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Optimal design of a redundant spherical parallel manipulator

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SUMMARY
A new kinematic design of a parallel spherical wrist with actuator redundancy is presented. A special feature of this parallel manipulator is the arrangement of co-axial actuators which allows unlimited rotation about any axis inside a cone-shaped workspace. A detailed kinematic analysis has shown that actuator redundancy not only removes singularities but also increases workspace while improving dexterity. The structure optimization has been performed with a global dexterity criterion. Using a conditioning measure, a comparison with a non-redundant structure of the same type was performed and shows that a significant improvement in dexterity has been obtained.

KEYWORDS: Parallel manipulator; Spherical wrist; Actuator redundancy

1 INTRODUCTION

A wrist is intended to modify the end-effector orientation around any direction in space. The performances of an ideal wrist would be constant and independent of its configuration. This property, known as isotropy, is not guaranteed. Indeed, many mechanisms have configurations in which large actuator motions result in small changes in end-effector orientation. However, isotropy is necessary in many practical applications as the same specifications must be satisfied in any direction, whatever the configuration of the manipulator may be.

Designing a wrist that exhibits isotropic properties over a given workspace first requires the selection of an appropriate mechanical structure. Second, the design parameters of the structure have to be optimized in order to satisfy one or several dexterity measures. For the mechanical structure, it is possible to choose between serial and parallel kinematic chains. Serial chains, in this case three consecutive rotational joints, are characterized by a large usable workspace but poor isotropy. They also have singular points for which the end-effector cannot be rotated around a particular direction. An additional joint can be used to avoid this singularity problem and improve the isotropy. This is called kinematic redundancy.

The main advantages of parallel kinematic chains are lightness and rigidity. The use of these chains in a robotic context is of recent date.[1–4] Their lightness, mainly due to the ability of bringing the actuators as close as possible to the fixed base is interesting in many applications requiring high rate of acceleration[5,6] or force control such as teleoperation with a force-reflecting controller.[7] In this latter case, the master-slave coupling must be transparent, which calls for the design of a master system with low inertia and high isotropy. The use of parallel structures as wrists is even more recent.[8-10] Opposite to the kinematic redundancy in serial chains is the actuator redundancy in parallel mechanisms, which has been studied particularly by Hayward. This means that the mechanism has more actuators than necessary, without increasing mobility. Actuator rates are uniquely determined by a given trajectory but actuator torques are undetermined. It can be used to increase dexterity and eliminate certain type of singularities.

Many authors have analysed the dexterity of manipulators and compared different measures, particularly Klein and Blaho,[11] and Park.[12] One of the first papers to consider dexterity was by Salisbury and Craig,[13] who introduced the condition number of the Jacobian J ($k = \|J\|\,\|J^{-1}\|$) which is simply the ratio of the radii of the largest and smallest principal axes of the manipulability ellipsoid described by Yoshikawa.[14] A direct physical significance of this local measure has also been shown.[11,15,16] Other measures taking into account inertial properties have been defined (generalized inertia ellipsoid by Asada,[15] dynamic manipulability measure by Yoshikawa[17]). Since they characterize the dexterity of a robot only at a given configuration, the above measures are local. But for design optimization, a global measure may be more desirable. In papers of Gosselin and Angeles,[18] Kurtz and Hayward,[16] global measures are defined by integrating local dexterity indices over the workspace.

The purpose of this paper is to present a new design of a redundantly actuated spherical parallel wrist, developed at CERT-ONERA by C. Reboulet.[19] To this end, the inverse kinematics, the Jacobian and the singularities of this mechanism are presented and discussed. Afterwards, a global dexterity measure is introduced. This measure, coupled with constraints on workspace volume, is used to optimize the geometrical parameters of the structure. Finally, the results are compared with another, similar, non redundant mechanism.

