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IBN RUSHD'S THEORY OF *MINIMA NATURALIA*

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The essence of the theory of *minima naturalia* is the contention that a physical body is not infinitely divisible *qua* that specific body. Having a form, it must have a "reasonable" size. The evolution of the theory of *minima naturalia* is an intriguing chapter in the history of science. It involves an on-going dialogue between the two diametrically opposed approaches to the understanding of the world – Aristotelianism and atomism – and illustrates how these two rival world views came somewhat closer to each other.

The theory is commonly considered to be a Scholastic one, which developed in the context of the interpretation of several statements in Aristotle. It was studied by the two pioneer students of Scholastic science, Pierre Duhem and Aneliese Maier. Neither ascribes any role in the development of this theory to Ibn Rushd. Duhem quotes Aristotle's statement that "flesh cannot increase or diminish in quantity beyond a certain limit",[1] and remarks "Ce texte ne semble guère avoir retenu l'attention d'Averroès […]. Mais très vite, la Scolastique latine s'est emparée de cette courte phrase et a développé l'idée qu'elle contenait en germe".[2] Maier, who usually studies the views of Avicenna and Averroes carefully before coming to those of the Latin scholars, only quotes the relevant passage of Aristotle and remarks that from these statements the Scholatics developed their thesis.[3] Several more recent studies do not correct the

[1] *Physics* I.4 187b35, Charlton's translation. The context is Aristotle's criticism of Anaxagoras' theory of matter.

[2] Duhem, *Le système du monde. Histoire des doctrines cosmologiques de Platon à Copernic* (Paris, 1954), vol. VII, p. 42. This attitude is typical of Duhem, who elsewhere comments "Il ne faut pas nous attendre à trouver chez les philosophes arabes la profondeur et l'originalité de pensée d'un Damasius ou d'un Simplicius" (*ibid*., p. 159).

[3] A. Maier, *Die Vorläufer Galileis im 14. Jahrhundert* (Roma, 1949), pp. 179-96, on p. 180.

impression that the theory is of a Scholastic origin.[4] The only scholar I found who "does justice" to Ibn Rushd and to his role in the development of the theory is Van Melsen,[5] and I shall refer to him several times in what follows.

Ibn Rushd's theory of *minima naturalia* was neglected, I believe, because it is most clearly presented in the middle commentary on the *Physics*, while the Scholastics were acquainted with his physics mainly or exclusively through the Latin translation of the long commentary.[6] There are a few brief recapitulations in the long commentary, but only when reading the middle commentary can one follow the course of the development of Ibn Rushd's theory and to understand what motivated it. The paper is a preliminary presentation of Ibn Rushd's theory of *minima naturalia*.[7] It does not deal with the question to what extent Scholastic scholars were acquainted with Ibn Rushd's theory, which is a subject for a separate study.

[I] ARISTOTLE'S TWO APPROACHES

The basic question with which we are dealing is whether a physical body is a continuum. In the writings of Aristotle, as often happens, two different answers can be found to this question, which are not easy to reconcile. Before coming to Aristotle's two answers, I shall briefly note that Aristotle offers two different definitions of continuity in the *Physics*: in book VI continuous "is that which is divisible into divisibles that are infinitely

[4] E.g. Wallace in *The New Catholic Encyclopedia* (New York, 1966), I, pp. 1020-4 discusses the theory of *minima naturalia* but does not mention the name of Ibn Rushd. Multhauf in his article "The science of matter" describes the theory of *minima naturalia* as being of a Scholastic origin. See D.C. Lindberg, *Science in the Middle Ages* (Chicago, 1978), pp. 369-90, on p. 385-6.

[5] A.G. Van Melsen, *From Atomos to Atom* (New York, 1960), pp. 58-60. Pyle follows Van Melsen on this subject. See A. Pyle, *Atomism and its Critics* (Bristol, 1995), pp. 217-19.

[6] The translation of the long commentary is attributed to Michael Scotus and was well known. The first three books of the middle commentary were translated from the Hebrew much later by Jacob Mantino. Of the Arabic original there is only an outline. See S. Harvey, "A unique Averroes MS in the British Museum", *Bulletin of the School of Oriental and African Studies*, 45 (1982): 571-4. The commentary was translated twice into Hebrew and is extant in many manuscripts.

[7] A more complete presentation of the theory and of its importance to the understanding of Ibn Rushd's natural philosophy is included in my forthcoming comparative study of Ibn Rushd's three commentaries on the *Physics*.

divisible";[8] in book V things are called continuous when they are contiguous and "the touching limits of each become one and the same and are contained in each other".[9] I have argued elsewhere that book VI is an early book and the definition of VI is the earlier.[10]

Aristotle's first answer is given in *Physics* VI, and assumes the definition of book VI: "body" by definition is infinitely divisible. This answer reflects a certain lack of distinction between "the mathematical" and "the physical", which is typical of Aristotle's early physics. The concept "physical body" is neither defined by Aristotle nor is it the subject of a systematic discussion.[11] Aristotle's definitions of "body" are mathematical: a body is a magnitude divisible in three dimensions or a magnitude which extends in three dimensions;[12] magnitude, in Aristotle's philosophical dictionary, is that which is divisible into continuous parts".[13]

