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THE PERIOD OF FINANCIAL DISTRESS IN SPECULATIVE MARKETS: INTERACTING HETEROGENEOUS AGENTS AND FINANCIAL CONSTRAINTS

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We investigate how stochastic asset price dynamics with herding and financial constraints explains the presence of a *period of financial distress* (PFD) following the peak and preceding the crash of a bubble [Charles P. Kindleberger, *Manias, Panics, and Crashes: A History of Financial Crisis*, 4th ed. (New York: Wiley, 2000, Appendix B)] as common among most major historical speculative bubbles. Simulations show that the PFD is due to (1) agents' wealth distribution dynamics and (2) positive and sufficiently high transaction costs generating losses for a significant mass of the agents' distribution after the peak of the bubble. The use of transaction costs to get the result is only a modeling tool. Many other mechanisms—able to generate losses for a large mass of the agents' distribution in periods in which financial constraints bind—can produce the same result. The paper also shows how the PFD is affected by a variation of the sensitivity of price to the excess demand and by the switching strategy.

Keywords: Period of Financial Distress, Financial Constraints, Herding Behavior

1. INTRODUCTION

In the fourth edition of his magisterial *Manias, Panics, and Crashes* (2000), Charles Kindleberger has an Appendix (B) that lists a series of famous speculative

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bubbles and crashes in world history.[1] The list begins with the 1622 currency bubble of the Holy Roman Empire during the Thirty Years War and ends with the Asian and Russian crises of 1997–1998. In his discussion of how speculative bubbles operate, drawing heavily on the work of Hyman Minsky (1972, 1982), Kindleberger identifies a general pattern followed by most of them. There is an initial displacement of the fundamental that begins the bubble, although not all have a well-defined such displacement. Later the bubble reaches a peak after a period of credit expansion and speculative euphoria. Then for most there is another date after the peak when there is a crash or crisis. Kindleberger calls the period in between these two dates *the period of financial distress*.[2] Of the 46 bubbles listed in this appendix, Kindleberger identifies 36 as having such a period, as indicated by having clearly distinct dates for a peak and a later crash, with a few others potentially having one.

One can argue with his list. Missing bubbles include the U.S. silver price bubble that peaked and crashed in 1980 and the U.S. NASDAQ bubble that peaked and crashed in March 2000, following the pattern set by the first two on his list, the Holy Roman Empire bubble and then the Dutch tulip mania that crashed suddenly on February 5, 1637 [Posthumus (1929); Garber (1989)], with the last one on his list from 1997 to 1998 also showing this pattern.[3] Nevertheless, to the extent that Kindleberger's list reasonably reflects historical patterns, it would appear that a solid majority of historically noteworthy speculative bubbles had such periods of financial distress, periods after the peaks of the bubbles in which the market declined somewhat gradually before it dropped more precipitously in a panic-driven crash. Even the most famous stock market crash (October 1929) followed a similar path: as Figure 1 shows, it peaked in August before eventually crashing two months later.

To date there have been only a few theoretical models that have been able to separate a peak from a crash [DeLong et al. (1990); Rosser (1991, 1997); Hong and Stein (2003); Föllmer et al. (2005)]. One problem has been the widespread reluctance of economic theorists to accept the reality that potentially speculative markets have heterogeneous agents, reflecting favoritism for representative agent models in which the agent in question has rational expectations. Indeed, under sufficiently strict conditions (a finite number of infinitely lived, risk-averse, rational agents, with common prior information and beliefs, trading a finite number of assets with real returns in discrete time periods), it can be shown that speculative bubbles are impossible [Tirole (1982)]. Influenced by the spectacular crashes in 1987 and 2000, economists have become increasingly willing to doubt the realistic applicability of such theorems to actual markets. DeLong et al. (1991) showed not only that "noise traders" could survive, but even that some of them might outperform the supposedly rational fundamentalist traders in the market. Such arguments have opened the door to studies that emphasize the roles of heterogeneous interacting agents [Day and Huang (1990); Chiarella (1992); Arthur et al. (1997); Brock and Hommes (1997); Lux (1998); Chiarella et al. (2001, 2003); Chiarella and He (2002); Kaizoji (2000); Bischi et al. (2006); Hommes (2006)],

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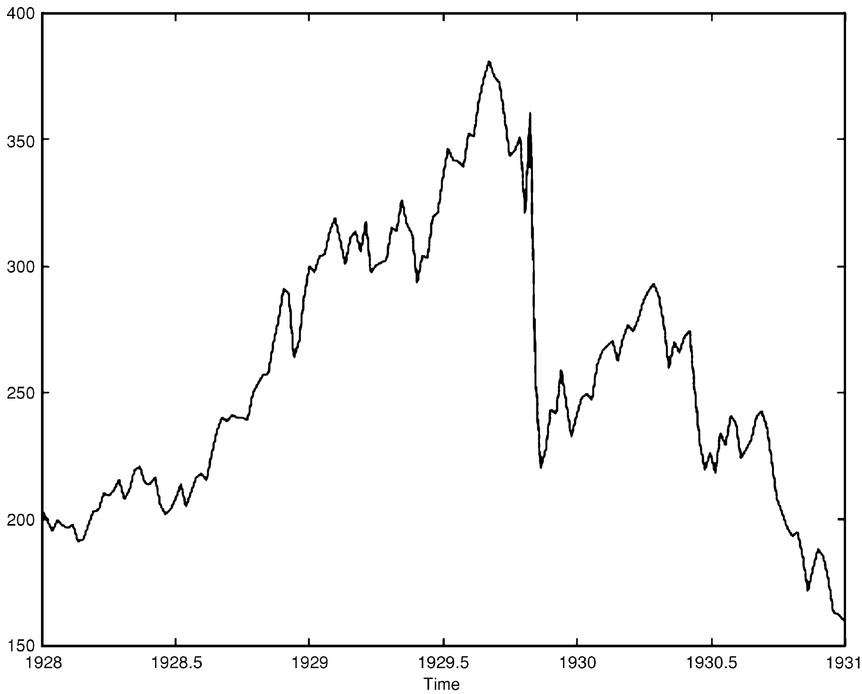

FIGURE 1. Stock market crash (October 1929). In the abscissa the ".5" year label means the end of June.

even as none of these have demonstrated the pattern described by Minsky and Kindleberger as the "period of financial distress."

