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repeated astonishment at a range of applications (in his expansive sense) of mathematics, but in expanding upon their diversity and philosophical interpretations he seems to lose the thread of why they should have been so central to the philosophy of mathematics or why (apart from some provisional hypotheses from cognitive science and a few other lightly developed rationales) they should be so astonishing in the first place.

Part of the trouble is the book's meditative genre, heavy on suggestive digressions but short on the sort of synthesis and argument for which Hacking is frequently and justly praised. Where in other texts Hacking's tangle of foreshadowing, deferral and cross-reference signals a dense and multi-layered explanation that rewards a reader's close attention, here it appears to reflect the book's piecewise elaboration through a series of shorter expositions in other formats. Hacking exhibits a frustrating habit of mentioning a provocative topic and then disavowing it as peripheral to his main quarry. A generous and scrupulous writer, Hacking devotes considerable attention to doing his contemporary and historical interlocutors justice, even when it sometimes comes at the expense of his own cogency.

There is much to praise here. Hacking's perambulatory reflections about the philosophy of mathematics teem with insight and provocation. He displays his habitually impeccable ear for delectable quotes and original aphorisms, enriched by his characteristic close attention to nuances of usage and interpretation. Hacking's perspective on the development of a range of themes in the philosophy of mathematics over the last century and a half is markedly well informed and frequently illuminating. His laudable commitment to engaging with recent and forbiddingly difficult mathematical work has mixed results, but stands out in a field whose practitioners (as Hacking discusses) often seem overly preoccupied with fanciful or simplistic examples that are scarcely connected to what mathematicians do. On their own, these features may satisfy a great many readers. As elements of a larger intervention in the history and theory of philosophy, they tend to accentuate the volume's disappointingly persistent lacunae.

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VICTOR J. KATZ and KAREN HUNGER PARSHALL, **Taming the Unknown: A History of Algebra from Antiquity to the Early Twentieth Century.** Princeton: Princeton University Press, 2014. Pp. xiii + 485. ISBN 978-0-691-14905-9. £34.95 (hardback). doi:10.1017/S0007087415000709

As with the technical terms of any field, the word 'algebra' has undergone several changes of meaning throughout its history, as the subjects to which it has been attached have developed. The book under review traces this process from ancient times up to the modern day. The word 'algebra', we are told, first emerged in western European languages as a corruption of part of the title of a ninth-century text by the Islamic scholar al-Khwārizmī (fl. 800–847). Since this work concerned the solution of polynomial equations (in modern notation, any equation of the form

 $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0 = 0,$

which is to be solved for x, and where n is an integer and a_n, \ldots, a_0 are known numbers), the term 'algebra' subsequently became the new name for the (much older) subject in which solutions of such equations are sought.

Although al-Khwārizmī limited himself to quadratic equations (those of degree 2: that is, n = 2 in the above equation), the centuries following his work saw the extension of his methods to equations of higher degree: solutions for degrees 3 and 4 were discovered in sixteenth-century Italy, for example. However, efforts to extend known methods still further failed, and, by the eighteenth

century, mathematicians were beginning to suspect that such higher-degree equations were not in fact soluble in general – at least not in the desired form; that is, in terms of elementary operations performed on the coefficients a_n, \dots, a_0 : addition, subtraction, multiplication, division and root extraction. Then, in 1824, the Norwegian mathematician Niels Henrik Abel (1802–1829) published a proof that there is indeed no such general solution for an equation of fifth degree. His findings gave impetus to a new direction in the study of polynomial equations: the determination of which equations may be solved in this way. A comprehensive answer to this question was provided by the French mathematician Evariste Galois (1811-1832) in 1831. Galois's approach was to study certain permutations of the solutions of a polynomial; in doing so, he noted that the collection of these permutations forms a structure that he termed a 'group'. This notion subsequently became the cornerstone of the theory of polynomial equations. Moreover, as the nineteenth century progressed, mathematicians began to notice that this same structure appears elsewhere in mathematics, and might therefore be studied in an entirely abstract setting, divorced from any specific interpretation: theorems proved about an abstractly defined group would then be applicable in any specific instance. Owing to its origins in the study of polynomial equations, the investigation of such abstract groups, and of the other similarly abstractly defined structures that emerged in the late nineteenth and early twentieth centuries, was given the name 'modern algebra', or 'abstract algebra', or, latterly, simply 'algebra'. Modern mathematicians thus employ the word 'algebra' in a variety of senses, ranging from the solution of equations to something rather more abstract.

The book under review, written by two leading authorities in the history of mathematics, is a history of algebra (in all its senses) from its origins in the solution of practical (and, indeed, notso-practical) word problems in the ancient world, through the works of al-Khwārizmī, Abel, Galois and many others, to the more recent development of abstract algebra. The reader requires a basic mathematical competence, but need not be an expert. Indeed, the book should be entirely accessible to a (mathematical) undergraduate readership. It will certainly be a useful resource for teaching courses in the history of mathematics, and will also supply the interested student reader with the historical context that straight mathematics courses often lack.

