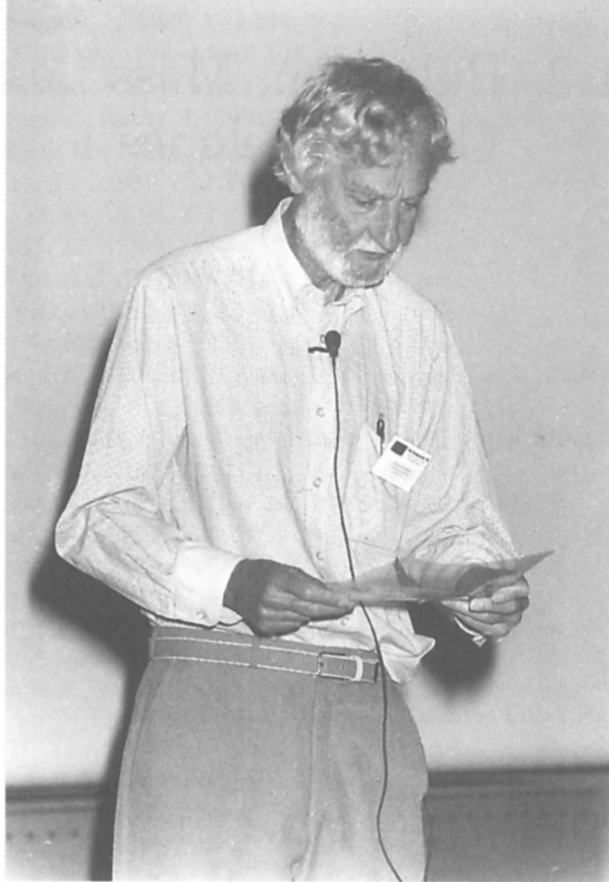


Part 2. Pulsation, Mass Loss, Cool Envelopes



**Michael Feast during a fundamental (or was it an overtone ?)
talk on the pulsation mode of Miras**

Pulsation Modes in Mira and Semiregular Variables

Michael Feast

*Department of Astronomy, University of Cape Town, Rondebosch, 7700,
South Africa*

Abstract. The most direct evidence for the pulsation mode of Mira and SR variables comes from the linear diameters derived from individual parallaxes and angular diameters, together with the observed periods. The data now available strongly suggest that most, if not all, Miras and SRs pulsate in the same mode. If comparison is made with conventional models, this mode is the first overtone. Uncertainties in deriving the pulsational radius of a Mira from the observations do not appear significant enough to affect this conclusion. Evidence from SRs and Miras in globular clusters supports this conclusion and shows that the pulsation mode does not depend on metallicity. Analysis of light curves leads to broadly the same conclusions, if less directly. The light curves indicate that periodicities other than the main one may also be present and that mode switching may occur in some stars.

1. Introduction

There has been a long debate regarding the mode of pulsation of Mira variables. Discussion has mainly focussed on whether they pulsate in the fundamental radial mode or in the first overtone. This question is closely related to the problem of the absolute magnitudes and effective temperatures of the Miras. There has been much less discussion regarding the mode of pulsation of the Semiregular (SR) variables. However in quite recent times there has been a good deal of work bearing on the question of pulsation modes for both the Miras and the SRs. The main pieces of new evidence come from the measurement of angular diameters of Miras and SRs by interferometry or occultations; parallaxes for individual members of these classes from HIPPARCOS; and, the detailed analysis of light curves.

2. Modes from linear diameters

The most direct way to determine the pulsational mode of a Mira or SR variable is to derive its linear diameter from its measured angular diameter and its parallax and then to use the $P\sqrt{\rho} = Q$ relation between the period (P) and the mean density (ρ) (or a more sophisticated form of this relation) to deduce the pulsational constant (Q) and to compare this with theoretical prediction. This procedure is possible because, as a class, the Miras and SRs are old, low

mass stars. Thus a mass near one solar mass can be adopted. Supergiant SRs (SRc variables) will not be discussed in this paper.

The first angular diameters of these stars were obtained from lunar occultation observations, but great progress has been made recently using interferometry. A problem arises at once with the Miras since they have very deep atmospheres and the measured diameter varies with wavelength. Models suggest that the measured angular diameter in the infrared (at about $2\ \mu\text{m}$), where most of the energy is being radiated, should approximate closely to that required in the pulsation analysis. Observations are being increasingly made in the infrared but the currently available data contains crucial observations which have been made at shorter wavelengths. These have been corrected for atmospheric extension using models. That these models (e.g. Bessell et al. 1996) are successful in predicting these corrections is shown by the general agreement of the radii of Miras determined in the infrared with those measured at shorter wavelengths and corrected (see Feast 1996).