Kindleberger (2000, p. 17) provides a stylized account of what has typically been involved in the process.

> As the speculative boom continues, interest rates, velocity of circulation, and prices all continue to mount. At some stage, a few insiders decide to take their profits and sell out. At the top of the market there is hesitation, as new recruits to speculation are balanced by insiders who withdraw. Prices begin to level off. There may then ensue an uneasy period of "financial distress." The term comes from corporate finance, where a firm is said to be in financial distress when it must contemplate the possibility, perhaps only a remote one, that it will not be able to meet its liabilities. For an economy as a whole, the equivalent is the awareness on the part of a considerable segment of the speculating community that a rush for liquidity—- to get out of other assets and into money—may develop, with disastrous consequences for the prices of goods and securities, and leaving some speculative borrowers unable to pay off their loans. As distress persists, speculators realize, gradually or suddenly, that the market cannot go higher. It is time to withdraw. The race out of real or long-term financial assets and into money may turn into a stampede.

In the next section we discuss the literature on stock market crashes. We then present a model (Section 3) that is similar to the one used by Chiarella et al.

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(2003) and Bischi et al. (2006) for a large set of heterogeneous agents who interact with each other, derived from work originally done by Brock (1993), Brock and Hommes (1997), and Brock and Durlauf (2001a, 2001b). We introduce a wealth constraint into such heterogeneous interacting agents models to study this period of financial distress. We show that during a bubble the existence of financial constraints, in such a framework, is sufficient to produce a period of financial distress provided transaction costs are sufficiently high. We present simulations in Section 4 that display the phenomenon [for general discussion about simulations in finance see LeBaron (2006)]. Section 5 concludes.

2. BUBBLES, CRASHES, AND FINANCIAL DISTRESS

Historical discussions of the most spectacular of the early bubbles, the closely intertwined Mississippi bubble of 1719–1720 in France and South Sea bubble of 1720 in Britain, show a standard pattern [Bagehot (1873); Oudard (1928); Wilson (1949); Carswell (1960); Neal (1990)]. Common to all these discussions are two groups of agents, a smart group of “insiders,” who buy into the bubble early and who get out early, usually near the peak, and a less well-informed (or intelligent or experienced) group of “outsiders” who do not get out in time. These are the agents who continue to prop the bubble up during the period of distress as the wiser insiders are selling out. The crash comes when this group of outsiders, for whatever reason, finally panic and sell. In discussing the British South Sea bubble, Wilson (1949, p. 202) characterizes this outsider group as including “spinsters, theologians, admirals, civil servants, merchants, professional speculators, and the inevitable widows and orphans.”

An important factor in many of the actual crashes, noted especially for the 1929 stock market crash by Minsky, Kindleberger, and also Galbraith (1954), is that investors can encounter wealth constraints, especially if they have borrowed on margin to buy assets. Actually in our work we want to show, in a simplified setting, that wealth constraints per se are able to explain the crash after a period of financial distress. The crash itself can be exacerbated by a mounting series of margin calls that force investors to sell to meet them, thereby pushing the price further down and triggering more such calls. These calls arise because many buyers only have put a small portion of money down to buy compared to the price (the “margin”), but then must put up more money if the price falls below a critical level based on the margin.

Efforts have long been made to model speculative bubbles using the interactions of such insiders and outsiders [Baumol (1957); Telser (1959): Farrell (1966)], although without showing such a period of distress. Others have simply shown interactions between fundamentalists who do not participate in the bubble and trend-chasing chartists who do, but without subdividing them [Zeeman (1974)]. However, all these incipient efforts to model using heterogeneous agents fell into disrepute as the rational expectations revolution gathered steam during the 1970s.

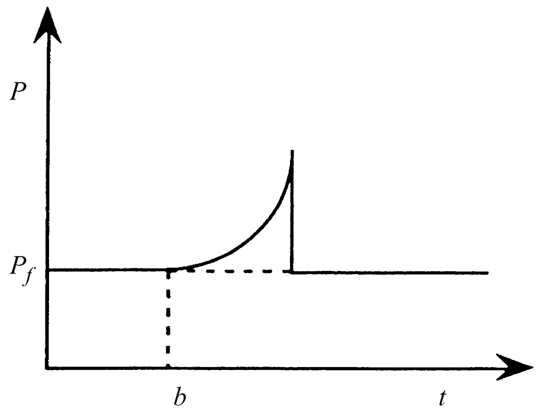

FIGURE 2. Stylized representation of a bubble produced by rational bubbles.

The first to revive such an effort, and also to show something like a period of financial distress, were DeLong et al. (1990). Following Black (1986), they were principally concerned with demonstrating the possibility of "rational speculation" in the presence of noise traders, with the rational speculators forecasting the future purchases of the noise traders and thereby making money by buying in advance of their purchases. This makes the "rational speculators" like the "insiders" from the older literature, whereas the noise traders are the "outsiders." However, they are not interested in a Minsky–Kindleberger period of financial distress as such, and their model shows more slowly rising prices after the noise traders enter the market, rather than actually falling prices. The trend chasing of the noise traders guarantees that the bubble continues to rise even as the rational speculators are selling, although at a lower rate, or at least does not decline.

The first to specifically model a period of financial distress was Rosser (1991, Chapter 5; 1997), who introduced multiple periods and a lag operator within a stochastically crashing bubble framework, following Blanchard and Watson (1982). Although this model allows rational speculation, the rationality assumption was relaxed. It was shown that the three basic cases discussed by Kindleberger and shown below in Figures 2–4 could occur, although the parameter set for the financial distress case was of measure zero and thus unable to explain the ubiquity of that historical phenomenon.

Both of these models involved strong assumptions with agents of extreme types, in contrast to those used in this paper. In the model used here, agents are allowed to be of intermediate types in terms of trend chasing and willingness to switch strategies, all operating within a wealth constraint. Although there are links to the Rosser approach, the greater flexibility and realism of the model used here is better able to model the financial distress phenomenon.