2 DESCRIPTION OF THE MECHANISM

The mechanism is composed of two pairs of sub-arms, pair (1) and (2), and pair (3) and (4), attached at one end to points $P_{1,2}$ and $P_{3,4}$ of the moving platform, respectively, and at the other two points of the fixed

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axis z_f. Each sub-arm consists of two spherical links. All the joints are revolute and their axes (active and passive) intersect at point O, defining the rotation center of the spherical wrist. As it is shown in Figure 1, this structure has all its actuator axes aligned to the z_f-axis.

This mechanism which is close to non-redundant structures studied by Asada and Cro Granito,[8] Cox and Tesar,[9] Gosselin and Angeles,[10] differs also in design as there are only two attachment points on the moving platform. In its nominal configuration (moving frame coinciding with the fixed frame), the structure is symmetrical about the $x_f y_f$ plane (Figure 1). An angle γ between the attachment points ($P_{1,2}$ and $P_{3,4}$) has been chosen equal to 90° in order to simplify the design. Computation of the conditioning measure for different values of γ showed that perfect isotropy is impossible and that the measure is divided by $(1+|\cos\gamma|)$, increasing the anisotropy of the mechanism. This justify the choice of 90° for angle γ.

Due to the four collinear actuators, unlimited rotation is possible around any axis inside a cone inscribed in the workspace. The opening angle of the cone is twice the maximum angle between the moving axis z and the fixed axis z_f, whatever the orientation of the platform may be. A cone as wide as possible is equivalent to maximizing the workspace.

3 KINEMATIC ANALYSIS

3.1 Inverse kinematic problem

The inverse kinematic problem for this manipulator consists in finding the joint variables α_i, $(i = 1, \ldots, 4)$ corresponding to a given orientation of the moving platform. As two configurations exist for each sub-arm, there are 16 solutions.

Orientation of the moving platform is specified by three Euler angles ψ, θ, φ, where ψ is the rotation around the z_f-axis, θ is the rotation around the new axis x and φ is the rotation around the new axis z. The rotation matrix with respect to the base coordinates is given by:

$$R = Rot_z(\psi)\cdot Rot_x(\theta)\cdot Rot_z(\varphi)$$

Let us consider the geometric parameters of the manipulator in Figure 2. Let us denote by a and b the link angles of the segments of each sub-arm and define $\vec{u}_i$ as the unit vector along the axis (OP_i) of the revolute joint connecting the moving platform to the adjacent link. In the chosen layout, the points P_1 and P_2 merge in $P_{1,2}$ and points P_3, P_4 into $P_{3,4}$. $[x_i, y_i, z_i]^T$ the components of $\vec{u}_i$ in the fixed base, are functions of the three Euler angles ψ, θ, φ. Moreover, $\vec{w}_i$ is defined as the unit vector along the axis of the intermediate revolute pair of each sub-arm. The components of vector $\vec{w}_i$ are given by:

$$\vec{w}_i(\alpha_i) = \begin{bmatrix} \sin a\cos\alpha_i \\ \sin a\sin\alpha_i \\ (-1)^i\cos a \end{bmatrix} \tag{1}$$

The solution of the inverse kinematics problem is obtained by solving for α_i each of the four equations:

$$\vec{u}_i\cdot\vec{w}_i(\alpha_i) = \cos b \tag{2}$$

For each sub-arm, this leads to:

$$X_i\cos\alpha_i + Y_i\sin\alpha_i = Z_i \tag{3}$$

where:

$$X_i = x_i\sin a$$
$$Y_i = y_i\sin a$$
$$Z_i = \cos b - (-1)^i z_i\cos a$$

This classic equation has a solution if:

$$X_i^2 + Y_i^2 \geq Z_i^2$$

For each sub-arm, it gives the two following solutions to the inverse kinematics problem:

