanation. In such a case, for the model in question, prices for the financial asset grow faster than the rate of return on capital for sufficiently small levels of the capital stock. This causes consumers to invest a smaller fraction of their wage income in real capital. This reinforces convergence to a zero capital stock, in spite of an unbounded rate of return on capital. Formally, one observes that the elasticity $\alpha(k)$ is a (first-order) measure for the curvature of the production function f, which determines simultaneously wages and returns in an additive way, $f(k) = w(k) + kr(k)$. Yet, the multiplicative form of the function ϕ involves also second-order properties of f, which are weak for $\alpha(k) < 0.5$. They become strong when $\alpha(k)$ is larger than one half.[3]

For the remainder of the analysis, Assumption 1 will be made throughout, because the purpose of the paper is to single out causes for multiplicity *induced* by asset market integration. Therefore, it is desirable to restrict the analysis to situations in which the closed economy with a domestic market for a financial asset has a unique interior steady state (implying the existence of a unique, interior, and symmetric steady state in the world economy). Then, if multiple SREEs and instability of the symmetric equilibrium in the world economy arise *after* combining the domestic asset markets, the integration can be identified as the cause of instability and of symmetry breaking.

3.2. Dynamics under Rational Expectations

The asset market clearing condition given in (5) together with the accumulation equation $k_{t+1} = w(k_t) - p_t x$ defines implicitly a two-dimensional dynamical system in asset prices and capital under rational expectations. The associated perfect foresight steady state is a saddle, a feature that is found in most macroeconomic models with perfect foresight dynamics. Therefore, in order to analyze the dynamics of the closed economy under rational expectations, we follow the standard procedure of the literature in such cases and analyze the so-called minimum state variable (MSV) solution. It describes the dynamics along the saddle path of the two-dimensional system.[4] The dynamic solution has some important specific features that stem directly from the structure of the model. Equation (5) implies that the equilibrium asset price in any *given* period is affected by the expectations about next periods asset price, the future capital return, and the moments of next periods random dividend payments. Normally, one would expect this to imply random capital accumulation. However, because the realizations of the random dividend affect old age consumption only *and* because dividends are i.i.d. (making the moments of the random dividend constant over time), it follows that capital accumulation under perfect foresight will be deterministic. As a consequence, consumers can choose consistent *deterministic* (point) forecasts for next periods capital stock and its return based on the current asset price.

The essential property of the MSV solution stipulates that the equilibrium asset price in any given period can be determined as a function of the current capital stock alone. If this is the case, the capital accumulation equation implies an explicit perfect predictor for next periods asset price and for the future capital return. In other words, assume for the moment that the asset market clearing price is a function of current capital alone, $p_t = \mathcal{P}(k_t)$. Then, the capital accumulation equation implies that next period's capital

$$k_{t+1} = \mathcal{G}(k_t) \equiv w(k_t) - \mathcal{P}(k_t) \quad \textbf{(15)}$$

is also a function of current capital alone. As a consequence, the prefect prediction for the price and for the interest rate can be chosen as $p_{t+1} = \mathcal{P}[\mathcal{G}(k_t)]$ and $r_{t+1} = r[\mathcal{G}(k_t)]$. For them to induce perfect foresight they must be consistent with the price law. In other words, they must satisfy the functional equation

$$\mathcal{P}(k_t) \equiv \mathcal{S}\{w(k_t), r[\mathcal{G}(k_t)], \mathcal{P}[\mathcal{G}(k_t)], 1\} \quad \textbf{(16)}$$

for any $k_t \in \mathbf{R}_+$. Thus, the pair of functions $(\mathcal{G}, \mathcal{P})$ satisfying the system of functional equations (15) and (16) completely describes the evolution of the economy under rational expectations, which induces the MSV solution. In fact, with one mild additional assumption on the technology a full characterization of the rational expectations dynamics is possible.

Assumption 3. The production function is such that $\lim_{k\to 0} -kf''(k) = \infty$.

Since $w'(k) = -kf''(k)$, Assumption 3 implies that the wage function has an unbounded slope at the origin.

PROPOSITION 3. *If Assumptions 1, 2, and 3 are satisfied, then the corner equilibrium is unstable, whereas the interior equilibrium is globally stable under rational expectations dynamics.*

Proposition 3 implies that for any economy of the given type, there exists a unique interior SREE in autarky that is globally stable under rational expectations dynamics. Thus, economies with the same characteristics of consumers and producers converge to the same positive steady state independently of initial conditions, implying identical income, identical capital returns, and an identical asset price in the long run.

4. A TWO-COUNTRY MODEL

Consider now a world economy composed of two identical economies of the above type, which are denoted by h (for home country) and by f (for foreign country). Consumers and firms in each country have identical characteristics. Factors of production, capital and labor, are immobile across countries. However, the market for the financial asset is integrated into a *unified international market*, where the asset is traded at a uniform price while the same dividend is paid in each country. Therefore, consumers from each country now diversify to invest in *domestic* capital and in a financial asset from an integrated *international* market.

The demographic structure of the model implies that all financial assets sold by old consumers of both countries are bought by young consumers. Because each country is endowed with one unit of the asset, it follows that the total number of available assets in the international financial asset market is now two. This implies that for a given nonnegative vector $(w_t^h, w_t^f, r_{t+1}^h, r_{t+1}^f, p_{t+1}) \geq 0$ of domestic and foreign wage incomes, next period's rates of returns on capital, and next period's asset price p_{t+1} (measured in units of the consumption good), an asset price p_t clearing the international asset market must solve the equation

$$\varphi\big(w_t^h, r_{t+1}^h, p_{t+1}, p_t\big) + \varphi\big(w_t^f, r_{t+1}^f, p_{t+1}, p_t\big) = 2. \tag{17}$$

Given our assumptions, (17) has a unique solution because the aggregate asset demand function is strictly decreasing in p_t. Let

$$p_t = \mathcal{S}\big(w_t^h, w_t^f, r_{t+1}^h, r_{t+1}^f, p_{t+1}\big) \tag{18}$$

denote the unique asset market clearing price.

4.1. Stationary Rational Expectations Equilibria

DEFINITION 2. *A SREE in the world economy is a triple* $(k^h, k^f, p) \in \mathbf{R}_+^3$ *such that*

- *given* (k^h, k^f)*, the price* p *clears the asset market under perfect foresight, i.e.,* p *is a fixed point of the temporary price law*

$$p = \mathcal{S}[w(k^h), w(k^f), r(k^h), r(k^f), p]; \tag{19}$$

- *given* p*, the pair* (k^h, k^f) *is a fixed point of each country's capital accumulation equation*

$$\begin{aligned} k^h &= A(k^h, p) := w(k^h) - p\varphi[w(k^h), r(k^h), p, p], \\ k^f &= A(k^f, p) := w(k^f) - p\varphi[w(k^f), r(k^f), p, p]. \end{aligned} \tag{20}$$

Because there are two steady states in each closed economy, 0 and k^* [where k^* solves (12) with $x = 1$], it follows that two symmetric steady states from autarky survive after integrating the asset markets, i.e., the points $(0, 0)$ and (k^*, k^*) are also stationary equilibria in the two-country world economy. In addition, there are two asymmetric steady states in which one country absorbs all assets with positive capital while the other deteriorates to zero levels of capital and income. Thus, $(0, \widetilde{k})$ and $(\widetilde{k}, 0)$ are two asymmetric steady states in the two-country economy, where $\widetilde{k}$ solves (12) with $x = 2$. The interesting issue to examine is whether, after integrating the asset markets internationally, there are additional equilibria in which both countries hold positive quantities of the asset at positive but *different* levels of capital. To study the existence of such *interior asymmetric* steady states, we introduce the following concepts and notation.