Some more recent efforts to model periods of financial distress have been carried out using insider–outsider models in models of financial crises in emerging market

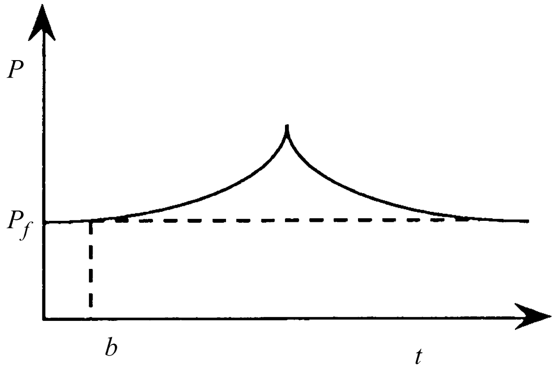

FIGURE 3. A stylized representation of a bubble produced by interacting heterogeneous agents. It can be asymmetric, but it falls much slower than the rational bubble.

foreign exchange rates, although without showing a period of declining currency value prior to a full crash, or "sudden stop" [Calvo and Mendoza (2000)]. Although it does not specifically focus on the period of distress, the model of Hong and Stein (2003) looks as if it could generate such a pattern and can be argued to fit the insider–outsider pattern, as it involves differing degrees of information among traders, with more pessimistic traders only buying after the price starts to decline and gets to a level they think is sustainable, with their buying helping to prop it up for a period of time.

The model of Föllmer et al. (2005) shows some patterns in its simulations that resemble periods of financial distress, with a gap between a peak and a crash. However, the dynamics involve a struggle between fundamentalists and chartists for domination of the market dynamics just prior to a switch from the chartists dominating to the fundamentalists dominating after the crash happens. The crash does not involve financial constraints specifically. Furthermore, the authors make

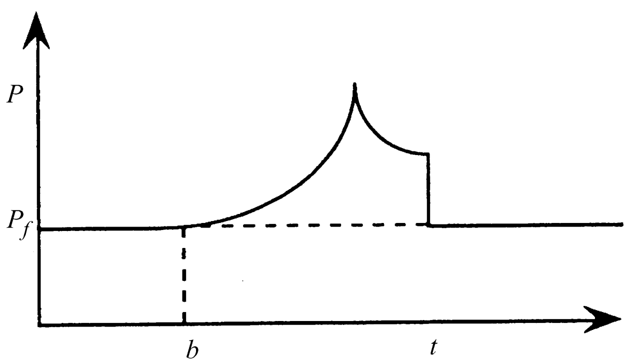

FIGURE 4. A stylized representation of a bubble with a crash preceded by a period of financial distress.

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no mention of these scattered appearances arising from their model or that it might possibly help explain a widely existing feature of most major historical bubbles.

Rosser (1991, Chapter 5) provides three canonical patterns for bubbles and crashes, drawing on the discussion by Kindleberger. The first is that of the accelerating bubble that is followed by a sudden crash, much like that of the Dutch tulip mania on February 5, 1637.[4] This is depicted in Figure 2. Most of the models of rational agent bubbles tend to follow this pattern [Blanchard and Watson (1982)].

Another is that of a bubble that decays more gradually after rising, as in France in 1866 or in Britain in 1873 and 1907. This is depicted in Figure 3. It is often argued that many bubbles follow an intermediate path between these two, with a decline that is not a discontinuous crash, but that asymmetrically declines more rapidly than it increased.[5] This has been studied using heterogeneous interacting agents models [Chiarella et al. (2003)]. The model used by the authors is similar to the framework in Bischi et al. (2006) but with a more complex timing mechanism. An other difference is that, in the Chiarella et al. work, the herding component in agents' decisions is not exogenous but chosen period by period using a *genetic algorithm*. That paper shows that (1) this kind of model may generate endogenous bubbles; (2) when a speculative bubble starts the herding component becomes positive and sufficiently high (the J parameter in Section 3); (3) herding behavior is rational (in line with the DeLong et al. results) because during the bubble it allows speculating agents to make more profits. To be precise, because their work shows that the distribution of profit using herding behavior strategies has greater variance than the fundamentalist one, the formers' becomes rational when the expected value of that strategy is sufficiently higher than that of the fundamentalist to compensate for the risk.

Finally, there is the pattern we are studying in this paper, the historically most common pattern according to Kindleberger, that of the bubble that exhibits a period of financial distress after the peak but prior to the crash. This is depicted in Figure 4.

In our project, we show the existence of a period of financial distress (as defined by Kindleberger)—given a bubble—using the Bischi et al. (2006) framework that generates bubbles according to the values of parameters because it is more computationally convenient, which in turn follows directly the work of Chiarella et al. (2003) for showing the emergence of endogenous bubbles. This framework will have some differences from the stylized story told above.

3. THE MODEL

In this section we will describe a model able to explain the existence of a period of distress during the bubble. In such a framework, we consider a population of investors facing a *binary choice problem*. The agents, at the beginning of each period of the simulation, choose a strategy $w_{it} \in \{-1, +1\}$, where -1 stands for "willing to sell," whereas $+1$ stands for "willing to buy" a unit of a given share. We do not model an optimal portfolio problem explicitly; rather, the trade decisions have to be interpreted as the marginal adjustment the agents make as they try to take advantage of profit opportunities arising due to continuous trading information diffusion. As a simplification, all agents trade in every period. The following assumptions are made:

(i) There exist two assets: a risk-free asset with a constant real return on investment r and a risky asset with price P_t that pays a constant dividend, say y.

(ii) Agents, whose number is N, observe past prices, the past relative excess demand, $w_{t-1} = N^{-1} \sum w_{it-1}$, and the real interest rate, r, and have rational expectations about the dividend process. Therefore the fundamental solution of the risky asset price is the ratio $F = y/r$.[6] The information set of the agent is the union of his or her private characteristics and the public information set; that is r, y, and past prices and excess demand. In our simulations y/r is held constant and determines the starting point of the simulation process. Also, N is held constant in our simulations, except for agents dropping out due to bankruptcy only to be replaced by new agents. As shown in Bischi et al. (2006), in principle N can follow a stationary process. Assuming N constant allows us to avoid the volumes dynamics problem, i.e., the relation between changes of prices and changes of volumes.

