Statistical Challenges in 21st Century Cosmology
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Statistics of cosmological fields

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Abstract. The general problem of the statistics of the primordial curvature perturbation field in cosmology is reviewed. The search for non-Gaussian signatures in cosmological perturbations, originated from inflation in the early Universe is discussed both from the theoretical point of view and in connection with constraints coming from recent observations and future prospects for observing/constraining them.

Keywords. Cosmology: early Universe: Inflation; Cosmic Microwave Background Radiation; Large Scale Structure of Universe; non-Gaussianity

1. Introduction

A very relevant fraction of the datasets used to obtain cosmological parameters and to understand the overall cosmological scenario relies on cosmological random fields, like the matter density fluctuation field, the gravitational potential field, the galaxy number density fluctuation field and their peculiar velocity field. These random fields are considered to be mutually connected either because of some common origin or owing to some dynamical mechanism.

In most theoretical treatments these fields are considered to be well described by Gaussian random fields prior to the action of gravity, that tends to create mode-coupling, owing to its intrinsically non-linear character. Of course, we see phase coherence (i.e. deviation from Gaussianity) in the sky, even on very large scales. A simple – and by many respects attractive – idea is that all the non-Gaussianity we observe is of gravitational origin and that perturbations (e.g. in the matter density field or in the so-called peculiar gravitational potential) were primordially Gaussian.

Historically, the first determination of the galaxy 3-point correlation function, i.e. the first evidence for a non-Gaussian signal in the galaxy distribution, was obtained in the late seventies, when Groth and Peebles (Groth & Peebles 1977) analysed the high-resolution Shane-Wirtanen catalog of galaxies and fitted the spatial three-point function ζ (indirectly obtained from angular correlations, thanks to the Limber equation) by the so-called hierarchical model: $\zeta(1,2,3) = Q\,(\xi(1,2) + \xi(1,3) + \xi(2.3))$, where $\xi(i,j)$ indicates the spatial two-point function at separation $|\mathbf{r}_i - \mathbf{r}_j|$ and Q is a phenomenological constant that they determined to be $Q = 1.29 \pm 0.21$.

The expectation was (and still is) that such a hierarchical formula arises due to the non-linear gravitational action. At that time, however, there also existed some radically alternative ideas on the origin of the large-scale structure of the Universe, which made of use of *strongly* non-Gaussian initial conditions for clustering: the so-called explosion scenario and various scenarios based upon the idea that cosmic defects (strings, textures) could have triggered structure formation in the Universe. Numerical N-body simulations of Large-Scale Structure (LSS), assuming some simple non-Gaussian models for the initial conditions were also performed (Moscardini *et al.* 1991; Weinberg & Cole 1992). Simple ideas were also proposed to study non-Gaussianity in Cosmic Microwave Background (CMB) temperature anisotropies (Coles & Barrow 1987).

Independently on the specific reasons which led to abandon these alternative ideas, we can say that the incredible improvement on the amount and quality of data on galaxy clustering and CMB temperature anisotropies (which had not been yet observed at that time) implied that the "rule of the game" on non-Gaussianity has completely changed! Nowadays we restrict ourselves to consider only slight variations on the Gaussian paradigm. Primordial fluctuations are assumed to be either exactly Gaussian or very mildly non-Gaussian, as described in the next section.

The plan of the paper is as follows. In Section 2 Ithe early-Universe model that is used to describe the mildly non-Gaussian random field considered in the cosmological framework will be introduced. Section 3 deals with CMB constraints on primordial non-Gaussianity. Section 4 briefly describes how LSS constrains primordial non-Gaussianity. Section 5 provides some general conclusions.

2. Origin of perturbations and primordial non-Gaussianity

It has now become standard practice to parametrise primordial non-Gaussianity by means of a Taylor expansion in powers of a Gaussian zero-mean field φ. One writes (Gangui *et al.* 1994; Wang & Kamionkowski 2000; Komatsu & Spergel 2001)

$$\Phi(\mathbf{x}) = \varphi(\mathbf{x}) + f_{\rm NL} \star \left(\varphi(\mathbf{x})^2 - \langle\varphi^2\rangle\right) + g_{\rm NL} \star \left(\varphi(\mathbf{x})^3 - \langle\varphi^2\rangle\varphi(\mathbf{x})\right) + \ldots \,. \tag{2.1}$$

