J. Plasma Physics (2013), *vol.* 79, *part* 6, *pp.* 1117–1121. © Cambridge University Press 2013
doi:10.1017/S0022377813001372

Dust grain surface potential in a non-Maxwellian dusty plasma with negative ions

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(Received 2 September 2013; revised 14 October 2013; accepted 15 October 2013)

Abstract. Dust charging processes involving the collection of electrons and positive/negative ions in a non-equilibrium dusty plasma are revisited by employing the power-law kappa (κ)-distribution function. In this context, the current balance equation is solved to obtain dust grain surface potential in the presence of negative ions. Numerically, it is found that plasma parameters, such as the κ spectral index, the negative ion-to-electron temperature ratio (γ), the negative–positive ion number density ratio (α), and the negative ion streaming speed (U_0) significantly modify the dust grain potential profiles. In particular, for large kappa values, the dust grain surface potential reduces to the Maxwellian case, and at lower kappa values the magnitude of the negative dust surface potential increases. An increase in γ and U_0 leads to the enhancement of the magnitude of the dust grain surface potential, while α leads to an opposite effect. The relevance of present results to low-temperature laboratory plasmas is discussed.

1. Introduction

A multi-component dusty plasma, or complex plasma, can comprise Maxwellian electrons and ions, as well as negatively/positively charged dust grains. The characteristic scale lengths associated with a dusty plasma involve the ordering of the form $r_d \ll d, \lambda_D \ll L$, where r_d is the radius of dust grains, d is the inter-grain distance, λ_D is the effective Debye length, and L is the dimension of the plasma system. Dust grains are quite common in space and astrophysical plasmas as well as in plasma processing industry (Shukla and Mamun 2002; Fortov et al. 2005). They can be found in various sizes and masses, and may be made of dielectric, metallic, and ice particles (viz. silicates, graphite, magnetite, amorphous carbons, etc.). The charge on dust grains is not fixed (constant) but fluctuates in space and time. Therefore, dust component and associated dust charge perturbations introduce new dusty modes and novel impact on the dynamics of collective modes and instabilities (Shukla and Mamun 2002). The most fundamental collective modes in a Maxwellian dusty plasma (Shukla 2001) are the dust-acoustic and dust-ion-acoustic waves, which have been investigated both theoretically (Rao et al. 1990; Shukla and Silin 1992) and experimentally by many authors (Barkan et al. 1995; Barkan et al. 1996; Pieper and Goree 1996; Prabhakara and Tanna 1996; Merlino et al. 1998; Molotkov et al. 1999).

Dust grains can be charged either negatively or positively depending upon charging mechanisms (Whipple et al. 1985; Northrop 1992; Choi and Kushner 1994; Young et al. 1994). The latter may comprise thermionic emissions, photo emissions, and secondary electron emissions, which may lead to constitute a positive charge on dust grains. On the other hand, in low-temperature experiments, the dust grains mostly acquire a negative charge because the electrons are more mobile compared with ions, and as a result the electron current flows to dust grain's surface. The dust grains become negatively charged as the electron current dominates over the ion current. However, the negative dust surface potential leads to a decrease in electron current and an increase in positive ion current until a balance is reached (viz. $|I_e| = |I_i|$, where I_e (I_i) is the electron (ion) current). In 1994, the dust charging phenomena was considered due to the bombardment of Maxwellian electrons and positive ions (Barkan et al. 1994) to investigate dust grain surface potential both theoretically and experimentally. Later on, the work was extended (Mamun and Shukla 2003) to highlight the role of negative ions in Maxwellian plasmas. It was numerically found that the variation in negative ion number density, negative ion charge, negative ion temperature, and negative ion streaming speed strongly affect the dust grain surface potential.

