

RESEARCH ARTICLE

Self-adaptive differential evolution-based coati optimization algorithm for multi-robot path planning

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Abstract

The multi-robot path planning problem is an NP-hard problem. The coati optimization algorithm (COA) is a novel metaheuristic algorithm and has been successfully applied in many fields. To solve multi-robot path planning optimization problems, we embed two differential evolution (DE) strategies into COA, a self-adaptive differential evolution-based coati optimization algorithm (SDECOA) is proposed. Among these strategies, the proposed algorithm adaptively selects more suitable strategies for different problems, effectively balancing global and local search capabilities. To validate the algorithm's effectiveness, we tested it on CEC2020 benchmark functions and 48 CEC2020 real-world constrained optimization problems. In the latter's experiments, the algorithm proposed in this paper achieved the best overall results compared to the top five algorithms that won in the CEC2020 competition. Finally, we applied SDECOA to optimization multi-robot online path planning problem. Facing extreme environments with multiple static and dynamic obstacles of varying sizes, the SDECOA algorithm consistently outperformed some classical and state-of-the-art algorithms. Compared to DE and COA, the proposed algorithm achieved an average improvement of 46% and 50%, respectively. Through extensive experimental testing, it was confirmed that our proposed algorithm is highly competitive. The source code of the algorithm is accessible at: <https://ww2.mathworks.cn/matlabcentral/fileexchange/164876-HDECOA>.

1. Introduction

Robot systems, primarily accomplishing desired tasks through close collaboration between individual or multiple robots and humans, have now become an indispensable part of the development in agriculture, industry, and service sectors. In this trend of development, robot path planning has become one of the most crucial issues in robot systems. Multi-robot path planning is an NP-hard problem, which describes the scenario where, in the same working environment, these robots need to move from their initial positions to their respective destinations at the minimum cost while avoiding collisions with other robots or obstacles during the movement. Multi-robot path planning involves multiple constraints, particularly in dynamically changing environments. Previous work has proposed several solutions to avoid collisions and coordinate between multiple robots [1]. Classical methods for solving robot path planning problems typically include potential field techniques [2], bounding box representations [3], Breadth-First Search and Depth-First Search [4], A* algorithm [5], and exploring random tree methods [6]. However, these methods often fail to effectively address multi-robot path planning as the scale of robots or obstacles increases. Current researchers primarily utilize artificial neural networks and metaheuristic algorithms to

solve such problems. Metaheuristic algorithms primarily mimic biological behaviors or natural phenomena as heuristic methods and are primarily categorized into four types. The first type is population-based metaheuristic algorithms, exemplified by particle swarm optimization (PSO) [7], which simulates the foraging behavior of bird flocks, and ant colony optimization (ACO) [8], which mimics the process where ant populations release pheromones along their paths during foraging to find optimal routes based on pheromone concentration. Similar algorithms include artificial bee colony (ABC) [9]. The second type is evolutionary-based metaheuristic algorithms, such as genetic algorithm (GA) [10] and differential evolution (DE) [11], which emulate the process of biological evolution where the most optimal genetic individuals are preserved over generations. The third type is based on chemical and physical phenomena, grounded in theoretical foundations, with representative algorithms including gravitational search algorithm (GSA) [12] and Archimedes optimization algorithm (AOA) [13]. The fourth type is inspired by human social behaviors and can draw inspiration from various aspects of daily life. Examples include lungs performance-based optimization (LPO) [14], which simulates human lung function activity, and football team training algorithm (FTTA) [15], which mimics the training process of a football team. However, due to the homogeneity of their update strategies, these algorithms tend to fall into local optima to some degree when solving problems, and their parameter settings are relatively uniform, making them unsuitable for solving diverse problems.

This article is a study on path planning problem based on metaheuristic algorithm. According to the No Free Lunch theorem, no single algorithm can solve all problems optimally. Therefore, researchers often integrate multiple strategies and algorithms to improve the performance of path planning algorithms. Geng et al. [16] combined sparrow search algorithm (SSA) with reverse learning strategy to solve a robot path planning problem on a grid map. Parhi et al. [17] merged the classical method of linear regression (LR) with GSA called RGSA, incorporating multiple chaos strategies to obtain optimal paths in multi-humanoids (Nao robots) movement problems. Li et al. [18] proposed a hybrid algorithm based on Improved GA and Dynamic Window Approach for solving mobile robot path planning problems. Xu et al. [19] combined a new fourth-order Bezier transition curve with an improved PSO algorithm, proposed a novel method for smooth path planning of mobile robots. Nazarahari et al. [20] proposed an innovative artificial potential field (APF) algorithm to find all feasible paths between start and end points in a discrete grid environment and developed an Enhanced GA to improve initial path and find the optimal path between start and end points in robot path planning problems. Zhang et al. [21] proposed a hybrid algorithm of GA and firefly algorithm to enhance the responsiveness and computational capability of mobile robots during movement. Dai et al. [22] introduced an Improved ACO utilizing characteristics of A* algorithm and MAX-MIN ant system to achieve efficient search capability for mobile robot path planning in complex maps. Chen et al. [23] addressed path planning problems for mobile robots in known environments, proposed a grid-based hybrid APF and ACO path planning method. Zhang et al. [24] proposed a turning point-based grey wolf optimizer (GWO) for solving the path planning problem of patrol robots. The algorithm utilizes the concepts of roulette wheel selection and crossover mutation to broaden the search scope of the initial population. Additionally, the convergence factor function varies with the number of obstacles, enhancing the performance of the proposed algorithm. Liu et al. [25] integrated the A* algorithm with an improved GWO to propose an A*-IGWO for solving parking lot path planning problems. The algorithm constructs a minimum cost equation using the A* algorithm on the basis of the population updating mechanism of the IGWO, fully leveraging the advantages of both algorithms to improve performance. Dai et al. [26] proposed an improved sparrow search algorithm (ISSA) for solving the path planning problem of cellular robots on large-scale three-dimensional truss. Julius et al. [27] proposed an improved self-adaptive learning PSO (ISALPSO) algorithm for solving the path planning problem of mobile robots in 2D lidar maps. The algorithm can fine-tune the lidar information using a binary occupancy grid method during the robot's movement, thereby adaptively adjusting the parameters by ISALPSO.

The coati optimization algorithm (COA) [28] is an efficient and novel metaheuristic algorithm proposed by Dehghani et al. in 2022, widely applied to various global optimization problems. However, due to its inherent mechanism issues, it tends to fall into the dilemma of premature convergence when solving

high-dimensional problems. Researchers have proposed some improvement ideas for COA. Hashim et al. [29] introduced an adaptive mutation strategy for the COA to solve feature extraction and global optimization problems. Baş et al [30] proposed an Enhanced COA for solving large-scale high-dimensional big data optimization problems (BOP). Yildizdan et al. [31] used a transfer function to transform the continuous optimization problem-solving COA into a binary optimization algorithm (BinCOA) to solve the Knapsack Problem (KP) and the Uncapacitated Facility Location Problem (UFLP). Hasanien et al. [32] introduced a new improved COA to find the optimal solution for the Probability Optimal Power Flow (POPF) problem. Jia et al. [33] proposed an improved COA based on the sound search envelope strategy to solve six engineering application problems. Although DE and COA have a wide range of applications and unique advantages, both algorithms exhibit poor solution quality when dealing with high-dimensional and multimodal problems, making them susceptible to local optima. To overcome the limitations of COA, this paper combines a cross-update strategy from some DE algorithm variants with the original COA to propose a new hybrid algorithm (SDECOA). When faced with different problems, self-adaptive differential evolution-based coati optimization algorithm (SDECOA) adaptively adjusts the use frequency of update strategies, fully utilizing the update mechanisms of both algorithms to enhance the algorithm's global search and local search capabilities.

The contributions of this paper are summarized as follows:

1. Combining the update strategies from variants of DE algorithms with COA, an SDECOA is proposed, enhancing the COA's global search and local search capabilities.
2. The crossover probability CR in the DE strategy adapts dynamically based on the algorithm's iterations, increasing the convergence speed of the algorithm.
3. The proposed algorithm successfully solved 48 real-world constrained optimization problems from various fields and achieved superior results compared to the top algorithms in the CEC2020 competition.
4. The multi-robot path planning problem is NP-hard, and the presence of multiple moving obstacles further complicates the problem. The proposed algorithm effectively addresses this issue.

The organization of this paper is as follows: Section 2 discusses the relevant definitions of DE, COA, real-world constrained optimization problems, and online multi-robot path planning problems. Section 3 provides a detailed explanation of SDECOA's definition. Section 4 showcases the experimental results of this study. Section 5 is the summary and future research objectives of this paper.

2. Preliminaries

In this section, COA and DE algorithm are introduced separately, along with the definition of real-world constrained engineering optimization problems and the definition of multi-robot path planning problems.

2.1. Coati optimization algorithm (COA)

This section describes the mathematical model of the original COA. The COA is a novel swarm intelligence algorithm proposed in 2022, which primarily simulates the behavior of coatis hunting iguanas. Initially, half of the population will climb trees to approach their food source, the iguana, and this behavior is defined by Eq. (1):

$$Xnew_i: xnew_{ij} = x_{ij} + r1 \cdot (I_g - I \cdot x_{ij}), i = 1, 2, 3, \dots \left\lfloor \frac{N}{2} \right\rfloor \quad (1)$$

In the above equation, $x_{i,j}$ represents the position of the i -th individual in the j -th dimension of the solution space, Ig denotes the position of the best solution in the current population, N represents the population size, r_1 is a random number between $[0, 1]$, and I takes a random value of either 1 or 2.

Faced with the siege of coatis, the iguana will randomly drop to the ground. Eq. (2) expresses this behavior in the solution space. Meanwhile, the other half of the coatis will search for their prey, as represented by Eq. (3):

$$Ig^G : Ig_j^G = lbj + r_2 \cdot (ubj - lbj), j = 1, 2, 3, \dots m \tag{2}$$

$$X_{new_i} : x_{new_{i,j}} = \begin{cases} x_{i,j} + r_3 \cdot (Ig_j^G - I \cdot x_{i,j}), F_{Ig}^G < F_i \\ x_{i,j} + r_3 \cdot (I \cdot x_{i,j} - Ig_j^G), else \end{cases}, i = \left\lfloor \frac{N}{2} \right\rfloor + 1, \dots N \tag{3}$$

where Ig_j^G represents the random dropping position of the iguana, with lb and ub denoting the upper and lower bounds of the solution space, respectively. r_2 and r_3 are both random numbers between $[0,1]$. In Eq. (3), F_{Ig}^G represents the fitness value of Ig and F_i denotes the current fitness value of the i -th individual. Subsequently, if the fitness value F_{new_i} of the newly generated individual is superior to the current individual, its position is replaced; otherwise, the original position is retained, as expressed in Eq. (4):

$$X_i = \begin{cases} X_{new_i}, F_{new_i} < F_i \\ X_i, else \end{cases} \tag{4}$$

After locating the iguana, all the coatis will slowly surround their prey. This biological characteristic is represented by equations (5) and (6):

$$lb_j^t = \frac{lbj}{t}, ub_j^t = \frac{ubj}{t}, t = 1, 2, 3, \dots T \tag{5}$$

$$X_{new_i} : x_{new_{i,j}} = x_{i,j} + (1 - 2r_4) \cdot (lb_j^t + r_5 \cdot (ub_j^t - lb_j^t)), i = 1, 2, 3, \dots N \tag{6}$$

where t denotes the current iteration number, while T represents the maximum iteration number, and r_4 and r_5 are both random numbers between $[0, 1]$. Subsequently, the new position of the current individual is also calculated using Eq. (4). The pseudocode of the basic framework of the COA is represented by Algorithm 1.

2.2. Differential evolution (DE)

This section describes the mathematical model of original DE. DE is a well-known classic metaheuristic algorithm, mainly consisting of three evolutionary processes: mutation, crossover, and selection.

2.2.1. Mutation

In DE, the mutation process involves randomly selecting two individuals and using the difference of their position vectors as the step size for updating the target carrier position, as represented by Eq. (7):

$$y_{ij} = x_{i,j} + F \cdot (x_{R1,j} - x_{R2,j}), i = 1, 2, 3, \dots, N \tag{7}$$

In the above equation, x_i represents the i -th individual in the population, x_{R1} and x_{R2} are two different individuals randomly selected from the population, j denotes the j -th dimension of the solution, and F is the scaling factor.