Here the potential Φ is defined in terms of the "comoving curvature perturbation" ζ on super-horizon scales by $\Phi \equiv (3/5)\zeta$. In matter domination, on super-horizon scales, Φ is equivalent to Bardeen's gauge-invariant gravitational potential (Bardeen 1980), and I adopt this notation for historical consistency. The non-linearity parameters $f_{\rm NL}$ and $g_{\rm NL}$ set the amplitude of quadratic and cubic non-Gaussianity are constants in the simplest case (dubbed *local* non-Gaussianity) or may depend on space themselves for more general shapes. In all generality the symbol $\star$ denotes a convolution which reduces to a product in the local case (see e.g. Bartolo *et al.* 2004). Notice that, owing to the smallness of the gravitational potential itself (and hence of the Gaussian field φ) whose r.m.s. is smaller than 10^{-5}, our Taylor expansion makes sense even for relatively large values of $f_{\rm NL}$ and even larger values of $g_{\rm NL}$. In other words the percent of quadratic non-Gaussianity in the model is $f_{\rm NL}\Phi_{\rm rms}$ (and $g_{\rm NL}\Phi^2_{\rm rms}$ for cubic non-Gaussianity).

Besides having the great advantage of simplicity, the model above is actually well-motivated in the frame of inflationary models for the early Universe (see e.g. Bartolo *et al.* 2004, Chen 2010 for a review), which indeed predict non-Gaussianity of this type. The shapes of non-Gaussianity are fully described by their impact on the bispectrum (we restrict ourselves to quadratic non-Gaussianity, for simplicity; the case of cubic NG follows very similar lines, making use of the trispectrum). which is defined as

$$\langle\Phi(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)B_\Phi(k_1, k_2, k_3), \tag{2.2}$$

The bispectrum $B_\Phi(k_1, k_2, k_3)$ measures the correlation among 3 perturbation modes. Assuming translational and rotational invariance, it depends only on the magnitudes of the three wave-vectors. In general, the bispectrum can be written as

$$B_\Phi(k_1, k_2, k_3) = f_{\rm NL} F(k_1, k_2, k_3)\,. \tag{2.3}$$

The bispectrum is measured by sampling triangles in Fourier space. The dependence of the function $F(k_1, k_2, k_3)$ on the type of triangle (i.e., the configuration) formed by the

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three wave-vectors describes the *shape* (and the scale-dependence) of the bispectrum, which encodes a lot of physical information. Different non-Gaussianity shapes are linked to distinctive physical mechanisms that can generate such non-Gaussian fingerprints in the early Universe. Notice that, according to the latter notation, we treat $f_{\rm NL}$ as a constant and ascribe the shape dependence to the function $F(k_1, k_2, k_3)$. Let us provide here the most important shapes. We have

$$\begin{aligned} B_\Phi^{\rm local}(k_1,k_2,k_3) &= 2 f_{\rm NL}^{\rm local}\Big[P_\Phi(k_1)P_\Phi(k_2) + P_\Phi(k_1)P_\Phi(k_3) \\ &+ P_\Phi(k_2)P_\Phi(k_3)\Big] \\ &= 2A^2 f_{\rm NL}^{\rm local}\left[\frac{1}{k_1^{4-n_s}k_2^{4-n_s}} + {\rm cycl.}\right] , \end{aligned} \tag{2.4}$$

for the local case, havine parametrised the Φ power-spectrum as $P_\Phi(k) = Ak^{n_s-4}$, with A is a constant amplitude and n_s is the spectral index of scalar perturbations.

"Equilateral" non-Gaussianity is described via

$$\begin{aligned} & B_\Phi^{\rm equil}(k_1,k_2,k_3) = 6A^2 f_{\rm NL}^{\rm equil} \\ \times & \left\{ -\frac{1}{k_1^{4-n_s}k_2^{4-n_s}} - \frac{1}{k_2^{4-n_s}k_3^{4-n_s}} - \frac{1}{k_3^{4-n_s}k_1^{4-n_s}} - \frac{2}{(k_1k_2k_3)^{2(4-n_s)/3}} \right. \\ & \left. + \left[\frac{1}{k_1^{(4-n_s)/3}k_2^{2(4-n_s)/3}k_3^{4-n_s}} + (5 \text{ permutations})\right]\right\} , \end{aligned} \tag{2.5}$$