When some or all plasma particles move faster than their thermal speeds, the usual Maxwellian distribution becomes inadequate to study the behavior of superthermal particles appropriately. In such a situation, a kappa distribution (Vasyliunas 1968) can be used to model the plasma behavior. The kappa distribution is a generalization of the Maxwellian distribution and can be formed due to wave–particle interactions and/or due to the external forces acting on space and astrophysical

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environments (Pierrad and Lazar 2010), e.g. in the solar wind, ionosphere, thermoshpere, interstellar medium, magnetoshpere, etc. Numerous investigations (Summers and Thorne 1991; Mace and Hellberg 1995; Baluku and Hellberg 2008; Hellberg et al. 2009) have been carried out to explain collective interactions in plasmas by employing the kappa (κ)-distribution function. In this context, Vasyliunas (1968) was the first to introduce kappa distribution to fit the empirical data obtained from solar wind and explained the reduction of the κ-distribution function to the Maxwellian distribution function. The power-law κ-distribution function (Vasyliunas 1968) can be expressed for jth ($j = e, i, n$) plasma species as $f_{j\kappa}(\mathbf{V}_j) = A_{j\kappa}(1 + V_j^2/\kappa\theta_j^2)^{-\kappa-1}$, where the spectral index κ measures the deviation from thermal equilibrium and the coefficient $A_{j\kappa} = (n_{j0}(\theta_j^2\pi\kappa)^{-3/2})$ $\Gamma(\kappa+1)/\Gamma(\kappa-1/2)$ with a gamma function Γ, where n_{j0} is the equilibrium number density, and $\theta_j = \{(2\kappa-3)/\kappa\}^{1/2}(T_j/m_j)^{1/2}$ is the effective thermal speed in the limit $\kappa > 3/2$. For $\kappa \to \infty$, the κ-distribution function reduces to the Maxwellian distribution function and for small κ-values, it represents a power-law distribution in velocity with high energy tails. The phenomenon of Debye shielding has already been explained in non-Maxwellian dusty plasmas (Rubab and Murtaza 2006a), making a comparison between the characteristics of kappa and r –q distributions. Furthermore, the expressions for equilibrium and fluctuating electron and ion currents (Rubab and Murtaza 2006b) have been derived for a negatively charged dust grain with high energy tails described by kappa or broad shouldered and high energy tails described by spectral indices r and q. It is shown that the dispersion properties of dust acoustic waves become significantly modified by taking into account these distributions. More recently, a dust charging process has been considered due to collections of electrons and ions in a non-equilibrium dusty plasma (Du 2004; Gong and Du 2012) to account for a new form of power-law q-distribution with non-extensive statistics. They examined numerically the effects of non-extensive power-law distributions caused by electrons and ions on the dust grain surface potential.

In this paper, we derive expressions for the dust grain surface potential by solving the current balance equation which contains non-Maxwellian currents due to electrons and positive/negative ions for the negatively charged dust grains. Numerical analyses have been performed to examine the effects of plasma parameters on the negative dust grain surface potential.

The paper is organized as follows: In Sec. 2, we study the non-Maxwellian currents attributed to electrons, positive/negative ions on the dust grain surface by using the power-law κ-distribution function. In a steady state, the dust charging equation reduces to the current balance equation, which can be employed to study the dust grain surface potential in the presence of negative ions. Numerical results are discussed and summarized in Sec. 3.

2. Dust grain surface potential with κ-distribution

We consider a non-Maxwellian dusty plasma whose constituents are electrons, positive ions, negative ions, and negatively charged dust grains. At equilibrium, the charge-neutrality condition demands $n_e - Z_i n_i + Z_n n_n - q_d n_d/e$, where n_s is the unperturbed number density of sth species ($s = e$ for electrons, i for positive ions, n for negative ions, and d for negatively charged dust grains). Z_i (Z_n) is the positive (negative) ion charging state and q_d represents the charge of dust grains. When the dust charge attains its steady state value (viz. q_d =const), the current balance equation comes into play caused by the electrons and positive/negative ions in non-Maxwellian plasmas (Rubab and Murtaza 2006b) and can be expressed as

$$\sum_{j=e,i,n} I_{j\kappa} = 0, \tag{2.1}$$

with

$$I_{j\kappa} = q_j \int V_j \sigma_j^d f_{j\kappa}\left(V_j\right) d^3\mathbf{V}_j. \tag{2.2}$$