2.2.2. Crossover

The crossover process involves exchanging the j -th dimension component between the mutated individual y_i and the target carrier x_i , generating a crossover individual z_i , as represented by Eq. (8):

Algorithm 1 Pseudocode of COA.

```

1: Set the number of iterations  $T$  and the number of coatis  $N$ .
2: Initialize population.
3: for  $t = 1:T$ 
4:   Update location of the iguana based on the location of the best member of the population.
5:   Phase 1: Hunting and attacking strategy on the iguana (Exploration Phase).
6:   for  $i = 1 : N / 2$ 
7:     Calculate new position for the  $i$ -th coati using Eq. (1).
8:     Update position of the  $i$ -th coati using Eq. (4).
9:   end for
10:  for  $i = 1 + N/2: N$ 
11:    Calculate random position for the iguana using Eq. (2).
12:    Calculate new position for the  $i$ -th coati using Eq. (3).
13:    Update position of the  $i$ -th coati using Eq. (4).
14:  end for
15:  Phase 2: The process of escaping from predators (Exploitation Phase).
16:  Calculate the local bounds for variables.
17:  for  $i = 1: N$ 
18:    Calculate the new position for the  $i$ -th coati using Eq. (6).
19:    Update the position of the  $i$ -th coati using Eq. (4).
20:  end for
21:  Save the best candidate solution found so far.
22: end for
23: Output of the best obtained solution by COA for given problem. End COA.

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$$Z_i: z_{ij} = \begin{cases} y_{ij}, & r < CR | j_r = j \\ x_{ij}, & \text{else} \end{cases} \quad (8)$$

where CR denotes the crossover probability, j_r represents a randomly selected number between 1 and the maximum dimensions, and r is a random number between $[0, 1]$.

2.2.3. Selection

Similar to Eq. (4), after comparing the fitness values of the target carrier and the crossover individual, one of them is randomly selected. The better individual is chosen as the position vector for the next generation, achieving the goal of evolution:

$$X_i = \begin{cases} Z_i, & F_{z_i} < F_i \\ X_i, & \text{else} \end{cases} \quad (9)$$

where F_{z_i} represents the fitness value of the crossover individual and F_i represents the fitness value of the target carrier.

2.3. Real-world constrained optimization problems

This paragraph introduces the basic definition of constrained optimization problems, which typically refer to minimization problems under several equality and inequality constraints. The real-world constrained optimization problems in this paper are all derived from CEC2020, covering various fields such as industrial chemical processes, synthesis and design, mechanical engineering, power systems, power electronics, and livestock feed optimization. The definitions of these problems are referenced

from literature [34] and all meet the following function definition.

$$\begin{aligned} &\text{Minimize } f(X), X = (x_1, x_2, \dots, x_n) && (10) \\ &\text{Subject to: } && \\ &\quad g_i(X) \leq 0, i = 1, \dots, n && \\ &\quad h_j(X) = 0, j = n + 1, \dots, m && \end{aligned}$$

In the above equation, X denotes the solution vector of the problem, g represents inequality constraints, n denotes the number of g , h represents equality constraints, and m signifies the total number of all constraints.

2.4. Multi-robot online path planning problem

This article studies the multi-robot path planning problem with multiple static and dynamic obstacles; therefore, it is necessary to consider the safe distance between robots and other robots, robots and static obstacles, and robots and dynamic obstacles. The mathematical model of this problem is provided in literature [35], where each robot has its own starting point and destination. During the robots movement, there are static and dynamic obstacles of various sizes and shapes. All robots not only need to avoid these obstacles but also must not collide with each other. The robots need to reach their destinations with the minimum cost step by step.

Minimize,

$$Fit = F_1 + F_2 + F_3 + F_4 \tag{11}$$

where F_1 represents the shortest distance, F_2 represents avoiding static obstacles, F_3 represents avoiding dynamic obstacles, and F_4 represents avoiding other robots. F_1 can be expressed as Eq. (12):

$$F_1 = \sum_{i=1}^{NR} (f_i + g_i) \tag{12}$$

where

$$f_i = \sqrt{(x_i^n - x_i^c)^2 + (y_i^n - y_i^c)^2} \tag{13}$$

$$g_i = \sqrt{(x_i^n - x_i^s)^2 + (y_i^n - y_i^s)^2} \tag{14}$$

where NR represents the number of robots and (x_i^s, y_i^s) represents the destination coordinates of the i -th robot. F_2 can be expressed as Eq. (15):

$$F_2 = \begin{cases} \varepsilon, d_i^s \leq d_s \\ 0, else \end{cases} \tag{15}$$

where ε is a large penalty value, d_s represent the safety distance between any two objects, and d_i^s is the sum of distances from all static obstacles to the i -th robot, as derived from Eq. (16):

$$d_i^s = \sum_{i=1}^{NR} \sum_{j=1}^{NS} \sqrt{(x_i^n - x_j^s)^2 + (y_i^n - y_j^s)^2} \tag{16}$$

where NS is the number of static obstacles and (x_j^s, y_j^s) is the coordinate position of the j -th static obstacle. F_3 can be expressed as Eq. (17):

$$F_3 = \begin{cases} \varepsilon, d_i^D \leq d_s \\ 0, else \end{cases} \tag{17}$$

where d_i^D is the sum of distances from all dynamic obstacles to the i -th robot, as obtained from Eq. (18):

$$d_i^D = \sum_{i=1}^{NR} \sum_{j=1}^{ND} \sqrt{(x_i^n - x_j^D)^2 + (y_i^n - y_j^D)^2} \tag{18}$$

where ND is the number of dynamic obstacles, (x_j^D, y_j^D) is the coordinate position of the j -th dynamic obstacle, and the movement of dynamic obstacles at each step is determined by equations (19) and (20):

$$x_j^D = x_j^D + v_j^D \cos(\alpha_j) \tag{19}$$

$$y_j^D = y_j^D + v_j^D \sin(\alpha_j) \tag{20}$$

In the above equation, v_j^D represents the movement speed of dynamic obstacle j and α_j represents its radial position relative to the target position. F_4 can be expressed as Eq. (21):

$$F_4 = \begin{cases} \varepsilon, & d_i^R \leq d_s \\ 0, & \text{else} \end{cases} \tag{21}$$

where d_i^R is the sum of distances between different robots and (x_j^R, y_j^R) is the coordinate position of the j -th robot, as obtained from Eq. (22):

$$d_i^R = \sum_{i=1}^{NR-1} \sum_{j=i+1}^{NR} \sqrt{(x_i^n - x_j^R)^2 + (y_i^n - y_j^R)^2} \tag{22}$$

3. The proposed SDECOA algorithm

This section will elaborate on the mathematical definition of SDECOA in detail. In the original COA algorithm, there existed an imbalance between global search capability and local search capability. To overcome this deficiency, we propose called SDECOA. For optimization problems with fewer extreme points, a mutation operator with stronger local search capability should be used. For optimization problems with more extreme points, a mutation operator with stronger global and local search capability should be employed. It is difficult for a single mutation operator to balance global search capability and local search capability, creating obstacles to achieving this balance. In SDECOA, two variants of DE are introduced in the update strategy, and these strategies are called upon based on a roulette wheel method. Initially, all strategies have the same probability of being used. However, during the algorithm iteration process, strategies that yield better results are given higher usage probabilities through learning. Learning is determined through points, where a strategy that achieves good results earns a point. As a result, satisfactory results can be achieved for different problems, overcoming the challenge of imbalance between local and global search in COA.

3.1. Crossing

In traditional DE algorithms, the crossing probability CR is a fixed value, with literature [36] suggesting a range of values for CR between $[0, 1]$. When CR approaches zero, the changes in individuals between adjacent generations are minimal, leading to a high presence of similar individuals in the population and potentially causing the algorithm to get stuck in local optima. Conversely, when CR is close to 1, the individuals update positions fluctuate greatly, hindering efficient convergence. To address these issues, this paper proposes an adaptive dynamic adjustment method, calculated as follows:

$$CR(t) = CR_{\min} + (CR_{\max} - CR_{\min}) \cdot \left(1 - \frac{t}{T}\right) \tag{23}$$

where CR_{\max} equals 0.95, CR_{\min} equals 0.05, t represents the current iteration number, and T is the maximum number of iterations.

3.2. Variant update strategies of DE

The first introduced update strategy, $rand < CR$, was proposed in literature [37], while the other part was inspired by the GWO [34] and is represented by Eq. (24):

$$X_{new_i}: x_{new_{i,j}} = \begin{cases} x_{i,j} + FF \cdot (x_{best,j} - x_{i,j} + x_{R1,j} - x_{R2,j}); & \text{if } rand < CR \\ \frac{1}{3} (x_{best,j} + x_{sec\ ond,j} + x_{third,j}), & \text{else} \end{cases} \quad (24)$$

In the above equation, x_{best} , x_{second} , and x_{third} represent the current population’s top three individuals, FF is the scaling factor with a value of 0.8, while $R1$ and $R2$ are randomly selected individuals that are different from the i -th individual.

The second introduced update strategy was proposed in literature [39] and is represented by Eq. (25):

$$X_{new_i}: x_{new_{i,j}} = x_{i,j} + L \cdot (x_{R1,j} - x_{i,j}) + FF \cdot (x_{R2,j} - x_{R3,j}) \quad (25)$$

where L is a random number between $[0, 1]$, and $R1$, $R2$, and $R3$ are also three distinct random individuals.

3.3. Algorithm implementation

In the algorithm proposed, during the exploration phase, Eqs. (1), (24), and (25) are randomly used, with their probabilities determined by Eq. (26). During the invocation process, a counter records which update strategy an individual has been used. If an individual achieves better results in this iteration, the score of the strategy used by that individual is incremented. The probabilities for the next roulette wheel selection are calculated based on the total score obtained by each strategy in this iteration:

$$p_j(t + 1) = \frac{S_j(t)}{\sum_{i=1}^{SN} S_i(t)}, \quad j \in \{1, 2, \dots, SN\} \quad (26)$$

where $p_j(t + 1)$ represents the probability of strategy j being used in the next iteration, $S_i(t)$ represents the score of strategy i in the current iteration, and SN represents the number of strategies, which is set to 3 in this paper. The update strategy for each individual is determined by a roulette wheel selection. The pseudocode and flow chart of SDECOA are, respectively, shown in Algorithm 1 and Figure 1.

3.4. Time complexity analysis

The operation of SDECOA includes population initialization, fitness calculation, population position update, individual probability update, and strategy selection, so by analyzing the complexity of each process, the time complexity of SDECOA can be obtained. Assuming the population size is N , the number of iterations is T , and the dimension of the problem is D , the time complexity of population initialization is $O(ND)$. In each iteration process, the time complexity of population position update is $O(ND+ND)$, the time complexity of fitness value calculation and comparison is $O(ND+ND)$, and the time complexity of individual probability update and strategy selection is $O(ND)$. Therefore, the time complexity of SDECOA can be concluded as $O(ND+T \times (ND+ND+ND+ND+ND+ND)) \approx O(TND)$.

4. Experiment results and analysis

4.1. Experimental configuration and design

The programming language used in this experiment is MATLAB, and the compilation software is MATLAB 2021a. The computer configuration consists of a 64-bit Windows 10 operating system, 8.0G RAM, and a processor with a base frequency of 2.10 GHz. To demonstrate the effectiveness of the improved algorithm, our algorithm was initially compared with several classic and advanced algorithms on the 20-dimensional CEC2022 benchmark functions. To further validate the performance of our algorithm, SDECOA was employed to solve 48 real-world constrained optimization problems from

Algorithm 2 Pseudocode of SDECOA.

```

1: Set the number of iterations  $T$ , the number of coatis  $N$  and the scale factor  $FF$ .
2: Initialize population.
3:  $p_i \leftarrow 1/3$ ,  $S_i \leftarrow 0$ ,  $i \in \{1,2,3\}$ .
4: Calculate the fitness value of the population.
5: for  $i = 1:N$ 
6:    $pp_i \leftarrow \text{RouletteWheelSelection}(p)$ .
7: end for
8: for  $t = 1:T$ 
9:   Calculate the crossover probability  $CR$  using Eq. (23).
10:  Calculate the optimal three individuals  $X_{best}$ ,  $X_{second}$ , and  $X_{third}$  in the population.
11:   $Iguana \leftarrow X_{besto}$ 
12:  for  $i = 1:N$ 
13:    Randomly select three individuals R1, R2, R3 that are different from individual  $i$ .
14:    if  $pp_i == 1$ 
15:      Calculate the new position  $X_{new}$  for the  $i$ -th coati using Eq. (1).
16:    elseif  $pp_i == 2$ 
17:      Calculate the new position  $X_{new}$  for the  $i$ -th coati using Eq. (24).
18:    elseif  $pp_i == 3$ 
19:      Calculate the new position  $X_{new}$  for the  $i$ -th coati using Eq. (25).
20:    end if
21:    Update position of the  $i$ -th coati using Eq. (4)
22:    if  $f(X_{new}) < f(X_i)$ 
23:       $S_j \leftarrow S_j + 1$ 
24:    end if
25:  end for
26:  Update the  $p$  using Eq. (26).
27:  for  $i = 1:N$ 
28:     $pp_i \leftarrow \text{RouletteWheelSelection}(p)$ .
29:  end for
30:  for  $i = 1:N$ 
31:    Calculate the new position for the  $i$ -th coati using Eq. (6).
32:    Update the position of the  $i$ -th coati using Eq. (4).
33:  end for
34:  Save the best candidate solution found so far.
35: end for
36: Output of the best obtained solution by SDECOA for given problem. End SDECOA.