$$\alpha_i = \beta_i \pm \arccos\left(\frac{Z_i}{d_i}\right) \tag{4}$$

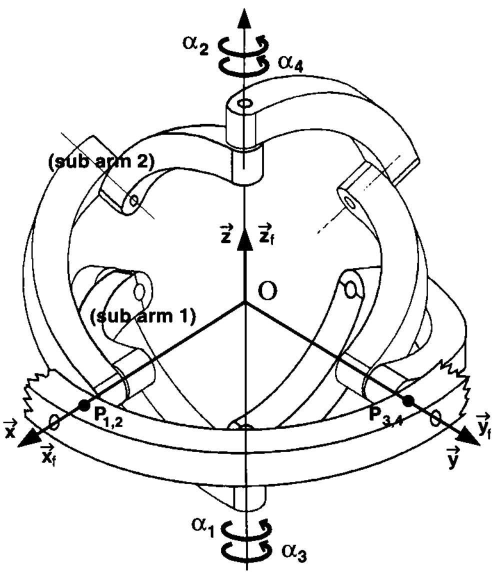

Fig. 1. The spherical wrist.

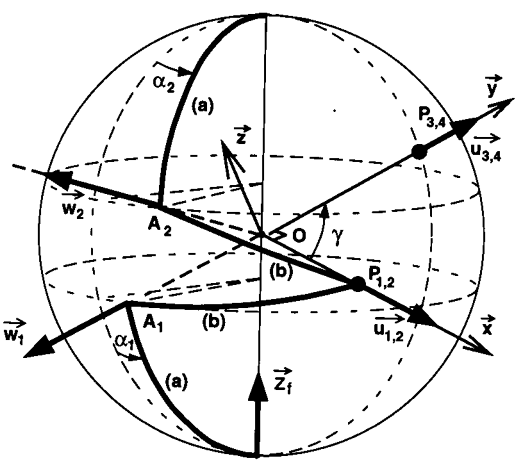

Fig. 2. Geometric parameters.

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with

$$d_i = \sqrt{X_i^2 + Y_i^2}$$

$$\beta_i = \arctan 2\left(\frac{Y_i}{X_i}\right)$$

For technical reasons of design, the layout of Figure 1 has been chosen. The corresponding solutions α_i $(i = 1, \ldots, 4)$ are selected by testing the sign of $\det(\vec{w}_i, \vec{u}_i, \vec{z}_f)$.

3.2 Jacobian matrix

The Jacobian matrix can be found by differentiation of equation (2) as shown by Gosselin,[10] namely:

$$\dot{\vec{u}}_i \cdot \vec{w}_i + \vec{u}_i \cdot \dot{\vec{w}}_i = 0 \quad (5)$$

where

$$\dot{\vec{u}}_i = \vec{\omega} \wedge \vec{u}_i$$

$\vec{\omega}$ being the angular velocity of the moving platform.

Equation (5) can be rewritten as:

$$b_i \dot{\alpha}_i + (\vec{u}_i \wedge \vec{w}_i) \cdot \vec{\omega} = 0 \quad (6)$$

with:

$$b_i = \vec{z}_f \cdot (\vec{w}_i \wedge \vec{u}_i)$$

which gives the 4×3 inverse of the Jacobian matrix J^{-1}:

$$\dot{\vec{\alpha}} = \begin{bmatrix} \cdots \\ \dfrac{-(\vec{u}_i \wedge \vec{w}_i)^T}{b_i} \\ \cdots \\ \cdots \end{bmatrix} \vec{\omega} = J^{-1}\vec{\omega} \quad (7)$$

$\dot{\vec{\alpha}}$ being the 4×1 vector of joint rates.

It is interesting to note that because the motor axes coincide with the z_f-axis, the choice of ψ, rotation around the same z_f-axis provides an unusual property. The condition number of the Jacobian matrix J does not depend on angle ψ, which allows to perform the optimization process with angles θ and φ only.