For any given interior level of asset holdings $x \in (0, 2)$, let $k = \pi(x)$ denote the unique interior solution of equation (12). Assumptions 1 and 2 together with Proposition 2 guarantee the existence and uniqueness of k solving the equation $\phi(k) = dxh(k, x)$ for any $x \in (0, 2)$. Then, for any distribution of asset holdings

$(x, 2-x)$ among the two countries, there exist associated SREE levels of capital in each country $k^h = \pi(x)$ and $k^f = \pi(2-x)$. Given these capital levels and the asset holdings $(x, 2-x)$, there are corresponding supporting asset market clearing prices $p^h = \Pi(x)$ and $p^f = \Pi(2-x)$ in each country. The function Π is defined as

$$\Pi(x) := P[\pi(x), x], \tag{21}$$

where $P(k, x)$ is the inverse demand function as defined in (11). Thus, Π has to be interpreted as the stationary inverse demand function under perfect foresight for given asset holdings x.

Finally, the asset price p at a SREE with asset distribution $(x, 2-x)$ after asset market integration must be the same as the two supporting asset prices in the two countries, i.e., $\Pi(x) = p^h = p^f = \Pi(2-x)$. Therefore, at an international SREE, asset holdings x by consumers in the home country must be such that

$$\Psi(x) := \Pi(x) - \Pi(2-x) = 0. \tag{22}$$

In other words, the asset holdings x (in the home country) at any stationary rational expectations equilibrium in the world economy must be a zero of the excess price map Ψ. To study the existence of asymmetric steady states, we first establish some properties of the continuous function Ψ, which is differentiable on $(0, 2)$. By construction, one has $\Psi(1) = 0$. Equation (12) implies that $\lim_{x\to 0} \pi(x) = \widehat{k}$, which, combined with equation (21), implies

$$\lim_{x\to 0} \Pi(x) = \lim_{k\to \widehat{k}} P(k, 0) = \lim_{k\to \widehat{k}} \frac{dq}{r(k) - 1} = \infty.$$

Moreover, since $\lim_{x\to 2} \Pi(x)$ is finite, one obtains as the boundary behavior for Ψ

$$\lim_{x\to 0} \Psi(x) = \infty \qquad \text{and} \qquad \lim_{x\to 2} \Psi(x) = -\infty. \tag{23}$$

In order to find a sufficient condition for the existence of at least two asymmetric steady states, we consider the stationary asset demand and its elasticity. Define the stationary asset demand function $X : [0, \widehat{k}] \times \mathbf{R}_+ \to \mathbf{R}_+ \cup \{\infty\}$ as

$$X(k, p) := \begin{cases} 0 & \text{if} \quad p \le \underline{p} \\ x & \text{if} \quad p \in \left(\underline{p}, \overline{p}\right) \\ \infty & \text{if} \quad p \ge \overline{p} \end{cases}, \tag{24}$$

where x is the solution of the equation $P(k, x) = p$ for a given pair (k, p) and the constants $\underline{p}$ and $\overline{p}$ are defined as $\underline{p} := P(k, 0)$ and $\overline{p} := P(k, \infty)$. The monotonicity of the function P (see Lemma 2) implies that stationary asset demand is monotonically increasing with respect to its first argument k and monotonically

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decreasing with respect to its second argument p. Define

$$\epsilon(k, p) := \frac{kX_k(k, p)}{X(k, p)} \quad \text{and} \quad \sigma(k) := -\frac{f'(k)\left[f(k) - kf'(k)\right]}{kf''(k)f(k)}, \tag{25}$$

as the elasticity of asset demand with respect to capital and the elasticity of factor substitution in production, respectively.

Now, consider the symmetric steady state $x^* = 1$, and let ϵ^*, α^*, σ^* and s^* denote the respective values of the elasticity of asset demand with respect to capital, the capital share in production, the elasticity of factor substitution, and the share of wage income spent on the asset market all evaluated at the symmetric steady state:

$$\epsilon^* = \epsilon(k^*, p^*),\ \alpha^* = \alpha(k^*),\ \sigma^* = \sigma(k^*) \quad \text{and} \quad s^* = p^*/w(k^*). \tag{26}$$

Then, one obtains the following sufficient conditions for the existence of interior asymmetric steady states.

PROPOSITION 4. *Let Assumptions 1 and 2 be satisfied. If*

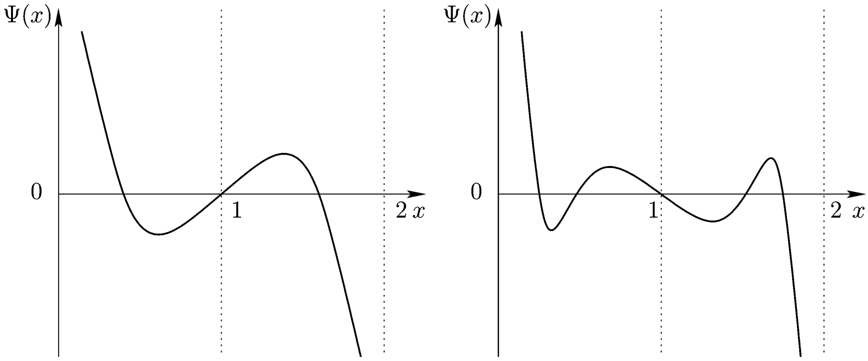
$$\delta^* := \epsilon^* - \frac{1}{s^*}\left(\frac{\alpha^*}{\sigma^*} - 1\right) - 1 < 0 \tag{27}$$

holds, there exist at least two interior asymmetric steady states in the world economy.

The condition $\delta^* < 0$ implies a positive slope of the function Ψ at the symmetric steady state. Multiplicity then follows from continuity and from the boundary behavior of Ψ. The proposition provides a sufficient condition for the existence of interior asymmetric steady states to hold *locally* at the symmetric steady state. It identifies how properties of the production function and the utility function must interact in each country separately to induce symmetry breaking. Therefore, it can always be verified using information of the autarkic economy only. Observe that (27) indicates that interior asymmetric steady states are more likely to coexist with the symmetric one whenever the elasticity of asset demand ϵ^* *and* the elasticity of factor substitution σ^* are both small at the same time. Figure 2(a) portrays the situation when $\Psi'(1) > 0$. Then, there exist at least two additional interior steady states in the world economy. Figure 2(b) shows that (27) is only a sufficient condition, because there may well exist asymmetric steady states even when $\Psi'(1)$ is negative and (27) fails to hold.