According to these definitions infinite divisibility is the essence of being a body. Is this equally true of a mathematical and of a physical body? The answer of *Physics* VI is decidedly positive: there is no difference between the continuity of a mathematical and of a physical body. In VI.1 Aristotle argues that nothing continuous is made up of indivisible parts, and comments: "the same reasoning applies equally to magnitude, to time and to motion: either all of them are composed of indivisibles and are divisible into indivisibles, or none".[14] If magnitude is composed of indivisibles "the being moved will also be composed of indivisibles".[15] Thus the "being moved", which is a physical body, and magnitude, which is basically a mathematical concept, have the same structure. The thesis of parallel divisibility of *Physics* VI is an important stage in the evolution of Aristotle's natural philosophy. As Waschkies has shown, before Aristotle only spatial magnitudes were considered to be

[8] *Physics* VI.2, 232b24-25.

[9] *Physics* V.3, 227a9-11.

[10] "The early stages in the development of Aristotle's *Physics*", forthcoming.

[11] See A. Edel, *Aristotle and his Philosophy* (London, 1982), p. 49.

[12] *De caelo* I.1, 268a8-9; I.7, 274b20; *Physics* III.5, 204b20; *Met*. V.6, 1016b28; XI.10, 1066b31-32; *Top*. IV.5, 142b24. Aristotle also gives another definition: "a body is that which is bounded by a surface" (*Physics* III.5, 204b5; *Met*. XI.2, 1060b15; XI.10,1066b23).

[13] *Metaphysics* V.13, 1020b12.

[14] *Physics* VI.1, 231b18-19.

[15] *Physics* VI.1, 231b26.

continuous.[16] Aristotle's major project in *Physics* VI was to generalize the concept of continuity and to apply it also to the physical world. This project, I believe, was important to him both as part of his anti-atomistic campaign and as part of his attempt to establish natural science as a scientific discipline which relies on the firm basis of the science of geometry. Thus, magnitude, time and motion are associated through the notion of continuity, continuity being understood as infinite divisibility.

In VI.4 Aristotle further develops the thesis of parallel divisibility in more detail and concludes: "time, the motion, the being in motion, the thing that is in motion and the sphere of motion must all be susceptible to the same divisions (though spheres of motion are not all divisible in a like manner: thus place is essentially, quality accidentally divisible)".[17] The comment in parentheses is already a slight drawback. Physical bodies are more complex than mathematical bodies, still, in *Physics* VI and in *De caelo* this does not affect their divisibility:[18]

> This matter has already been considered in our discussion of movement,[19] where we have shown that an indivisible length is impossible. But with respect to natural bodies there are impossibilities involved in the view which asserts indivisible lines, which we may briefly consider at this point. For the impossible consequences which result from this view [that there are indivisible lines] will reproduce themselves when applied to physical bodies, but there will be difficulties in physics which are not present in mathematics [...] There are many attributes necessarily present in physical bodies which are necessarily absent from indivisibles [...] Attributes of bodies are all divisible in one of two ways. They are divisible into kinds [...] and they are divisible *per accidens* when that which has them is divisible".[20]

This passage asserts that physical attributes are infinitely divisible and that natural bodies, like the mathematical, are infinitely divisible. In the same book, he uses the divisibility of the elements as an argument against Plato's geometrical account of the structure of the elements: "the element air is divisible and

[16] H.J. Waschkies, "Mathematical continuity and continuity of movement", in F. De Gant and P. Souffrin (eds.), *La Physique d'Aristote et les conditions d'une science de la nature* (Paris, 1991), pp. 151-79. This view is assumed in Aristotle's early philosophical dictionary, *Metaphysics* V 13, 1020a10-12.

[17] *Physics* VI.4, 235a15-17.

[18] Similarly in I.4 a complete separation is impossible "both in respect of quantity and of quality – of quantity because there is no minimum magnitude, and of quality, because affections are inseparable" (*Physics* I.4, 188a10-13).

[19] Probably referring to *Physics* VI.1.

[20] *De caelo* III.1, 299a9-22.

the same could be shown of fire and of all elements and of all bodies whose parts are relatively fine".[21]

Thus, Aristotle's first answer to the question whether a physical body is a continuum is positive: physical and mathematical bodies have the same continuous structure, continuity being defined as infinite divisibility. In *Physics* VI.4 and in *De caelo* III.1 the qualities of physical bodies do not affect their divisibility, still, the distinction between the divisibility of the qualitative and the quantitative aspects was a first step towards the second answer.

Aristotle's second answer to the question whether a physical body is a continuum reflects, I believe, a more mature stage in the development of his *Physics*. After the introduction of the concepts of matter and form in *Physics* I,[22] the isomorphism thesis became less evident. Consisting of matter and form, physical body is a complex structure, and the difference between mathematical and physical bodies becomes clearer.[23] This consideration led Aristotle to his second answer: a physical body must have a definite size – neither too big nor too small. In *Physics* I, arguing against Anaxagoras' theory of matter he remarks:

> It is necessary that if a part of a thing can be as large or small as you please, then so can the whole, and if it is not possible for any animal or plant to be as large or small as you please, it is not possible that any part should be either, for if it could, so could the whole. Now flesh and bone are parts of animals, and fruits are parts of plants. Clearly then neither flesh nor bone nor anything of the sort can proceed indefinitely far either in enlargement or in diminution.[24]

Physics I.4 is usually considered to be the main text in Aristotle which leads in the direction of a theory of *minima naturalia*.