There are certainly many problems remaining to be sorted out regarding the measured angular diameters of Miras. Different workers do not always agree too well with one another. Part of this might be observational. But there appears to be considerable variations in diameter with time for some of the variables, at least at the shorter (visual) wavelengths. Some of these changes appear to be unrelated to pulsation (or at least to the main pulsational period) (e.g. Tuthill et al. 1995). There is also considerable evidence for non-circularly symmetrical discs, at least at optical wavelengths. This may well be associated with large star spots or, perhaps, either non-radial oscillations or stellar rotation (see e.g. Haniff 1995 and references there, Lattanzi et al. 1997, and papers in the present volume). There is also the question of the variation of the radius with phase in the pulsation cycle itself. This is only just beginning to be studied though it is not expected to be large in the $2\ \mu\text{m}$ region. All these factors certainly imply that the estimate of the true mean angular diameter of a Mira will have a greater uncertainty than the individual measurements from which it is derived. Nevertheless the available data should give a set of angular diameter measurements suitable for the analysis of pulsational modes, especially if the stars are considered as a group rather than individually.

Robertson & Feast (1981) used two infrared occultation diameters of Miras together with distances derived from a period-luminosity (PL) relation, to deduce that Miras were overtone pulsators. First overtone pulsation was assumed although it is in fact difficult to distinguish between this and higher overtones. Haniff et al. (1995) used interferometric diameters of 10 Miras together with a PL relation to confirm this conclusion. Tuthill et al. (1994) used an accurate ground based parallax of R Leo (Gatewood 1992) to reach to same result.

It is now possible to make progress both for the Miras and the SRs using individual parallaxes from the HIPPARCOS catalogue. Perhaps the main concern here is that one might get spurious parallaxes due to star spots on the Miras. The angular diameter of a Mira is typically about five times that of its parallax. If star spots cause the apparent photocentre of the star to move around then this might affect the derived parallax. Unless the motion of the photocentre mimicked the parallactic motion, this would simply be reflected in a larger than normal parallax error. Even if the parallax itself were affected this could either

result in an overestimate or an underestimate of the true value and this would average out in a set of Miras. This does however suggest that care is necessary in dealing with individual stars.

Early release of HIPPARCOS data allowed van Leeuwen et al. (1997) to derive the linear diameters of eight Miras using the angular diameters of Haniff et al. (1995). Their results indicated that Miras with periods less than 400 days are overtone pulsators but that at least some of the Miras with longer periods pulsate in the fundamental. This result (i.e. both overtone and fundamental pulsators) was rather unexpected. Whitelock et al. (1998) have now discussed all the Miras and SRs which have measured angular diameters and also HIPPARCOS parallaxes. A little consideration shows that it is best to work with the reciprocal of the linear diameter (D) rather than with D directly. In the sample considered the percentage error in the parallax (π) can, in the best cases, be about the same as that in the angular diameter (ϕ), but it is often much greater. In these circumstances the uncertainty in $1/D (= \pi/\phi)$ remains approximately gaussian whilst that of D itself becomes quite skew and diagrams of D against $\log P$, for instance, can be somewhat misleading. Working with $1/D$ means that one can use all the parallaxes (including negative ones, if they occur). This is important since provided the stars are not chosen by measured parallax, and since the uncertainty in $1/D$ depends on the error in the parallax (σ_π) and not on σ_π/π , one can combine the results for a sample of stars without introducing significant statistical bias (Lutz-Kelker type bias etc.).

Whitelock et al. (1998) use angular diameters from a variety of sources. Early data were summarized by Feast (1996) and to this has been added data from van Belle et al. (1996, 1997) and Bedding et al. (1998). Where necessary the angular diameters have been reduced to effective infrared diameters and corrected at least approximately for limb darkening. A comparison of theoretical predictions with a plot of $1/R$ (where R is the linear radius) against $\log P$ for Miras and SRs, and including both oxygen-rich and carbon-rich stars, indicates that at least the bulk of the stars are pulsating in the first overtone. With a first overtone relation from Fox & Wood (1982) viz.:

$$\log P = 1.5 \log R - 0.5 \log M + \log Q, \quad (1)$$

and adopting Q (the pulsation constant) = 0.04, one finds a mean mass (M) of 1.3 ± 0.2 solar masses. The actual value of the mean mass is of course sensitive to the adopted pulsation constant but the small uncertainty is an indication that the bulk of the stars are pulsating in the same mode whether they are oxygen or carbon rich, or Miras or SRs. The residuals in $1/R$ from the theoretical overtone relation are quite gaussian. The 40 stars¹ involved in the discussion include the Mira, R Cas, which was the clearest example of a long-period fundamental pulsator in the analysis of van Leeuwen et al. (1997). R Cas has indeed the largest fractional residual from the solution ($\Delta/\sigma = 2.8$ where Δ is the residual and σ the standard error of $1/R$ for R Cas). However in a sample of 40 stars, such a residual is not particularly abnormal. Whitelock et al. conclude that their analysis does not compel one to believe that either R Cas or χ Cyg ($\Delta/\sigma = 2.0$)