Inequality (27) reveals that asset demand needs to be sufficiently inelastic at the symmetric steady state to guarantee the existence of interior asymmetric steady states. To study further the role of the assumptions needed for the above result, notice that the definition of asset demand given in equation (24) implies the identity

$$P[k, X(k, p)] \equiv p. \tag{28}$$



(a) Two asymmetric SREE (b) Four asymmetric SREE

FIGURE 2. Existence of asymmetric steady states.

Applying the implicit function theorem to equation (28) reveals that the elasticity of asset demand at the symmetric steady state can be represented as the ratio of elasticities of the inverse demand function with respect to capital and with respect to asset holdings:

$$\epsilon^* = \frac{k^* P_k(k^*, x^*)}{x^* P_x(k^*, x^*)}, \tag{29}$$

both evaluated at $(k, x) = (k^*, x^*)$. Thus, asset demand is inelastic when the inverse demand function P is very sensitive with respect to asset holdings and insensitive with respect to capital.

One immediate consequence of (29) is that the randomness of dividends *and* the strict concavity of the utility function are both necessary for (27) to hold. To see this, suppose that there is no uncertainty in dividend payments. Then, $q = 1$ and the inverse demand function given in (11) becomes

$$P(k, x) = \frac{d}{r(k) - 1}. \tag{30}$$

This implies an infinitely elastic asset demand, because $P_x(k, x) = 0$ for any $(k, x) > 0$. When $\epsilon^* = \infty$, (27) fails to be satisfied for any $(k, x) > 0$. In other words, asset price equalization from equation (30) implies equalization of capital stocks as well as inducing convergence to the symmetric steady state. Thus, symmetry breaking cannot occur. The same implication follows when agents are risk neutral. In this case, equation (11) implies that the risk-adjusted probability of a good state realization is constant and independent of the pair (k, x) with $h(k, x) = q$. This again implies immediate equalization of capital stocks and convergence of the world economy to the symmetric equilibrium after asset market integration, because the asset demand is infinitely elastic. Thus, (27) is never satisfied.

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Summarizing this discussion, when there is no uncertainty or when agents are risk neutral, the equilibrium asset price does not depend on the level of asset holdings in the two countries. The price must be equal to the discounted value of the expected dividend, as in equation (30). This means that asset price equalization implies the equalization of capital returns and the equalization of capital stocks. When $q \neq 1$ and agents are risk averse, the inverse demand function $P(k, x)$ depends positively on k and negatively on x. Therefore, after asset market integration, returns on capital are equalized with *risk-adjusted returns* on financial assets within each country. However, risk-adjusted returns can differ in stationary equilibria, which implies the possibility of asymmetric steady states and of symmetry breaking.

4.2. Dynamics under Rational Expectations

To analyze the dynamics of the world economy, we proceed as in the case of autarky and use the MSV solution. Suppose that there exists a function $\mathcal{P} : \mathbf{R}_+^2 \to \mathbf{R}_+$ such that the uniform asset price can be determined by the capital stocks in each country, $p_t = \mathcal{P}(k_t^h, k_t^f)$. Then, the capital accumulation equations imply that (k_{t+1}^h, k_{t+1}^f) should satisfy

$$\begin{aligned} k_{t+1}^h &= w\big(k_t^h\big) - s\big(k_t^h, k_{t+1}^h, k_{t+1}^f\big), \\ k_{t+1}^f &= w\big(k_t^f\big) - s\big(k_t^f, k_{t+1}^f, k_{t+1}^h\big), \end{aligned} \quad \textbf{(31)}$$

where $s(k_t^h, k_{t+1}^h, k_{t+1}^f)$ and $s(k_t^f, k_{t+1}^f, k_{t+1}^h)$ are total spending on the international financial market by young agents of countries h and f. These functions are defined as

$$\begin{aligned} s\big(k_t^h, k_{t+1}^h, k_{t+1}^f\big) &:= \varphi\Big[w\big(k_t^h\big), r\big(k_{t+1}^h\big), \mathcal{P}\big(k_{t+1}^h, k_{t+1}^f\big), \mathcal{P}\big(k_t^h, k_t^f\big)\Big]\mathcal{P}(k_h, k_f), \\ s\big(k_t^f, k_{t+1}^f, k_{t+1}^h\big) &:= \varphi\Big[w\big(k_t^f\big), r\big(k_{t+1}^f\big), \mathcal{P}\big(k_{t+1}^h, k_{t+1}^f\big), \mathcal{P}\big(k_t^h, k_t^f\big)\Big]\mathcal{P}(k_h, k_f). \end{aligned} \quad \textbf{(32)}$$

For the price predictor to be perfect, it must be consistent with the price law, i.e., it must satisfy the functional equation

$$\mathcal{P}\big(k_t^h, k_t^f\big) \equiv \mathcal{S}\big[w\big(k_t^h\big), w\big(k_t^f\big), r\big(k_{t+1}^h\big), r\big(k_{t+1}^f\big), \mathcal{P}\big(k_{t+1}^h, k_{t+1}^f\big)\big]. \quad \textbf{(33)}$$

As a consequence of the symmetry of equation (31) together with (32) and (33), one can write the symmetric solutions of capital accumulation as

$$k_{t+1}^h = \mathcal{G}\big(k_t^h, k_t^f\big) \quad \text{and} \quad k_{t+1}^f = \mathcal{G}\big(k_t^f, k_t^h\big), \quad \textbf{(34)}$$

which now defines the time one map of capital accumulation with perfect foresight dynamics for the world economy as a two-dimensional system $\mathcal{F} : \mathbf{R}_+^2 \to \mathbf{R}_+^2$

given by

$$\begin{pmatrix} k^h_{t+1} \\ k^f_{t+1} \end{pmatrix} = \mathcal{F}\left(k^h_t, k^f_t\right) := \begin{pmatrix} \mathcal{G}\left(k^h_t, k^f_t\right) \\ \mathcal{G}\left(k^f_t, k^h_t\right) \end{pmatrix}. \tag{35}$$

We begin the dynamic analysis by showing that the boundary steady states are not stable under rational expectation dynamics.

PROPOSITION 5. *If Assumptions 1, 2, and 3 are satisfied, then the symmetric steady state* $(0, 0)$ *is a source and the asymmetric steady states* $(\widetilde{k}, 0)$ *and* $(0, \widetilde{k})$ *are unstable saddles.*

The instability of the corner steady states follows from Assumption 3, which makes $k = 0$ locally unstable for the closed economy. Therefore, no matter what the initial distribution of capital in the world economy is, the steady states $(0, 0)$, $(\widetilde{k}, 0)$, and $(0, \widetilde{k})$ cannot be reached from interior initial distributions of capital.