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CEC2020, and comparisons were made with several algorithms that won in the CEC2020 competition. Finally, our algorithm was applied to the problem of multi-robot online path planning.

4.2. SDECOA for CEC2022 benchmark functions

Solving benchmark test functions is a common method for testing algorithm performance, and this section presents the experimental results of SDECOA solving this test set. The CEC2022 benchmark test functions are the latest test functions, and basic information is shown in Table 1. This test suite mainly includes three dimensions ($dim = \{2, 10, 20\}$), with function types primarily including single-peak functions (F1), multimodal functions (F2–F5), hybrid functions (F6–F8), and composite functions (F9–F12). To validate the algorithm's effectiveness, we compare SDECOA with the original DE and COA.

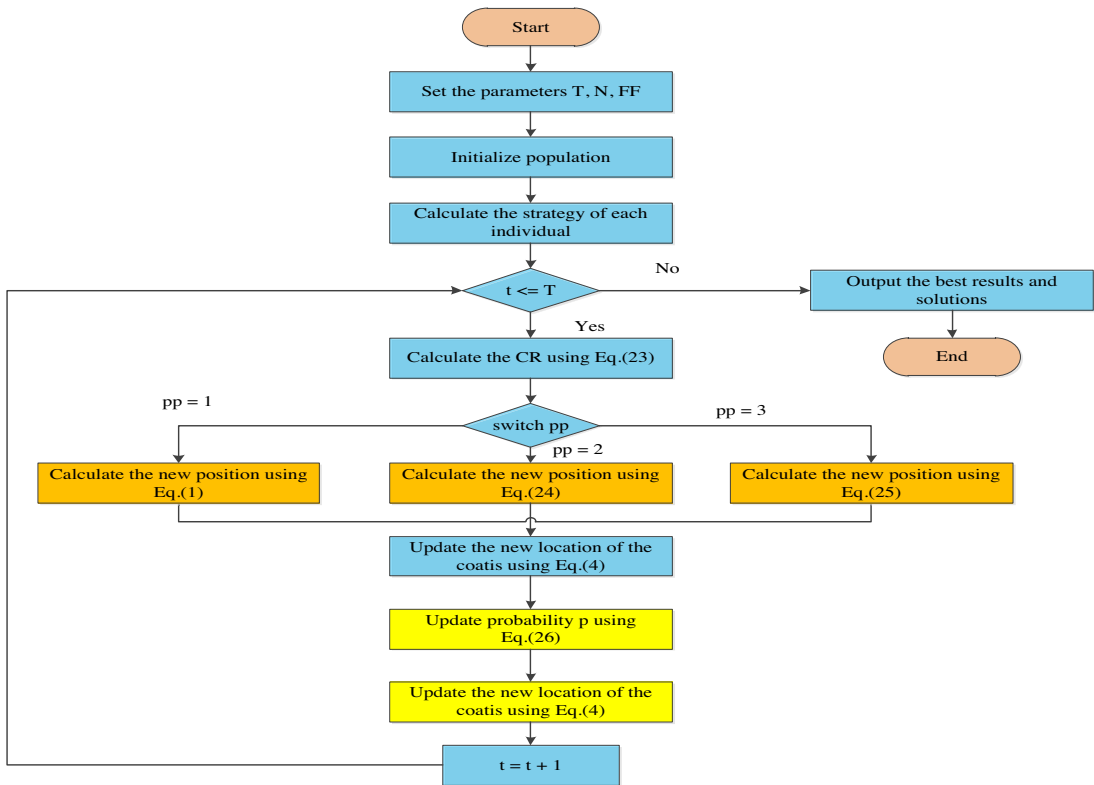


Figure 1. Flow chart of SDECOA.

Furthermore, to evaluate its performance, we also test and analyze it against some well-known classical algorithms and recent advanced algorithms. The algorithms involved in testing the CEC2022 benchmark functions include simulated annealing (SA) [40], PSO [7], GWO [38], sine-cosine algorithm (SCA) [41], whale optimization algorithm (WOA) [42], Harris hawk optimizer (HHO) [43], SSA [44], modified adaptive sparrow search algorithm (MASSA) [41], self-adaptive differential sine-cosine algorithm (sdSCA) [35], hybrid algorithm of differential evolution and flower pollination (HADEFP) [45], etc. In this set of experiments, all algorithms were independently run 25 times. Except for SA, the iteration count for all algorithms was set to 1000 iterations, with a population size of 50. The other parameters for each algorithm are shown in Table 2.

As shown in Table 3, the means and standard deviations of different algorithms are recorded. In the experimental results across all test functions, the proposed algorithm outperforms the original COA and DE by a significant margin, particularly achieving the best results in solving F1, F6, F7, F8, and F9. Among all the compared algorithms, SDECOA proves to be the most competitive. Table 3 also presents the rankings of all algorithms based on the Friedman rank-sum test, with the data confirming that the proposed algorithm achieved the best performance in solving the CEC2022 test functions.

The convergence curves for different CEC2022 test functions are shown in Figure 2. From these curves, it is evident that SDECOA exhibits substantial improvements in both optimization capability and convergence speed compared to DE and COA. Compared with other algorithms, SDECOA shows the best convergence speed and results for most test functions. The boxplots for all test functions are shown in Figure 3, where it can be seen that SDECOA performs better in terms of median values and lower variance for most functions. Compared to other algorithms, SDECOA is more stable, thus demonstrating the superior robustness of the proposed algorithm.

Table I. Basic information of CEC2022 benchmark test functions.

	No.	Functions	f^*
Unimodal Function	F1	Shifted and full Rotated Zakharov Function	300
Basic Functions	F2	Shifted and full Rotated Rosenbrock's Function	400
	F3	Shifted and full Rotated Expanded Shaffer's f6 Function	600
	F4	Shifted and full Rotated Non-continuous Rastrigin's Function	800
	F5	Shifted and full Rotated Levy Function	900
Hybrid Functions	F6	Hybrid Function 1 ($N = 3$)	1800
	F7	Hybrid Function 2 ($N = 6$)	2000
	F8	Hybrid Function 3 ($N = 5$)	2200
Composition Functions	F9	Composition Function 1 ($N = 5$)	2300
	F10	Composition Function 2 ($N = 4$)	2400
	F11	Composition Function 3 ($N = 5$)	2600
	F12	Composition Function 4 ($N = 6$)	2700
Search range: $[-100,100]^D$			

Table II. Parameter settings of different algorithms.

Algorithm	Parameter
SDECOA	F = 0.8
DE	F = 0.4, CR = 0.5
COA	–
SA	Initial temperature = 100, cooling rate = 0.99
PSO	c1 = 2, c2 = 2, v0 = 0, w = 0.7
GWO	–
SCA	–
WOA	–
HHO	–
SSA	P = 0.2, ST = 0.8, SP = 0.2
MASSA	P = 0.2, ST = 0.8, SP = 0.15
sdSCA	CR = 0.6, FF = 0.8
HADEFP	$\alpha = 0.01$, MaxRuntime = 30, a = 4

4.3. SDECOA for CEC2020 problems

To further demonstrate the effectiveness of SDECOA, this section presents the experimental results and data analysis of SDECOA applied to real-world constrained optimization problems. The real-world constrained problems used in this study are based on the CEC2020 real-world constrained optimization problems. The mathematical models for all problems in this study are derived from reference [46]. Table 4 displays the basic information of 48 problems, where D represents the dimension of the problem, g represents the number of inequality constraints, h represents the number of equality constraints, and f^* represents the known optimal value. These problems are mainly designed in fields such as Industrial Chemical Processes, Process Synthesis and Design, Mechanical Engineering, Power Systems, Power Electronics, and Livestock Feed Ration Optimization, with dimensions ranging from 2 to 118 and varying numbers of constraints from 1 to 108.

Table III. Experimental results of different algorithms on CEC2022 benchmark functions in 20 dimensions.

Function	Metric	SDECOA	DE	COA	SA	PSO	GWO	SCA	WOA	HHO	SSA	MASSA	sdSCA	HADEFP
F1	Avg.	3.0000E + 02	5.8850E + 04	3.7666E + 04	3.5105E + 04	1.7048E + 05	1.0387E + 04	1.3565E + 04	1.5429E + 04	2.1594E + 03	1.0080E + 04	2.1201E + 04	3.9211E + 02	1.6934E + 04
	SD.	3.4094E-03	3.1172E + 04	9.5610E + 03	1.5593E + 04	4.6289E + 04	4.2106E + 03	2.9376E + 03	3.9526E + 03	1.5431E + 03	5.4612E + 03	1.2747E + 04	2.3339E + 02	5.0041E + 03
F2	Avg.	4.4199E + 02	4.8547E + 03	2.5294E + 03	4.5374E + 02	2.1734E + 04	4.8140E + 02	6.8423E + 02	5.2963E + 02	4.7717E + 02	4.4902E + 02	4.6164E + 02	4.4808E + 02	4.4745E + 02
	SD.	2.0764E + 01	1.8155E + 03	4.8823E + 02	2.1086E + 01	7.2428E + 03	2.3937E + 01	7.2110E + 01	5.0224E + 01	2.8911E + 01	9.9530E + 00	1.4380E + 01	1.8259E + 00	2.1303E + 00
F3	Avg.	6.0173E + 02	6.9760E + 02	6.7513E + 02	6.0000E + 02	7.6729E + 02	6.0357E + 02	6.4424E + 02	6.5949E + 02	6.5991E + 02	6.5002E + 02	6.3845E + 02	6.0000E + 02	6.0046E + 02
	SD.	3.1131E + 00	1.4915E + 01	1.0789E + 01	9.6371E-03	1.6745E + 01	2.7836E + 00	4.8728E + 00	1.5219E + 01	9.0051E + 00	1.0890E + 01	1.6219E + 01	4.2874E-04	2.0633E-01
F4	Avg.	8.5616E + 02	1.0420E + 03	9.6756E + 02	9.1241E + 02	1.2533E + 03	8.5370E + 02	9.3862E + 02	9.2024E + 02	8.8049E + 02	8.9022E + 02	8.8467E + 02	9.1549E + 02	9.0727E + 02
	SD.	1.2321E + 01	5.2455E + 01	1.6928E + 01	2.4606E + 01	4.8872E + 01	2.7739E + 01	1.2515E + 01	2.9567E + 01	1.2061E + 01	5.4475E + 00	1.5268E + 01	1.0550E + 01	1.6138E + 01
F5	Avg.	1.3809E + 03	7.7792E + 03	3.2282E + 03	4.7036E + 03	2.5857E + 04	1.0436E + 03	2.1384E + 03	4.0273E + 03	2.7385E + 03	2.4796E + 03	2.2327E + 03	9.0000E + 02	9.0051E + 02
	SD.	6.1203E + 02	3.1415E + 03	3.4212E + 02	1.6448E + 03	4.1328E + 03	1.2666E + 02	2.3011E + 02	1.3920E + 03	2.7645E + 02	6.9158E + 01	4.9697E + 02	1.0766E-04	5.4120E-01
F6	Avg.	1.8628E + 03	2.9397E + 09	1.8023E + 09	1.7458E + 04	1.2439E + 10	9.3085E + 05	1.3671E + 08	1.9680E + 05	8.1438E + 04	6.6033E + 03	5.0013E + 03	4.0734E + 03	6.9989E + 05
	SD.	2.8380E + 01	1.5603E + 09	8.4583E + 08	8.5441E + 03	6.2024E + 09	2.8487E + 06	6.9288E + 07	3.6333E + 05	3.3276E + 04	6.1194E + 03	4.1507E + 03	1.1220E + 04	7.1453E + 05
F7	Avg.	2.0427E + 03	2.3026E + 03	2.1906E + 03	2.1161E + 03	2.4045E + 03	2.0750E + 03	2.1445E + 03	2.2173E + 03	2.1777E + 03	2.1671E + 03	2.1407E + 03	2.0638E + 03	2.0684E + 03
	SD.	1.9283E + 01	8.6210E + 01	3.7285E + 01	5.1414E + 01	4.8177E + 01	3.9807E + 01	1.9773E + 01	7.0364E + 01	5.4260E + 01	7.2444E + 01	4.3666E + 01	9.7407E + 00	1.7894E + 01
F8	Avg.	2.2310E + 03	2.7497E + 03	2.3917E + 03	2.2970E + 03	1.6854E + 05	2.2540E + 03	2.2631E + 03	2.2779E + 03	2.2691E + 03	2.2955E + 03	2.2613E + 03	2.2373E + 03	2.2380E + 03
	SD.	2.0676E + 00	4.3058E + 02	1.1877E + 02	7.0938E + 01	1.2529E + 05	4.7243E + 01	1.4034E + 01	5.4650E + 01	5.8601E + 01	8.5724E + 01	4.5755E + 01	2.7204E + 00	2.1052E + 00
F9	Avg.	2.4808E + 03	3.5370E + 03	3.2232E + 03	2.4855E + 03	5.2114E + 03	2.5091E + 03	2.5850E + 03	2.5391E + 03	2.4902E + 03	2.4809E + 03	2.4808E + 03	2.4808E + 03	2.4808E + 03
	SD.	6.2957E-13	3.2405E + 02	2.5265E + 02	4.6754E + 00	8.1657E + 02	2.1480E + 01	2.8873E + 01	3.6123E + 01	6.5441E + 00	1.5576E-01	2.6014E-01	1.1235E-08	1.4916E-03
F10	Avg.	2.9324E + 03	5.9777E + 03	5.7749E + 03	2.4170E + 03	7.3046E + 03	2.9286E + 03	3.0549E + 03	4.5989E + 03	3.6398E + 03	3.0940E + 03	3.5958E + 03	3.3921E + 03	2.5260E + 03
	SD.	9.9669E + 02	1.5928E + 03	1.6254E + 03	2.4782E + 01	2.6820E + 02	5.9806E + 02	1.2903E + 03	1.1633E + 03	7.0158E + 02	4.4200E + 02	9.1648E + 02	7.0045E + 02	7.2782E + 01
F11	Avg.	2.9040E + 03	6.8477E + 04	8.3588E + 03	2.9000E + 03	4.5224E + 05	3.5956E + 03	5.9828E + 03	3.1928E + 03	2.9867E + 03	2.9280E + 03	2.9732E + 03	2.9000E + 03	2.9543E + 03
	SD.	2.0000E + 01	6.2575E + 04	8.6341E + 02	3.9076E-05	1.2018E + 05	2.8798E + 02	4.4265E + 02	9.9508E + 01	1.0610E + 02	4.5823E + 01	1.6668E + 02	2.0972E-05	5.2048E + 01
F12	Avg.	2.9740E + 03	3.7834E + 03	3.5750E + 03	2.9630E + 03	3.1357E + 03	2.9639E + 03	3.0482E + 03	3.0619E + 03	3.1186E + 03	3.0147E + 03	3.0857E + 03	2.9405E + 03	2.9396E + 03
	SD.	3.2828E + 01	3.5057E + 02	2.1345E + 02	1.7314E + 01	8.8622E + 01	1.6969E + 01	2.7432E + 01	8.6321E + 01	8.3877E + 01	6.0802E + 01	1.6208E + 02	3.5849E + 00	3.8063E + 00
Friedman mean rank		2.3333	12.0833	10.8333	5.5833	12.8333	5.2500	7.7500	8.7500	7.1667	5.8333	6.0000	2.7500	3.8333
Rank Statistic		1	12	13	6	4	10	11	9	7	8	3	2	5
								107.21978						

