Macroeconomic Dynamics, **12** (Supplement 1), 2008, 90–99. Printed in the United States of America.
DOI: 10.1017/S1365100507070083

RAMSEY MONETARY POLICY WITH CAPITAL ACCUMULATION AND NOMINAL RIGIDITIES

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Recent literature on the design of optimal monetary policy has shown that deviations from price stability are small whenever prices are sticky. This paper reconsiders this issue by introducing capital accumulation in the model. Optimal monetary policy in this setup implies small deviations from price stability. The monetary authority optimally uses inflation as an explicit tax on monopolistic profits to reduce the price markup across states. Variable markup is achieved in this setup because the share of investment demand over output varies across states and in response to TFP shocks.

Keywords: Optimal Monetary Policy, Capital Accumulation, Nominal Rigidities

1. INTRODUCTION

The analysis of the foundations of optimal monetary policy recently has been the object of an intense research program in macroeconomics. Systematic attention has been devoted to the optimality of price stability policies. Zero inflation is the core result in the analysis of Woodford (2003) and Clarida, Gali, and Gertler (2000), who consider a monopolistic competitive framework with sticky prices á la Calvo (1983). Those authors assume the existence of a complementary policy instrument (e.g., a fiscal subsidy) that offsets the wedge represented by the monopolistic markup and analyze optimal monetary policy by resorting on log-linear approximation of the competitive equilibrium conditions and on a quadratic approximation of the households' utility function. In this context, the monetary authority optimally sets zero inflation to eliminate relative price dispersion. Recently, Khan, King, and Wolman (2003) and Schmitt-Grohe and Uribe (2004) have shown, using the Ramsey approach, that in the presence of sticky prices optimal policy implies small deviations from price stability and departure from the Friedman rule. Finally, Adao, Correia, and Teles (2003) have shown, by using a model with prices set one period in advance, that zero inflation is the optimal policy under a certain class of preferences.

I thank Albert Marcet, Peter Sorensen, Pedro Teles, two anonymous referees for useful suggestions and participants at the conference on Dynamic Macroeconomic Theory at the Institute of Economics of the University of Copenhagen. I gratefully acknowledge financial support from the DSGE grant of the Spanish Ministry of Education and the Marie Curie fellowship. All errors are my own responsibility. Address correspondence to: Ester Faia, Department of Economics, Universitat Pompeu Fabra, Ramon Trias Fargas 25-27, 08005, Barcelona, Spain; e-mail: ester.faia@upf.edu.

 1365-1005/08 $18.00

This paper examines this issue in a model with sticky prices and capital accumulation. Optimal monetary policy is studied using the Ramsey approach. The introduction of capital accumulation is essential because it accounts for a big portion of business cycle fluctuations and because investment is an important determinant of the monetary transmission mechanism. Optimal monetary policy in this setup implies small deviations from price stability for any class of preferences. The monetary authority optimally uses inflation as an explicit tax on monopolistic profits to reduce the price markup across states. Variable markup is achieved in this setup because the share of investment demand over output varies across states and in response to TFP shocks. Quantitative responses also show that the optimal volatility of inflation increases when the markup increases. The main results in our context hinge on the assumption that the fiscal system is incomplete; hence, it does not have access to a distortionary tax rate on profits.[1]

2. STRUCTURE OF THE DISTORTED COMPETITIVE ECONOMY

Agents maximize the following discounted sum of utilities:[2]

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \tag{1}$$

where C_t denotes aggregate consumption. The households receive at the beginning of time t a real labor income $\frac{W_t}{P_t} N_t$. To ensure their consumption pattern against random shocks at time t, they decide to spend $\nu_{t,t+1} B_{t+1}$ in real-state contingent securities, where $\nu_{t,t+1} \equiv \nu(s^{t+1}|s^t)$ is the pricing kernel of the state contingent portfolio. Each state-contingent asset B_{t+1} pays one unit of domestic currency at time $t+1$ and in state s^{t+1}. Agents also invest in new physical capital, K_{t+1}, and rent it to the production sector at a rate Z_{t+1} one period later. Capital gets depreciated at a rate δ. Agents also receive transfers from the government, T_t, and profits as owner of the monopolistic sector, $\frac{\Theta_t}{P_t}$. Hence, the sequence of budget constraints in real terms reads as follows:

$$C_t + \nu_{t,t+1} B_{t+1} + K_{t+1} - (1-\delta) K_t \leq \frac{W_t}{P_t} N_t + T_t + \frac{\Theta_t}{P_t} + Z_t K_t + B_t. \tag{2}$$