Here $\sigma_j^d = \pi r_d^2(1 - 2q_j\varphi_d/m_jV_j^2)$ is a cross section for charging collisions between the dust grains and plasma species of charge q_j with φ_d being the surface potential of dust grains of radius r_d. To proceed, we simplify (2.2) by integrating over the non-Maxwellian velocity distribution function $[f_{j\kappa}(V_j)]$ and express the volume element as $d^3\mathbf{V}_j = V_j^3 dV_j d\mu d\phi$ in spherical coordinates to obtain the currents due to electrons, positive ions, and negative ions for negatively charged dust grains (Rubab and Murtaza 2006b), respectively, as

$$I_{e\kappa} = -2\sqrt{\pi} r_d^2 B_\kappa \left(1 - \frac{2e\varphi_d}{\kappa m_e \theta_e^2}\right)^{-\kappa+1} e n_e \theta_e, \tag{2.3}$$

$$I_{i\kappa} = 2\sqrt{\pi} r_d^2 B_\kappa \left\{1 - \frac{2(\kappa-1)e\varphi_d}{\kappa m_i \theta_i^2}\right\} Z_i e n_i \theta_i, \tag{2.4}$$

and

$$I_{n\kappa} = -2\sqrt{\pi} r_d^2 B_\kappa \left\{1 + \frac{2(\kappa-1)}{\kappa}\frac{Z_n e\varphi_d}{m_n \theta_n^2}\right\} Z_n e n_n \theta_n, \tag{2.5}$$

where $B_\kappa = \left\{\Gamma(\kappa+1)/\kappa^{3/2}\Gamma\left(\kappa - \frac{1}{2}\right)\right\}\frac{\kappa}{\kappa-1}$. The corresponding effective thermal speeds for electrons, positive ions, and negative ions are, respectively, given by

$$\theta_e = \left(\frac{2\kappa-3}{\kappa}\right)^{1/2}\left(\frac{T_e}{m_e}\right)^{1/2},$$

$$\theta_i = \left(\frac{2\kappa-3}{\kappa}\right)^{1/2}\left(\frac{T_i}{m_i}\right)^{1/2},$$

and

$$\theta_n = \left(\frac{2\kappa-3}{\kappa}\right)^{1/2}\left(\frac{T_n}{m_n}\right)^{1/2}.$$

Here m_j is the mass, n_j is the plasma number density, and T_j is the temperature of jth species ($j = e, i, n$).

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Substituting (2.3)–(2.5) into (2.1), the equilibrium dust grain surface potential in the presence of kappa distribution for a negatively charged dust grains can be determined as

$$\sqrt{\sigma}-\frac{a_\kappa Z_i U}{\sqrt{\sigma}}-\mu\left(1-\alpha\frac{Z_n}{Zi}+Z_iPU\right)b_\kappa$$
$$=\alpha\beta\sqrt{\gamma}\frac{Z_n}{Z_i}\left(1+a_\kappa\frac{Z_nU}{\gamma}\right), \quad (2.6)$$

with

$$a_\kappa=\frac{2(\kappa-1)}{2\kappa-3},\quad b_\kappa=\left(1-\frac{2U}{2\kappa-3}\right)^{-\kappa+1},$$
$$\alpha=\frac{n_n}{n_i},\ \beta=\left(\frac{m_i}{m_n}\right)^{1/2},\ \gamma=\frac{T_n}{T_e},$$
$$\sigma=\frac{T_i}{T_e},\ \text{ and } \mu=\left(\frac{m_i}{m_e}\right)^{1/2}.$$

The normalized dust grain potential is denoted by $U=e\varphi_d/T_e$ and the normalized dust number density by $P=4\pi n_d r_d \lambda_0^2$ with $\lambda_0=(T_e/4\pi n_i Z_i^2 e^2)^{1/2}$. In deriving (6), we have used the charge-neutrality condition, $n_e-Z_in_i+Z_nn_n-q_dn_d/e=0$. When $\kappa\to\infty$, we have $a_\kappa\sim 1$ and $b_\kappa\sim\exp(U)$. As a consequence, (2.6) exactly reduces to the earlier results (Mamun and Shukla 2003) showing the dust grain potential in a Maxwellian dusty plasma. Equation (2.6) can be solved numerically to study the relation between the dust grain surface potential and the dust-to-ion number density ratio when the plasma system is far away from thermal equilibrium.