3.3 Discussion on singularities

As shown by Gosselin and Angeles,[20] equation (7) can be written as:

$$\begin{bmatrix} b_1 & & 0 \\ & \ddots & \\ 0 & & b_4 \end{bmatrix} \dot{\vec{\alpha}} = \begin{bmatrix} \cdots \\ -(\vec{u}_i \wedge \vec{w}_i)^T \\ \cdots \\ \cdots \end{bmatrix} \vec{\omega}$$

$$B\dot{\vec{\alpha}} = A\vec{\omega} \quad \text{with} \quad J^{-1} = B^{-1}A \quad (8)$$

which leads to the following mechanism singularities:

• Singularities of the first kind,[20] (or serial-type singularities), that appear when $\det(J) = 0$ which corresponds to $\det(B) = 0$. They consist of the set of points where different branches of the inverse kinematics problem meet and are known to lie on the boundary of the workspace. As the matrix B is of a diagonal form, this happens if one of the b_i is equal to zero, i.e.:

$$\vec{z}_f \cdot (\vec{w}_i \wedge \vec{u}_i) = 0 \quad i = 1, \ldots, 4 \quad (9)$$

This equation states that the two links of a sub-arm are coplanar, i.e. the corresponding sub-arm is totally unfolded or folded, which can be expressed as:

$$\vec{u}_i \cdot \vec{z}_f = (-1)^i \cos(a \pm b) \; i = 1, \ldots, 4 \quad (10)$$

which leads to:

$$\begin{aligned} \sin\theta \sin\varphi &= \pm\cos(a \pm b) \\ \sin\theta \cos\varphi &= \pm\cos(a \pm b) \end{aligned} \quad (11)$$

Hence it is possible to plot the locus of first type singularities in the $\theta - \varphi$ plane.

• Singularities of second kind[20] (or parallel-type singularities) that correspond to configurations in which the gripper is locally movable even when all the motors are locked. As opposed to the first kind, this kind of singularity lies inside the workspace and corresponds to a set of points where different branches of the direct kinematics problem meet. These singularities appear when $\det(J) \to \infty$, i.e. $\det(A) = 0$. Since $\vec{u}_1 = \vec{u}_2$ and $\vec{u}_3 = \vec{u}_4$, the matrix A can be written:

$$A = -\begin{bmatrix} (\vec{u}_1 \wedge \vec{w}_1)^T \\ (\vec{u}_1 \wedge \vec{w}_2)^T \\ (\vec{u}_3 \wedge \vec{w}_3)^T \\ (\vec{u}_3 \wedge \vec{w}_4)^T \end{bmatrix} \quad (12)$$

A is singular if its rank becomes less or equal to two, then if its four line vectors are coplanar. As sub-arm (1) is coupled with sub-arm (2) (respectively (3) with (4)), the first two line vectors being collinear implies:

$$\vec{w}_1 = \pm\vec{w}_2 \quad (13)$$

– $\vec{w}_1 = \vec{w}_2$ is possible only if $a = 90°$.

– $\vec{w}_1 = -\vec{w}_2$ is impossible without crossing a singularity of the first kind, as the chosen layout yields:

$$\begin{aligned} \det(\vec{w}_1, \vec{u}_1, \vec{z}_f) > 0 \\ \det(\vec{w}_2, \vec{u}_1, \vec{z}_f) > 0 \end{aligned} \quad (14)$$

The argument is the same for (3) and (4). Then, if $a \neq 90°$, the first two line vectors of A always define a plane perpendicular to $\vec{u}_1$ and the last two line vectors a second plane perpendicular to $\vec{u}_3$. Since $\vec{u}_1$ and $\vec{u}_3$ are always perpendicular, it is impossible for the two planes to merge and no parallel-type singularities exists in the workspace.

If $a = 90°$, $\vec{w}_1 = \vec{w}_2$ and $\vec{w}_3 = \vec{w}_4$ then rank $(A) = 1$; The nominal configuration is singular and uncontrolled degrees-of freedom appear.

4 DEXTERITY MEASURE AND OPTIMAL DESIGN

The notion of optimality is difficult to incorporate in the design process as there exists no global criteria including all objective functions. It is particularly difficult to take into account simultaneously geometrical, inertial and dynamic performance aspects and combine them into a single performance measure.

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Another goal of design, particularly for parallel mechanisms, is the maximisation of the usable workspace. It is also desirable to have the best conditioning inside the workspace, but these two goals are often conflicting.