(iii) In order to take their buy/sell decision, the agents evaluate an expected benefit function, V_{it}, that will depend on their private beliefs about what price will prevail in the market. We assume that the agents engage in rational herd behavior; i.e., they expect that V_{it} will be positively related to the other agents' buy/sell decisions.

(iv) Price dynamics—not known by the agents—are assumed to follow the difference equation (tâtonnement process)

$$p_t = p_{t-1} + f(w_t) + \sigma_1 z_{1t}, \tag{1}$$

where p_t is the logarithm of the asset price, and $f(w)$ is a deterministic term that measures the influence of excess demand on current price variations, with properties $f(0) = 0$, $f'(w) > 0$. The stochastic component of price dynamics is captured by $\sigma_1 z_1$, where z_1 is a NID(0, 1) process, so that σ_1 is the standard deviation of the shocks. Note that, when the excess demand is zero, both the conditional and the unconditional distribution of price changes follow a Gaussian process with zero mean and variance σ_1^2. With out-of-equilibrium dynamics ($w \neq 0$), the conditional distribution will have a different mean, although remaining Gaussian by definition, whereas the unconditional distribution may not belong to the normal distributions family [see Leombruni et al. (2003)].

(v) Agents have homogeneous expectations regarding the relative excess demand at period t, say w_t^e. Following Brock and Durlauf (2001b), agents' static expectations with respect to their information set are assumed; i.e., the excess demand expected is that previously observed, $w_t^e = w_{t-1}$.

The agent's choice is modeled as a binary random variable that describes, from the point of view of the modeler, the choice of agent i at time t between the two strategies. In other terms, the random variable w_i gives the probability distribution of agents' decisions conditional on their expectations. With perfect information, no social interactions, and perfect market efficiency, the relevant statistic to compute would be the ratio between the expected value of the fundamental solution of price dynamics and the actual price that measures the expected rate of profit (loss) when the price reaches the fundamental.

However, we assume that the herding behavior undermines the ability to calculate this for the actual assets' price dynamics. The rationale of this imitative behavior is that the agents try to extrapolate from the observed choices of the others, and exploit, the piece of information they are lacking.

Following the social interaction literature, a convenient way to model the herd component in the behavior of the investors is by means of a binary choice framework with interaction. Namely, we assume that the *non–financially constrained* benefit function for the strategy w_i is

$$U_{it} = (\bar{p}_t - p_{t-1})w_{it} + J w_{it} w_t^e + \varepsilon_{it}. \qquad \textbf{(2)}$$

The equation above is a standard assumption in the social interaction literature [see Brock and Durlauf (2001a, 2001b)]. It implies that the utility or benefit function is affected by three additive components. The first component gives the private benefit in choosing strategy w_i after having observed the price. This is done by comparing observed past log price p with log expected price $\bar{p}$ (see below). The second component is an interaction term (*proportional spillovers*) measuring the benefit of that choice in a situation where the expected average choice is w^e. Finally, the last term introduces, stochastically and from the point of view of the modeler, idiosyncratic factors and private information affecting agents' decisions.

To be precise, the first term of the right hand side is the benefit of the strategy "buy one unit of share" or "sell one unit of share" in case the agent would consider only the adaptive expectation of the price without social interaction. The second term captures the positive spillover agent i expects in following others' expected choices. It captures the interaction among investors, in the form of a proportional spillover $J w_i w^e$. In other words, the benefit expected by agent i depends on his or her expectation as to the average choice of the market, w^e. The positive parameter J measures the weight given to the choices of other agents. We assume a constant strength of the interaction because the aim of this work is not to prove that models of this kind may produce bubbles, as this is exactly what it is shown by the authors in Chiarella et al. (2003). Rather, we show the existence of a period of financial distress *given the bubble*. To be precise, the analysis in the following is parametric, with key positive parameters J and β (defined below), because models of this kind [see also Kaizoji (2000)] may produce bubbles (i.e., a "large" distance between actual price and the fundamental solution, e.g., 50% or 100%).

The discrete choice literature calls the term $(\bar{p}_t - p_{t-1})w_{it} + J w_{it} w_t^e$ the "deterministic component of the benefit function." The third term, ε_{it}, represents random variables that may have different distributions under the two choices and, in this setting, captures the source of heterogeneity between agents' decisions. As said above, the random variables capture agents' unknown (to the modeler) features.

As in Brock and Durlauf (2001a) and Bischi et al. (2006), we assume that the difference of the random components under the two choices (-1 and $+1$) is a logistic distribution with parameter β. This parameter, in the *binary choice with interaction framework*, has two interpretations: (1) the importance of the part of agents' decisions not known by the modeler and (2) the velocity at which agents switch their strategies when profitability changes [see Brock and Durlauf (2001a, 2001b)]. We note here that the two parameters, J and β, the herding parameter and the propensity–to switch strategies parameter, play the central roles in determining the bifurcation structure of the dynamical system.

The key additional assumption of this model is the presence of a financial constraint on agents' decision process. In other words, agents start the continuous trading with a given initial wealth and remain in the market only if their losses are not too high. Formally, an agent's benefit function with liquidity constrains may be expressed by the equation

$$V_{it} = \{U_{it} \text{ if } W_{it-1} > \theta W_{i0}, -\infty \cdot w_{it} \text{ if } W_{it-1} \leq \theta W_{i0}\}, \qquad (3)$$

with $0 < \theta < 1$ measuring the fraction of the initial wealth below which the agent sells with probability 1. W_i is the wealth of the agent.

In this framework, agents' stochastic decisions regarding whether to buy or to sell could be described by the probability that the benefit function will yield a higher benefit than the other choice [see Brock and Durlauf (2001a, 2001b) for surveys of the methodology]; that is,

$$\Pr(w_{it}) = \Pr\{V_{it}(w_{it}) > V_{it}(-w_{it})\}. \qquad (4)$$

So far, we have not considered any dynamics in the priors about the expected price. Actually, when an investor follows the herd because of the (assumed) presence of information asymmetries, he or she should coherently revise his or her priors. For instance, if he or she follows the herd during a bull market, we should expect that he or she will contextually increase his or her prior on the fundamental. More generally, we can model the priors revision assuming that the agents adjust their private expectations by comparing them with the public information that is currently mirrored in the price level. That is, we can assume the following adaptive learning mechanism, homogeneous across agents, for the priors on the log-expected price:

$$\bar{p}_{t+1} = \bar{p}_t - \rho(\bar{p}_t - p_t) + \sigma_2 z_{2t+1}, \qquad (5)$$

where $\rho \in [0, 1]$ is a measure of the adaptive speed of adjustment. The adaptive mechanism given in the equation above can be described by saying that the new expected price is a convex combination of the previous expected price and the previous realized price (ρ being the relative weight of the realized price) plus a stochastic component $\sigma_2 z_2$, where z_2 is a NID(0, 1) process, so that σ_2 determines the variance of the shocks. This hypothesis, made for the sake of simplicity, means that in the original variables the updating rule is not linear but geometric.