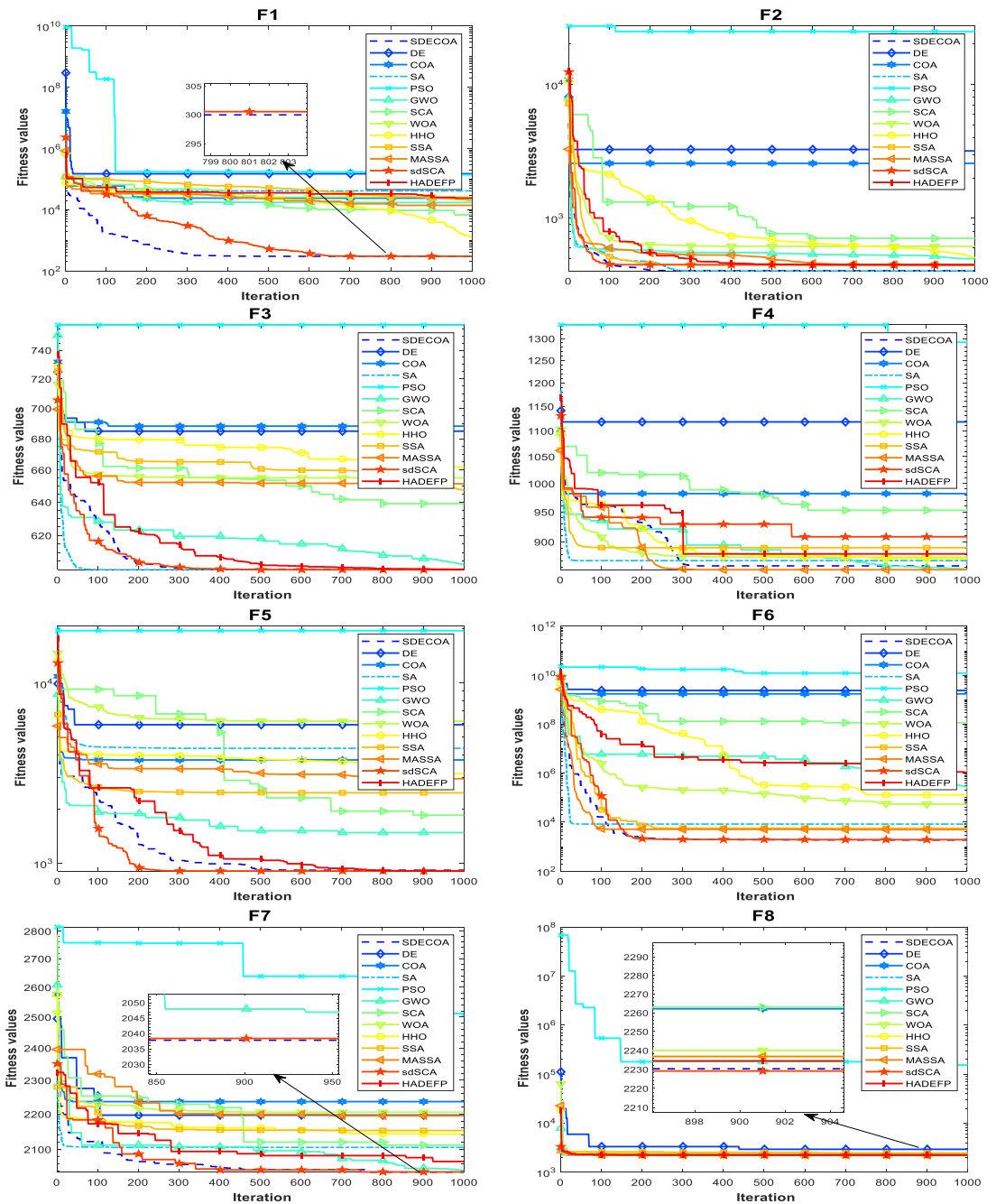


Figure 2. Convergence curves of different algorithms.

In this set of experiments, SDECOA is compared with several advanced algorithms that won in the CEC2020 competition. These top-ranking algorithms, in descending order, include SASS [47], COLSHADE [48], sCMaGES [49], EnMODE [50], and BpMaGES [51]. Each algorithm is independently run 25 times, with 1000 iterations, a population size of 60, and other control parameters consistent with the literature [52]. A dynamic adaptive penalty function is used. Table 5 presents the optimal values, mean values, and standard deviations of these algorithms. It is worth noting that shaded cells in the

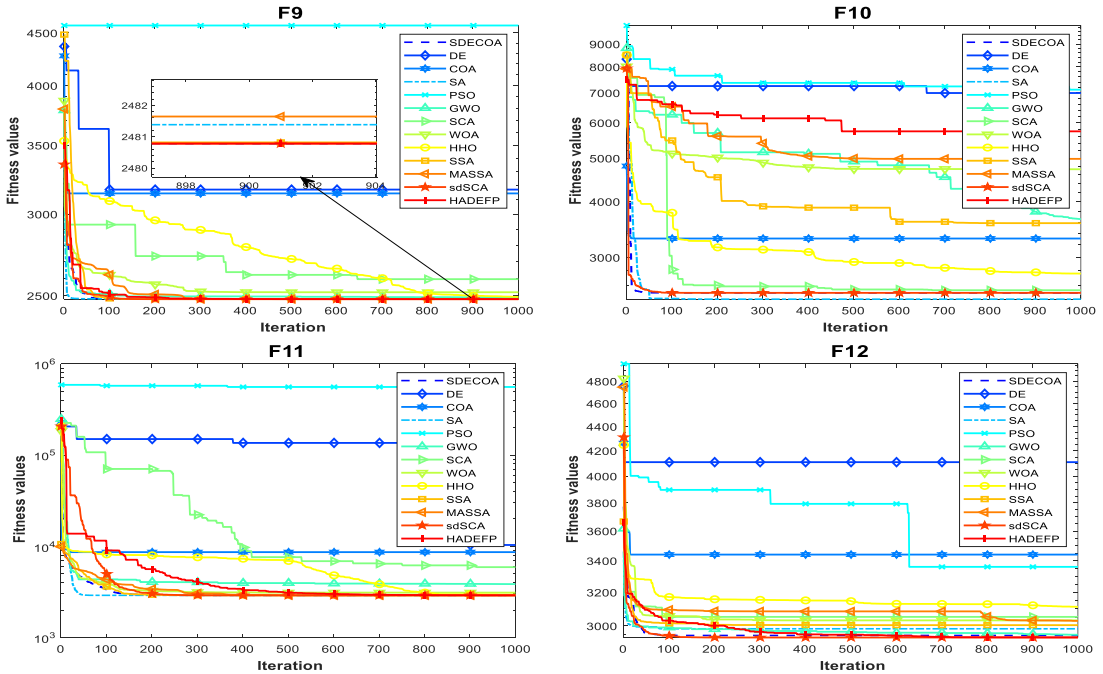


Figure 2. (Continued)

table indicate that the proposed algorithm’s best result exceeds the f^* for that problem. In this paper, the Friedman rank-sum test is also conducted on the average values of these 48 optimization problems, and the ranking of the proposed algorithm, obtained through calculation, is superior to the top five algorithms that won the CEC2020 competition.

The introduction in the literature [53] determines the quality of the solutions by calculating the percentage deviation of the mean with respect to the known optimal solution (PD_{mean}). As can be seen from Eq. (27), the smaller the average value obtained from 25 runs, the smaller the PD_{mean} value, indicating higher solution quality. Table 6 presents the PD_{mean} values when different algorithms solve different problems:

$$PD_{mean} = \begin{cases} \frac{mean - f^*}{f^* + \varepsilon}; & \text{if } f^* \geq 0 \\ \frac{f^* - mean}{f^*}, & \text{else} \end{cases} \quad (27)$$

where f^* represents the best know solution, $mean$ represents the average value of the test results, and ε represents a very small constant. An analysis of Table 6 shows that the proposed algorithm generally demonstrates superior solution quality compared to these advanced algorithms. Based on the experimental results, the proposed algorithm achieved ideal results in most cases, with results for 26 problems surpassing the known optimal solution. These experimental results are sufficient to demonstrate that the proposed algorithm not only performs well but also excels in addressing real-world constrained optimization problems using SDECOA. Therefore, in comparison with these state-of-the-art algorithms, our algorithm demonstrates excellent competitiveness.

4.4. SDECOA for multi-robot path planning problem

In this section, the experimental content applies SDECOA to the multi-robot path planning problem, the mathematical model code of which can be obtained from reference [35].

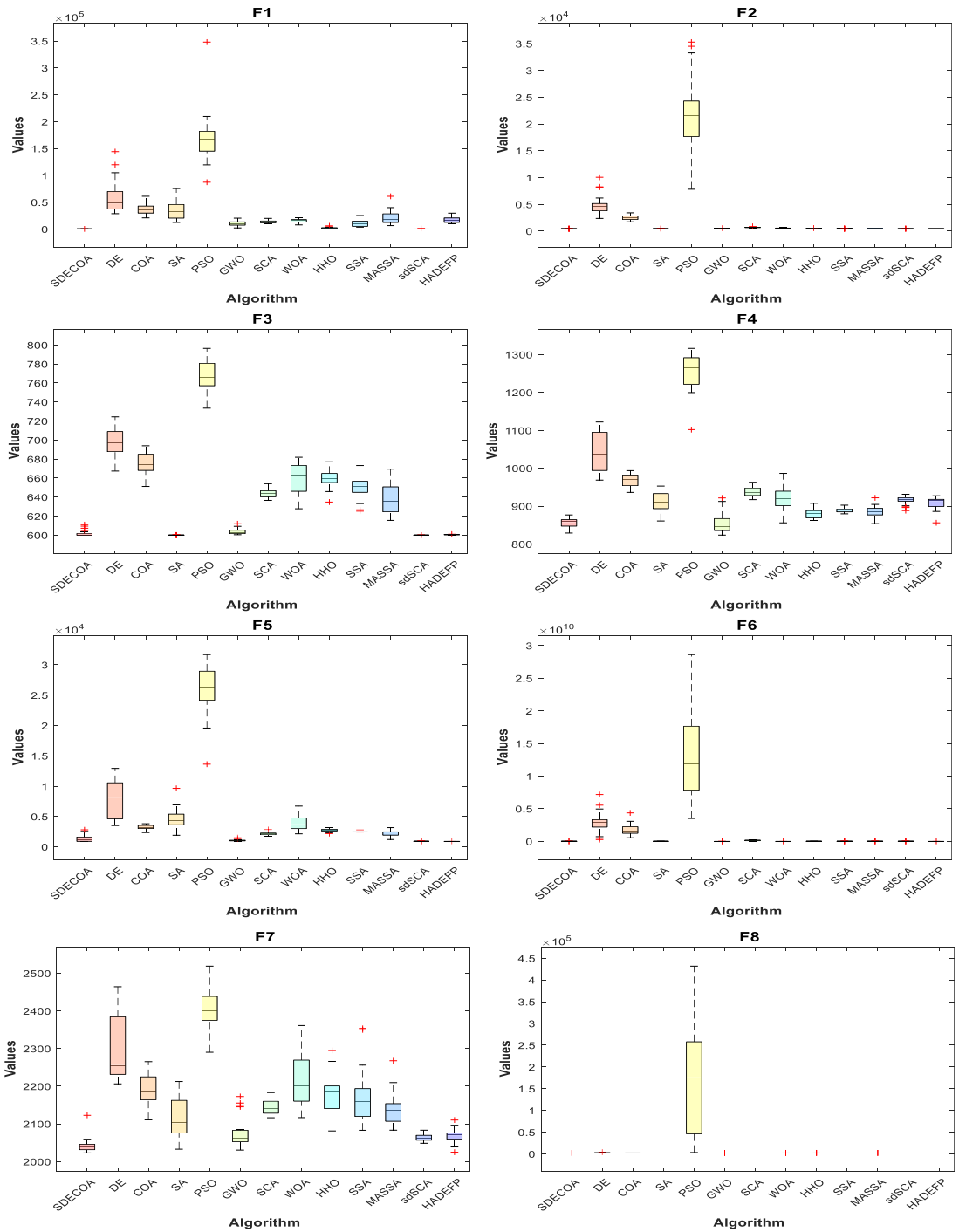


Figure 3. Boxplots of different algorithms.