Households choose the set of processes $\{C_t, N_t\}_{t=0}^{\infty}$ and assets $\{B_{t+1}, K_{t+1}\}_{t=0}^{\infty}$, taking as given the set of processes $\{P_t, W_t, Z_t, \nu_{t,t+1}\}_{t=0}^{\infty}$ and the initial wealth $B_0 + K_0$ so as to maximize (1) subject to (2). The following optimality conditions hold:

$$\frac{W_t}{P_t} = -\frac{U_{n,t}}{U_{c,t}}, \tag{3}$$

$$\beta \frac{U_{c,t+1}}{U_{c,t}} = \nu_{t,t+1}, \tag{4}$$

$$U_{c,t} = \beta E_t \{[Z_{t+1} + (1-\delta)] U_{c,t+1}\}. \tag{5}$$

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Equation (3) gives the optimal choice for labor supply. Equation (4) gives the price of the Arrow-Debreu security. Equation (5) is the optimality condition with respect to capital. Optimality requires that the first-order conditions and a No-Ponzi game conditions are simultaneously satisfied.

2.1. Monopolistic Production Sector

Each monopolistic firm assembles labor and capital to operate a constant return to scale production function for the variety i of the intermediate good, $Y_t(i) = A_t F(N_t(i), K_t(i))$, where A_t is a common productivity shock. Varieties are aggregated according to a Dixit-Stiglitz function, $Y_t = \int_0^1 [Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di]^{\frac{\varepsilon}{\varepsilon-1}}$, which implies the following optimal demand for variety: $Y_t(i) = (\frac{P_t(i)}{P_t})^{-\epsilon}(C_t + I_t + G_t)$, where G_t represents government expenditure and $I_t = K_{t+1} - (1-\delta)K_t$ represents investment. Each firm i has monopolistic power in the production of its own variety and therefore has leverage in setting the price. In so doing, it faces a quadratic cost equal to $\varkappa_t(i) \equiv \frac{\theta}{2}(\frac{P_t(i)}{P_{t-1}(i)} - 1),^2$ where the parameter θ measures the degree of nominal price rigidity. The problem of each domestic monopolistic firm is the one of choosing the sequence $\{K_t(i), N_t(i), P_t(i)\}_{t=0}^{\infty}$, in order to maximize the sum of the expected discounted real profits, $\frac{\Theta_t}{P_t} \equiv \frac{P_t(i)Y_t(i) - (W_t N_t(i) + Z_t K_t(i)) - \varkappa_t(i)}{P_t}$, subject to the demand constraint for each variety. Let us define mc_t as the lagrange multiplier on the constraint. The first-order conditions read as follows:

$$\frac{W_t}{P_t} = mc_t A_t F_{n,t}; \qquad \frac{Z_t}{P_t} = mc_t A_t F_{k,t}, \tag{6}$$

$$0 = \frac{P_t(i)}{P_t}^{-\varepsilon} \frac{Y_t}{P_t} \left\{ (1-\varepsilon) + \varepsilon mc_t \left[\frac{P_t(i)}{P_t} \right]^{-1} \right\} - \theta \left[\frac{P_t(i)}{P_{t-1}(i)} - 1 \right] \frac{1}{P_{t-1}(i)}$$
$$+ \beta\theta E_t \left\{ \left[\frac{P_{t+1}(i)}{P_t(i)} - 1 \right] \frac{P_{t+1}(i)}{P_t(i)^2} \right\}. \tag{7}$$

2.2. The Government

The government has to finance an exogenous stream of government purchases, G_t,[3] with lump-sum taxes. As government debt is irrelevant in this environment, we can write the government budget constraint as a balance budget constraint. Therefore, $G_t = T_t$.