¹R Aql and U Ori were in each case included twice, once with the Haniff et al. (1995) diameters and once with those of van Belle et al. (1996) which are rather different.

are anything other than overtone pulsators. Individual fundamental pulsators might exist in the present sample but only improved data can show this. The evidence now seems rather strong that at least the vast majority of the Miras and SRs pulsate in the first overtone.

3. Miras and SRs in globular clusters and metallicity effects

Feast (1996) argued that the SRs and Miras in the globular cluster 47 Tuc must all lie on a single evolutionary track. This can then be combined with a pulsation equation to derive the change of bolometric magnitude with period. Consistency of observations and theory can only be obtained if the SRs and Miras pulsate in the same mode.

Whitelock (1986) assembled the data (including infrared photometry) of SR and Mira variables in globular clusters. Her results show that the SR variables from metal-rich clusters have a distinctly different period-temperature relation (or equivalently, a different period- $(J - K)_0$ relation) from that in metal-poor clusters. This seemed a remarkable result at the time and possibly suggested that the metal-poor and metal-rich SRs pulsated in different modes. However one can now understand the result in terms of Wood's (1990) parameterization of the AGB. Combining his relation with the first overtone pulsation equation one obtains (e.g. Feast 1996):

$$\log T_{eff} = -0.13 \log P - 0.073 \log Z + 0.038 \log M + const \quad (2)$$

where T_{eff} is the effective temperature. This is a useful equation since the dependence on mass (M) is very small. The equation shows that a dependence on metallicity (Z) is predicted and that the metal independent quantity should be, $\log T_{eff} + 0.073 \log Z$. Whitelock et al. (1998) show that the observations bear out this prediction indicating that both metal poor and metal rich SRs pulsate in the same mode.

4. The relative positions of SRs and Miras in the PL plane

Wood & Sebo (1996) discovered a number of SR variables in the LMC. These lie above the Mira PL relation in a luminosity - $\log P$ diagram (that is, at a given luminosity, the SRs have shorter periods than the Miras). Recently, Bedding et al. (1998) have used HIPPARCOS parallaxes of galactic SR variables to show that they too lie above the Miras in the PL plane. Wood & Sebo interpret their results as showing that the SRs and Miras pulsate in different modes, which they suggest are the first overtone and the fundamental. However one cannot necessarily argue that, because two stars have the same luminosity but different periods, they pulsate in different modes. Even if the masses of the stars are nearly the same, the two stars may have different surface temperatures and hence different radii. That such is indeed the case for at least the SRs and Miras in metal-rich globular clusters is shown by the work of Whitelock (1986). She found that in these clusters the SR variables defined a line in the PL plane which had a shallower slope than the Mira PL relation and lay above it. The two relations intersected at the position of the Miras in the clusters. It is quite

clear in this case that the Whitelock track for the SRs is an evolutionary track ending at the position of the cluster Miras. The Mira relation on the other hand is a sequence of variables of different mass and/or metallicity. Thus the relative positions of SR and Mira variables in the PL plane do not require that these two classes of variables are pulsating in different modes. Furthermore, at least in the case of our Galaxy, SRs of period near 100 days cannot be considered as overtones of Miras with periods near 200 days since the galactic kinematics of these two groups are quite different (see Feast et al. 1972).

5. Evidence from light curves

Finally, on the observational side there is the direct analysis of the light curves. There are various problems here. Long series of observations are required and the analysis (especially in the case of SR variables) sometimes reveals many frequencies, the long term stability of which is uncertain. In addition the interpretation of the data can be uncertain since it depends on a comparison of observed period ratios with theoretical predictions and the latter are sensitive to the models (see for instance Barthès & Mattei 1997).

An extensive analysis of α Ceti itself has been published by Barthès & Mattei (1997). The main period is 332.9 days and the next most important 1503.8 days with evidence for many combinations of these two periods. They conclude that 332.9 days is probably the first overtone and 1503.8 days the fundamental although they do not entirely rule out other alternatives. Barthès & Tuchman (1994) found that the dominant mode in the Miras S CMi and χ Cyg was also the first overtone.