[21] *De caelo* III.5, 304b1-2.

[22] In my forthcoming paper "The early stages in the development of Aristotle's *Physics*" I argue that parts of the *Physics* (notably book VI) and parts of *De caelo* predate the "hylomorphic turn" and are earlier than the other books of the *Physics*.

[23] It has been suggested that Aristotle regarded also mathematical body as consisting of form and a kind of matter – intelligible matter. See I. Mueller, "Aristotle on geometrical objects", *Archiv für Geschichte der Philosophie*, 52 (1970): 156-71, on p. 164; T.C. Anderson "Intelligible matter and the objects of mathematics in Aristotle", *New Scholasticism*, 43 (1969): 1-28. This interpretation was, however, rejected by most commentators. See for example, J. Annas, *Aristotle's Metaphysics books M and N*. (Oxford, 1976), pp. 30-1.

[24] *Physics* I.4, 187b14-20, Charlton's translation. A similar, but somewhat vague statement appears also in *Physics* VI.10, 241a32-b2.

There are a few statements in other places suggesting that physical body cannot be infinitely divisible and still maintain its physical properties.[25] For example: "a drop of wine does not mix with ten thousand pitchers full of water for its form dissolves".[26] There is a minimal force that can cause movement. Therefore: "Zeno's reasoning is false when he argues that there is no part of the millet that does not make a sound".[27]

In *Physics* III, discussing the subject of infinite by addition and by division, Aristotle suggests that the matter-form structure renders only the second kind of infinite possible, because matter is infinitely divisible, while the form is associated with the whole. The sentence is somewhat vague (as the many brackets added by the translator testify):

> There seems to be no infinite by addition such as to exceed any magnitude, while there is [an infinite] by division [of that kind] – this too is a reasonable result. For it is matter and [so] the infinite, that is surrounded [and] inside, and it is the form that surrounds.[28]

In summary, Aristotle offers two answers to the question whether a physical body is a continuum: According to the first a physical body, *qua* being a three-dimensional magnitude, is infinitely divisible (mathematical and physical magnitudes being equally divisible); according to the second a physical body, *qua* having form or qualities, is not infinitely divisible and must have a definite size. This second answer is the Aristotelian basis of the theory of *minima naturalia*. I agree with Van Melsen that the theory "is found in Aristotle only in an embryonic state".[29]

[II] THE TWO TRADITIONS

Aristotle's two answers to the question whether a physical body is a continuum led, I suggest, to the development of the theory of *minima naturalia* along two different channels.

One channel was direct. Commentators on the *Physics* elaborated on Aristotle's second answer, namely that physical body is

[25] See also Van Melsen, *From Atomos to Atom*, pp. 41-4.
[26] *De gen. et cor.* I.10, 328a26-8. See also *De sensu* 6, 446a8-9.
[27] *Physics* VII.5, 250a20.
[28] *Physics* III.7, 207a33-6; Hussey's translation.
[29] Van Melsen, *From Atomos to Atom*, p. 44.

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not infinitely divisible, and further developed this idea. I shall mention two examples:

Commenting on the two passages quoted above from *Physics* I.4 and III.7 Philoponus writes:

There must be a lower limit too: no man has the size of a fist or a finger or a grain, because if something is too small it cannot receive a form either; a carpenter can only make the form of a boat from a piece of wood when it is large enough, and a potter cannot make a jar from one grain of clay.[30]

The division of the form often comes to an end because it has a certain limit, with regard to the smaller as to the greater (for it has been said that forms are not naturally able to remain in every magnitude), but because of matter the cutting does not come to an end.[31]

Commenting on the same passage from *Physics* I.4 quoted above Thomas Aquinas writes:

It must be pointed out that a body, considered mathematically, is divisible to infinity. For in a mathematical body nothing but quantity is considered [...] But in a natural body form also is considered.[32]

According to these interpretations matter is infinitely divisible, but the body composed of matter and form not necessarily.

The other channel through which the theory of *minima naturalia* developed was indirect. We shall see that both Epicurus' concept of minimum and Ibn Rushd's theory of *minima naturalia* were responses to the thesis of parallel divisibility of *Physics* VI, and particularly to the statement of VI.4 that everything that changes must be divisible.

In order to better understand this indirect process let us distinguish between atoms, which are elementary unsplittable particles, and *minima*, which are in some theoretical sense the smallest thing possible, but are not necessarily well-distinguished particles. Things are made up or composed of atoms, divisible into *minima*. Scholars agree that Presocratic atomism conceived atoms both as unsplittable particles and as theoretical

[30] A paraphrase of Philoponus on *Physics* I.4 187b7ff.: P. Lettinck, *Aristotle's Physics and its Reception in the Arabic World* (Leiden, 1994), p. 56.

[31] Philoponus, *In Phys.* 481, 3-6; translation: On Aristotle's *Physics* 3, translated by M.J. Edwards (Ithaca, 1994), p. 143.

[32] St. Thomas Aquinas, *Commentary on Aristotle's Physics*, translated by R.J. Blackwell, R.J. Spath and W.E. Thirlkel (New Haven, 1963), book I lecture 9, p. 34.