3. OPTIMAL MONETARY POLICY PROBLEM

The optimal policy is determined by a monetary authority that maximizes the discounted sum of utilities of all agents given the constraints of the competitive economy. I assume that *ex-ante commitment* is feasible. The first task is to select the minimal set of competitive equilibrium conditions that represent the relevant constraints in the planner's optimal policy problem following the primal approach described in Lucas and Stokey (1983).[4] The constraints for the monetary authority can be summarized as follows:

$$U_{c,t} - \beta E_t \left\{ \left[-\frac{U_{n,t+1} F_{k,t+1}}{F_{n,t+1} U_{c,t+1}} + (1-\delta) \right] U_{c,t+1} \right\} = 0, \tag{8}$$

$$\theta U_{c,t} \pi_t (\pi_t - 1) - \beta \theta U_{c,t+1} \pi_{t+1} (\pi_{t+1} - 1) + U_{c,t} \varepsilon A_t F(N_t, K_t) \left(-\frac{U_{n,t}}{U_{c,t} A_t F_{n,t}} - \frac{\varepsilon - 1}{\varepsilon} \right) = 0, \tag{9}$$

$$A_t F(N_t, K_t) - C_t - K_{t+1} + (1-\delta) K_t - G_t - \varkappa_t = 0. \tag{10}$$

The monetary authority will choose the policy instrument, the nominal interest rate, to implement the optimal allocation obtained as a solution to the following Lagrangian problem.

DEFINITION 1. *Let* $\lambda_{1,t}$, $\lambda_{2,t}$, $\lambda_{3,t}$ *represent the Lagrange multipliers on the constraints* (8), (9), *and* (10), *respectively. For given* B_0, K_0 *and processes for the exogenous shocks* $\{A_t, G_t\}_{t=0}^{\infty}$, *the allocations plans for the control variables* $\Xi_t \equiv \{C_t, N_t, K_{t+1}, \pi_t\}_{t=0}^{\infty}$ *and for the co-state variables* $\Lambda_t \equiv \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}\}_{t=0}^{\infty}$ *represent a first best constrained allocation if they solve the following maximization problem:*

$$Min_{\{\Lambda_t\}_{t=0}^{\infty}} Max_{\{\Xi_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right\}, \tag{11}$$

subject to (8), (9), *and* (10).

Notice that constraints (8) and (9) exhibit future expectations of control variables. For this reason, the maximization problem is intrinsically nonrecursive.[5] As shown by Marcet and Marimon (1999), a formal way to rewrite the same problem in a recursive stationary form is to enlarge the planner's state space with additional (pseudo) co-state variables, which bear the meaning of tracking, along the dynamics, the value to the planner of committing to the preannounced policy plan. The co-state variables $\chi_{1,t}$ and $\chi_{2,t}$ obey to the following law of motions, $\chi_{1,t+1} = \lambda_{1,t}$, $\chi_{2,t+1} = \lambda_{2,t}$. The first-order conditions of the maximization problem described earlier are in Part B of the technical appendix.

3.1. Long-Run Behavior Under Optimal Policy

To assess the optimal monetary policy design in the long run, a distinction must be made between the constrained and the unconstrained optimal inflation rate. The former is the inflation rate that maximizes households' instantaneous utility under the constraint that the steady-state conditions are imposed ex-ante.[6] In dynamic

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economies with discounted utility, the golden rule does not necessarily coincide with the unconstrained optimal long-run rate of inflation, which is the one to which the planner would like the economy to converge to if allowed to undertake its optimization unconditionally. The latter is obtained by imposing steady state conditions ex-post on the first-order conditions of the Ramsey plan.

I first analyze the long-run constrained optimal policy, or golden rule. Before deriving this policy formally, a few comments are worth to describe the long run trade-offs faced by the monetary authority. The policymaker faces a tension between the sticky prices distortion, which calls for price stability to close the gap given by $\frac{\theta}{2}(\pi_t - 1)^2$, and the markup distortion that by reducing output calls for variable inflation across states and times. To analyze this tension, it is instructive to examine the steady-state version of the optimal pricing condition (the long-run Phillips curve):

$$\mu(\pi, N) = \frac{\varepsilon F(K, N)}{\theta\pi(\pi - 1)(1 - \beta) + (\varepsilon - 1)F(K, N)}. \qquad \textbf{(12)}$$

Under costly adjustment, $\theta > 0$, an increase in inflation reduces the markup distortion. Optimality requires that a policymaker, endowed with a single instrument, must trade off between the two distortion and set the inflation rate between zero and the level that would force $\mu(\pi, N)$ to zero.

Formally, the golden rule inflation rate is obtained by maximizing the per period utility under the constraint (12) and the steady state version of the resource constraint. Figure 1 shows the changes in the optimal inflation rate, employment, investment and consumption to a change in the elasticity of demand, ε.[7] The inflation rate is always positive. Furthermore, an increase in the elasticity of demand, which corresponds to a decrease in markup, implies a decrease in the optimal inflation rate.[8] Next, I move to analzye the uncosntrained long run optimal policy.