"Orthogonal" non-Gaussianity can be described by the template

$$\begin{aligned} & B_\Phi^{\rm ortho}(k_1,k_2,k_3) = 6A^2 f_{\rm NL}^{\rm ortho} \\ \times & \left\{ -\frac{3}{k_1^{4-n_s}k_2^{4-n_s}} - \frac{3}{k_2^{4-n_s}k_3^{4-n_s}} - \frac{3}{k_3^{4-n_s}k_1^{4-n_s}} - \frac{8}{(k_1k_2k_3)^{2(4-n_s)/3}} \right. \\ & \left. + \left[\frac{3}{k_1^{(4-n_s)/3}k_2^{2(4-n_s)/3}k_3^{4-n_s}} + (5 \text{ perm.})\right]\right\} . \end{aligned} \tag{2.6}$$

Other bispectrum shapes are of course allowed. We refer to ... for a general discussion of this important problem, as well as to the general discussion of trispeectrum shapes.

3. Cosmic Microwave Background constraints on primordial non-Gaussianity

As described above, quadratic inflationary non-Gaussianity in Bardeen's gravitational potential can be characterised by the dimensionless non-linearity parameter $f_{\rm NL}$, for any given non-Gaussianity shape. The *Planck* collaboration (see Planck Collaboration 2013a, for general overview) estimated $f_{\rm NL}$ for various non-Gaussianity shapes – including the three fundamental ones, local, equilateral, and orthogonal – predicted by different classes of inflationary models, using nominal mission CMB temperature maps.

Results for these three fundamental shapes are $f_{\rm NL}^{\rm local} = 2.7 \pm 5.8$, $f_{\rm NL}^{\rm equil} = -42 \pm 75$, $f_{\rm NL}^{\rm ortho} = -25 \pm 39$ (Planck Collaboration 2013b). These results were obtained using a suite of optimal bispectrum estimators. The reported values are obtained after marginalising over the bispectrum contribution of diffuse point-sources – assumed to be Poissonian – and subtracting the bias due to the secondary bispectrum arising from the coupling of the Integrated Sachs-Wolfe (ISW) effect and the weak gravitational lensing of CMB photons.

The *Planck* collaboration (Planck Collaboration 2013b) also obtained constraints on key, primordial, non-Gaussian models and provided a survey of scale-dependent features and resonance models of inflation.

It is worth mentioning here that the *Planck* collaboration will soon release results based on full mission CMB temperature data plus CMB polarisation. The analysis of such an extended dataset is expected to improve accuracy in constraining the non-linearity parameter $f_{\rm NL}$ by a factor $\sim 30\%$.

Let me now very synthetically describe the way the analysis of CMB data – to the goal of searching for primordial non-Gaussiani signals described by Eq. (2.1) – is performed. The CMB temperature field can be characterised using the multipoles of a spherical harmonic decomposition of the CMB temperature map

$$\frac{\Delta T}{T}(\mathbf{x}_O, \hat{n}) = \sum_{\ell m} a^T_{\ell m}(\mathbf{x}_O) Y_\ell^m(\hat{n}) \;, \tag{3.1}$$

where Y_ℓ^m are spherical harmonics and $\mathbf{x}_O$ is the observer's position. At linear order, the relation between the primordial perturbation field and the CMB multipoles reads

$$a_{\ell m}(\mathbf{x}_O) = 4\pi(-i)^\ell \int \frac{d^3k}{(2\pi)^3}\, e^{\mathbf{k}\cdot\mathbf{x}_O}\, \Phi(\mathbf{k}) Y_\ell^m(\hat{\mathbf{k}}) \Delta_\ell(k) \;, \tag{3.2}$$

where Φ is our primordial (Bardeen's) gravitational potential and Δ_ℓ the linear CMB radiation temperature transfer function and, with our any loss of generality, we can set the observer's position in the origin $\mathbf{x}_O = \mathbf{0}$.

The CMB angular bispectrum is defined as

$$B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} \equiv \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle \;. \tag{3.3}$$

If the CMB sky is rotationally invariant the angular bispectrum can be factorised as

$$\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle = G^{\ell_1 \ell_2 \ell_3}_{m_1 m_2 m_3}\, b_{\ell_1 \ell_2 \ell_3} \;, \tag{3.4}$$

where $b_{\ell_1 \ell_2 \ell_3}$ is the so-called *reduced bispectrum*, and $G^{\ell_1 \ell_2 \ell_3}_{m_1 m_2 m_3}$ is the Gaunt integral, defined as the integral over the solid angle of the product of three spherical harmonics.

$$G^{\ell_1 \ell_2 \ell_3}_{m_1 m_2 m_3} \equiv \int Y_{\ell_1 m_1}(\hat{n})\, Y_{\ell_2 m_2}(\hat{n})\, Y_{\ell_3 m_3}(\hat{n})\, d^2\hat{n} \;. \tag{3.5}$$

The Gaunt integral, which can also be written in terms of Wigner 3j-symbols, enforces rotational symmetry, and allows us to restrict attention to a tetrahedral domain of multipole triplets $\{\ell_1, \ell_2, \ell_3\}$, satisfying both a triangle condition and a limit given by some maximum resolution $\ell_{\rm max}$ of the given experiment.