In the case of three-dof parallel manipulators with identical actuators, the Jacobian matrix J is involved in kinematic, force and inertial relations. Therefore, using the condition number of J (or its inverse, called dexterity) seems appropriate to perform optimization even if it is not the only criterion for mechanism design. The condition number is defined as the ratio between the largest and the smallest singular values of J:

$$k(J) = \frac{\sigma_{max}(J)}{\sigma_{min}(J)}$$

For the spherical wrist, it is possible to have isotropy at nominal configuration ($\psi, \theta, \varphi = 0$). The condition ($k(J) = 1$) can be written as an equation function of angles a and b:

$$3\cos^2 a + 2\cos^2 b = 2 \qquad (15)$$

But the opening angle of the inscribed cone is then of 70° maximum. Therefore, it may be more interesting to accept some deviation from isotropy at nominal configuration in order to obtain a larger workspace.

As the minimization of the condition number of J is only a local criterion, the choice of values for a and b is difficult. Gosselin and Angeles[18] and Kurtz and Hayward[16] defined an interesting global measure D_g, which consists in integrating the dexterity over the workspace and then normalizing by the volume N_w of the workspace:

$$D_g = \frac{1}{N_w}\int_W \frac{1}{k(J)}\,dw \qquad N_w = \int_W dw \qquad (16)$$

with dw, element of volume of the workspace W. Expressing dw for an orientation device is not as simple as it is for a positioning device. The workspace W and its limits are well-known in terms of Euler angles ψ, θ and φ but the preceding integral has to be computed with an element of volume dw that is homogeneous in all the orientation workspace. This is not the case for the volume generated by elementary angles $d\psi$, $d\theta$, $d\varphi$.

Finite rotations can be conveniently interpolated using quaternion coordinates q_0, q_1, q_2, q_3 (also known as quadratic invariants or Euler parameters) defined by:

$$\begin{aligned} q_0 &= \cos\frac{\alpha}{2} \\ \vec{q} &= [q_1, q_2, q_3]^T = \sin\frac{\alpha}{2}\,\vec{u} \end{aligned} \qquad (17)$$

where $\vec{u}$ is the unit vector of the axis of rotation and α is the angle of rotation. Any rotation is represented by a quaternion of unit magnitude, thus by a point on the hypersphere S_3 of radius 1 in the four-dimensional quaternion space.

Let us consider the rotation R, function of time, mapping a fixed base B_0 into a moving base B_M, and $\vec{\omega}$, the associated instantaneous angular velocity. The rotation R can be represented by a quaternion of unit magnitude $\tilde{q}$. The relation between R and the skew-symmetric matrix Ω associated to $\vec{\omega}$ is:

$$\Omega = \dot{R}R^T$$

A relation between $\tilde{q}$ and $\vec{\omega}$ can be derived in the same way:

$$\tfrac{1}{2}\vec{\omega} = \dot{\tilde{q}}\tilde{q}^* \qquad (18)$$

(with $\tilde{q}^* = [q_0, -\vec{q}]$, conjugate quaternion of $\tilde{q}$, $\dot{\tilde{q}}\tilde{q}^*$ is a pure quaternion, equivalent to a vector). From equation (18), it can be derived that:

$$\tfrac{1}{2}\|\vec{\omega}\| = \|\dot{\tilde{q}}\| \qquad (19)$$

The velocity along a trajectory on the unit hypersphere is then always equal to half of the instantaneous angular velocity, independently of the parameters used to specify the rotation. This relation is of basic importance as it proves that an element of volume on the unit hypersphere corresponds to an elementary volume homogeneous in all the orientation workspace.