LEMMA 1. *The (net) inflation rate associated with the unconstrained long-run optimal policy is zero.*

Proof. Consider the steady-state version of the first-order condition with respect to inflation of the Ramsey plan described in definition 1 (the appendix is available at http://www.econ.upf.es/~faia/). Because in steady-state $\lambda_2 = \chi_2$, and given that $\theta > 0$ and that $\lambda_1 > 0$, it follows that $\pi = 1$. ■

3.2. Nonoptimality of the Zero Inflation Policy in Response to Shocks

Under flexible prices, the wedge between the marginal rate of substitution between labor and consumption and the marginal rate of transformation is constant and equal to the markup. Under sticky prices, this wedge is constant on average but can vary across states. This is so because the share of investment demand over output changes in response to TFP shocks. This variable wedge can then be used to boosts demand in response to shocks.

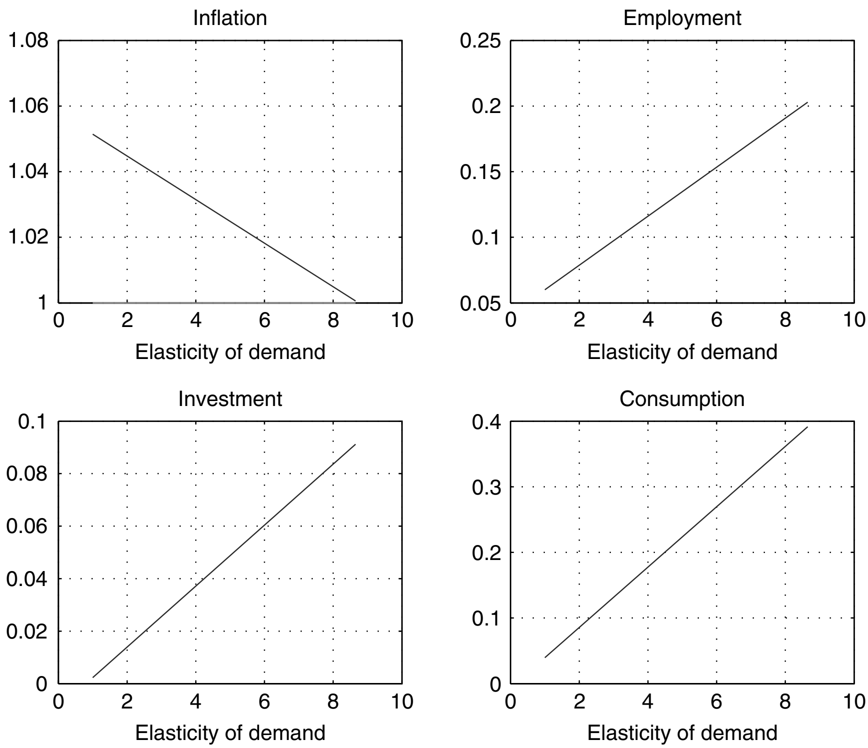

FIGURE 1. Golden rule in the Ramsey policy: Effects of varying the elasticity of demand.

LEMMA 2. *The set of implementable allocations under sticky prices contains the corresponding set under flexible prices. Therefore, the optimal allocation under sticky prices make the households at least as well off as under flexible prices.*

Proof. The feasibility constraint, equation (10), and the intertemporal condition on consumption, given by equation (8), are the same in the two environments. If we impose a zero inflation policy, the pricing condition for firms under sticky prices, equation (9), replicates the following pricing condition in the flexible price environment:

$$-\frac{U_{n,t}}{U_{c,t}A_t F_{n,t}} = \frac{\varepsilon - 1}{\varepsilon}. \quad \textbf{(13)}$$

LEMMA 3. *The zero inflation policy is not an optimal solution to the Ramsey plan under sticky prices unless* $\lambda_{2,t} = \chi_{2,t}$.