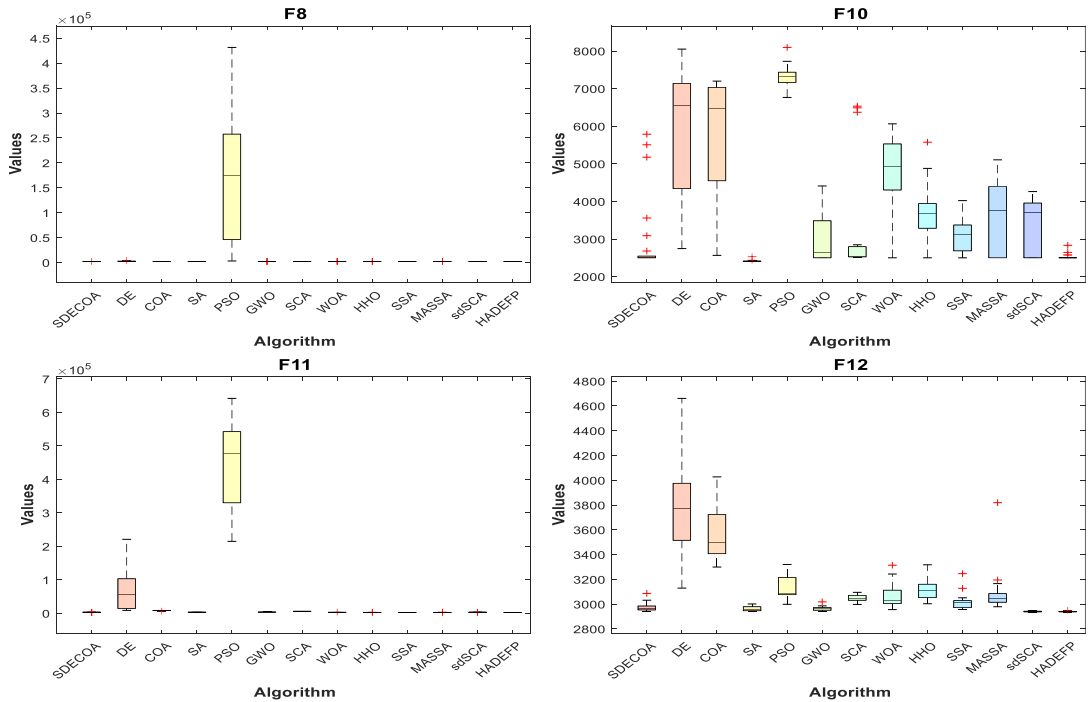


Figure 3. (Continued)

4.4.1. Design of the scenario for multi-robot path planning problem

This section introduces the simulation scenario layout for the robot path planning problem. In the simulation model of this problem, all robots are circular with equal radii, and collisions between them are not allowed. Each robot has its own starting point and end point. As for static obstacles, each one has a different size and shape. Similarly, each dynamic obstacle is also circular with equal radii and has its own starting point and end point. Each dynamic obstacle moves at a constant speed without deviating during the motion. In this study, three different scenarios are created, with the numbers of robots, static obstacles, and dynamic obstacles shown in Table 7. Except for Scenario 1, which references literature [35], the other scenarios are arranged independently in this study. The layouts of these three scenarios are shown in Figures 4, 5, and 6. In these scenarios, the black paths represent the planned paths of robots, the blue paths represent the paths of dynamic obstacles, and ‘×’ indicates the respective end points of objects.

In the experiments of the three scenarios mentioned above, in addition to the proposed algorithm, the comparison algorithms include DE, PSO, SCA, COA, and sdSCA, with a population size of 30, and the parameter settings are consistent with Section 4.2. It is worth noting that in this experiment, the iteration of all algorithms stops when all robots reach their end points. Each algorithm runs independently 20 times.

4.4.2. Simulation experiment in Scenario 1

In the simulation experiment in Scenario 1, Figure 7 illustrates the travel paths of each algorithm. In solving the multi-robot path planning problem, the stronger the algorithm’s performance, the smoother the robot’s path and the shorter the average travel distance. As seen in Figure 7, the path corresponding to SDECOA is notably the smoothest. Table 8 records the average travel distance and standard deviation for each robot from the starting point to the destination and also calculates the total average travel distance for the six robots in Scenario 1. For ease of comparison, a histogram of the average travel distances

Table IV. Basic information of 48 CEC2020 real-world constrained optimization problems.

Problems	Name	<i>D</i>	<i>g</i>	<i>h</i>	<i>f</i> [*]
Industrial Chemical Processes					
Pro1	Heat Exchanger Network Design (case 1)	9	0	8	1.8931163E + 02
Pro2	Heat Exchanger Network Design (case 2)	11	0	9	7.0490370E + 03
Pro3	Optimal Operation of Alkylation Unit	7	14	0	-4.5291197E + 03
Pro4	Reactor Network Design (RND)	6	1	4	-3.8826044E-01
Pro5	Haverly's Pooling Problem	9	2	4	-4.0000560E + 02
Pro6	Blending-Pooling-Separation problem	38	0	32	1.8638304E + 00
Pro7	Propane, Isobutane, n-Butane Nonsharp Separation	48	0	38	2.1158628E + 00
Process Synthesis and Design Problems					
Pro8	Process synthesis problem	2	2	0	2.0000000E + 00
Pro9	Process synthesis and design problem	3	1	1	2.5576546E + 00
Pro10	Process flow sheeting problem	3	3	0	1.0765431E + 00
Pro11	Two-reactor Problem	7	4	4	9.9238464E + 01
Pro12	Process synthesis problem	7	9	0	2.9248306E + 00
Pro13	Process design Problem	5	3	0	2.6887000E + 04
Pro14	Multi-product batch plant	10	10	0	5.3638943E + 04
Mechanical Engineering Problems					
Pro15	Weight Minimization of a Speed Reducer	7	11	0	2.9944245E + 03
Pro16	Optimal Design of Industrial refrigeration System	14	15	0	3.2213001E-02
Pro17	Tension/compression spring design (case 1)	3	3	0	1.2665233E-02
Pro18	Pressure vessel design	4	4	0	5.8853328E + 03
Pro19	Welded beam design	4	5	0	1.6702177E + 00
Pro20	Three-bar truss design problem	2	3	0	2.6389584E + 02
Pro21	Multiple disk clutch brake design problem	5	7	0	2.3524246E-01
Pro22	Planetary gear train design optimization problem	9	10	1	5.2576871E-01
Pro23	Step-cone pulley problem	5	90	3	1.6069869E + 01
Pro24	Robot gripper problem	7	7	0	2.5287918E + 00
Pro25	Hydro-static thrust bearing design problem	4	7	0	1.6254428E + 03
Pro26	Four-stage gear box problem	22	86	0	3.5359232E + 01
Pro27	10-bar truss design	10	3	0	5.2445076E + 02
Pro28	Rolling element bearing	10	9	0	1.4614136E + 04
Pro29	Gas Transmission Compressor Design (GTCD)	4	1	0	2.9648954E + 06

Table IV. (Continued)

Problems	Name	D	g	h	f^*
Pro30	Tension/compression spring design (case 2)	3	90	0	2.6138841E + 00
Pro31	Gear train design Problem	4	1	1	0.0000000E + 00
Pro32	Himmelblau's Function	5	6	0	-3.0665539E + 04
Pro33	Topology Optimization	30	30	0	2.6393465E + 00
Power System Problems					
Pro34	Optimal Sizing of Single Phase Distributed Generation with reactive power support for Phase Balancing at Main Transformer/Grid	118	0	108	0.00000000E + 00
Pro35	Wind Farm Layout Problem	30	91	0	-6.2607000E + 03
Power Electronic Problems					
Pro36	SOPWM for 3-level Inverters	25	24	1	3.8029251E-02
Pro37	SOPWM for 5-level Inverters	25	24	1	2.1215000E-02
Pro38	SOPWM for 7-level Inverters	25	24	1	1.5164538E-02
Pro39	SOPWM for 9-level Inverters	30	29	1	1.6787536E-02
Pro40	SOPWM for 11-level Inverters	30	29	1	9.3118742E-03
Pro41	SOPWM for 13-level Inverters	30	29	1	1.5096451E-02
Livestock Feed Ration Optimization					
Pro42	Beef Cattle (case 1)	59	14	1	4.5508511E + 03
Pro43	Beef Cattle (case 2)	59	14	1	3.3489821E + 03
Pro44	Beef Cattle (case 3)	59	14	1	4.9976069E + 03
Pro45	Beef Cattle (case 4)	59	14	1	4.2405483E + 03
Pro46	Dairy Cattle (case 1)	64	0	6	6.6964145E + 03
Pro47	Dairy Cattle (case 2)	64	0	6	1.4748933E + 04
Pro48	Dairy Cattle (case 3)	64	0	6	3.2132917E + 03

for different algorithms is shown in Figure 8. From this, it is evident that SDECOA demonstrates the best performance. Table 9 lists the number of steps taken by each robot, with fewer steps indicating a smoother path.

The experimental results from Scenario 1 show that, compared to DE and COA, the algorithm's performance improved by 33.2% and 22.4%, respectively, while the average number of steps taken was reduced by 26.9% and 15.8%.

4.4.3. Simulation experiment in Scenario 2

Figure 9 shows the travel paths of each algorithm in Scenario 2. It is clearly observed in Figure 9 that, as the number of robots increases, the differences in path smoothness between the algorithms become more pronounced. Compared to the other algorithms, the path corresponding to SDECOA is the smoothest. Table 10 records the average travel distance and corresponding standard deviation for the 12 robots from the starting point to the destination. Figure 10 presents a histogram of the average travel distances for different algorithms. From the analysis of Figure 10, it can be seen that, despite the increased complexity of the problem, SDECOA still demonstrates strong performance. Table 11 records the number of steps taken by each robot.

The experimental results from Scenario 2 show that, compared to DE and COA, the algorithm's performance improved by 46.1% and 50.2%, respectively, while the average total number of steps taken was reduced by 40.0% and 48.3%.

Table V. Experimental results of different algorithms solving 48 CEC2020 real-world constrained optimization problems (shaded areas indicate results superior to the known optimal solution).