To the goal of extracting the non-linearity parameter $f_{\rm NL}$ from the data, for different primordial shapes, one fits a theoretical CMB bispectrum $b_{\ell_1 \ell_2 \ell_3}$ to the observed 3-point function (see e.g. Liguori *et al.* 2010, for a general introduction). Theoretical predictions for CMB angular bispectra arising from inflation models can be obtained by applying the relation between $a_{\ell m}$ and Φ to the primordial bispectra of the previous section.

A very general expression for the optimal estimator of non-linearity parameter has been recently derived by (Verde *et al.* 2013). Such an expression is based on a second-order Edgeworth expansion for the multivariate PDF of the harmonic coefficients $a_{\ell m}$; it reads

$$\mathcal{P}(a|f_{\rm NL}) = \frac{(\det C^{-1})^{1/2}}{(2\pi)^{n/2}} \exp\left[-\frac{1}{2}\sum_{\ell\ell' m m'} a^{*m}_\ell (C^{-1})_{\ell m \ell' m'} a^{m'}_{\ell'})\right] \times$$

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$$
\Bigg\{1+\frac{1}{6}\sum_{\text{all}\,\ell_i m_j}\langle a^{m_1}_{\ell_1}a^{m_2}_{\ell_2}a^{m_3}_{\ell_3}\rangle\Big[(C^{-1}a)^{m_1}_{\ell_1}(C^{-1}a)^{m_2}_{\ell_2}(C^{-1}a)^{m_3}_{\ell_3}-3(C^{-1})^{m_1 m_2}_{l_1,l_2}(C^{-1}a)^{m_3}_{l_3}\Big]+
$$
$$
\frac{1}{24}\sum_{\text{all}\,\ell m}\langle a^{m_1}_{\ell_1}a^{m_2}_{\ell_2}a^{m_3}_{\ell_3}a^{m_4}_{\ell_4}\rangle\big[3(C^{-1})^{m_1 m_2}_{\ell_1\ell_2}(C^{-1})^{m_3 m_4}_{\ell_3,\ell_4}
$$
$$
-6(C^{-1})^{m_1 m_2}_{\ell_1,\ell_2}(C^{-1}a)^{m_3}_{\ell_3}(C^{-1}a)^{m_4}_{\ell_4}+(C^{-1}a)^{m_1}_{\ell_1}(C^{-1}a)^{m_2}_{\ell_2}(C^{-1}a)^{m_3}_{\ell_3}(C^{-1}a)^{m_4}_{\ell_4}\big]+
$$
$$
\frac{1}{72}\sum_{l_1,\ldots,l_6}\langle a^{m_1}_{\ell_1}a^{m_2}_{\ell_2}a^{m_3}_{\ell_3}\rangle\langle a^{m_4}_{\ell_4}a^{m_5}_{\ell_5}a^{m_6}_{\ell_6}\rangle\times
$$
$$
\big[(C^{-1}a)^{m_1}_{\ell_1}(C^{-1}a)^{m_2}_{\ell_2}(C^{-1}a)^{m_3}_{\ell_3}(C^{-1}a)^{m_4}_{\ell_4}(C^{-1}a)^{m_5}_{\ell_5}(C^{-1}a)^{m_6}_{\ell_6}
$$
$$
-15(C^{-1})^{m_1 m_2}_{\ell_1\ell_2}\left((C^{-1}a)^{m_3}_{\ell_3}(C^{-1}a)^{m_4}_{\ell_4}(C^{-1}a)^{m_5}_{\ell_5}(C^{-1}a)^{m_6}_{\ell_6}+(C^{-1})^{m_3 m_4}_{\ell_3\ell_4}(C^{-1})^{m_5 m_6}_{\ell_5\ell_6}\right)
$$
$$
+45(C^{-1})^{m_1 m_2}_{\ell_1\ell_2}(C^{-1})^{m_3 m_4}_{\ell_3\ell_4}(C^{-1}a)^{m_5}_{\ell_5}(C^{-1}a)^{m_6}_{\ell_6}\big]\Bigg\}, \tag{3.6}
$$

where C^{-1} is the inverse of the covariance matrix $C_{\ell_1 m_1,\ell_2 m_2}\equiv\langle a_{\ell_1 m_1}a_{\ell_2 m_2}\rangle$,.