The model can be implemented (after an initialization of parameters and starting values of all variables) by dividing period t into four steps as follows:

Start Period t
(Step 1) Agents decide w_{it} [using equations (2), (3), (4)]
(Step 2) Asset price changes [using equation (1)]
(Step 3) Agents realize profit/losses and update wealth using the profit equation (7) (see next section)
(Step 4) Agents compute $\bar{p}_{t+1}$ [using equation (5)]
End period t

In Step 1 agents decide w_{it} observing p_{t-1}, w_{t-1}, $\bar{p}_t$, and W_{it-1}. In Step 2 there is the computation of price p_t after the realization of w_t. In Step 3 agents compute Π_{it} and W_{it}. Finally, in Step 4 agents compute $\bar{p}_{t+1}$.

4. SIMULATION RESULTS

In this section the model is analyzed by simulations (performed using GNU Octave; see www.octave.org) that are implemented assuming an annual dividend with mean $d = 1$ and $r = 0.1$; therefore the fundamental solution F is equal to 10. Although in the short run the expected price will deviate from this fundamental, in the long run the mean of the expected price will equal this fundamental. We specify a linear-in-log-price dynamics with $f(wt) = k\,wt$; that is,

$$p_t = p_{t-1} + kw_t + \sigma_1 z_{1t}, \tag{6}$$

where $k = 0.4$ and the standard deviation $\sigma_1 = 2 \times 10^{-6}$, whereas $\sigma_2 = 4 \times 10^{-2}$. The initial log expected price is $\bar{p}_0 = \ln(10.1)$, whereas $p_0 = \ln(10)$. J and β were set equal to respectively 0.5 and 0.1. For every agent $i = 1, \ldots, 100$ the buy/sell decision is made according to the probability measure described in Section 3. To keep things simple, agents buy and sell in every period; $w = 1$ means to buy one unit at the beginning of period and sell it at the end; $w = -1$ is a one-period short position. To summarize, losses and profits are realized in every period. So in simulations agents do not collect shares. The following realized profit is computed:

$$\Pi_{it} = w_{it}(\Delta p_t + y) - c, \tag{7}$$

where y is the part of the dividend attributed to the time interval simulative step. The wealth in each period changes by adding the profit in equation (7). During the simulations the constant value 1/36,000 for y and an interest rate such that the fundamental solution is 10 were used. Finally, c is a transaction cost [as in Chiarella et al. (2003)] assumed constant and equal to 0.8. We will see that such high levels of transaction costs are necessary to produce a distribution dynamics of wealth generating the PFD. The parameter ρ of equation (5) is set to 0.7 during simulations. The simulations were started with the initial wealth of every agent, W_0, equal to \$1,000. In the simulations with financial constraint and positive

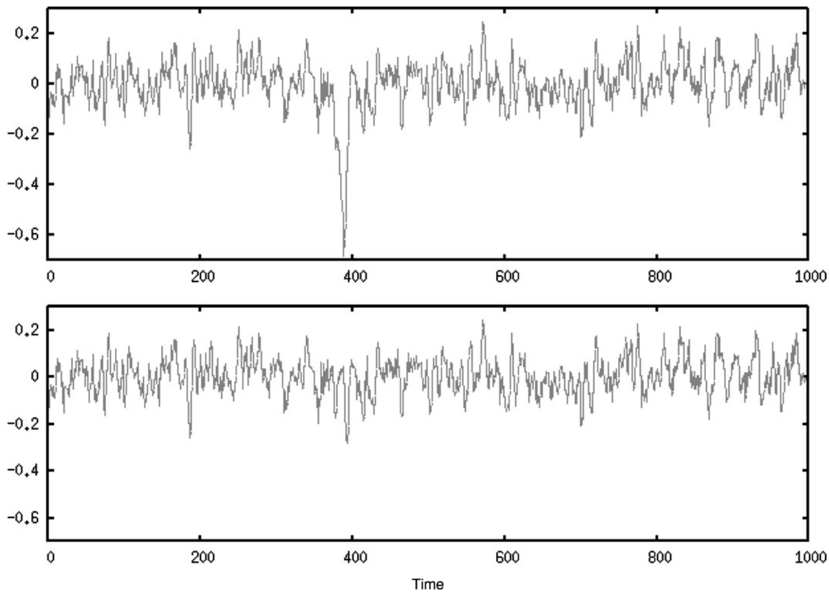

FIGURE 5. Asset price rate of growths with positive transaction costs (top panel) and with zero transaction costs (bottom panel).

transaction costs we set $\theta = 0.7$. To avoid the problem "how to reinvigorate the market after the crash," we replace the agents with new ones when the priors on the log expected price go below 60% of the fundamental solution. The hypothesis is made for computational convenience, but it can be interpreted as "new people enter the market to speculate after the crash."

With zero transaction costs we observe bubbles of the second type such as the one stylized in Figure 3. The numerical analysis shows that the time series bubbles look like Figure 3. Even though fluctuations may be asymmetric and may show a change of slope during downfall, they do not exhibit a crash preceded by a period of financial distress (i.e., an outlier rate of growth; see below).

If we allow transaction costs to be positive and the financial constraints to bind we may observe simulations such as Figure 5 in which a period of financial distress appears. To identify such phenomena we use the Kindleberger definition in the Introduction: *time between the peak of the bubble and the crash or crisis (at least 10 periods of decline preceding a crash).*

In the simulations the crash is identified by an accelerating (outlier rate of growth) fall after the peak (Figure 6).