Propositions 4 and 5 together imply that an instability of the symmetric steady states induces the appearance of asymmetric steady states. In other words, using the information contained in the characteristics of the symmetric steady state, the conditions for its *instability* must be related to those for the *existence of asymmetric steady states*. Our main result consists of a description of the role of the parameters characterizing the "Lucas tree" in causing symmetry breaking. Let $\Omega := \mathbf{R}_{++} \times (0, 1)$ denote the space of parameters (d, q) characterizing the exogenous production process.

PROPOSITION 6. *Let Assumptions 1, 2, and 3 be satisfied.*

1. *The interior and symmetric steady state* (k^*, k^*) *has two positive real roots.*
2. *There exists a nonempty set* $\Omega^s \subsetneq \Omega$ *such that* (k^*, k^*) *is asymptotically stable only if* $(d, q) \in \Omega^s$.
3. *As the parameters* (d, q) *leave the region* Ω^s, *the symmetric steady state loses its stability by undergoing a fold bifurcation.*

To investigate the stability of the symmetric interior steady state requires a standard but tedious argument of evaluating the relationship between the trace and determinant of the Jacobian matrix of the system (35), which depends heavily on the symmetry of the mapping. For an intuitive understanding of the proof of the result, consider the sets

$$\Omega^u := \{(d, q) \in \Omega | \delta^* < 0\}, \quad \Omega^c := \{(d, q) \in \Omega | \delta^* = 0\},$$
$$\Omega^s := \{(d, q) \in \Omega | \delta^* > 0\},$$

where δ^* is the critical value defined in equation (27). The function Ψ defined in equation (22) and displayed in Figure 2 slopes upward (downward) at $x = 1$ when $(d, q) \in \Omega^u$ [when $(d, q) \in \Omega^s$], whereas it is tangent to the zero line at $x = 1$ when $(d, q) \in \Omega^c$. It is evident from the inverse demand function, the stationarity condition given in (11) and (12), that the parameter values of the random process (d, q) interact in an important nonlinear way with the production function and the utility function determining the critical slope of the excess price map Π in each country. Thus, as soon as the parameters do not belong to the region Ω^s, the values of (d, q) also induce a nonlinear impact on the local stability of the mapping $\mathcal{G}$ at the symmetric steady state, a feature that is common to many symmetric dynamical systems of the form under consideration here.

The main result of this section can be summarized as follows. Given Assumptions 1–3, symmetry breaking in the sense of Matsuyama (1996) occurs whenever the parameters of the exogenous production process leave a certain well-defined set Ω^s. Propositions 4 and 6 together imply that the set Ω^u is nonempty and that noncyclical divergence occurs in the neighborhood of the symmetric steady state. Thus, the capital stocks of any two countries in a world economy with capital endowments arbitrarily close to the symmetric steady state will *not converge* to the symmetric steady state but rather diverge to an asymmetric stationary allocation of capital. As a consequence, output per capita, wages, and rates of return on capital will differ in the two countries.

Finally, one may also ask in which way asset market integration affects the welfare in each country. Suppose $\overline{c}^i$ and $\underline{c}^i$ denote the steady-state consumption levels in good and bad states in countries $i = h, f$, respectively. Then, equation (10) implies that

$$\underline{c}^i = f(k^i) - k^i \text{ and } \overline{c}^i = f(k^i) - k^i + x^i d. \tag{36}$$

When the world economy converges to a symmetric steady state, then $(k^h, x^h) = (k^f, x^f) = (k^*, 1)$, no matter how unequal the capital endowments in the two countries are when asset markets are integrated. Therefore, the steady-state welfare levels are identical, implying neither gain nor loss of welfare due to the integration of the asset markets. In contrast, when the conditions of symmetry breaking hold with unequal capital endowments at the time of integration, the steady-state welfare level will be lower in the initially poor country, whereas it will reach a higher level in the initially rich country as a result of asset market integration. This may be seen from the following argument.

Suppose that symmetry breaking occurs and the world economy converges to an asymmetric steady state (k^h, k^f), $k^h \neq k^f$. Then, $k^h > k^f$ $(k^h < k^f)$ if and only if $k_t^h > k_t^f$ $(k_t^h < k_t^f)$ at the time of asset market integration. Moreover, asset price equalization implies that equilibrium asset holdings satisfy $x^h > x^f$ if and only if $k^h > k^f$, because the equilibrium asset demand function, defined in (24), is monotonic with respect to steady-state capital. Because the steady-state capital satisfies the inequality $r(k^i) > 1$, it follows from (36) that $(\underline{c}^h, \overline{c}^h) > (\underline{c}^f, \overline{c}^f)$ only if $k_t^h > k_t^f$ at the time of asset market integration. Thus, under the conditions of symmetry breaking, the initially poor country will never improve its steady-state welfare level, whereas the initially rich country will never lose steady-state welfare as a result of asset market integration.

During the transition phase, however, the development of welfare is not necessarily monotonic. With unequal capital levels under autarky initially, the immediate equilibrium asset price after integration will be between the two expected prices under autarky. Therefore, the old generation in the poor country will gain in welfare while the one of the rich country loses. It is unclear for how long this effect can be maintained in the transient phase. In the long run, however, it will not be maintained, leading to lower welfare levels in the poorer country eventually associated with lower levels of capital and of income.

5. A NUMERICAL EXAMPLE

This section presents a parameterized version of the economy described above, providing a large and robust class of examples of economies with stable asymmetric steady states for an admissible configuration of parameters. The example also reveals further insight into the qualitative features of the nonlinearities appearing, giving evidence of the occurrence of symmetry breaking for such economies.

Let the production function be isoelastic and of the form $f(k) := Ak^{\alpha}$, and assume that the utility function u is such that its first derivative is

$$u'(c) := \begin{cases} \exp\left(-a\dfrac{c^{1-b}}{1-b}\right) & \text{if} \quad b \neq 1, \\ c^{-a} & \text{if} \quad b = 1. \end{cases} \tag{37}$$

The derivative form (37) implies a function of absolute risk aversion given by

$$T(c) = -\frac{u''(c)}{u'(c)} = ac^{-b}. \tag{38}$$

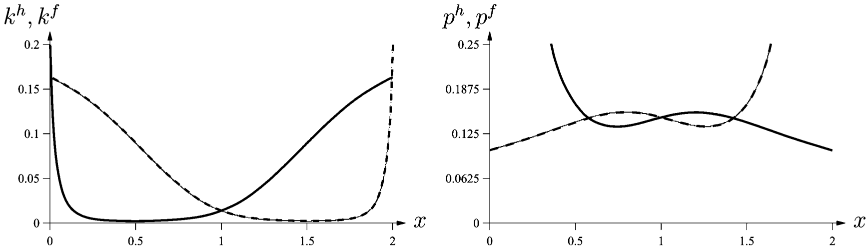
This function has a constant elasticity equal to $-b$. The parameter a measures the scale, and b measures the curvature of absolute risk aversion. When $b = 0$, the absolute risk aversion is constant, $T(c) = a$, whereas it is $T(c) = ac^{-1}$ for $b = 1$, implying constant relative risk aversion. Therefore, this specification includes both the constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) utility functions as special cases.