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minima.[33] Recent studies have shown that Epicurus distinguished between the two, and introduced a concept of *minimum* (*elachiston*),[34] which is different from that of the atom. The atoms of Epicurus are not indivisible; they have parts which are indivisible, but are not free-standing entities.[35] Thus Epicurus distinguishes between "being composed of" and "being divisible into", and presents a new conception of atomism, more sophisticated than that of his Presocratic predecessors.

Why did he introduce the concept of minimal part in addition to that of atom?[36] An answer to this question was suggested by Furley in 1967 and was widely accepted: Epicurus introduced *minima* into atomic theory as a response to Aristotle's criticism of early atomism in *Physics* VI. According to the analysis of motion of *Physics* VI "everything that changes[37] must be divisible"; "that which is without parts cannot be in motion except accidentally".[38] According to the analysis of contact a continuum cannot be composed of indivisibles.[39] These arguments apparently threaten the atomistic theory which was based on the conception of the atom as indivisible. Furley suggests that Aristotle's arguments led Epicurus to introduce indivisible *minima* which have dimensions, but unlike atoms cannot stand apart.[40]

Did the atomistic tradition influence the Aristotelian in antiquity? Alexander of Aphrodisias, commenting on *Physics* I.4, writes: "in every separation a certain number of *elachista* is

[33] D.J. Furley, "Indivisible magnitudes", in *Two Studies in the Greek Atomists* (Princeton, 1967), p. 111; D. Konstan, "Ancient atomism and its heritage: minimal parts", *Ancient Philosophy*, 2 (1982): 60-75, on p. 62.

[34] Epicurus, Letter to Herodotus, see A.A. Long and D.N. Sedley, *The Hellenistic Philosophers* (Cambridge, 1987), 9a, Greek text vol. II, p. 32; English translation vol. I, p. 40; commentary p. 42.

[35] D. Konstan, "Points, lines and infinity: Aristotle's *Physics* Zeta and Hellenistic philosophy", in J. Cleary (ed.), *Proceedings of the Boston Area Colloquium in Ancient Philosophy*, III (1988), pp. 1-32, on p. 5; "Ancient atomism and its heritage", p. 63.

[36] See for example G. Vlastos, "Minimal parts in Epicurean atomism", *Isis*, 56 (1965): 121-45, on p. 122. Vlastos approaches this question from a different direction than Furley.

[37] On Aristotle's use of the terms "move" and "change" see section IV below.

[38] *Physics* VI.4, 234b10-20; VI.10 240b8.

[39] *Physics* VI.1, 231a21-b17.

[40] Furley, *Two Studies*, ch. 8. The concept of extremity plays an important part in the development of this conception. An extremity does not stand by itself, yet according to Epicurus it has dimensions (*Two Studies*, p. 115). Konstan emphasizes the role of Aristotle's analysis of contact in the development of Epicurus' ideas. See D. Konstan, "Problems in Epicurean *Physics*", *Isis*, 70 (1979): 397-417, on p. 403.

separated".[41] It has been suggested that the word *elachista* is used in "a special technical sense".[42] It is possible that Alexander had Epicurus' *elachista* in mind, but he did not further contribute to the development of the theory.

It is accepted today, however, that Epicurus' theory of *minima* influenced medieval Arabic atomism. In 1936 Pines proposed that the atomism of Epicurus might have served as a major link towards the theory of Kalām "provided that it included the so-called theory of *minima*",[43] suggesting that there may be a resemblance between the *minima* (not the atoms!) of Epicurus and the *ajzā'* of the mutakallimūn. He adds, however that "the scantiness and uncertainty of our knowledge of that side of Epicurus' theory do not allow us to press this analogy any further with the hope of arriving at any sound conclusions".[44]

A few years ago Dhanani reexamined the question in the light of recent research on both ancient and Kalām atomism. He did not find an explicit statement of a theory of minimal parts analogous to that of Epicurean, but he concludes that there is sufficient evidence on which one can argue that such minimal parts were upheld by many of the mutakallimūn.[45] There is evidence that Ibn Rushd was acquainted with both conceptions of *minima*, that of the Kalām and that of Epicurus. I shall quote one example:

> And the mutakallimūn[46] of our nation, as they considered division to be division in act, denied that division can go on infinitely and contended that division terminates with something indivisible, and this is what they meant by indivisible parts. But the ancient upholders of indivisible parts accepted infinite division, as this is one of the postulates of geometry, and assumed that this is so in act, and according to them in one finite magnitude there are infinitely many indivisible parts.[47]

[41] Quoted from Van Melsen, *From Atomos to Atom*, p. 47.

[42] Van Melsen, *ibid*.; Wallace, *New Catholic Encyclopedia*, p. 1021 col. b.

[43] S. Pines, *Studies in Islamic Atomism* (Jerusalem, 1997 [Berlin, 1936]) pp. 112.

[44] *Ibid*., p. 113. In the comment that follows this passage Pines refers to Baily's book, *The Greek Atomists and Epicurus* of 1928. He is not yet acquainted with Luria's 1933 paper "Die Infinitisimaltheorie der antiken Atomisten", that had made some progress in the study of Epicurus' atomism.

[45] A. Dhanani, *The Physical Theory of Kalām* (Leiden, 1994), p. 106.