Note that if the negative ion streaming speed V_0 is much larger than the negative ion thermal speed, then the negative ion current can be expressed in the following form:

$$I_{n\kappa}=-2\sqrt{\pi}r_d^2B_\kappa\left\{1+\frac{2(\kappa-1)}{\kappa}\frac{2Z_ne\varphi_d}{m_nV_0^2}\right\}Z_nen_nV_0. \quad (2.7)$$

Using (2.3), (2.4), and (2.7) into (2.1), we obtain

$$\sqrt{\sigma}-\frac{a_\kappa Z_i U}{\sqrt{\sigma}}-\mu\left(1-\alpha\frac{Z_n}{Zi}+Z_iPU\right)b_\kappa$$
$$=\alpha\beta\sqrt{c_\kappa}\sqrt{U_0}\frac{Z_n}{Z_i}\left(1+d_\kappa\frac{Z_nU}{U_0}\right), \quad (2.8)$$

where

$$c_\kappa=\frac{\kappa}{2\kappa-3},\ d_\kappa=\frac{2(\kappa-1)}{\kappa},\ \text{and } U_0=\frac{m_nV_0^2}{2T_e}.$$

Equation (2.8) represents a relation between the dust grain surface potential U and the variable P in the presence of negative streaming ions when the system is far away from thermal equilibrium. For taking the spectral index $\kappa\to\infty$, we have $a_\kappa\sim 1$, $b_\kappa\sim\exp(U)$, $c_\kappa\sim 0.5$, and $d_\kappa\sim 1.9$. Hence, (2.8) exactly coincides the results of Mamun and Shukla (2003). Furthermore, neglecting the negative ion number density and streaming speed through the parameters α and U_0 and assuming the positive ion-to-electron temperature ratio to be unity (viz. $\sigma\sim 1$), we may reproduce (2.3) of Barkan et al. (1994) for isolated dust grains.

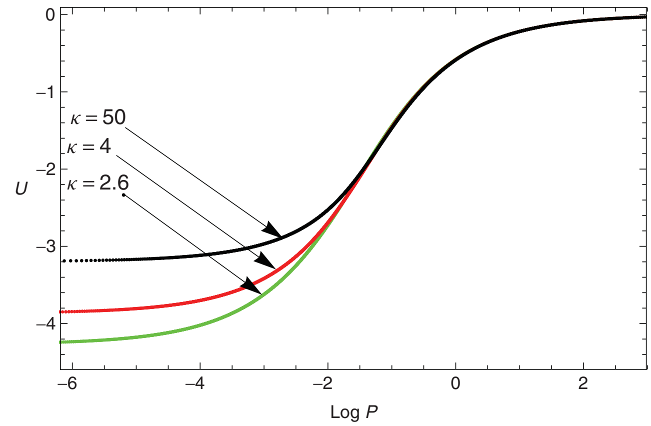

Figure 1. (Colour online) Normalized dust surface potential ($U=e\varphi_d/T_e$) versus normalized dust number density parameter ($P=4\pi n_d r_d\lambda_0^2$) (as given in (2.6)) for different values of spectral index κ (=2.6,4,50). Other numerical values used are: $\sigma=1$, $\alpha=0.4$, $\beta=1$, $\gamma=0.3$, $\mu=242.8$, and $Z_i=1=Z_n$.

3. Numerical results and discussion

For numerical illustration, we have solved (2.6) and (2.8) numerically to study the dust surface potential U as a function of dust parameter P (dust-to-ion number density ratio) and to examine the effects of various plasma parameters in a non-Maxwellian dusty plasma. The latter contains negative and positive ions (O_2^- and O_2^+ ions) in addition to electrons and negatively charged isolated dust grains. To find numerical solutions to (2.6) and (2.8), the Wolfram Mathematica version 8 is used for programming, even though it is a time-consuming and tedious task. We have also considered some normalized values, which are consistent with low-temperature laboratory plasmas (Amemiya et al. 1998, 1999; Vyas et al. 2002; Mamun and Shukla 2003), namely $\alpha=0-0.6$, $\beta=1$, $\gamma=0.1-1$, $\sigma=0.1-1$, $U_0=0.1-1$, and $Z_n=1=Z_n$.