It is straightforward to verify that the production function satisfies Assumptions 1 and 3 when $0 < \alpha < 0.5$ and that the utility function satisfies Assumption 2 when $a > 0$ and $b \geq 0$. In this case, Proposition 2 implies that for any $x \in (0, 2)$ there exists a unique k satisfying (12). To obtain and analyze the functions $\pi(x)$ and $\Pi(x)$, we use numerical procedures for which we choose the parameter values given in Table 1. Figure 3 displays the graphs of the two functions, which have been calculated numerically for the standard parameter set. The diagram provides the basic intuition of the role of the nonmonotonicity of the function Π for the existence of an asymmetric SREE.

Given the values of the parameters, one finds that the level of stationary capital described by the function $\pi(x)$ is first decreasing steeply for low values of x and increasing for large x; see Figure 3(a). This reversal describes the optimal trade-off between holding the two assets, which derives from the interaction of a strong

TABLE 1. Standard set of parameters

Parameter	Value
A	1.00
α	0.33
d	6.00
q	0.95
a	0.90
b	0.10



(a) $k^h = \pi(x)$ (solid); $k^f = \pi(2-x)$ (broken) (b) $p^h = \Pi(x)$ (solid); $p^f = \Pi(2-x)$ (broken)

FIGURE 3. Existence of asymmetric steady states.

wealth effect induced at low levels of the risky asset and the price effect induced from asset market equilibrium. The primary effect of an increase in asset holdings is a decrease in the equilibrium asset price. As asset holdings increase, young consumers have to bear more risk. However, despite the decrease in the asset price, their willingness to pay for the asset declines more quickly than the increase in their asset demand. As a result, their asset demand becomes relatively inelastic, causing asset market spending to decline and demand for capital to increase again. This in turn causes the stationary capital level $k = \pi(x)$ to rise again. In other words, as asset holding increases, the induced change of the stationary level of capital reverses and increases again, caused by a decrease in spending on assets.

The reversal effect on stationary capital is reinforced when transmitted into the inverse demand function $P(k, x)$, as can be seen from the definition of $\Pi(x) := P[\pi(x), x]$, equation (21). Under the conditions of symmetry breaking, the function Π is no longer monotonic. Given its steep negative slope at low levels of the risky asset, it becomes an *increasing* function at the symmetric steady state $x = 1$, while reverting to a negative slope again for larger values of the risky asset. Combined with the symmetry and the boundary behavior of the function Π, this causes the occurrence of asymmetric steady states, as shown in Figure 3(b).

In addition to the graphical representation, it may be informative to compute the numerical values at the steady states for the parameters chosen. At the symmetric

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steady state, investment in physical capital and spending on the asset market account for 8.3% and 91.7% of the wage income, respectively. The annual rate of return on capital is 5.2%, and the elasticity of asset demand with respect to capital is 0.208. This together with inequality (27) implies a critical value $\delta^* = -0.06$, which is a sufficient condition for symmetry breaking. At the asymmetric steady state, the initially rich and the initially poor countries hold $x^h = 1.42$ and $x^f = 0.58$ units of the asset, respectively. Investment in physical capital is 26.2% and 2.4% of their wage income, respectively, while annual rates of return on capital in the rich and in the poor country are 1.82% and 9.05%, respectively. Steady-state levels of the capital stock, of wage income, and of asset holdings are higher in the initially rich country, implying that the country with a high stationary level of capital attains a high level of welfare as well.

6. SUMMARY AND CONCLUSIONS

The paper analyzes possible implications of unifying a market of a financial paper asset internationally in a two-country model of economic growth. The standard neoclassical growth model of two *identical* countries with capital accumulation and OLG consumers is extended to include uncertainty and a market of a financial asset, enabling the transfer of ownership among generations of an exogenous *random production process*. Given the uncertainty and risk aversion of consumers in an otherwise standard convex environment with perfect competition and rational expectations, the model describes growing economies with two distinct investment opportunities that are not perfect substitutes. Therefore, consumers hold mixed portfolios in general with endogenous substitution effects between the financial asset and real capital.

Under complete separation and autarky of such economies, with perfect competition and rational expectations, the dynamics of capital accumulation is given by a *deterministic* solution with endogenously determined asset prices, in which both economies converge under general assumptions to the *same globally attracting steady state* with identical capital levels, incomes, consumptions, and asset returns. Thus, when the asset markets are separate, capital accumulation and asset price development adjust independently in each country, leading to intercountry income convergence, regardless of whether the countries start at different levels (poor or rich) of initial capital.

However, when a joint asset market is created (or equivalently when the two asset markets are integrated) *prior to reaching stationarity*, market forces generate an unequalizing mechanism that may prevent convergence to identical capital levels in both countries, corresponding to a symmetric steady state of the world economy. The creation of a joint asset market (or equivalently the integration of the two asset markets) between two such economies neutralizes all size and randomness effects between the two economies (assuming that the random process in both countries pays the same dividend). This implies that an asymmetry in the long-run development of the world economy must be attributed to the integration of the asset market alone. The paper identifies two-sided spillover effects between real markets and asset markets induced by portfolio behavior of rational consumers as the major villain of an unequalizing force of growth between otherwise identical countries. Although under autarky these forces are stabilizing within each country, they can create diverging effects after allowing trade and the integration of markets for financial assets.

The paper shows that there exist general conditions of consumer preferences and of production technologies such that additional stable *asymmetric steady states* appear, causing the symmetric steady state to become unstable endogenously. In this case, any heterogeneous initial capital endowments at the time of asset market integration become crucial in determining the long-run development of the otherwise identical economies. Thus, although the clearing of the international asset market still guarantees a uniform asset price and return in that market, capital accumulation and income processes in the two countries diverge, making long-run incomes, consumption levels, and rates of return on capital unequal. The result is shown to be robust and to occur for a general setting, in particular for economies with isoelastic production and isoelastic absolute risk aversion.

NOTES

1. The model introduces the same type of financial asset as in Böhm, Kikuchi, and Vachadze (2007) and Kikuchi (2008).
2. Strict monotonicity and concavity of the production function and the Inada conditions imply the existence of a unique $\widehat{k} > 0$ solving the equation $r(k) = 1$.
3. This feature can be verified directly when production is isoelastic, e.g., for $f(k) = k^{\alpha}$.
4. From the dynamical point of view the MSV solution corresponds to the associated functional rational expectations equilibrium discussed and used in the literature in such cases; see, for example, Spear (1988), McCallum (1998, 1999), and Böhm and Wenzelburger (2004).

REFERENCES

Bianchi, Marco (1997) Testing for convergence: evidence from non-parametric multimodality tests. *Journal of Applied Econometrics* 12, 393–409.

Böhm, Volker, Tomoo Kikuchi, and George Vachadze (2007) Risk, factor substitution, and asset market integration. In Wolfgang Franz, Hans Jürgen Ramser, and Manfred Stadler (eds.), *Dynamik Internationaler Märkte, Wirtschaftswissenschaftliches Seminar Ottobeuren*, pp. 49–69. Tübingen, Germany: Mohr Siebeck.