[46] Following here Zerahya ben Yishaq's translation (see next note). The two manuscripts of Qalonimus' translation listed below read "the ancients of our nation". The following sentence (not quoted here) confirms Zerahya's reading. It repeats the argument and all manuscripts read "mutakallimūn".

[47] *Middle Commentary on the Physics*, Hebrew translation by Qalonimus ben Qalonimus, Hamburg Staats- und Universitaetsbibliothek, MS Cod. hebr. 264, VI.1,

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[III] IBN RUSHD'S THEORY OF *MINIMA NATURALIA*

Ibn Rushd's statement of the theory of *minima naturalia* is more explicit and more elaborate than that of Aristotle or Philoponus. There are several statements of the theory and references to it in the commentaries of Ibn Rushd. I shall quote a few examples:

In the epitome on *Physics* VI "This division indeed occurs to the continuous thing *qua* being continuous, not *qua* being sensible body actually existing".[48] In the middle commentary on *Physics* III:

> And to him who would argue that the line is infinitely divisible *qua* a pure line, but *qua* a line of fire or a line of water it is not infinitely divisible, but divisible into *indivisible magnitude* which is the smallest magnitude which can assume the form of fire, as this magnitude is naturally bounded [...] – we say that the physicist[49] indeed studies magnitudes *qua* boundaries of natural enmattered bodies. Magnitude is infinitely divisible *qua* matter, not *qua* form; *qua* form its divisibility is limited. This is why a magnitude of fire cannot be infinitely divisible *qua* a magnitude of fire; it can be infinitely divisible *qua* pure magnitude, not *qua* being a natural body.[50]

Commenting on the passage from *Physics* III.7 quoted above (note 28) he adds: "The division of magnitude is in the matter, not in the form, as the form remains what it is".[51] There are a few references to the theory in the long commentary on the *Physics*:

> When we remove a part of fire and repeat this action again and again we finally reach a quantity which is such that by a further division the fire would perish, because there is a certain minimal quantity of fire.
>
> A line as a line can be divided infinitely. But such a division is impossible if the line is taken as made of earth.

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fol. 67b26-68a7. The arguments goes on to 68a15; Paris, MS BN héb. 941, fol. 92b8-15; Hebrew translation by Zerahya ben Yishaq She'altiel, Oxford Bodl., MS 1386, fol 83a19-b2. On the question whether Epicurus considered the number of *minima* in an atom is finite or infinite see Konstan, "Ancient atomism", pp. 63-6.

48 *Epitome on the Physics, Corpus commentariorum Averrois in Aristotelem*, ed. J. Puig (Madrid, 1983) p. 88.3-4; *Rasā'il Ibn Rushd* (Hyderabd, 1947), p. 73; Spanish translation: J. Puig, *Averroes: Epítome de fisica, traducción y estudio* (Madrid, 1987), p. 187.

49 Literally "the owner of this wisdom".

50 Ibn Rushd, *Middle Commentary on the Physics*, III.3.5, Hamburg, MS Cod. hebr. 264, fol. 31a22-25, b4-10.

51 *Ibid.*, fol. 32b12-13.

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> It is impossible for something to increase or decrease infinitely, because if the quantity determined by nature is passed, whether by increase or decrease, the being perishes.[52]

The question arises as to what made Ibn Rushd adopt an interpretation of the *Physics* which, in a sense, is non-Aristotelian. Aristotle's text by no means calls for it. I have suggested above that Ibn Rushd might have been acquainted with the Epicurean concept of *minimum*, but the question still remains to be answered: why did he adopt a concept of a school which he often criticized, and whose views he mostly rejected. My suggestion is that Ibn Rushd developed his theory not as an elaboration of Aristotle's second answer to the question whether a physical body is a continuum, but rather as an attempt to solve the difficulties raised by the first answer. The issue was not Aristotle's theory of *minima naturalia*, but his theory of continuity developed in *Physics* VI. In attempting to "save" this theory Ibn Rushd found a certain compromise with atomism unavoidable. I shall explain the difficulties in Aristotle's presentation and present Ibn Rushd's solution.

The association between body and motion is a central issue in Aristotle's early *Physics*. Aristotle develops this conception in *Physics* VI and VII. In VI.4 he states that everything that moves must be divisible,[53] and argues that motion and the moved body are equally divisible.[54] In VII.1 he suggests that a body moves *qua* being divisible, namely *qua* having parts. Nothing is moved *per se* and not by one of its parts being moved.[55] Both the arguments of VI.4 and VII.1 rely on the premise that every motion is from something to something.[56] Both arguments associate the possibility of motion and the continuity of motion with the divisibility of the moved body. The conception of motion as a continuous entity, however, is difficult, and both arguments

[52] *Long Commentary on the Physics*, Latin translation: *Aristotelis opera cum Averrois commentariis* vol. IV (Venice, 1562), VIII, comm. 44 fol. 384K; IV comm. 72 fol. 163H; VI comm. 32 fol. 267D; Van Melsen's translation, *From Atomos to Atom*, pp. 59-60. See also I comm. 37 fol. 25A; VI comm. 59 fol. 285C.

[53] *Physics* VI.4, 234b10-20.

[54] *Physics* VI.1, 231b18-232a18; VI.4, 234b20-235a10.

[55] *Physics* VII.1, version a 241b37-49; version b 241b27-242a15.