Hence, dw is defined as an elementary volume of the unit hypersphere S_3 even though the computation of the integral is performed with Euler angles. As the three Euler angles specify a trajectory on the unit hypersphere, a radius r is introduced which allows to go through the entire four-dimensional space $\Re^4$. The mapping from quaternions space to the four-dimensional space (r, ψ, θ, φ) is derived using the determinant of the Jacobian. As $\tilde{q}$ represent a rotation, r is taken equal to 1:

$$K(\psi, \theta, \varphi) = \left|\frac{\partial\tilde{q}}{\partial r}\,\frac{\partial\tilde{q}}{\partial\psi}\,\frac{\partial\tilde{q}}{\partial\theta}\,\frac{\partial\tilde{q}}{\partial\varphi}\right|_{r=1}$$

Therefore the measure D_g takes the form:

$$\begin{aligned} D_g &= \frac{1}{N_w}\iiint_{\psi,\theta,\varphi} \frac{1}{k(J)} K(\psi, \theta, \varphi)\, d\psi\, d\theta\, d\varphi \\ N_w &= \iiint_{\psi,\theta,\varphi} K(\psi, \theta, \varphi)\, d\psi\, d\theta\, d\varphi \end{aligned} \qquad (20)$$

In our case, a rotation specified by Euler angles ψ, θ, φ, ($R_{/z}, R_{/x}, R_{/z}$) is represented by a quaternion $\tilde{q}$ which is computed in terms of ψ, θ, φ using the multiplication in the quaternion algebra[21] as:

$$\tilde{q} = \tilde{q}_\psi \times \tilde{q}_\theta \times \tilde{q}_\varphi$$

$$= \begin{pmatrix} \cos\left(\frac{\psi}{2}\right) \\ 0 \\ 0 \\ \sin\left(\frac{\psi}{2}\right) \end{pmatrix} \times \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} \cos\left(\frac{\varphi}{2}\right) \\ 0 \\ 0 \\ \sin\left(\frac{\varphi}{2}\right) \end{pmatrix}$$

$$= \begin{bmatrix} \cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\psi+\varphi}{2}\right) \\ \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\psi-\varphi}{2}\right) \\ \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi-\varphi}{2}\right) \\ \cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi+\varphi}{2}\right) \end{bmatrix}$$

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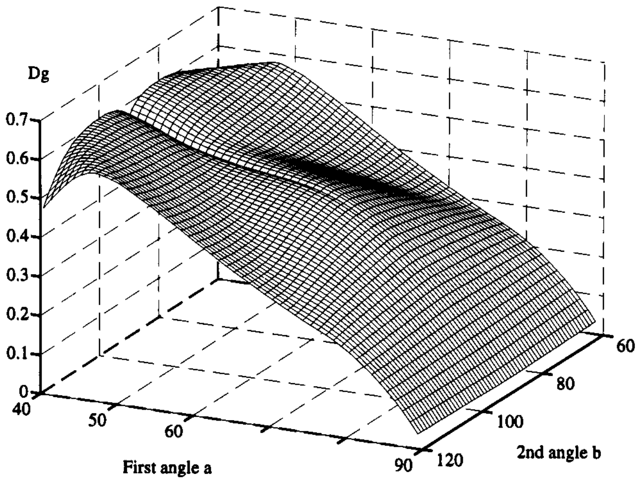

Fig. 3. Global dexterity measure for redundantly actuated wrist.

Which yields that:

$$K(\psi, \theta, \varphi) = \frac{\sin \theta}{8}$$

As stated before $k(J)$ is independent of the angle ψ, allowing a simpler form of D_g:

$$D_g = \frac{1}{N_w} \iint_{\theta,\varphi} \frac{1}{k(J)} \sin \theta \, d\theta \, d\varphi \tag{21}$$

Figure 3 shows the value of this global criterion for the redundant wrist as a function of the two characteristic parameters of the mechanism, the angles a and b. It is possible to determine values of a and b providing an optimal value of D_g.

It is also interesting to use this graph to compare the mechanism presented in this paper with an equivalent non redundant spherical wrist like the one examined by Asada and Cro Granito,[8] Cox and Tesar,[9] and Gosselin and Angeles.[10] This mechanism is of the same type, with collinear actuators but only three arms set at 120° from each other (for purpose of symmetry). Figure 4 shows the value of the same criterion for this wrist, as a function of link angles a and b. From this analysis, substantial differences between the two mechanisms can be seen. However, optimal values of parameters a and b for this measure don't ensure a large workspace.