In the second line of the latter equation one can recognise the standard formulation for approximating the PDF (Babich 2005) which is the starting point to derive the standard $f_{\rm NL}$ estimator (see e.g. Komatsu 2010). applied by the *Planck* collaboration (see Planck Collaboration 2013b, for details), which can be written as (Babich 2005; Creminelli *et al.* 2006; Yadav *et al.* 2008; Senatore, Smith & Zaldarriaga 2010)

$$
\hat{f}_{\rm NL}=\frac{1}{N}\sum_{\ell_i,m_i}G^{\,\ell_1\,\ell_2\,\ell_3}_{m_1 m_2 m_3}b^{\rm th}_{\ell_1\ell_2\ell_3}\times \tag{3.7}
$$
$$
\left[C^{-1}_{\ell_1 m_1,\ell_1' m_1'}a_{\ell_1' m_1'}C^{-1}_{\ell_2 m_2,\ell_2' m_2'}a_{\ell_2' m_2'}C^{-1}_{\ell_3 m_3,\ell_3' m_3'}a_{\ell_3' m_3'}-3\,C^{-1}_{\ell_1 m_1,\ell_2 m_2}C^{-1}_{\ell_3 m_3,\ell_3' m_3'}a_{\ell_3' m_3'}\right],
$$

where N is a suitable normalisation chosen to produce unit response to $b^{\rm th}_{\ell_1\ell_2\ell_3}$.

It should be mentioned here that such a standard estimator can be expressed in different ways, depending on how one expands the reduced bispectrum: KSW (Komatsu, Spergel & Wandelt 2005), modal (Fergusson, Liguori & Shellard 2010, 2012), binned (Bucher, Van Tent & Carvalho 2010), skew-C_ℓ (Munshi & Heavens 2010), needlet (Donzelli *et al.* 2012) and wavelet (Curto, Mafrtinez-Gonzalez & Barreiro 2009) bispectrum algorithms represent different representations of the same underlying optimal bispectrum estimator.

In the following lines of Eq. (3.6) one can recognise Eq. (32), but more specifically Eq. (158), of (Regan *et al.* 2010) as well as "new" terms that arise from expanding the exponential to second order in $f_{\rm NL}$, thus involving a term proportional to the bispectrum squared. Interpreting this as a likelihood for $f_{\rm NL}$ enables one to combine optimally bispectrum and trispectrum measurements and obtain both best-fit value and confidence intervals for the non-Gaussianity parameter.

This expression is valid for general non-Gaussianity shapes, as long as deviations from Gaussianity are small. This expression is strictly speaking second order in $f_{\rm NL}$. Within a given non-Gaussian model where the bispectrum and trispectrum amplitudes are specified by a single parameter, this PDF can be used to constrain such a parameter. Moreover, in a model-independent approach one could use the above PDF to find joint constraints on the amplitude of the bispectrum and trispectrum for comparison with theory.

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4. Large-Scale Structure constraints on primordial non-Gaussianity

The search for signatures of primordial non-Gaussianity in LSS data is made more complex by the very fact that non-linear gravity adds up its own non-Gaussian imprints on LSS observables during the evolution of perturbations. A second complication arises from the unavoidably "biased" which connects the galaxy to the matter distributions. Such a relation (called "galaxy bias" by cosmologists) may well involve non-linear, hence non-Gaussian terms (e.g. Verde *et al.* 2000). On the other hand, the LSS datasets has the obvious statistical advantage of being intrinsically 3-dimensional, contrary to the CMB pattern, which is restricted to a sphere around the observer; this fact can be in principle exploited to improve our determination of e.g. $f_{\rm NL}$ (e.g. Sefusatti & Komatsu 2007).

This field, however, experienced a remarkable boost (see e.g. Komatsu *et al.* 2009; Liguori *et al.* 2010; Verde 2010, for recent reviews on the field) when theoretical cosmologists started to work-out the consequences of a well-known fact: cosmic objects preferentially form on matter density fluctuation *peaks*. See also Licia Verde's contribution to these proceedings.

This idea can be traced back to the Press & Schechter model (Press & Schechter 1974), as well as to the Kaiser model for the formation of galaxy clusters (Kaiser 1984), later extended to the galaxies themselves (Bardeen *et al.* 1986).