[56] Both demonstrations explicitly refer to this premise. See *Physics* VI.4, 234b10; VII.1, 242a6.

involve serious difficulties.[57] Ibn Rushd develops his theory of *minima naturalia* in his middle commentary on VI.4 and VII.1, as an answer to these difficulties.

Before examining Aristotle's arguments and Ibn Rushd's presentations, it should be noted that in *Physics* VI Aristotle usually uses the term "motion", but in several passages he uses the term "change" instead.[58] In the present context, the terms can be understood as interchangeable.

Aristotle's argument in *Physics* VI.4 is:[59]

- Every change is from something to something.
- When a thing is at that to which it was changing it is no longer changing.
- When it and all its parts are at that from which it was changing it is not changing (for that which is in whole and part in an unvarying condition is not in a state of change).
- It follows that part of that which changes must be at the starting point and part at the goal. (Here by 'goal of change' I mean that which comes first in the process of change: e.g. in the process of change from white the goal in question will be grey, not black: for it is not necessary that that which is changing should be at either of the extremes).
- It is evident, therefore, that everything that changes must be divisible.

The argument is dubious, and according to Bostock "must be dismissed as worthless".[60] A major problem is the problematic concept of "that which comes first in the process of change" or a "first goal of change". This concept is not well-defined, and conflicts with the main thesis of *Physics* VI, namely that change is a continuous process. The reader of book VI, therefore, has two alternatives: either to reject the concept of a first goal of change, or to adopt this concept, at the cost of giving up the conception

[57] Without modern calculus and the conception of instantaneous velocity, it is hard to understand what happens at the point of division, or at the endpoints of a motion interval. Aristotle's comment that a thing cannot "both be in motion and at the same time have completed its motion" (*Physics* VI.1, 231b27), reflects this difficulty. The argument of VI.5 that in a process of motion (or change) there is a last, but no first element also illustrates the difficulty (*Physics* VI.5, 236a10 ff).

[58] In my forthcoming paper "The early stages in the development of Aristotle's *Physics*" I try to explain why in these passages Aristotle uses the term "change".

[59] *Physics* VI.4, 234b10-20.

[60] D. Bostock, "Aristotle on continuity in *Physics* VI", in L. Judson (ed.), *Aristotle's Physics* (Oxford, 1991), pp. 179-212, on p. 201.

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of change as a continuous process. Ibn Rushd prefers the second way. It is in this context that his theory of *minima naturalia* comes in and replaces Aristotle's theory of continuous motion.

Ibn Rushd's argument follows Aristotle but the concept of "first goal of change" is more explicitly adopted and used. To make the argument more readable I shall abbreviate the two phrases 'that towards which the change is' and 'that from which the change is', which occur frequently, as T and F respectively.

- Every change is from something to something.
- When the changing body is at T it is at rest.
- When a thing is at F it is at rest.
- The changing body actually changes,[61] when a part of it is at F and a part of it at the first T.
- It cannot be as a whole in either of them.
- It is also impossible for it to be in none of them, because between F and the first T there is no intermediate. For if there was an intermediate, the first T would not be first. Indeed the intermediate [is[62]] between F and the final T, and there is no motion to the final T, but through the first T. For example if the change is from white to black it first changes from white to yellow.[63]
- Since everything that changes – a part of it is in F and part in the first T, it follows that whatever changes is divisible.[64]

Ibn Rushd is aware of the implications of Aristotle's argument and does not hesitate to accept them: between F and the final T

[61] In *Physics* VI Aristotle does not yet distinguish between an actual and a potential division. Ibn Rushd, here uses the term "anachronistically".

[62] Written above the line in Paris, BN MS héb. 941, fol. 98a1.

[63] Aristotle refers to the intermediate colour between black and white as *phaion*, namely gray (234b18). A word for gray did not exist in medieval Hebrew. Qalonimus in his translation of Ibn Rushd's Middle commentary refers to this color as yellow. Yellow and green were interchangeable terms in medieval Hebrew, so one often finds both yellow and green in this context. Gersonides uses yellow or *qamely*. This word can be derived from the root *qml* – to become dry (referring to plants), See *Commentary on the Middle Commentary on the Physics*, Paris, MS BN héb. 964/1, fol. 111b1. The use of yellow or green as intermediate colours between white and black is explained by the Aristotelian theory of the generations of colours. See on this subject R. Fontaine, "Red and yellow, blue and green: The colours of the rainbow according to medieval Hebrew and Arabic scientific texts", in Y. Toby (ed.), *'Ever and 'Arav Contrasts between Arabic Literature and Jewish Literature in the Middle Ages and Modern Time* (Tel-Aviv, 1998), pp. VII-XXV, on p. VIII.