Looking at the four-period rate of growth of the asset price (Figure 5) computed with the log differences for the cases with positive transaction costs (top panel) and zero transaction costs (bottom panel), we see that a crash is identifiable by an outlier rate of growth. Using the standard one-period rate of growth, $\Delta \log p$,

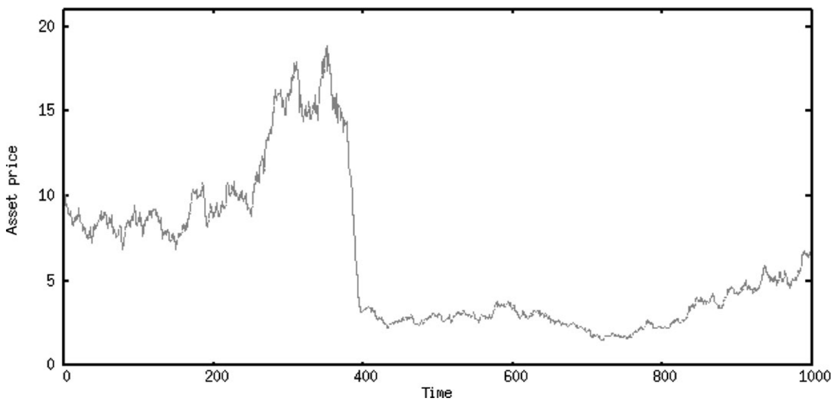

FIGURE 6. Introduction of positive transaction costs with liquidity constraints.

nothing changes qualitatively, but the result is less evident. This is because it an anomalous rate of growth with zero transaction costs may occur, but the probability of observing a four-period outlier is negligible.

Regarding the test, we use the *Grubbs outliers test*, but the result is robust with respect to non-Gaussianity of the rate of growth (the excess kurtosis is 0.38) because the four-period rate of growth before iteration 400 is more than four standard deviations higher. The two series were obtained using the same *seed* for the *GNU Octave* random number generator (the *seed* is 4 in a Debian Stable operative system). To be precise, analyzing situations with $c = 0, 0.1, 0.2, \dots,$ the minimum level able to produce the bubble is the value $c = 0.8$.

Performing 200 Monte Carlo simulations with 100 agents, each of 1,000 runs with parameters as above, but changing the random seed in the random number generator, we get 84 crashes preceded by a PFD. We consider an episode a PFD only if its duration is at least 10 periods and the 4-period rate of growth is an outlier of the empirical distribution. During simulations the maximum (minimum) value of the duration of the PFD was 158 (11); the mean duration was 61.23 with standard deviation 29.12 (Table 1, first row).

Because we do not obtain such phenomena without the introduction of the financial constraint (or put differently, when the θ parameter is sufficiently low so that the constraint is not binding), the following conjectures can be asserted for this model.

TABLE 1. Period of financial distress statistics

β	Min	Max	Mean	Stand. dev.	Reduction in amplitude (%)
0.1	11	158	61.23	29.12	22.71
0.7	19	202	72.69	37.64	

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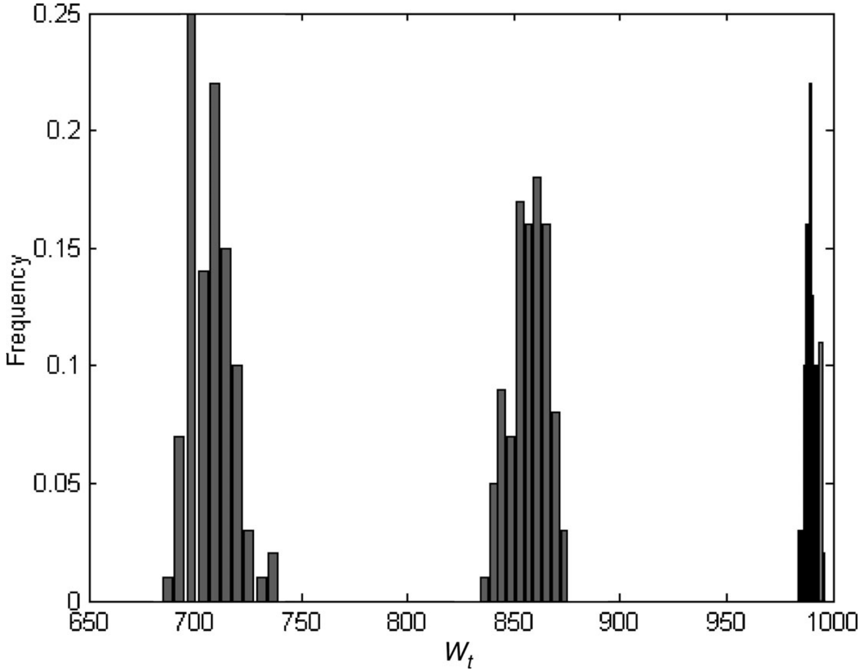

FIGURE 7. Wealth distribution dynamics.

RESULT 1. *Bubbles characterized by a PFD (type 3 bubble, Figures 1 and 4) are generated when the agents are financially constrained with sufficiently high transaction costs.*

The explanation of type 3 bubbles is evident when looking at the evolution of the wealth distribution of agents (Figure 7). The three densities (from right to left) describe the situation at the beginning (= 20), after 200 periods and last just before the crash, where it is possible to see that a large tail of the distribution is at or below the threshold $0.7 \times 1{,}000 = 700$.[7] In other words,

RESULT 2. *The PFD is the time necessary, after the maximum of the bubble, to have a sufficient mass of agents needing to sell because of financial constraints.*

The relatively high transaction cost is necessary to have a PFD in simulations. The distribution of wealth seem to spread over time. With zero transaction cost, the distribution may become so wide that the tail crossing the critical level (700 in simulations) has too little to generate the PFD. In other words, the financial constraint binds for too few agents. It must be stressed that the positive transaction cost device to get the PFD was chosen for its simplicity from both a modeling and a computational point of view. Many other mechanisms—able to generate losses for a big mass of the agents' distribution in periods in which financial constraints bind—can produce the same result.