Böhm, Volker and George Vachadze (2008) Capital accumulation with tangible assets. *Journal of Economic Behavior and Organization* 68, 248–257.

Böhm, Volker and Jan Wenzelburger (2004) Expectational leads in economic dynamical systems. In William A. Barnett, Christophe Deissenberg, and Gustav Feichtinger (eds.), *Economic Complexity: Non-linear Dynamics, Multi-agents Economies, and Learning*, pp. 333–361. Amsterdam: Elsevier.

Durlauf, Steven N. and Paul Johnson (1995) Multiple regimes and cross-country growth behaviour. *Journal of Applied Econometrics* 10, 365–384.

Jones, Charles I. (1997) On the evolution of the world income distribution. *Journal of Economic Perspectives* 3, 19–36.

Kikuchi, Tomoo (2008) International asset market, nonconvergence, and endogenous fluctuations. *Journal of Economic Theory* 139, 310–334.

Matsuyama, Kiminori (1996) Why are there rich and poor countries?: Symmetry-breaking in the world economy. *Journal of the Japanese and International Economies* 10, 419–439.
Matsuyama, Kiminori (2004) Financial market globalization, symmetry-breaking and endogeneous inequality of nations. *Econometrica* 72, 853–884.
McCallum, Bennett T. (1998) Solutions to linear rational expectations models: A compact exposition. *Economics Letters* 61, 143–147.
McCallum, Bennett T. (1999) Role of the minimal state variable criterion in rational expectations models. *International Tax and Public Finance* 6, 621–639.
Quah, Danny (1997) Empirics for economic growth and distribution: stratification, polarization, and convergence club. *Journal of Economic Growth* 2, 27–59.
Spear, Stephen E. (1988) Existence and local uniqueness of functional rational expectations equilibria in dynamic economic system models. *Journal of Economic Theory* 44, 124–155.

APPENDIX

LEMMA 1. *Let $\widehat{k}$ denote the unique solution of $r(k) = 1$ and define the function $\phi : [0, \widehat{k}] \to \mathbf{R}_+$ as*

$$\phi(k) := [w(k) - k]\,[r(k) - 1]\,, \tag{A.1}$$

where w and r are the functions of the wage and the interest rate, respectively. If Assumption 1 is satisfied, then $\phi(0) = \infty$, $\phi(\widehat{k}) = 0$, and ϕ is nonnegative and strictly decreasing.

Proof. Since $r(\widehat{k}) = 1$, it follows that $\phi(\widehat{k}) = 0$. Assumption 1 implies that $kr(k) < w(k)$ on the interval $[0, \widehat{k}]$. This with inequality $r(k) \geq 1$ implies that $w(k) > k$ and thus $\phi(k) \geq 0$ for all $k \in [0, \widehat{k}]$.

To show that $\lim_{k\to 0} \phi(k) = \infty$, we use the following argument. On the one hand, $\lim_{k\to 0} \phi(k) = \lim_{k\to 0} w(k)r(k)$. On the other hand, Assumption 1 implies that for sufficiently small k, $f(k) = Ck^{\alpha(0)}$, where $C > 0$ is some constant and $\alpha(0)$ is the elasticity of production at $k = 0$. This implies that for sufficiently small k, $r(k) = C\alpha(0)k^{\alpha(0)-1}$ and $w(k) = C[1 - \alpha(0)]k^{\alpha(0)}$. Combined with the inequality $\alpha(k) < 0.5$, this implies

$$\lim_{k\to 0} w(k)r(k) = C^2\alpha(0)[1 - \alpha(0)] \lim_{k\to 0} k^{2\alpha-1} = \infty. \tag{A.2}$$

To show that ϕ is a strictly decreasing function, we rewrite equation (A.1) as follows:

$$\phi(k) = w(k)r(k) - kr(k) - w(k) + k = w(k)r(k) - [f(k) - k]. \tag{A.3}$$

On the one hand, $r'(k) < 0$ and $w(k) > kr(k)$ implies

$$\begin{aligned}[w(k)r(k)]' &= w'(k)r(k) + w(k)r'(k) = -kr'(k)r(k) + w(k)r'(k)\\ &= r'(k)\,[w(k) - kr(k)] < 0.\end{aligned} \tag{A.4}$$

On the other hand, $r(k) \geq 1$ for $k \in [0, \widehat{k}]$ implies

$$-\,[f(k) - k]' = -r(k) + 1 < 0. \tag{A.5}$$

It follows from inequalities (A.4) and (A.5) and from equation (A.3) that $\phi' < 0$ for $k \in [0, \widehat{k}]$. ■

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LEMMA 2. *Define the function* $h : [0, \widehat{k}] \times \mathbf{R}_+ \to [0, 1]$ *as*

$$h(k, x) := \frac{qu'[g(k) + xd]}{qu'[g(k) + xd] + (1 - q)u'[g(k)]}, \tag{A.6}$$

where u *is the utility function,* $g(k) := f(k) - k$*, and* f *is the production function. If Assumption 2 is satisfied, then* h *is nondecreasing with respect to its first and nonincreasing with respect to its second argument.*

Proof. Continuous differentiability of h follows, because u and g are twice continuously differentiable. Differentiating (A.6) with respect to k implies

$$h_k(k, x) = q(1 - q)\frac{u''[g(k) + xd]u'[g(k)] - u'[g(k) + xd]u''[g(k)]}{\{qu'[g(k) + xd] + (1 - q)u'[g(k)]\}^2}g'(k). \tag{A.7}$$

The numerator of (A.7) can be further simplified:

$$u''[g(k) + xd]u'[g(k)] - u'[g(k) + xd]u''[g(k)]$$

$$= \frac{u'[g(k)]}{u'[g(k) + xd]}\{T[g(k)] - T[g(k) + xd]\}. \tag{A.8}$$

Because $T(c)$ is nonincreasing and $g'(k) > 0$ on the interval $[0, \widehat{k}]$, (A.7) and (A.8) imply that $h_k(k, x) \geq 0$.

To show that $h_x(k, x) \leq 0$ for a given $k \in [0, \widehat{k}]$, we take the natural logarithm on both sides of (A.6) and then differentiate it. We obtain

$$\frac{h_x(k, x)}{h(k, x)} = -T[g(k) + xd]\,[1 - h(k, x)]\,g'(k). \tag{A.9}$$

(A.9) implies that

$$h_x(k, x) = -T[g(k) + xd]\,[1 - h(k, x)]\,h(k, x)g'(k). \tag{A.10}$$

Since $h(k, x) \in [0, 1]$ and $g'(k) \geq 0$, (A.10) with Assumption 2 implies that $h_x(k, x) \leq 0$. ■

Proof of Proposition 1. Rewrite the consumer's optimization problem as

$$\max_{x \in B(w,p)} qu(\overline{c}) + (1 - q)u(\underline{c}), \tag{A.11}$$

where $\overline{c}_1 = wr_1 + xd - x(pr_1 - p_1)$ and $\underline{c}_1 = wr_1 - x(pr_1 - p_1)$. Let $\mathcal{L}(x) := qu(\overline{c}_1) + (1 - q)u(\underline{c}_1)$; then $\mathcal{L}'(x) = qu'(\overline{c}_1)[d - (pr_1 - p_1)] - (1 - q)u'(\underline{c}_1)(pr_1 - p_1)$. From the concavity of the utility function, it follows that $\mathcal{L}''(x) < 0$ for any $x \geq 0$.