[64] Ibn Rushd, *Middle Commentary on the Physics*, Hamburg, MS Cod. hebr. 264, fol. 72a5-19.

there is an intermediate, but not between F and the first T. Thus the structure of the part is not identical to that of the whole. Commenting on this passage, Moshe Narboni, Ibn Rushd's fourteenth century Hebrew follower, further explains the conception of "quanta of change":

> For every green, one can imagine a green lesser than it, and this [process] can be continued indefinitely. But in actuality a definite limit is achieved, such that a lesser [green] does not exist in actuality. And the black, when it first changed into green, which is intermediate between white and black – first it changed into a part [of greenness] so small that a smaller part does not exist in actuality, though in potentiality it is possible. This way the change is a part of the body[65] which is so small, that it is impossible that a part smaller than it would be actually green, as it is impossible that greenness less than it exists in actuality. Therefore, in actuality a first T is possible but not in potentiality, since potentially division can be carried on indefinitely".[66]

Commenting on *Physics* VI.4 Ibn Rushd suggests that motion, or change, is divisible into indivisible minimal parts. In his commentaries on VII.1 he argues further that physical body is divisible into indivisible parts. In *Physics* VII.1 Aristotle proves that there is nothing that is "moved *per se* and not by one of its parts being moved".[67] He does not explain this statement further nor specify what the moving part is. The main point for him is the general association of motion with divisibility: a body does not move as a whole but as Helen Lang puts it, as "a whole of parts".[68] Ibn Rushd is more specific and suggests a positive interpretation of Aristotle's statement: a body is divisible into minimal parts and the minimal part is, in a sense, the mover. Let us look at the significant difference between the demonstrations of Aristotle and of Ibn Rushd.

Aristotle's argument briefly paraphrased is: Suppose that a body AB is moved *per se*, not by one of its parts.[69] Everything moved must be divisible [according to VI.4]. So let AB be divided at C. If CB does not move, then AB [the whole] will not move, because had it moved it would have been moved by the

[65] Should be, perhaps, part of the change.

[66] Moshe Narboni, *Commentary on the Middle Commentary on the Physics*, Paris, MS BN héb. 967, fol. 66a6-16.

[67] *Physics* VII.1, 241b34-242a49 (version a), 241b24-242a15 (version b).

[68] H.S. Lang, "Parts, wholes, and motion: *Physics* 7.1", in H.S. Lang, *Aristotle's Physics and its Medieval Varieties* (New York, 1992), pp. 35-62, on pp. 36, 41.

[69] *Physics* VII.1 version a, 241b38.

part AC, contrary to the assumption that it moves *per se*. Therefore if CB rests, the whole AB rests.[70]

The main point of the argument is that the state of motion or rest of the whole is determined by the state of motion or rest of the part. Aristotle's argument has been subjected to much criticism. Ibn Rushd does not criticize the argument but reinterprets it. He starts with a new distinction which Aristotle does not suggest:

- That which is moved essentially is either first or nonfirst.
- The 'first-moved' is that which is not moved because a part of it moves or can move essentially. For instance, the motion of a part of earth or water, namely this part that a part smaller than it cannot assume the form of water, as such magnitude is defined[71] for the natural bodies.
- The nonfirst is that which moves as a whole because a part of it moves essentially. For instance, the motion of any magnitude greater than the smallest part of earth, or the local motion of animals which is due to the vital heat that is in it.[72]

We see that the argument of VII.1 continues that of VI.4. In his commentary on VI.4 Ibn Rushd introduced the concept of 'a first that towards which the motion is' (a first T), namely a minimal indivisible part of motion. Commenting on VII.1, he introduces a concept of 'first-moved part', which is a minimal indivisible part of a moved body. The conceptions that body is divisible into indivisible parts and that motion is divisible into indivisible parts apparently complement each other. At this point comes the main point of the theory:

- The nonfirst-moved [magnitude] is moved by something else, which is the first-moved [part]. It is established that if a self-mover exists, namely that the mover and the moved are one and the same thing, it must necessarily be first.[73]

Ibn Rushd's argument is very different from Aristotle's. Aristotle's argument is indirect. He assumes the existence of

[70] *Physics* VII.1 version a, 242a38-49.
[71] Following Zerahya's translation; 'listed' or 'limited' in different manuscripts of Qalonomus' translation.
[72] Ibn Rushd, *Middle Commentary on the Physics*, Hamburg, MS Cod. hebr. 264, fol. 83a2-10.
[73] *Ibid.*, fol. 83a10-13.

"that which is moved *per se* and not by one of its parts being moved" as a hypothesis in order to rule out such a possibility. Ibn Rushd's demonstration is direct and constructive. He introduces a concept of minimal part and identifies the minimal part as that part which moves the whole. According to Aristotle a self-mover does not exist; according to Ibn Rushd that which is first-moved is a self-mover and also the mover of the whole.

Following the introduction of these terms Ibn Rushd presents Aristotle argument summarized above as an argument about first-moved bodies: "Having verified this premise [that there are Primary-Moved parts] we can establish that everything that is moved primarily and *per se* is moved by something else.[74] The outline of the argument is: there are two categories of moved objects, first-moved and nonfirst-moved. The latter are not self movers by definition, but are moved by the former. Aristotle's argument establishes that the former are not self movers either.[75]

The theory is presented also in the epitome on *Physics* VII.1:

> The 'first-moved', that which is not moved because of a part of it, is that which is moved essentially. And in these simple bodies (about which the doubt is) it is the minimal possible magnitude of fire that moves upwards or the minimal possible magnitude of earth that moves downwards, because the moved [part] of earth and of fire which is of such a character is a first-moved [part] because this movement cannot occur to a part of it, as there is no smaller part of fire, because the magnitudes of existing things are limited.[76]