Proof. From the first-order condition with respect to inflation of the Ramsey plan described in definition 1 (the appendix is available at: http://www.econ.upf.es/~faia/), it is immediate to see that the solution $\pi = 1$ for the gross inflation rate cannot be a solution to Ramsey plan, unless $\lambda_{2,t} = \chi_{2,t}$. ■

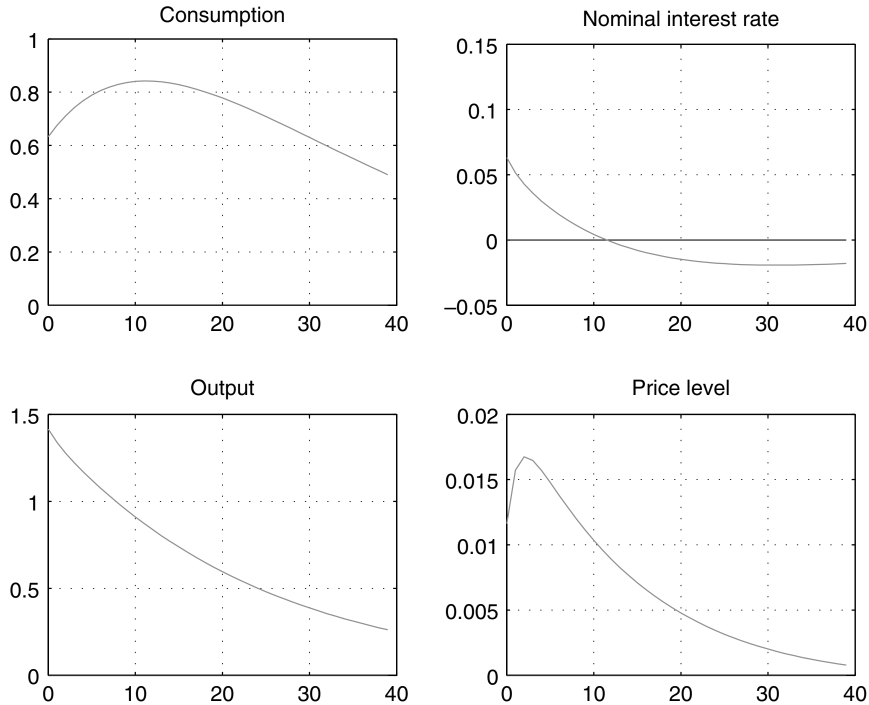

FIGURE 2. Impulse responses under optimal policy to a 1% increase in productivity.

3.3. Optimal Stabilization Policy in Response to Shocks

Let's now analyze the dynamic properties of the Ramsey plan in a calibrated version of the model.[9] Period utility function takes the form $U(C_t,\ N_t) = \log(C_t) + \tau \log(1 - N_t)$ and τ is chosen so as to generate a steady-state level of employment of 0.3. The discount factor β is set to 0.99, so that the annual real interest rate is equal to 4%. The share of capital in the production function, α, is 0.35, the quarterly depreciation rate, δ, is 0.025. Following Basu and Fernald (1997), the value added markup of prices over marginal cost is set equal to 0.2. This generates a value for the price elasticity of demand, ε, of 6. Given the assigned value for the price markup and consistently with Sbordone (1998) the price adjustment cost parameter is set equal to $\theta = 17.5$. The technology process follows an AR(1) with persistence equal to 0.9. Log-government consumption evolves according to the following exogenous process, $\ln(G_t/G) = \rho_g \ln(G_{t-1}/G) + \varepsilon_t^g$, where the steady-state share of government consumption, G, is set so that $G/Y = 0.25$ and ε_t^g is an i.i.d. shock with standard deviation σ_g. Empirical evidence for the United States in Perotti (2004) suggests $\sigma_g = 0.008$ and $\rho_g = 0.9$.

Figure 2 shows impulse response functions to a 1% positive productivity shock for consumption, nominal interest rate, output, and the price level. Because of the