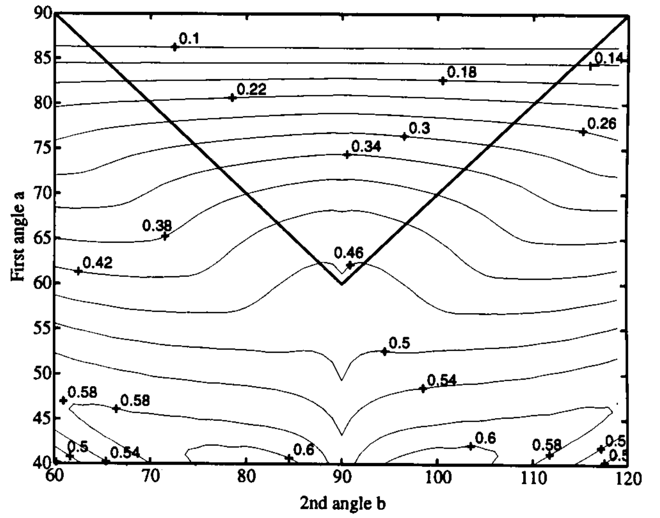

Fig. 5. Contour line of global measure for redundantly actuated wrist.

It seems necessary to add a constraint on workspace limits. This consists of specifying a minimal opening λ for the cone inscribed in the workspace inside which unlimited rotation is possible. The limits of the workspace are given by singularities of the first kind when two links are totally folded or unfolded. The opening angle λ is limited by the values of a and b as:

$$\frac{\lambda}{2} + 90° \leq a + b$$
$$\frac{\lambda}{2} - 90° \leq a - b \tag{22}$$

Figures 5 and 6 show the contour lines of D_g and the geometrical constraints added, with a value of 120° for λ. It can be seen that the values of the criterion satisfying this constraint are much higher in the case of the redundant robot.

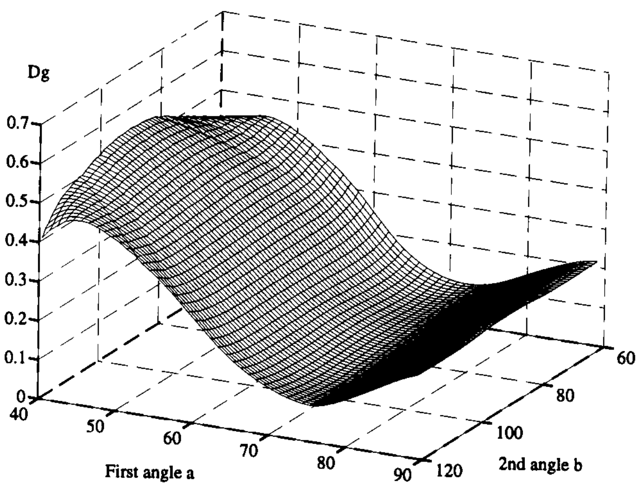

Fig. 4. Global dexterity measure for three-actuators wrist.

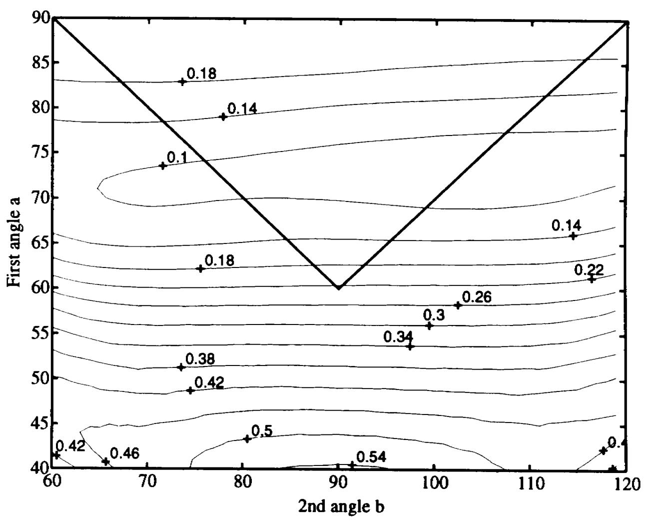

Fig. 6. Contour line of global measure for three-actuators wrist.