Again the 'first-moved' is the minimal part, namely the most elementary mover-moved unit. Ibn Rushd presents the argument of VII.1 as an argument about the first-moved body and identifies the part that is at rest and the part that is not at rest with the form and the matter of the first-moved body. When the first-moved body rests "the mover in it becomes inactive", while "the remaining part of the first-moved body does not rest, and this is because it [the remaining part] did not loose the faculty (*maʿnā*) by which it is moved". The fact that the first-moved body can either move or be at rest points out to an inner differentiation, which turns out to be that between matter and form. The mover and the moved cannot coincide because when the first-moved body is at rest one faculty, that of the mover (namely the form) is idle while the other, that of being moved (namely

[74] *Ibid*., fol. 83a21-22.
[75] *Ibid*., fol. 83a23-b12.
[76] Ibn Rushd, *Epitome on the Physics* VII, Puig's edition p. 114.3-9.

the matter), remains unchanged. The faculty for being moved (matter) is also for being a magnitude and divisible;[77] the faculty for moving (form) is indivisible, and it is precisely this indivisibility which defines the minimal part as such.[78] Ibn Rushd concludes the discussion:

And in the moved body there are necessarily two faculties: one of them by which it is divided, and this is the faculty by which it is moved, and the second is indivisible, and it is the faculty that when lost – the movement would be lost, and this is necessarily the mover. Therefore bodies, similar to these simple bodies, the first-moved in them is divided *qua* being moved and indivisible *qua* being a mover, and therefore the mover in them is necessarily other than the moved. And divisibility occurs to them by their matter and indivisibility by their form. And the form is the mover and is other than the moved.[79]

While Aristotle's argument in VII.1 is an exercise in logic, Ibn Rushd's is an analysis of the structure of the first-moved body, namely the minimal part that maintains the body's form and motion. The argument of the epitome and the middle commentary is briefly recapitulated in the long commentary on this chapter:

And it is necessary that there be things that are first-moved (*prima mota*), as the natural bodies are not infinitely divisible in act[80] *qua* natural bodies. For instance the first-moved in the case of fire is a part such that a part smaller than it would not be fire in act.[81]

In itself this passage is not sufficiently clear, and the Scholastics, who did not read the epitome or the middle commentary, could not have fully understood Ibn Rushd's theory.

[IV] CONCLUSION

In conclusion I would like to summarize briefly the history of the concept of *minima naturalia*, emphasizing the dialogue between the two great and rival systems of thought, the Aristotelian and the atomistic.

The atomistic theory of Leucippus and Democritus offers an answer to the Eleatic impasse, and to Zeno's paradoxes in

[77] According to *Physics* VI.4 everything that is moved must be divisible.
[78] Ibn Rushd, *Epitome on the Physics* VII, Puig's edition pp. 114.16-115.6.
[79] Ibn Rushd, *Epitome on the Physics* VII, Puig's edition p. 115.6-12.
[80] "In act" in the Hebrew translation, "in eo" in the Latin.
[81] Ibn Rushd, *Long Commentary on the Physics* VII. Latin translation *Aristotelis opera*, fol. 307 I.

particular. Aristotle's analysis of the continuum in *Physics* VI offers a different answer, involving serious criticism of atomism, to Zeno's paradoxes. "Aristotle was emphatically not an atomist", as Molland writes, "He had no unique set of immutable and indivisible constituents of the universe, but a world in which all bodies were divisible, and for this reason did not form a constant set".[82] Epicurus, according to Furley, reconsidered the indivisibility of the atom in the light of Aristotle's criticism in *Physics* VI, and consequently modified classical atomism, distinguishing between the atoms which are the constituents of the universe and the *minima*, which are indivisible but not independently existing.[83]

Aristotelianism could not come to terms with the first concept, but could seriously consider the second. Ibn Rushd rejects the concept of atom, but adopts the concept of *minimum*, taking it out of the atomistic context and adjusting it to the "Aristotelian environment". *Minima* for Ibn Rushd are neither atoms (as they were for some Islamic atomists) nor parts of atoms (as they were for Epicurus), but rather parts of the continuum. Maier describes the *minima naturalia* as a kind of atoms, which also have the structure of the continuum.[84]

According to this account, the concept of *minimum* which was introduced by Epicurus as a response to Aristotle's theory of continuity developed in *Physics* VI, served Ibn Rushd to modify this very theory (of *Physics* VI) which contains inner difficulties. Ibn Rushd's theory of *minima naturalia* was only partly influential. Jewish fourteenth-century Aristotelians, who studied physics from Ibn Rushd's epitome and middle commentary, took it for granted. The Scholastic commentators were not acquainted with Ibn Rushd's argument but with a few statements of the theory and allusions to the argument in the long commentary. They developed a theory of *minima naturalia* that did not follow the specific direction indicated by Ibn Rushd, and focused on different questions. The atomistic aspects of the theory of *minima naturalia* gradually became better understood and eventually contributed to the rise of atomism in the seventeenth century.[85]

[82] G. Molland, "The atomization of motion: A facet of the scientific revolution", *Studies in the History and Philosophy of Science*, 13 (1982): 31-54, on p. 31.

[83] Furley, *Two Studies*, ch. 8.

[84] Maier, *Die Vorläufer Galileis*, p. 179.

[85] Wallace, *The New Catholic Encyclopedia*, p. 1022; Pyle, *Atomism and its Critics*, ch. 5.