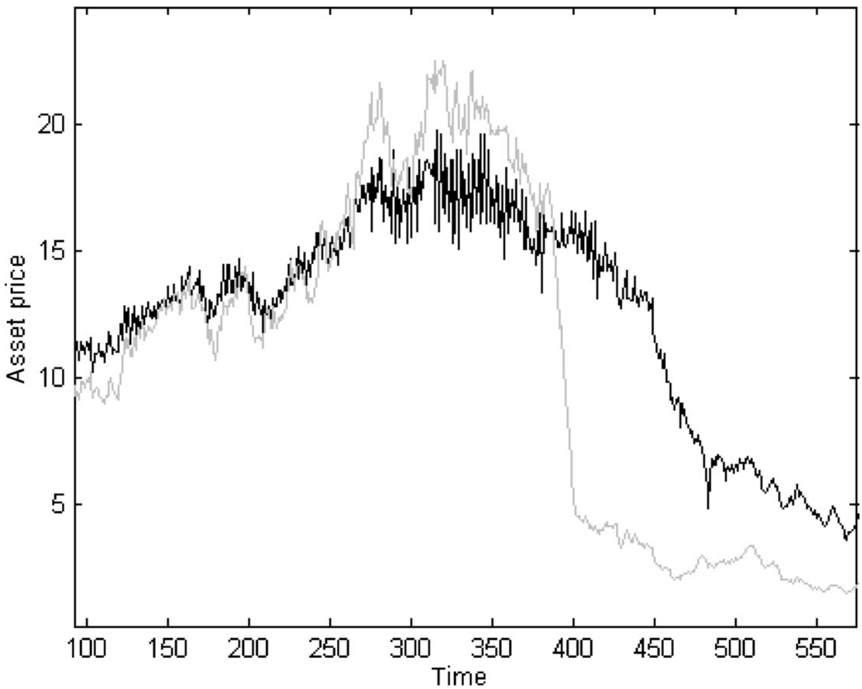

FIGURE 8. An increase in β. The two time series share the same random numbers and the same parameters other than β. In the gray time series $\beta = 0.1$; in the black time series $\beta = 0.7$.

An increase of the parameter β from 0.1 to 0.7 produces an increase in the mean and the variance of the PFD distribution (Table 1, second row). An example of this result is shown in Figure 8. Table 1 (second row) summarizes a Monte Carlo simulation with parameters as above, but β increased from 0.1 to 0.7. During simulations the maximum (minimum) value of the duration of the PFD was 202 (19); the mean duration was 72.69 with standard deviation 37.64. Furthermore, in every run on the second set ($\beta = 0.7$), the amplitude of the bubble decreases. The average reduction is 22.71%. This value was computed evaluating the reduction in percentage of the maximum preceding the crash. Summarizing the results, the following conclusion can be asserted.

RESULT 3. *An increase of the parameter β increases the PFD and decreases the amplitude of the bubble.*

An interpretation of this result might be that as β represents the willingness of agents to change their strategies, it can be interpreted as an index of the rationality of the agents.[8] Thus the amplitude of the bubble is kept down when agents are more likely to switch away from the herd-driven upward surges, and the crash is delayed when the same tendency to switch in response to profitability maintains the liquidity of more traders for a longer period.

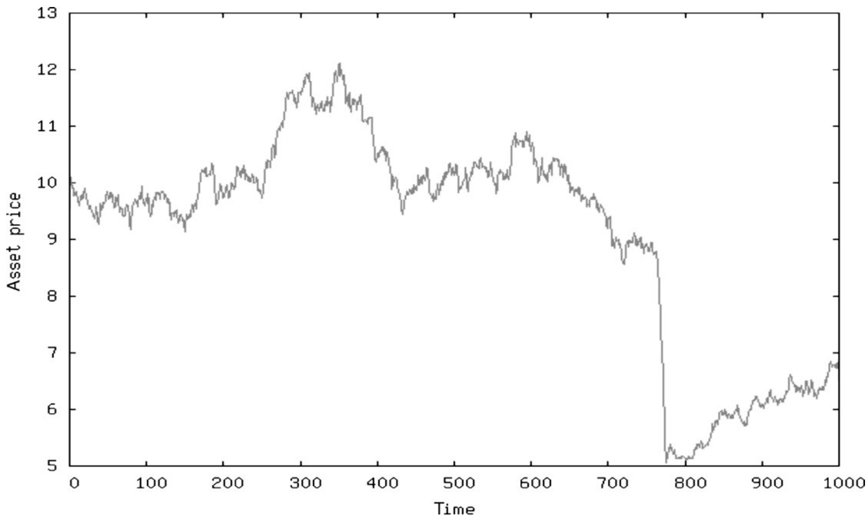

FIGURE 9. A decrease of k and c. The time series is produced with $k = 0.1$ and $c = 0.4$.

Regarding the role of the k parameter, simulations show the following:

RESULT 4. *A decrease of the parameter k decreases the minimum transaction cost able to produce the PFD.*

In other terms, analyzing situations with a low level of k, we found that lower values of c produce the PFD compared to the case $k = 0.4$. To be precise, with $k = 0.1$ and with $c = 0, 0.1, 0.2, \ldots$ the minimum level able to produce the bubble, with other parameters unchanged, is the value $c = 0.4$ (see Figure 9).

The result is due to the fact that a reduction in k reduces price volatility and so the possibility to make profits. In such situations, the reduction of wealth produced by c affects more the probability that wealth will reach the critical level below which agents sell with probability one.

Finally, a set of simulations is performed with the same random numbers as the first one, except the parameter J, which increases from 0.5 to 3. Every run results in a greater amplitude of the bubble (the distance of actual price, at the maximum, and the fundamental solution more than tripled), but with no significant change in duration. An example of such a simulation is shown in Figure 10. Table 2

TABLE 2. Effect of an increase in J from 0.5 to 3

J	Mean (PFD)	Stand. dev. (PFD)	Increase in amplitude (%)
0.5	61.23	29.12	56.87
3.0	62.18	30.23	

Note: The last column shows the average percentage increase at the maximum of the bubble.