(1) Suppose that $pr_1 \geq p_1 + qd$, then the optimal asset demand is zero. Since

$$\mathcal{L}'(0) = u'(wr_1)\,[qd - (pr_1 - p_1)] \leq 0 \tag{A.12}$$

and $\mathcal{L}'' < 0$, it follows from the Kuhn–Tucker conditions that $x = 0$ is an optimal solution.

(2) Suppose that $pr_1 \leq p_1$. Then consumptions in good and bad states, $\overline{c}$ and $\underline{c}$, are both increasing functions of x. Because the utility function is strictly increasing, it follows that

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the agent will invest all his wage income in the asset market and make no investment in physical capital, and thus the optimal demand is $x = w/p$.

(3) Suppose that $p_1 < pr_1 < p_1 + qd$, and let $\overline{x} := wr_1/(pr_1 - p_1) > 0$. Then, depending on whether $\mathcal{L}'(\overline{x})$ is positive or negative, we can have either a corner or an interior solution. A unique corner solution $x = w/p$ exists when

$$\mathcal{L}'(\overline{x}) = qu'(\overline{x}d)\,[d - (pr_1 - p_1)] - (1-q)u'(0)\,(pr_1 - p_1) > 0. \qquad \textbf{(A.13)}$$

Otherwise, there exists a unique and interior solution solving the equation $\mathcal{L}'(x) = 0$. Let $x = \varphi_m(p_1 - pr_1, wr_1)$ denote the solution. Applying the implicit function theorem, one finds that φ_m is increasing with respect to both arguments. In addition, asset demand satisfies the boundary condition. As $p \downarrow p_1^* := p_1/r_1$, then the asset demand grows unboundedly. This implies that there exists a constant $p_2^* \in (p_1^*, p_3^*)$ such that

$$p_2^* \varphi_m(p_1 - p_2^* r_1, wr_1) = w \qquad \blacksquare$$

.

Proof of Proposition 3. Let us first show that $\mathcal{G}' \leq 0$ implies a contradiction. Differentiating the price law (16), we obtain that at an SREE, $k = \mathcal{G}(k)$, the following equation should be satisfied:

$$\mathcal{P}'(k) - \mathcal{S}_1 w'(k) = \mathcal{S}_2 r'(k)\mathcal{G}'(k) + \mathcal{S}_3 \mathcal{P}'(k)\mathcal{G}'(k), \qquad \textbf{(A.14)}$$

where $\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{S}_3$ are the partial derivatives of the function $\mathcal{S}$ with respect to its first, second, and third arguments, respectively.

Since $\mathcal{S}_1 \in [0, 1]$ and $\mathcal{G}' < 0$, it follows that the left-hand side of (A.14) is positive because

$$\mathcal{P}'(k) - \mathcal{S}_1 w'(k) = w'(k) - \mathcal{G}'(k) - \mathcal{S}_1 w'(k) = (1 - \mathcal{S}_1)w'(k) - \mathcal{G}'(k) > 0. \qquad \textbf{(A.15)}$$

The inequalities $\mathcal{S}_2 < 0$, $\mathcal{S}_3 > 0$, $r' < 0$, $\mathcal{P}' = w' - \mathcal{G}' > 0$, and $\mathcal{G}' < 0$ imply that the right-hand side of (A.14) is nonpositive, because

$$\mathcal{S}_2 r'(k)\mathcal{G}'(k) + \mathcal{S}_3 \mathcal{P}'(k)\mathcal{G}'(k) = \left[\mathcal{S}_2 r'(k) + \mathcal{S}_3 \mathcal{P}'(k)\right]\mathcal{G}'(k) \leq 0. \qquad \textbf{(A.16)}$$

But the inequalities (A.15) and (A.16) contradict (A.14), and thus $\mathcal{G}' > 0$.

Now, let us show that $0 < \mathcal{G}'(0) = \gamma < \infty$ implies a contradiction. By dividing both sides of (A.14) by $w'(k)$, we obtain

$$\frac{\mathcal{P}'(k)}{w'(k)} - \mathcal{S}_1 = \left[\mathcal{S}_2 \frac{r'(k)}{w'(k)} + \mathcal{S}_3 \frac{\mathcal{P}'(k)}{w'(k)}\right]\mathcal{G}'(k). \qquad \textbf{(A.17)}$$

Taking the limit of both sides of (A.17) as $k \to 0$, we obtain

$$\lim_{k\to 0} \frac{\mathcal{P}'(k)}{w'(k)} - \mathcal{S}_1 = 1 - \mathcal{S}_1 \in [0, 1] \qquad \textbf{(A.18)}$$

and

$$\lim_{k\to 0}\left[\mathcal{S}_2 \frac{r'(k)}{w'(k)} + \mathcal{S}_3 \frac{\mathcal{P}'(k)}{w'(k)}\right]\mathcal{G}'(k) = \lim_{k\to 0}\left(-\mathcal{S}_2 \frac{1}{k} + \mathcal{S}_3\right)\gamma = \infty. \qquad \textbf{(A.19)}$$

(A.17), (A.18), and (A.19) imply a contradiction, and thus $\mathcal{G}'(0) = \infty$.

Because the time-one map of capital accumulation is a strictly increasing function with two fixed points $k = 0$ and $k = k^*$, $\mathcal{G}'(0) = \infty$ implies the instability of the corner steady state and the stability of the interior SREE. ■

Proof of Proposition 4. To show the existence of interior asymmetric steady states, we rely on the property of the function Ψ. Since $\Psi(1) = 0$, $\Psi(0) = \infty$, and $\Psi(2) = -\infty$, it follows that the condition $\Psi'(1) > 0$ is sufficient for the existence of interior asymmetric steady states. (22) implies that $\Psi'(1) = 2\Pi'(1)$, and thus $\Pi'(1) > 0$ is sufficient for the existence of interior asymmetric steady states.