As regards the SR variables, particularly, it has to be borne in mind that brightness variations on characteristic time scales may well occur due to the effects of star spots or large convection cells on the stellar surface. In the case of the SRa variable V Boo (spectral type Me) Szatmáry et al. (1996) find a main periodicity of 258 days which they take as the fundamental, with a secondary period of 137 days which is then interpreted as an overtone. On the other hand for BS Lyr, Mantegazza (1996) finds a dominant period of 304.7 days which is taken as the first overtone with a secondary period of 272.3 days which is interpreted as the second overtone. Percy & Desjardins (1996) find three periods for W Boo, 25, 35 and 50 days. They suggest these are the 4th, 3rd and 2nd overtones with an interpretation as the 3rd, 2nd and 1st overtones less likely. There is also evidence for power switching between the modes. A very extensive analysis of the light curve of R Dor has recently been published by Bedding et al. (1998). The star shows two periods, 332 days, which they interpret as the 1st overtone, and 175 days, which they believe to be the 3rd overtone. They find that power shifts back and forth between these two modes. They also find evidence for weak chaos in the light curve.

6. Summary

The direct empirical evidence suggests that the main periodicity of most, perhaps all, Miras corresponds to the first overtone at least if classical pulsation models are considered. Other modes may also be present. The same is true for the SR

variables with, in this case, evidence for exchange of power between modes (e.g R Dor).

A problem raised by these conclusions is that theorists have so far had considerable difficulty in producing strong shock waves in model atmospheres with radii as large as those implied by the angular diameter measurements. Such shocks are generally considered to be necessary to explain the observed splitting of the CO absorption lines at certain phases in the light curve and also the excitation of the characteristic Mira emission-line spectrum. This remains an unsolved theoretical problem whether or not we interpret the data (as in this paper) using classical pulsation models or whether one adopts the model of Tuchman (this volume).

7. Acknowledgements

I am grateful to Dr Patricia Whitelock for many helpful discussions and for unpublished data. I am also grateful to several astronomers for preprints.

References

- Barthès D., Mattei J.A., 1997, *AJ* 113, 373
 Barthès D., Tuchman Y., 1994, *A&A* 289, 429
 Bedding T.R., Zijlstra A.A., von der Lühe O., Robertson J.R., Marson R.G., Barton J.R., Carter B.S., 1997, *MNRAS* 286, 957
 Bedding T.R., Zijlstra A.A., 1998, *ApJ* 506, L47
 Bedding T.R., Zijlstra A.A., Jones A., Foster G., 1998, *MNRAS* 301, 1073
 Bessell, M.S., Scholz M., Wood P.R., 1996, *A&A* 307, 481
 Feast M.W., 1996, *MNRAS* 278, 11
 Feast M.W., Woolley R., Yilmaz N., 1972, *MNRAS* 158, 23
 Fox M.W., Wood P.R., 1982, *ApJ* 259, 198
 Gatewood G., 1992, *PASP* 104, 23
 Haniff C., 1995, in *Astrophysical Applications of Stellar Pulsation*, R.S. Stobie & P.A. Whitelock (eds.), *ASP Conf. Ser.* 83, p. 270.
 Haniff C.A., Scholz M., Tuthill P.G., 1995, *MNRAS* 276, 640
 Lattanzi M.G., Munari U., Whitelock P.A., Feast M.W., 1997, *ApJ* 485, 328
 Mantegazza L., 1996, *A&A* 315, 481
 Percy J.R., Desjardins A., 1996, *PASP* 108, 847
 Robertson B.S.C., Feast M.W., 1981, *MNRAS* 196, 111
 Sztatmáry K., Gál J., Kiss L.L., 1996, *A&A* 308, 791
 Tuthill P.G., Haniff C.A., Baldwin J.E., Feast M.W., 1994, *MNRAS* 266, 745
 Tuthill P.G., Haniff C.A., Baldwin J.E., 1995, *MNRAS* 277, 1541
 van Belle G.T., Dyck H.M., Benson J.A., Lacasse M.G., 1996, *AJ* 112, 2147
 van Belle G.T., Dyck H.M., Thompson R.R., Benson J.A., Kannappan S.J., 1997, *AJ* 114, 2150

- van Leeuwen F., Feast M.W., Whitelock P.A., Yudin B., 1997, MNRAS 287, 955
Whitelock P.A., 1986, MNRAS 219, 525
Whitelock P.A., et al., 1998, in preparation
Wood P.R., 1990, in *From Miras to Planetary Nebulae: Which Path for Stellar Evolution ?*, M.O. Mennessier & A. Omont (eds.), Editions Frontières, Gif-sur-Yvette, p. 67
Wood P.R., Sebo K.M., 1996, MNRAS 282, 958



**Marie-Odile Mennessier, from HIPPARCOS to GAIA:
which path for stellar evolution ?**