Figure 1 plots the normalized dust surface potential ($U=e\varphi_d/T_e$) as a function of normalized dust density parameter ($P=4\pi n_d r_d\lambda_0^2$) (as given by (2.6), which is now modified with the power-law κ-distribution) for varying the spectral index $\kappa\sim$ (2.6, 4, 50) with fixed values of $\mu=242.8$, $\alpha=0.4$, $\beta=1$, $\gamma=0.3$, $\sigma=1$, and $Z_i=1=Z_n$. See that as we increase the value of kappa, the curves tend to approach the Maxwellian case. For $\kappa\sim 50$, our result is exactly in agreement with earlier studies (Mamun and Shukla 2003), which is in thermodynamic equilibrium. Interestingly, at lower κ-values, the magnitude of the negative dust surface potential increases and the effect becomes significant when $\log P$ is less than −1.75. The dependence of negative ion temperature γ (=0.1, 0.5, 1) is shown on

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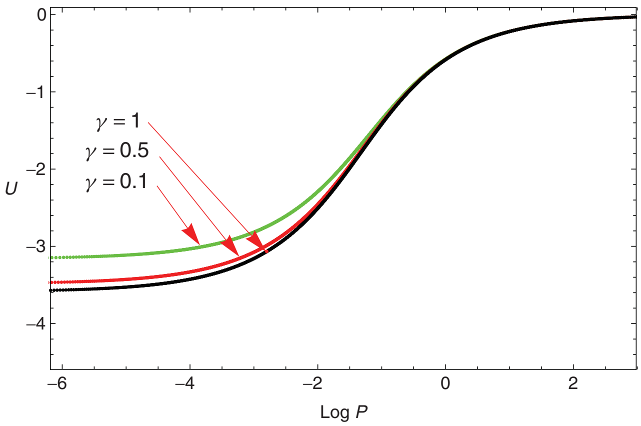

Figure 2. (Colour online) Normalized dust surface potential (U) versus normalized dust number density parameter (P) (as given in (2.6)) for different values of the negative ion-to-electron temperature ratio γ (=0.1, 0.5, 1) with fixed $\kappa = 3$ and $\sigma = 0.2$. Other numerical values are the same as in Fig. 1.

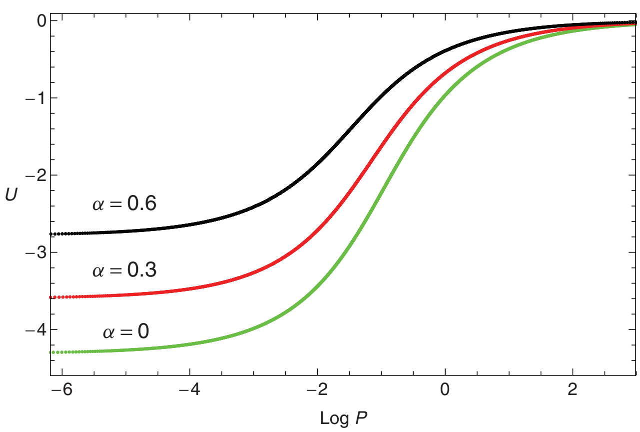

Figure 3. (Colour online) Normalized dust surface potential (U) versus normalized dust number density parameter (P) (as given in (2.6)) for different values of the negative ion number density α ($=0$, 0.3, 0.6) with $\kappa = 4$, $\gamma = 0.5$, and $\sigma = 0.2$. Other numerical values are the same as in Fig. 1.