Problems	Metric	SDECOA	SASS	COLSHADE	sCNAgES	EnMODE	BPMaGES
Pro1	Best	1.90136494E + 02	1.89311639E + 02	1.89484062E + 02	1.89311647E + 02	1.89311726E + 02	1.89558919E + 02
	Avg.	4.97261246E + 02	1.89311639E + 02	2.17274353E + 02	1.89311647E + 02	1.89311726E + 02	2.03797339E + 02
	SD.	2.52789483E + 02	5.80155714E-14	2.59764421E + 01	1.32042156E-09	2.90077857E-14	1.40425132E + 01
Pro2	Best	7.04903695E + 03	7.04903770E + 03	7.04903707E + 03	7.04903760E + 03	7.04903882E + 03	7.04966344E + 03
	Avg.	7.04903695E + 03	7.04903770E + 03	7.04903707E + 03	7.04903760E + 03	7.04903882E + 03	1.58370588E + 04
	SD.	2.35563225E-12	9.28249143E-13	2.78474743E-12	2.57538931E-08	9.28249143E-13	1.06825558E + 04
Pro3	Best	-4.52918561E + 03	-1.42719318E + 02	-4.52911969E + 03	-4.52776573E + 03	-4.52911967E + 03	-4.52911969E + 03
	Avg.	-4.34475603E + 03	-1.42719316E + 02	-4.36667729E + 03	-4.32491082E + 03	-4.35366365E + 03	-4.75240766E + 02
	SD.	4.27626238E + 02	4.08248291E-06	3.31584160E + 02	2.69220987E + 02	8.77280074E + 02	1.35106780E + 03
Pro4	Best	-3.88084571E-01	-3.88260420E-01	-3.88260436E-01	-3.88244310E-01	-3.88260375E-01	-3.86980632E-01
	Avg.	-3.73479442E-01	-3.88260153E-01	-3.87465209E-01	-3.87155644E-01	-3.75738899E-01	-3.83121793E-01
	SD.	1.35723172E-02	5.08049538E-07	2.93352081E-03	3.70189247E-03	4.69053171E-03	3.05014349E-03
Pro5	Best	-2.25000000E + 01	-4.00005532E + 02	-4.00005600E + 02	-4.00003872E + 02	-4.00005600E + 02	-1.96391022E + 02
	Avg.	-5.69700000E + 00	-4.00002118E + 02	-3.40830432E + 02	-3.97683750E + 02	-3.37813699E + 02	-3.05838116E + 01
	SD.	6.58700710E + 00	8.21440903E-03	1.17179814E + 02	8.00812181E + 00	1.31367249E + 02	1.38292570E + 02
Pro6	Best	1.10307109E + 00	1.86386710E + 00	1.91762880E + 00	1.86390003E + 00	1.08353014E + 00	2.12172047E + 00
	Avg.	3.13774007E + 00	1.87233338E + 00	2.06339806E + 00	2.00520226E + 00	1.08701830E + 00	2.44867119E + 00
	SD.	2.63577263E + 00	1.86575140E-02	1.05771577E-01	1.32012583E-01	7.43075221E-03	1.99250965E-01
Pro7	Best	9.81574349E-01	1.56705180E + 00	1.38382662E + 00	1.75241050E + 00	9.51040119E-01	1.36257235E + 00
	Avg.	2.03920690E + 00	1.57650206E + 00	1.84002526E + 00	2.02137918E + 00	1.16170831E + 00	1.86600951E + 00
	SD.	5.44598091E-01	1.98224027E-02	1.99058990E-01	1.12402096E-01	1.94983718E-01	2.96719631E-01
Pro8	Best	1.99999000E + 00	2.00000010E + 00	2.00000002E + 00	2.00000015E + 00	2.00000118E + 00	2.00000475E + 00
	Avg.	1.99999000E + 00	2.00000010E + 00	2.00000002E + 00	2.00000015E + 00	2.00000118E + 00	2.01942277E + 00
	SD.	9.06493304E-16	0.00000000E + 00	4.53246652E-16	1.28197512E-15	1.35973996E-15	6.52581319E-02
Pro9	Best	2.55780959E + 00	2.55765484E + 00	2.55765461E + 00	2.55765478E + 00	2.55765487E + 00	2.55765815E + 00
	Avg.	2.55780959E + 00	2.55765484E + 00	2.55765461E + 00	2.55765478E + 00	2.55765487E + 00	2.55766153E + 00
	SD.	1.35973996E-15	0.00000000E + 00	9.06493304E-16	2.77135304E-09	9.06493304E-16	7.68697762E-06
Pro10	Best	1.07652094E + 00	1.07654317E + 00	1.07654310E + 00	1.07654317E + 00	1.07654383E + 00	1.07655076E + 00
	Avg.	1.07652094E + 00	1.07654317E + 00	1.10429621E + 00	1.07654317E + 00	1.15286487E + 00	1.07655076E + 00
	SD.	6.79869978E-16	2.26623326E-16	6.49016291E-02	1.94632366E-15	8.78772212E-02	2.26623326E-16
	Best	9.92395162E + 01	9.92384667E + 01	1.07781781E + 02	9.92384845E + 01	9.92385128E + 01	1.00285320E + 02

Table V. (Continued)

Problems	Metric	SDECOA	SASS	COLSHADE	sCNAgES	EnMODE	BPMAgES
Pro11	Avg.	9.92441965E+01	1.01516778E+02	1.47815320E+02	9.92388702E+01	1.05096920E+02	1.12439915E+02
	SD.	3.22686049E-03	3.72876276E+00	2.12144612E+01	4.39439234E-04	3.72869337E+00	8.99546453E+00
	Best	2.92482984E+00	2.92483078E+00	2.92483061E+00	2.92483069E+00	2.92483153E+00	2.92483930E+00
Pro12	Avg.	3.09538650E+00	2.92483078E+00	2.92483061E+00	2.93621432E+00	2.92483153E+00	3.98163035E+00
	SD.	4.27634626E-01	0.00000000E+00	9.06493304E-16	1.12765320E-02	1.35973996E-15	7.67243223E-01
	Best	2.68680210E+04	2.68874239E+04	2.68874223E+04	2.68874245E+04	2.68874303E+04	2.68874756E+04
Pro13	Avg.	2.68680210E+04	2.68874239E+04	2.68874223E+04	2.68874245E+04	2.68874303E+04	2.68874756E+04
	SD.	1.48519863E-11	7.42599314E-12	7.42599314E-12	7.42599314E-13	7.42599314E-12	3.71299657E-12
	Best	5.36349646E+04	5.85054547E+04	5.85054503E+04	5.36511215E+04	5.85054523E+04	5.85562988E+04
Pro14	Avg.	5.81089147E+04	5.85054620E+04	5.85054503E+04	5.66204061E+04	5.85054523E+04	7.59334196E+04
	SD.	1.34654051E+03	1.64760432E-02	0.00000000E+00	3.02847983E+03	1.48519863E-11	1.32694478E+04
	Best	2.99423425E+03	2.99442454E+03	2.99442452E+03	2.99442449E+03	2.99442605E+03	2.99443489E+03
Pro15	Avg.	2.99423425E+03	2.99442454E+03	2.99442452E+03	2.99442449E+03	2.99442605E+03	3.00645526E+03
	SD.	1.31274253E-13	9.28249143E-13	9.28249143E-13	2.50627269E-12	0.00000000E+00	1.54021266E+01
	Best	3.22129956E-02	3.22130012E-02	3.22130012E-02	3.35503836E-02	3.22130314E-02	5.03410999E-01
Pro16	Avg.	3.22144449E-02	3.22130012E-02	3.22130012E-02	3.64187643E-02	3.22130314E-02	3.86462552E+03
	SD.	8.76687054E-07	7.08197893E-18	1.41639579E-17	1.76241034E-03	7.08197893E-18	1.33483803E+04
	Best	1.26652310E-02	1.26652336E-02	1.26652330E-02	1.26652339E-02	1.26652367E-02	1.26653573E-02
Pro17	Avg.	1.26652310E-02	1.26652336E-02	1.26652578E-02	1.26676230E-02	1.27104463E-02	1.28854549E-02
	SD.	3.99049247E-18	3.54098947E-18	1.08445455E-07	4.63566637E-06	2.01379742E-05	3.24045831E-04
	Best	5.74301891E+03	6.05971451E+03	6.05971435E+03	6.05971480E+03	6.05971433E+03	6.05978255E+03
Pro18	Avg.	5.74301891E+03	6.05971451E+03	6.06217930E+03	6.08860116E+03	6.05971433E+03	6.48270988E+03
	SD.	2.78474743E-12	0.00000000E+00	8.53142879E+00	6.63653093E+01	9.28249143E-13	4.67137882E+02
	Best	1.67021511E+00	1.67021785E+00	1.67021771E+00	1.67021783E+00	1.67021878E+00	1.67022763E+00
Pro19	Avg.	1.67021511E+00	1.67021785E+00	1.67021771E+00	1.67022849E+00	1.67021878E+00	1.67316894E+00
	SD.	2.07703709E-16	2.26623326E-16	4.53246652E-16	5.33094634E-05	2.26623326E-16	1.00550187E-02
	Best	2.63852346E+02	2.63895853E+02	2.63895841E+02	2.63895854E+02	2.63895854E+02	2.63896890E+02
Pro20	Avg.	2.63852346E+02	2.63895853E+02	2.63895841E+02	2.63895854E+02	2.63895854E+02	2.63896890E+02
	SD.	0.00000000E+00	5.80155714E-14	0.00000000E+00	2.17058886E-12	1.16031143E-13	1.16031143E-13

Table V. (Continued)

Problems	Metric	SDECOA	SASS	COLSHADE	sCNAgES	EnMODE	BPMaGES
Pro21	Best	2.35242458E-01	2.35242480E-01	2.35242461E-01	2.35242468E-01	2.35242614E-01	2.35242568E-01
	Avg.	2.35242458E-01	2.35242480E-01	2.35242461E-01	2.35242468E-01	2.35242614E-01	2.35242568E-01
	SD.	5.66558315E-17	0.00000000E+00	5.66558315E-17	2.83279157E-17	5.66558315E-17	5.66558315E-17
Pro22	Best	5.25768707E-01	5.25769005E-01	5.25768715E-01	5.25967410E-01	5.25769222E-01	5.27588075E-01
	Avg.	5.33188822E-01	1.11140461E+00	5.41026211E-01	5.30809304E-01	5.26911917E-01	8.31511745E-01
	SD.	5.27604610E-03	8.80794458E-01	4.34505158E-02	4.34837419E-03	1.44018181E-03	4.74691245E-01
Pro23	Best	1.60439153E+01	1.60698706E+01	1.60698691E+01	1.60699252E+01	1.60698700E+01	1.60700105E+01
	Avg.	1.60439153E+01	1.60698706E+01	1.60698691E+01	1.62086769E+01	1.60698700E+01	1.60868794E+01
	SD.	5.84670613E-15	7.25194643E-15	1.08779196E-14	2.05466006E-01	1.08779196E-14	2.93226549E-02
Pro24	Best	2.54378561E+00	2.54378579E+00	2.54378561E+00	2.58013916E+00	2.54378761E+00	2.79518644E+00
	Avg.	2.54393667E+00	2.54378579E+00	2.54378561E+00	2.82978402E+00	2.54378761E+00	3.82988988E+00
	SD.	7.40084735E-04	1.35973996E-15	4.53246652E-16	2.20002973E-01	9.06493304E-16	1.19218086E+00
Pro25	Best	1.24535162E+02	1.61611987E+03	1.61611981E+03	2.28447644E+03	1.61612138E+03	1.69106118E+03
	Avg.	4.25061575E+02	1.61612031E+03	1.63903736E+03	3.02213559E+03	1.61612138E+03	2.98208911E+03
	SD.	6.07854479E+02	1.14749001E-03	1.02805302E+02	3.95554565E+02	2.32062286E-13	1.16144570E+03
Pro26	Best	3.62784048E+01	3.62504032E+01	3.53592322E+01	3.62485386E+01	3.53592664E+01	4.20781016E+01
	Avg.	3.62802229E+01	3.86662117E+01	3.66109756E+01	5.37101361E+01	3.57284192E+01	6.82199553E+01
	SD.	8.81651318E-04	2.46204679E+00	1.39591172E+00	1.78764043E+01	5.99105585E-01	1.78825339E+01
Pro27	Best	5.22383263E+02	5.24457638E+02	5.24450764E+02	5.24508224E+02	5.24451210E+02	5.24471114E+02
	Avg.	5.22383263E+02	5.24469909E+02	5.24450764E+02	5.24740013E+02	5.24451210E+02	5.28368863E+02
	SD.	3.00890125E-07	6.99795427E-03	2.32062286E-13	1.91943684E-01	0.00000000E+00	2.92877989E+00
Pro28	Best	1.46078146E+04	1.46141364E+04	1.69582022E+04	1.46141363E+04	1.69582084E+04	1.46241749E+04
	Avg.	1.46078146E+04	1.46141364E+04	1.69582022E+04	1.46141363E+04	1.69582084E+04	1.50840552E+04
	SD.	4.06738396E-12	3.71299657E-12	0.00000000E+00	1.35257081E-11	7.42599314E-12	9.69919837E+02
Pro29	Best	1.67775928E+06	2.96489564E+06	2.96489542E+06	2.96489558E+06	2.96489619E+06	2.96490195E+06
	Avg.	1.67807821E+06	2.96489564E+06	2.96489542E+06	2.96491252E+06	2.96489619E+06	2.96507075E+06
	SD.	1.59468037E+03	1.42579068E-09	4.75263561E-10	3.48061660E+01	1.42579068E-09	5.80107481E+02
Pro30	Best	2.65855791E+00	2.65855930E+00	2.65855921E+00	2.85234342E+00	2.65856093E+00	2.61390821E+00
	Avg.	2.65855791E+00	2.65855930E+00	2.66183397E+00	4.23690707E+00	2.81494085E+00	2.62819070E+00
	SD.	1.35973996E-15	4.53246652E-16	1.13342498E-02	1.05662763E+00	3.65700052E-01	3.30602014E-02