Needless to say, peaks represent *rare events* in the underlying dark matter density field: such peaks, while having the apparent disadvantage of being rare they have the obvious advantage of probing the tails of the underlying PDF, hence being more sensitive to deviations from Gaussianity of the underlying matter distribution. If we make the rough approximation that all existing galaxies reside In suitably high peaks of the dark matter density field, there is no loss of information arising from considering these "rare" events (unless one can somehow observe the underlying gravitational potential field itself, as is the case of gravitational lensing).

Moreover, on suitably large scales, one should expect that matter density peaks are less affected by non-linear gravitational evolution than the underlying (smoothed) matter density field, hence preserving a better memory of their initial conditions, including their primordial statistical distribution.

It is this very fact which makes the analysis of galaxy clustering a potential gold mine from the point of view of the search for primordial non-Gaussian signatures. The effects of primordial non-Gaussianity on the clustering of peaks was studied in the eighties (Grinstein & Wise 1986; Matarrese, Lucchin & Bonometto 1986; Lucchin & Matarrese 1987; Lucchin, Matarrese & Vittorio 1988), when very general relations were obtained. The implementation of the physically motivated non-Gaussian model of Eq.(2.1) to analyse the clustering of peaks led to some very interesting results for observables such as the mass-function of cosmic objects (Matarrese, Verde & Jimenez 2000; Verde *et al.* 2001) the linear bias of dark mater halos (Dalal *et al.* 2008; Matarrese & Verde 2008), as appearing in the galaxy power-spectrum, as well as some promising results on higher-order correlations (Giannantonio & Porciani 2010; Baldauf, Seljak & Senatore 2011).

A very promising technique to constrain primordial non-Gaussianity is that of studying the large-scale limit of galaxy biasing, which, in the presence of primordial non-Gaussianity as described by the model of Eq. (2.1) gets an extra contribution, linearly proportional to the parameter $f_{\rm NL}$. In the local case, one indeed gets a scale-dependent term which, for small wave numbers k goes like $\Delta b_{\rm NG} \propto f_{\rm NL} k^{-2}$ (Dalal *et al.* 2008; Matarrese & Verde 2008).

Analyses of available LSS datasets have so far led to interesting and promising results (see, e.g. Slosar *et al.* 2008; Xia *et al.* 2011, Giannantonio *et al.* 2014; Karagiannis, Shanks

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& Ross 2014; Leistedt, Peiris & Roth 2014). Owing to uncertainties on systematics affecting galaxy surveys, the present limits on $f_{\rm NL}$ are still uncertain. For instance, Leistedt *et al.* (Leistedt, Peiris & Roth 2014), analysing the clustering of 800,000 photometric quasars from the Sloan Digital Sky Survey in the redshift range $0.5 < z < 3.5$ obtain $-49 < f_{\rm NL} < 31$ (95% CL) for the local case.

One should mention here that future prospects in this field are extremely exciting. Future galaxy surveys are indeed expected to provide constraints to values of local $f_{\rm NL}$ around unity (e.g. Carbone, Verde & Matarrese 2008), hence opening the window to the possibility of probing signatures of General Relativity on LSS (Verde & Matarrese 2009; Bruni, Hidalgo & Wands 2014).

5. Conclusions

The analysis of primordial non-Gaussianity in cosmology proved to be an extremely relevant source of information on the physics of the early Universe. Indeed, contrary to earlier naive expectations of the eighties, some level of non-Gaussianity is generically present in all inflation models. The level of non-Gaussianity predicted in the simplest (single-field, slow-roll, canonical kinetic term, "Bunch-Davies" initial state, General Relativity as the correct theory of gravitation up to the energy scale at which inflation took place) inflation is below the minimum value detectable by *Planck*. However, even the expected amount of non-Gaussianity of the simplest inflation models is at reach of future galaxy surveys, if one accounts for general relativistic effects which held a contribution to $f_{\rm NL}$ of order unity.

Constraining/detecting non-Gaussianity is a powerful tool to discriminate among competing scenarios for perturbation generation some of which imply large non-Gaussianity. Thanks to the analysis of *Planck* data, non-Gaussianity has become the smoking-gun for non-standard inflation models and a powerful tool to probe fundamental physics and the highest energy scales.

Primordial non-Gaussianity appears in a surprisingly large variety of cosmic phenomena, hence opening the possibility to constraining it by several complementary techniques.

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