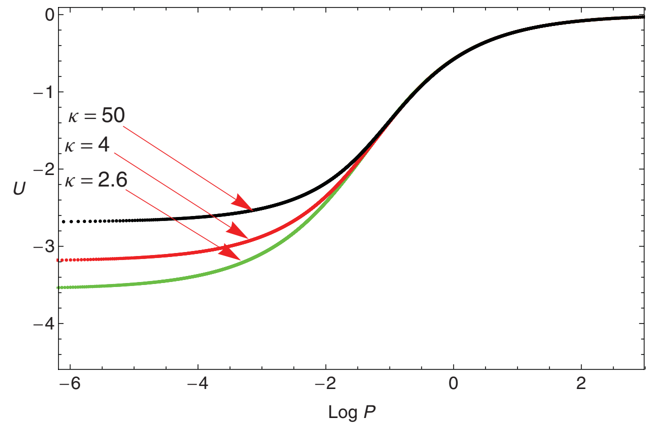

Figure 4. (Colour online) Normalized dust surface potential (U) versus normalized dust number density parameter (P) (as given in (2.8)) for different values of $\kappa(= 2.6, 4, 50)$ with $U_0 = 0.1$. Other numerical values are the same as in Fig. 1.

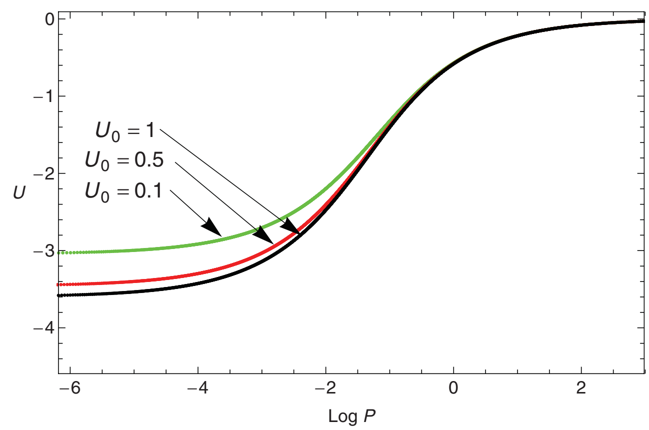

Figure 5. (Colour online) Normalized dust surface potential (U) versus normalized dust number density parameter (P) (as given in (2.8)) for different values of the negative ion streaming speed $U_0(= 0.1, 0.5, 1)$ with $\kappa = 2.6$ and $\sigma = 0.2$. Other numerical values are the same as in Fig. 1.

the curves of U and (P) (given by (2.6)) in Fig. 2. It is evident from the plot that the magnitude of the dust surface potential increases with the increase of negative ion temperature effect. However, the negative ion number density would lead to decrease the dust surface potential at a low kappa value (viz. $\kappa = 4$) and affects significantly in the range $-6 < \text{Log}\, P < 2$. In Figure 3, we note that increase in the negative ion number density causes a significant reduction of electrons via the charge-neutrality condition.

On the other hand, Figs. 4 and 5 represent how the power-law κ-distribution and the negative ion streaming speed effects (given by (2.8)) modify the dust surface potential or dust charge. The magnitude of the latter decreases (increases) as we increase the value of kappa parameter (the negative ion streaming speed U_0).

To summarize, we have presented the dust charging process associated with the collection of electrons and positive/negative ions in a non-Maxwellian dusty plasma, accounting for the power-law κ-distribution. We have considered the current balance equation when the dust charge attains its steady state value (viz. $q_d = \text{const}$) and can be solved numerically to find the dust surface potential in the presence of negative ions. In other words, we have derived the expressions for dust grain surface potential by solving the current balance equation containing non-Maxwellian currents due to electrons and positive/negative ions for negatively charged dust grains. Numerical analyses reveal that the variation of plasma parameters, such as the κ spectral index, the negative ion-to-electron temperature ratio (γ), the negative–positive ion density ratio (α), and the negative ion streaming speed (U_0) significantly modify the profiles of the equilibrium dust surface potential. For increasing the κ-values, the magnitude of the equilibrium dust surface potential decreases and approaches to the Maxwellian case (Mamun and Shukla 2003), while the negative ion number density also shows similar effect. However, the negative ion-to-electron temperature ratio and the negative ion streaming speed would lead to

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enhance the dust grain surface potential or dust charge. Our results can be helpful for studying dust charging process in low-temperature plasmas, where the plasma particles are far away from thermal equilibrium.

Acknowledgement

This paper is dedicated to Professor Padma Kant Shukla for his devoted and unlimited contributions in science.

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