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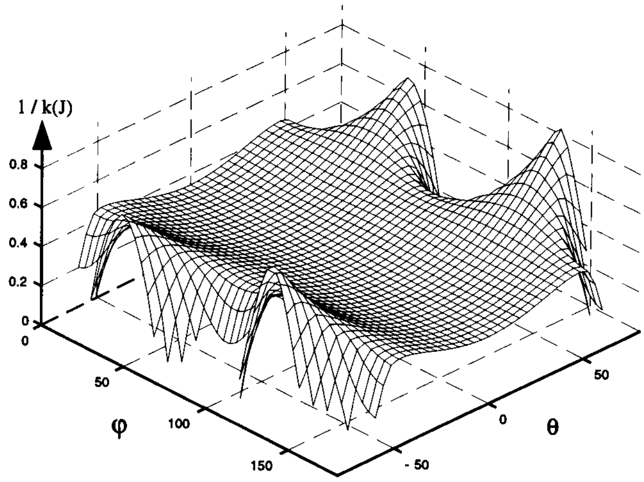

Fig. 7. Local dexterity measure 1/k(J) for redundantly actuated wrist.

The preceding graphs allows us to choose optimal values for the parameters a and b. It is interesting to visualise the inverse of the condition number with these values for the two mechanisms. The dexterity $1/k(J)$ is plotted as a function of only two Euler angles: θ and φ. As it has been shown, this is enough to cover the entire workspace as $k(J)$ is independent of the angle ψ.

Figure 7, for the redundant system, and Figure 8, for the non redundant system show the dexterity for chosen optimal values of a and b. θ represents the angle of inclination of axis z of the moving platform and φ represents rotation around this axis. A substantial degradation in performance can be seen for the non redundant mechanism.

It can also be seen that for the point corresponding to $\theta = 0$, both types of mechanism have the same value of dexterity. When θ is increased thus the moving platform axis is inclined, performance improves for the redundant mechanism whereas it worsens for the non redundant one.

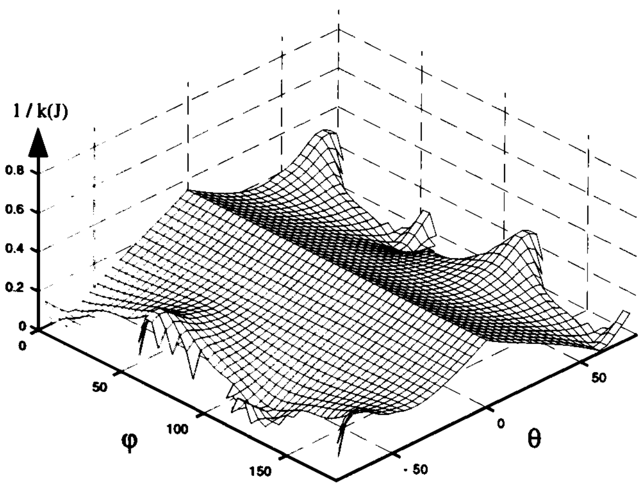

Fig. 8. Local dexterity measure 1/k(J) for three-actuators wrist.

5 CONCLUSION

A new kinematic design of a parallel spherical wrist with actuator redundancy has been presented. Kinematic analysis has shown that actuator redundancy not only removes singularities but also improves dexterity in an enlarged workspace. The method of optimisation adopted is based on a global criterion which covers both the notion of isotropy and the workspace volume. Supplementary constraints concerning the workspace volume have been introduced, mainly the possibility of unlimited rotation around any axis situated within a 120° cone (a property which is not common in most known parallel mechanism). The structure which has been presented guarantees dexterity superior to 1/3 over the whole workspace. Comparisons with another equivalent non-redundant structure were carried out with the aid of condition number surfaces thus allowing a visualisation of dexterity. They show a notable improvement in performance in terms of uniformity of dexterity for the redundant structure.

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