Since $P[k, X(k, p)] \equiv p$, we obtain that $X_k(k^*, p^*) = -P_k(k^*, 1)/P_x(k^*, 1)$, and inequality (27) implies

$$\epsilon^* < \frac{1}{s^*}\left(\frac{\alpha^*}{\sigma^*} - 1\right) + 1 \Leftrightarrow -\frac{kP_k(k^*, 1)}{P_x(k^*, 1)} < \frac{w(k^*)}{p^*}\frac{w'(k^*)k^* - k^*}{w(k^*)}. \tag{A.20}$$

Inequality (A.20) implies

$$-\frac{P_k(k^*, 1)}{P_x(k^*, 1)} < \frac{w'(k^*) - 1}{p^*} \Leftrightarrow p^*P_k(k^*, 1) + [w'(k^*) - 1]P_x(k^*, 1) < 0. \tag{A.21}$$

The two identities $\Pi(x) \equiv P[\pi(x), x]$ and $w[\pi(x)] - \pi(x) \equiv \Pi(x)x$ imply that

$$\Pi'(1) = P_k(k^*, 1)\pi'(1) + P_x(k^*, 1) \text{ and } \left[w'(k^*) - 1\right]\pi'(1) = \Pi(1) + \Pi'(1). \tag{A.22}$$

By solving the above system with respect to $\pi'(1)$ and $\Pi'(1)$ we obtain

$$\Pi'(1) = \frac{p^*P_k(k^*, 1) + [w'(k^*) - 1]P_x(k^*, 1)}{w'(k^*) - 1 - P_k(k^*, 1)} \text{ and } \pi'(1) = \frac{p^* + P_x(k^*, 1)}{w'(k^*) - 1 - P_k(k^*, 1)}. \tag{A.23}$$

Stability of the unique interior steady state in the closed economy implies that $w'(k^*) - 1 - P_k(k^*, 1) < 0$. Combined with inequality (A.23), this implies $\Pi'(1) > 0$, if inequality (A.22) is satisfied. ■

Proof of Proposition 6. Evaluating the Jacobian matrix at the symmetric steady state, one finds that the trace T and the determinant D are related by

$$T = 2\mathcal{G}_1 \text{ and } D = \mathcal{G}_1^2 - \mathcal{G}_2^2, \tag{A.24}$$

where $\mathcal{G}_1 \equiv \mathcal{G}_1(k^*, k^*)$ and $\mathcal{G}_2 \equiv \mathcal{G}_2(k^*, k^*)$ are the derivatives of the function $\mathcal{G}$ with respect to its first and second arguments, respectively, evaluated at the symmetric steady state. Since $T^2 - 4D = 4\mathcal{G}_2^2 > 0$, it follows that both roots of the characteristic polynomial

$$\lambda^2 - T\lambda + D = 0 \tag{A.25}$$

are real with $\lambda_1 = \mathcal{G}_1 + \mathcal{G}_2$ and $\lambda_2 = \mathcal{G}_1 - \mathcal{G}_2$. Equations (31) and (32) imply that the functions $\mathcal{G}_1$ and $\mathcal{G}_2$ satisfy the system of equations

$$\begin{cases} \mathcal{G}_1 \equiv w' - \left[\varphi_1 w' + \varphi_2 r'\mathcal{G}_1 + \varphi_3\left(\mathcal{P}_1\mathcal{G}_1 + \mathcal{P}_2\mathcal{G}_2\right) + \varphi_4\mathcal{P}_1\right]\mathcal{P} - \varphi\mathcal{P}_1, \\ \mathcal{G}_2 \equiv -\left[\varphi_2 r'\mathcal{G}_2 + \varphi_3\left(\mathcal{P}_1\mathcal{G}_2 + \mathcal{P}_2\mathcal{G}_1\right) + \varphi_4\mathcal{P}_2\right]\mathcal{P} - \varphi\mathcal{P}_2, \end{cases} \tag{A.26}$$

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where

$$\begin{cases} \mathcal{P}_1 \equiv \mathcal{S}_1 w' + \mathcal{S}_3 r' \mathcal{G}_1 + \mathcal{S}_4 r' \mathcal{G}_2 + \mathcal{S}_5 \left(\mathcal{P}_1 \mathcal{G}_1 + \mathcal{P}_2 \mathcal{G}_2\right), \\ \mathcal{P}_2 \equiv \mathcal{S}_2 w' + \mathcal{S}_3 r' \mathcal{G}_2 + \mathcal{S}_4 r' \mathcal{G}_1 + \mathcal{S}_5 \left(\mathcal{P}_1 \mathcal{G}_2 + \mathcal{P}_2 \mathcal{G}_1\right). \end{cases} \quad \textbf{(A.27)}$$

Clearly, at the symmetric steady state, $\mathcal{S}_1 = \mathcal{S}_2$, and $\mathcal{S}_3 = \mathcal{S}_4$, and therefore, $\mathcal{P}_1 = \mathcal{P}_2$ holds. This property together with equation (A.27) implies that

$$\mathcal{P}_1 = \mathcal{S}_1 w' + \mathcal{S}_3 r'(\mathcal{G}_1 + \mathcal{G}_2) + \mathcal{S}_5 \mathcal{P}_1 (\mathcal{G}_1 + \mathcal{G}_2). \quad \textbf{(A.28)}$$

Applying arguments similar to those used in the proof of Proposition 3, one finds that λ_1 satisfies $\lambda_1 = \mathcal{G}_1 + \mathcal{G}_2 \in (0, 1)$. From the system of equations (A.26), one obtains that the second root of the characteristic equation satisfies

$$\lambda_2 = \mathcal{G}_1 - \mathcal{G}_2 = \frac{w'\left(1 - \varphi_1 \mathcal{P}\right)}{1 + \varphi_2 r' \mathcal{P}} > 0, \quad \textbf{(A.29)}$$

since $\varphi_1 \mathcal{P} < 1$, $\varphi_2 < 0$, and $r' < 0$. Therefore, $\lambda_1 \in (0, 1)$ and $\lambda_2 > 0$ imply that the symmetric steady state can lose its stability only by undergoing a fold bifurcation. This proves properties 1 and 3.

To show property 2, we first show that when $\delta^* = 0$; then λ_2 defined in equation (A.29) satisfies $\lambda_2 = 1$. Equation $\delta^* = 0$ implies that

$$\epsilon^* = \frac{1}{s^*}\left(\frac{\alpha^*}{\sigma^*} - 1\right) + 1 \Leftrightarrow k^* X(k^*, p^*) = \frac{w^*}{p^*}\left[\frac{k^* f'(k^*)}{f(k^*)} \frac{f(k^*) w'(k^*)}{f'(k^*) w(k^*)} - 1\right] + 1. \quad \textbf{(A.30)}$$

(A.30) implies

$$k^* X(k^*, p^*) = \frac{k^* w'(k^*) - w(k^*) + p^*}{p^*} \Leftrightarrow p^* X(k^*, p^*) = w'(k^*) - 1. \quad \textbf{(A.31)}$$

Since $X(k, p) = \varphi[w(k), r(k), p, p]$, it follows from (A.31) that

$$p^* \left[\varphi_1 w'(k^*) + \varphi_2 r'(k^*)\right] = w'(k^*) - 1. \quad \textbf{(A.32)}$$

Combined with (A.29), this implies $\lambda_2 = 1$. Therefore, from (A.30) and (A.31) one has that $(d, q) \in \Omega^u := \{(d, q) \in \Omega | \delta^* < 0\}$ implies $\lambda_2 > 1$ and $(d, q) \in \Omega^s$ implies $\lambda_2 < 1$. ∎