Table V. (Continued)

Problems	Metric	SDECOA	SASS	COLSHADE	sCNAgES	EnMODE	BPMAgES
Pro31	Best	0.0000000E + 00	2.88828925E-24	1.15940219E-23	0.0000000E + 00	0.0000000E + 00	0.0000000E + 00
	Avg.	6.07355164E-16	3.60715445E-18	1.88066664E-16	0.0000000E + 00	0.0000000E + 00	0.0000000E + 00
	SD.	9.86793323E-16	1.24318244E-17	3.89233887E-16	0.00000000E + 00	0.00000000E + 00	0.00000000E + 00
Pro32	Best	-3.06675816E + 04	-3.06655384E + 04	-3.06655388E + 04	-3.06655365E + 04	-3.06655389E + 04	-3.06653717E + 04
	Avg.	-3.06675816E + 04	-3.06655384E + 04	-3.06655388E + 04	-3.06655365E + 04	-3.06655389E + 04	-3.06652139E + 04
	SD.	7.38876989E-12	1.11389897E-11	1.11389897E-11	3.14181826E-11	7.42599314E-12	6.84538805E-01
Pro33	Best	2.63934650E + 00	2.63934651E + 00	2.63934651E + 00	2.63934659E + 00	2.63934847E + 00	2.63934979E + 00
	Avg.	2.63934650E + 00	2.63934651E + 00	2.63934651E + 00	2.63934659E + 00	2.63934847E + 00	2.71534725E + 00
	SD.	9.06493304E-16	0.00000000E + 00	0.00000000E + 00	1.10279670E-15	9.06493304E-16	4.95061367E-02
Pro34	Best	2.26165480E-01	9.88911680E-10	1.24281937E + 00	3.26365242E-01	5.85369556E-06	4.66777204E-01
	Avg.	6.68733178E-01	1.18911605E-03	4.95481969E + 00	1.22930749E + 00	2.93235236E + 00	9.19360334E-01
	SD.	2.71788762E-01	3.37986831E-03	2.05741650E + 00	5.73900722E-01	3.50850049E + 00	3.20170970E-01
Pro35	Best	-5.91780159E + 03	-6.21107481E + 03	-6.19728069E + 03	-6.22250950E + 03	-6.18684896E + 03	-5.94718427E + 03
	Avg.	-5.48530985E + 03	-6.09804308E + 03	-6.03241907E + 03	-6.08570456E + 03	-6.08384270E + 03	-5.61595587E + 03
	SD.	1.40015782E + 02	8.72131893E + 01	1.08443435E + 02	8.01671935E + 01	5.34931484E + 01	3.41836051E + 02
Pro36	Best	2.38528540E-01	3.14100567E-02	3.43680404E-02	6.22341065E-02	7.43337014E-02	7.36185387E-02
	Avg.	4.88500237E-01	5.26138823E-02	4.27949467E-02	8.56817526E-02	1.43239307E-01	8.52844563E-01
	SD.	1.94719298E-01	1.02129100E-02	5.63196460E-03	1.68405113E-02	5.43043874E-02	4.56265888E-01
Pro37	Best	1.06153620E-01	3.73003372E-02	2.02403353E-02	2.70744540E-02	5.15082868E-02	1.89802144E-01
	Avg.	2.37090658E-01	5.55104749E-02	2.60819335E-02	4.87614434E-02	6.35807994E-02	4.46911854E-01
	SD.	5.85386432E-02	1.14318075E-02	5.79570143E-03	1.55700510E-02	5.17854199E-03	2.91587900E-01
Pro38	Best	1.03678745E-01	2.10371838E-02	1.27830682E-02	2.18177361E-02	3.81537441E-02	4.58417421E-02
	Avg.	1.03682993E-01	4.90818129E-02	1.82115863E-02	3.32366546E-02	6.43661599E-02	4.77230818E-01
	SD.	2.40870981E-06	2.96663478E-02	3.26107960E-03	6.12417268E-03	1.68774575E-02	4.31820157E-01
Pro39	Best	3.01636422E-01	2.16197578E-02	1.68273483E-02	2.50975344E-02	4.03635428E-02	2.64446615E-01
	Avg.	7.33351813E-01	5.99707655E-02	2.18758966E-02	1.33218921E-01	7.00629241E-02	5.58620218E-01
	SD.	2.41776910E-01	2.26586180E-02	4.07834543E-03	1.96907336E-01	1.00227369E-01	3.43224331E-01
Pro40	Best	1.06883388E-01	2.29202003E-02	2.17177091E-02	4.39293017E-02	3.73134549E-02	1.15740197E-01
	Avg.	2.98612456E-01	3.77571453E-02	3.25815543E-02	1.80061930E-01	9.41501471E-02	3.58325090E-01
	SD.	1.00324480E-01	9.86770922E-03	4.15778833E-03	9.00013900E-02	4.80954246E-02	1.95955310E-01

Table V. (Continued)

Problems	Metric	SDECOA	SASS	COLSHADE	sCNAgES	EnMODE	BPMaGES
Pro41	Best	1.52823432E-01	1.51151681E-02	2.04502232E-02	1.79851064E-02	1.36541786E-01	8.19117274E-02
	Avg.	3.38185299E-01	2.26010046E-02	6.50911405E-02	8.38493866E-02	3.27215685E-01	2.09060523E-01
	SD.	6.19263210E-02	7.12209847E-03	4.91924745E-02	2.69146197E-02	7.07874248E-02	7.94531560E-02
Pro42	Best	4.50901256E + 03	4.55091819E + 03	4.55089302E + 03	4.55085207E + 03	4.45822736E + 03	3.65750914E + 03
	Avg.	4.50922981E + 03	4.55098126E + 03	4.55094514E + 03	4.55100407E + 03	4.50300277E + 03	5.80365461E + 03
	SD.	1.33101751E-01	7.16906300E-02	6.92445105E-02	1.04759010E-01	1.81981927E + 01	2.26501373E + 03
Pro43	Best	3.58129428E + 03	3.75088403E + 03	3.35225321E + 03	3.58929248E + 03	3.34971895E + 03	4.85718147E + 03
	Avg.	4.26834374E + 03	4.19099441E + 03	3.37212476E + 03	3.63393847E + 03	3.36815435E + 03	5.92391310E + 03
	SD.	4.54325201E + 02	2.81548423E + 02	1.32470824E + 01	5.60554017E + 01	1.51924733E + 01	1.14774009E + 03
Pro44	Best	5.18652247E + 03	5.00123129E + 03	5.02952031E + 03	5.15773450E + 03	4.34524830E + 03	4.43794820E + 03
	Avg.	5.18678939E + 03	5.25637355E + 03	5.10949966E + 03	5.46661366E + 03	4.67610664E + 03	5.67576975E + 03
	SD.	1.63985769E-01	1.54961347E + 02	5.76759871E + 01	1.15641334E + 02	4.31813269E + 02	9.80924315E + 02
Pro45	Best	3.59624113E + 03	4.24054431E + 03	4.24166502E + 03	4.24608539E + 03	3.19205843E + 03	2.46363995E + 03
	Avg.	3.59640071E + 03	4.24153092E + 03	4.24593645E + 03	4.27310050E + 03	3.33474725E + 03	5.52347602E + 03
	SD.	1.09825017E-01	2.97891024E + 00	3.47940766E + 00	8.56411494E + 00	3.00734113E + 01	3.35382785E + 03
Pro46	Best	3.48444483E + 03	6.69707089E + 03	6.69800955E + 03	6.72712930E + 03	1.69427926E + 03	3.53979682E + 03
	Avg.	4.96537944E + 03	6.70061243E + 03	6.73250538E + 03	6.72737549E + 03	4.93755208E + 03	6.09141713E + 03
	SD.	1.00978715E + 03	2.52978893E + 00	5.57923062E + 01	2.83448059E-01	1.68757424E + 03	1.89471056E + 03
Pro47	Best	1.03918716E + 04	1.47465804E + 04	1.40125061E + 04	1.47534654E + 04	9.04906878E + 03	1.08239247E + 04
	Avg.	1.36187779E + 04	1.47518351E + 04	1.46466559E + 04	1.47647685E + 04	1.14192669E + 04	1.35606103E + 04
	SD.	1.74646343E + 03	3.92778171E + 00	2.08954188E + 02	3.30313032E + 00	1.21731025E + 03	1.87793072E + 03
Pro48	Best	2.57867513E + 03	3.21329173E + 03	3.29508063E + 03	3.24899607E + 03	1.96266818E + 03	3.29133159E + 03
	Avg.	5.19002448E + 03	3.21330864E + 03	3.62823993E + 03	3.31230400E + 03	2.46861402E + 03	7.46744502E + 03
	SD.	2.14966966E + 03	3.99869908E-02	2.99257683E + 02	4.51513658E + 01	3.74142744E + 02	2.35293584E + 03
Friedman mean rank		2.6771	3.2917	2.8542	3.8646	3.4479	4.8646
Rank		1	3	2	5	4	6

Table VI. Percentage deviation values (PDmean) obtained through solutions using different algorithms.

Problems	SDECOA	SASS	COLSHADE	sCNAgES	EnMODE	BPMaGES
Pro1	162.668	0.000	14.771	0.000	0.000	7.652
Pro2	0.000	0.000	0.000	0.000	0.000	124.670
Pro3	4.071	96.849	3.587	4.509	3.874	110.493
Pro4	3.807	0.000	0.205	0.285	3.225	1.324
Pro5	98.576	0.001	14.794	0.580	15.548	92.354
Pro6	68.349	0.456	10.707	7.585	-41.678	31.378
Pro7	-3.623	-25.491	-13.037	-4.465	-45.095	-11.809
Pro8	-0.001	0.000	0.000	0.000	0.000	0.971
Pro9	0.006	0.000	0.000	0.000	0.000	0.000
Pro10	-0.002	0.000	2.578	0.000	7.090	0.001
Pro11	0.006	2.296	48.950	0.000	5.903	13.303
Pro12	5.831	0.000	0.000	0.389	0.000	36.132
Pro13	-0.071	0.002	0.002	0.002	0.002	0.002
Pro14	8.333	9.073	9.073	5.558	9.073	41.564
Pro15	-0.006	0.000	0.000	0.000	0.000	0.402
Pro16	0.004	0.000	0.000	13.056	0.000	0.000
Pro17	0.000	0.000	0.000	0.019	0.357	1.739
Pro18	-2.418	2.963	3.005	3.454	2.963	10.150
Pro19	0.000	0.000	0.000	0.001	0.000	0.177
Pro20	-0.016	0.000	0.000	0.000	0.000	0.000
Pro21	0.000	0.000	0.000	0.000	0.000	0.000
Pro22	1.411	111.387	2.902	0.959	0.217	58.152
Pro23	-0.162	0.000	0.000	0.864	0.000	0.106
Pro24	0.599	0.593	0.593	11.903	0.593	51.451
Pro25	-73.849	-0.574	0.836	85.927	-0.573	83.463
Pro26	2.605	9.353	3.540	51.898	1.044	92.934
Pro27	-0.394	0.004	0.000	0.055	0.000	0.747
Pro28	-0.043	0.000	16.040	0.000	16.040	3.216
Pro29	-43.402	0.000	0.000	0.001	0.000	0.006
Pro30	1.709	1.709	1.834	62.092	7.692	0.547
Pro31	0.000	0.000	0.000	0.000	0.000	0.000
Pro32	-0.007	0.000	0.000	0.000	0.000	0.001
Pro33	0.000	0.000	0.000	0.000	0.000	2.880
Pro34	0.000	0.000	0.000	0.000	0.000	0.000
Pro35	12.385	2.598	3.646	2.795	2.825	10.298
Pro36	1184.538	38.351	12.532	125.305	276.656	2142.602
Pro37	1017.561	161.657	22.941	129.844	199.697	2006.584
Pro38	1907.595	850.362	252.628	543.555	1146.310	9140.532
Pro39	4268.430	257.234	30.310	693.559	317.351	3227.589
Pro40	3106.792	305.473	249.893	1833.681	911.076	3748.045
Pro41	2140.164	49.711	331.168	455.424	2067.501	1284.832
Pro42	-0.915	0.003	0.002	0.003	-1.051	27.529
Pro43	27.452	25.142	0.691	8.509	0.572	76.887
Pro44	3.785	5.178	2.239	9.385	-6.433	13.570

Table VI. (Continued)

Problems	SDECOA	SASS	COLSHADE	sCNAgES	EnMODE	BPMaGES
Pro45	-15.190	0.023	0.127	0.768	-21.360	30.254
Pro46	-25.850	0.063	0.539	0.462	-26.266	-9.035
Pro47	-7.663	0.020	-0.693	0.107	-22.576	-8.057
Pro48	61.517	0.001	12.913	3.081	-23.175	132.392

Table VII. Number of robots and obstacles in different scenarios.