An impressive feature of this book is its comprehensiveness, not only in time span but also in subject matter and in geography. With regard to subject matter, for example, it is good to see linear algebra (originally the solution of systems of simultaneous linear equations) being treated here. The history of this subject can be rather difficult to trace: in certain respects, it is so basic that it has arisen independently within a broad range of apparently unrelated mathematical contexts; moreover, the topics that linear algebra treats are so varied that they have not historically been connected, and were only brought together under the heading 'linear algebra' in the twentieth century. Thus the subject's somewhat tortuous history means that it does not always feature (at least, not in any great detail) in general histories, so the authors of the book under review are to be congratulated for providing an accessible integrated overview. In relation to geography, we are not simply presented with a Eurocentric view of the subject (although the later developments certainly took place in Europe): we learn also of ancient and medieval Indian, Chinese and Arabic developments. It is nice, for example, to see the so-called Chinese remainder theorem, a result well known to mathematicians, in its Chinese context. Another highlight of the book is Chapter 8, in which the authors give an overview of the transmission of ancient mathematical knowledge via the Islamic world to Renaissance Europe.

All of the mathematical content of the book is, quite self-consciously, converted into modern terms: for example, symbolic notation is used in the description of problems that pre-date the introduction of such symbolism by several centuries. Generally speaking, such conversions of notation can have the effect of disguising the thought processes of the original authors, and of inadvertently imposing modern ideas onto historical mathematics. In the case of the present book, however, I believe that this is entirely justified: this is not a book to go to in order to find original formulations of historical mathematics. Instead, it is an accessible introduction, which provides enough references for the interested reader to pursue matters further. The balance of references seems about right for a book written at this level, with a mixture of primary and secondary sources cited. In summary, this is a very readable introduction to an important topic within the history of mathematics.

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WILLIAM E. BURNS, **The Scientific Revolution in Global Perspective**. New York: Oxford University Press, 2015. Pp. 216. ISBN 978-0-19998-933-1. £16.99 (paperback). doi:10.1017/S0007087415000710

Isaac Newton never left Britain. Nonetheless, his *Principia* (1687) was undoubtedly the product of an increasingly globalized world. William Burns's *The Scientific Revolution in Global Perspective* opens with a map detailing the different locations on which Newton relied for his astronomical accounts. It stretches from St Kitts across the Atlantic to St Helena; through Europe to Lisbon and Danzig; and then down into Asia, ending in the Gulf of Tonkin. Newton, many scholars now accept, cannot be treated as a man bounded by the walls of Trinity College. And what goes for Newton goes for the Scientific Revolution in general. Burns looks to bring all this into the classroom, beginning by asking, 'Was the Scientific Revolution a World Revolution?' (p. 2). However, despite a promising introduction, *The Scientific Revolution in Global Perspective* fails to deliver. There are two major flaws that run throughout.

First, there is almost no analytic framing. Burns does not tell us exactly what constitutes the 'global perspective' in the title. Instead, terms like 'global', 'globalization' and 'world' are used interchangeably with little reflection. He also fails to suggest how the 'global', whatever it might be, affects major debates in the historiography of early modern science. Instead, 'global' seems to be left as a catchall description for people and places outside Europe. When Burns does venture some analysis, it is sporadic and out of date. The language is of 'origins' and 'impact'. The thrust is broadly diffusionist, with the reader told in the conclusion that 'Western science triumphed not simply because of Western power but also because it simply worked better' (p. 162). Burns also has an annoying habit of using the word 'scientist' to describe men like Newton, Galileo Galilei and Robert Boyle. The fact that this book is designed for an undergraduate survey course is no excuse. The best existing introductions to the period, Steven Shapin's *The Scientific Revolution* (1996) and Peter Dear's *Revolutionizing the Sciences* (2001), might be Eurocentric, but at least they are historiographically grounded and offer a strong analysis.

Second, despite all its pretensions to offering a 'global perspective', the first seven chapters focus almost exclusively on Europe. In fact, they read very much like existing accounts of the Scientific Revolution. The first two chapters retrace the standard story of the development of ancient Greek and medieval Arabic science alongside the recovery of these texts in the European humanist tradition. Mentions of 'Aztec medicine' and the 'yin–yang school' are token at best. Chapter 3 is a brief improvement, explaining how European colonial expansion motivated cartographic and collecting projects. Burns also rightly points out how the ideology of 'scientific progress' underlying the work of Francis Bacon and his followers was itself a product of colonialism. From here the chapters are organized thematically. Once again, the world outside Europe does not get much of a look in. Burns's account of astronomy simply proceeds through the intellectual development of Nicolaus Copernicus, Tycho Brahe, Galileo Galilei and Johannes Kepler. We are briefly reminded that Newton deployed astronomical observations from across the world. However, who made these observations, why, and how the information got to Cambridge remain for the reader to guess. Chapters on religion,