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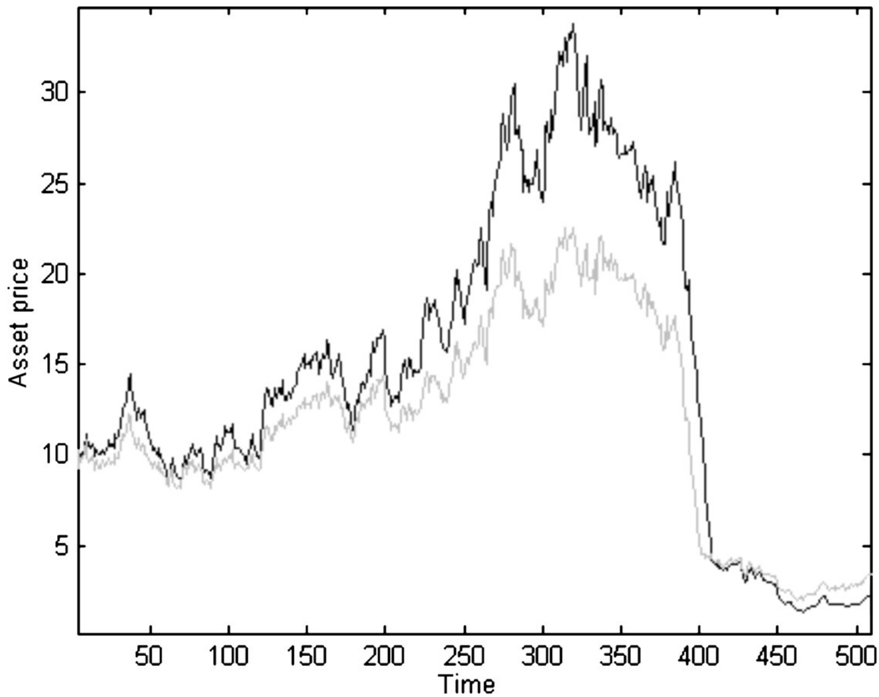

FIGURE 10. Increase in J. The two time series share the same random numbers and the same parameters except for J. In the gray time series $J = 0.5$; in the black time series $J = 3$.

summarizes the results of the third simulation (second row) compared to the first one (first row). The second set of simulations show a 56.87% average increase in the mean value of bubble amplitude.

RESULT 5. *An increase of the parameter J increases the amplitude of the bubble.*

An interpretation of this is fairly intuitive in that one can easily expect that an increase in the strength of herding will increase the size of the bubble. As argued earlier, it is this herding or interaction parameter J that fundamentally lies behind the emergence of bubbles in this model, just as is widely thought to be the case in real markets. Trend-chasing speculators imitate each other and push the price upward.

5. CONCLUDING REMARKS

In this work, we have considered how introducing a financial constraint into a framework characterized by herding and switching of priors about the fundamentals by heterogeneous investors can per se explain the appearance of a PFD between the peak and the crash of a speculative bubble. The incompleteness of the agents' information set is sufficient for nonfundamentalist dynamics in the agents' decision process, modeled in a binary choice setting with interacting agents.

The model is built along the line of Chiarella et al. (2003) and Bischi et al. (2006), which are able to produce bubbles but unable to explain a period of financial distress preceding a crash unless we introduce a financial constraint on agents' decisions.

Simulations show that the PFD can arise from agents' wealth distribution dynamics in situations characterized by sufficiently high transaction costs, although we recall that our model involves a number of simplifying assumptions such as a constant number of agents and that they all must trade in each period. The PFD is the time necessary, after the bubble's peak, for a sufficient mass of agents coming to need to sell because of financial constraints. An increase in the switching strategy velocity (the intensity of choice in the social interaction literature β) increases the PFD's length and decreases the bubble's amplitude.

Furthermore, a decrease in k (the sensitivity of prices to the excess demand) decrease the minimum level of c able to produce the PFD. The result is due to the fact that a reduction in k reduces price volatility and so the possibility of making profits.

Finally, an increase of the strength of the interaction parameter J increases the amplitude of the bubble.

NOTES

1. The earlier editions were in 1978, 1989, and 1996. All of them had this Appendix, to which Kindleberger kept adding more bubbles with each edition. In the first edition he listed 37, with 30 of them having clearly distinct dates for the peak and the subsequent crash.

2. The term for a time when a firm may not be able to meet its liabilities originally comes from corporate finance [Gordon (1971)]. In this paper investors in an asset market face liquidity constraints that become more severe as prices decline. Kindleberger (2000, p. 94) also notes that this period is sometimes called any of uneasiness, apprehension, tension, stringency, pressure, uncertainty, ominous conditions, fragility, an ugly drop in the market, or a thundery atmosphere, with these more colourful later expressions dating back to the South Sea Bubble of 1720.

3. One can argue that the 1997–1998 episodes were really two bubbles and crashes, which would make the numbers 36 of 47. Adding the U.S. silver and NASDAQ bubbles would make this 36 of 49, still a solid majority. Even the NASDAQ bubble arguably exhibited a period of financial distress, as the day of its most rapid decline occurred nearly a month after its peak in March 2000.

4. Actually what happened during the tulip mania was that on the very next day after it started to crash, the market was shut down for several months, thereby making it unclear what would have happened if it had remained open [Posthumus (1929)].

5. The pattern in which a rapid price increase is followed by a long decline is very rarely observed, with perhaps the most prominent recent example being that of Japanese real estate [Land Information Division (2002)]. The authors thank the late Charles Kindleberger for his personal discussion of the historically unusual nature of the relationship between the 1990 crash of the Japanese stock market and the slow decline of Japanese real estate prices after 1991.

6. This fundamental does not directly drive the dynamics in the model in this paper. In that regard, the model in this paper does not conform precisely to the implied story depicted in Figures 2–4, where price follows the fundamental, only to deviate from it during a definite bubble, and then to return to it afterward.

7. Although agents drop out of the market as their wealth falls below the 700 cutoff, they are not replaced by new agents before the crash. After the crash, agents below are replaced by new agents with a wealth of 1,000 in order to avoid the problem of "how to reinvigorate the market after the crash." The hypothesis is made for computational convenience, but it can be interpreted as "new people enter the market after the crash."

8. Buz Brock has argued to the authors that an infinite value for β is equivalent to "Chicago rationality," where agents in effect have no tendency to stick with a strategy at all and immediately switch to the current best one.

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