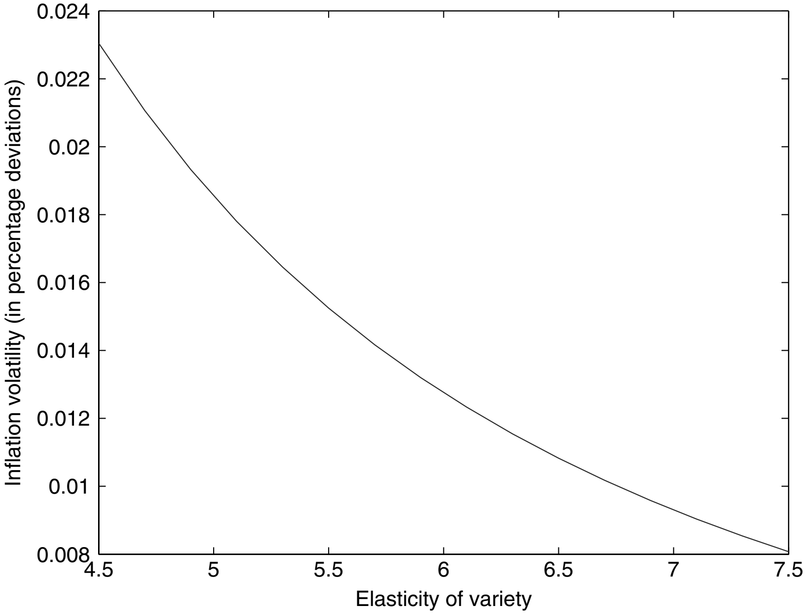

FIGURE 3. Optimal inflation volatility: Effect of varying elasticity of variety.

increase in the marginal productivity of capital, output and consumption increase. Optimal monetary policy is pro-cyclical because under sticky prices an increase in inflation by boosting demand reduces the markup. We also observe nonstationarity of the price level, which is a typical feature of history-dependent policies. Indeed, given forward looking policy, commitment induces expectations of future overshooting in the path of inflation. The impulse responses also show that the nominal interest rate moves significantly from zero implying the nonoptimality of the Friedman rule. Figure 3 shows that the optimal volatility of inflation decreases when the elasticity of demand increases (the markup decreases). This is so because a lower markup reduces the desire of the policymaker to inflate the economy and to boost demand.

In response to government expenditure shocks, optimal monetary policy implies a fall in consumption and in the price level.[10] This is consistent with the findings of Khan, King, and Wolman (2003).[11] In order to generate a fall in consumption, the government increases the nominal interest rate and this also implies a fall in the price level. Overall, however, the deviations of the price level from the full-price stability case are rather small.

4. CONCLUSIONS

This paper analyzed optimal monetary policy in a model with nominal rigidities and capital accumulation. The full-price stability across states and times is not

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optimal. Deviations from zero inflation are related to the size of the monopolistic distortion. Throughout this paper, I remain consistent to a public finance approach by an explicit consideration of all the distortions that are relevant to the Ramsey planner.

NOTES

1. The assumption of fiscal incompleteness embeds the idea that implementability delays and uncertainty in the political process render the fiscal policy less effective than the monetary policy.

2. Let $s^t = \{s_0, \dots s_t\}$ denote the history of events up to date t, where s_t is the event realization at date t. The date 0 probability of observing history s^t is given by $\rho(s^t)$. The initial state s^0 is given so that $\rho(s^0) = 1$. Henceforth, and for the sake of simplifying the notation, let us define the operator $E_t\{.\} \equiv \sum_{s_{t+1}} \rho(s^{t+1}|s^t)$ as the mathematical expectation over all possible states of nature conditional on history s^t.

3. These purchases are obtained by aggregating different varieties with a Dixit-Stiglitz aggregator.

4. See Part A of the appendix available at http://www.econ.upf.es/~faia/ to see how to cast the competitive equilibrium relations of the present model into the primal form, which involves a minimal set of constraints for the monetary authority.

5. See Kydland and Prescott (1980).

6. Following King and Wolman (1999) this can be defined as the policymaker's golden rule.

7. Calibration of the remaining parameters is described in Section 3.3 of this paper.

8. It is worth noticing that varying the size of the distortion from zero to empirically plausible values has big impacts on consumption, hence on welfare. This is so because long-run effects of Ramsey policy (mostly under the golden rule) are typically big. Indeed, the reason for which the Ramsey method has an advantage over other approximated methods for calculating optimal policy is that it allows to account for the big welfare effects of the distortions over the long run. On the contrary, welfare effects of changing the distortions over the business cycle are typically small.

9. Technically, I compute the stationary allocations that characterize the deterministic steady-state of the first-order conditions to the Ramsey plan. I then compute a second-order approximation of the respective policy functions in the neighborhood of the same steady-state. This amounts to implicitly assuming that the economy has been evolving and policy been conducted around such a steady-state already for a long period of time.

10. Results are not reported in the text for brevity but are available in Part B of the appendix available at http://www.econ.upf.es/~faia/.

11. They argue that the government will want to have less consumption when government purchases are high because this makes the contingent claims value of the public spending high, making it easier to satisfy monopoly producers. This argument is valid when the utility of the representative agent is separable so that the price of the state-contingent security only depends on consumption.

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