Scenario	Robots	Static obstacles	Dynamic obstacles
Scenario 1	6	7	3
Scenario 2	12	9	5
Scenario 3	20	13	7

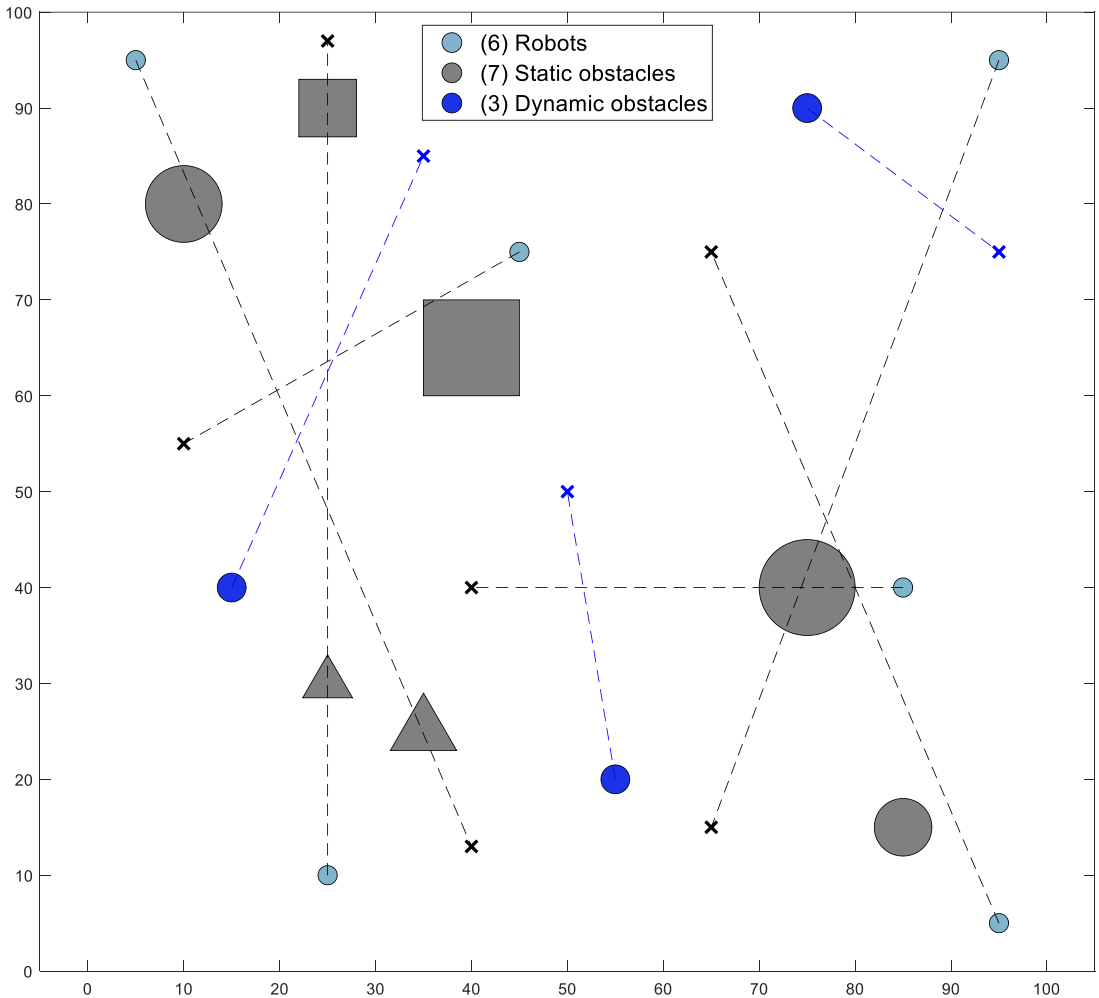


Figure 4. Scenario 1.

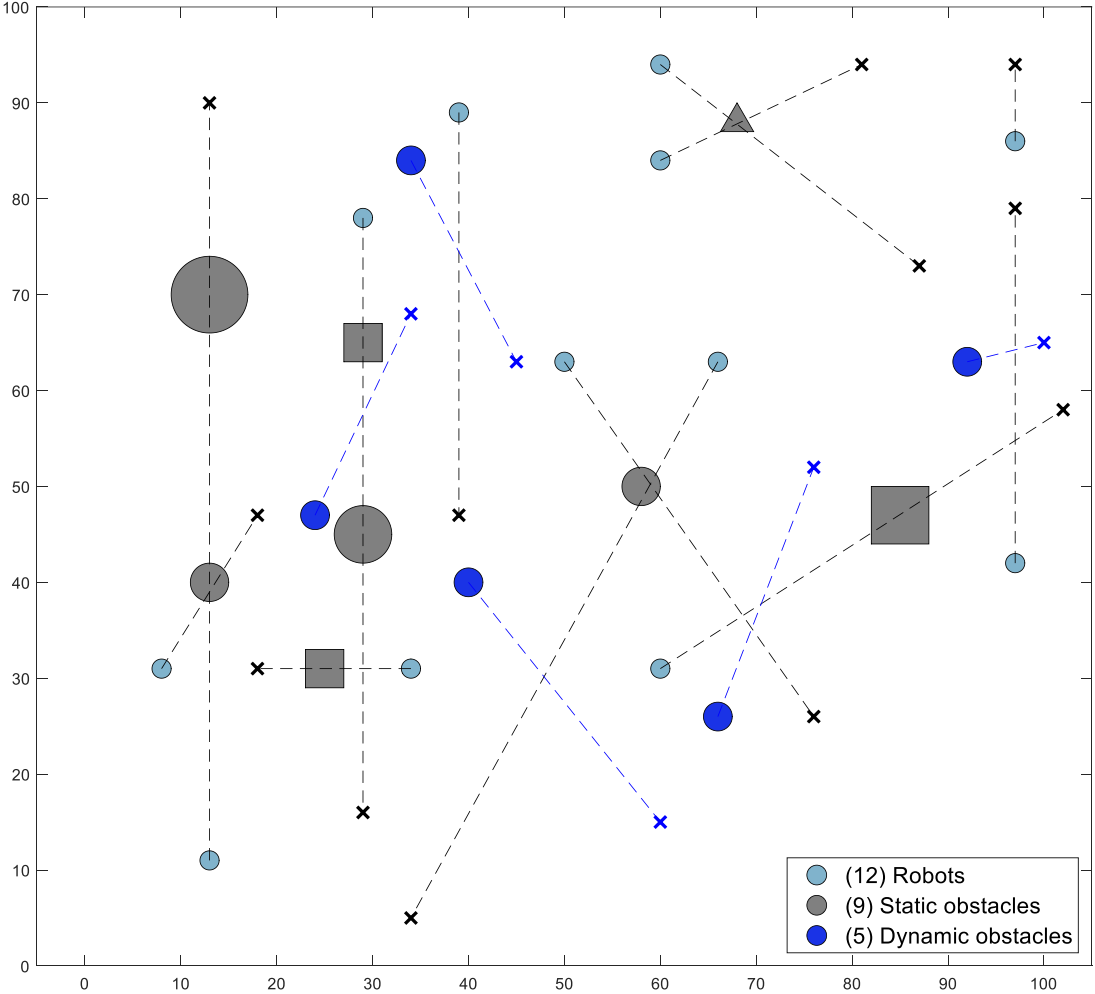


Figure 5. Scenario 2.

4.4.4. Simulation experiment in Scenario 3

In Scenario 3, the number of robots was increased to 20, and the quantities of both dynamic and static obstacles were significantly raised, making the path planning problem more complex. It should be noted that in this experiment, COA led to the phenomenon of robots getting lost, demonstrating subpar performance. Therefore, the experimental results involving COA are excluded from consideration in this section. In this experiment, Figure 11 illustrates the trajectories of each algorithm in Scenario 3. From Figure 11, it is clearly observable that, even as the problem scale increased, the proposed algorithm still managed to find effective paths compared to the other algorithms. Table 12 records the average travel distance and corresponding standard deviation for 20 robots from the starting point to the destination, while Figure 12 shows the histogram of the average travel distances for different algorithms. The histogram clearly indicates that the average travel distance of SDECOA is significantly lower than that of the other algorithms. Table 13 documents the number of steps taken by each robot.

From the experimental results in Scenario 3, it can be concluded that, compared to DE, the proposed algorithm improved performance by 45.3%, with the total average movement steps reduced by 37.0%.

Through these experimental data, it can be seen that regardless of the number of robots, the proposed algorithm always achieves the best performance. Moreover, as the number of robots increases, the values

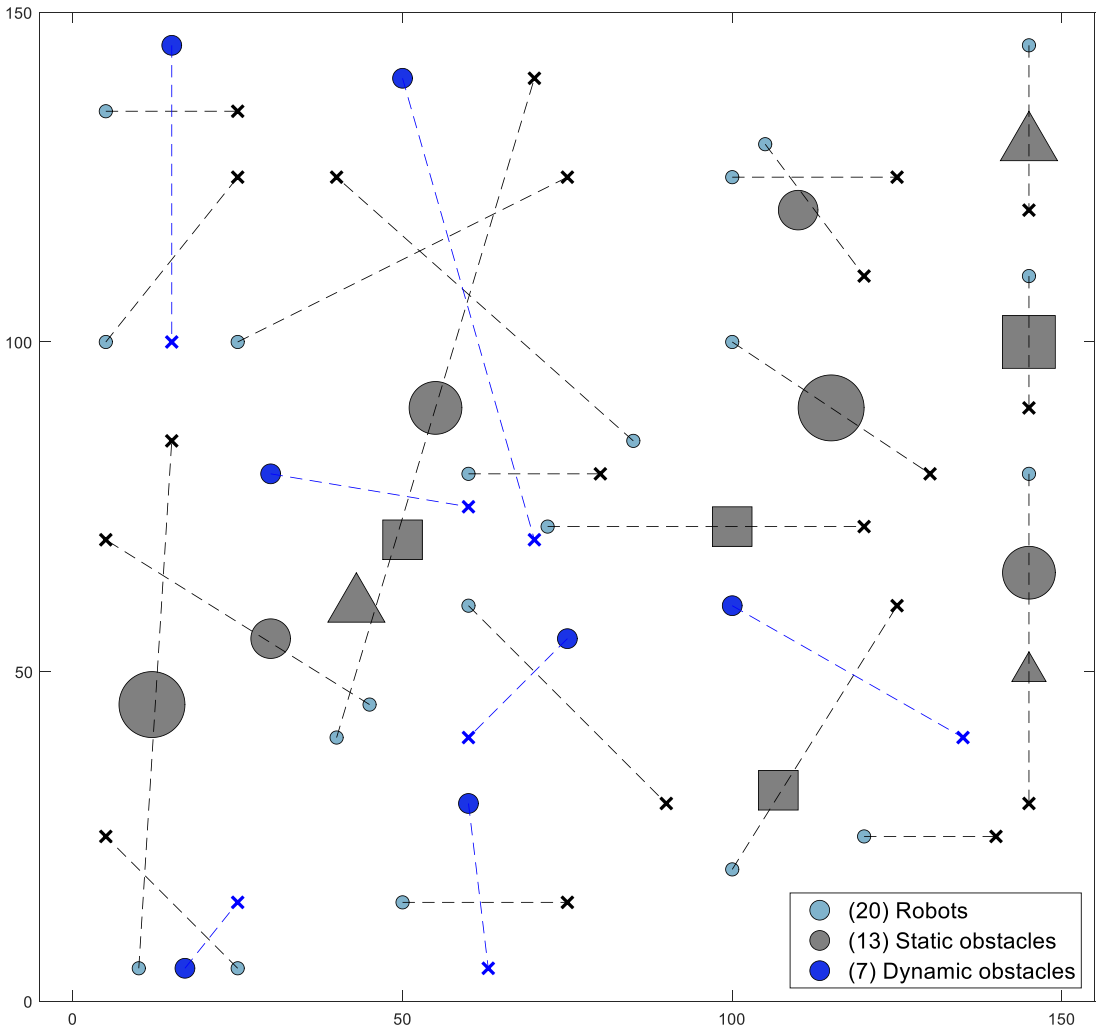


Figure 6. Scenario 3.

of other algorithms become very large, while SDECOA can still manage reasonably. The path diagrams in the three scenarios clearly show that the proposed algorithm's results in a shorter travel distance and smoother paths for the robot, particularly demonstrating significant advantages when the robot operates in more complex environments. Therefore, it can be proven that our algorithm outperforms these algorithms in terms of performance and competitiveness.

5. Conclusion and future work

Current research on robot path planning mainly considers environments with static obstacles or involves a very small number of robots. However, in this study, we consider a scenario with 20 robots and 7 dynamic obstacles, significantly increasing the complexity of the problem. To address the issue of multi-robot online path planning in environments with both static and dynamic obstacles, this paper proposed a SDECOA. The proposed algorithm extends the original COA by incorporating two DE update strategies. These strategies are adaptively selected based on a learning mechanism, which eliminates the

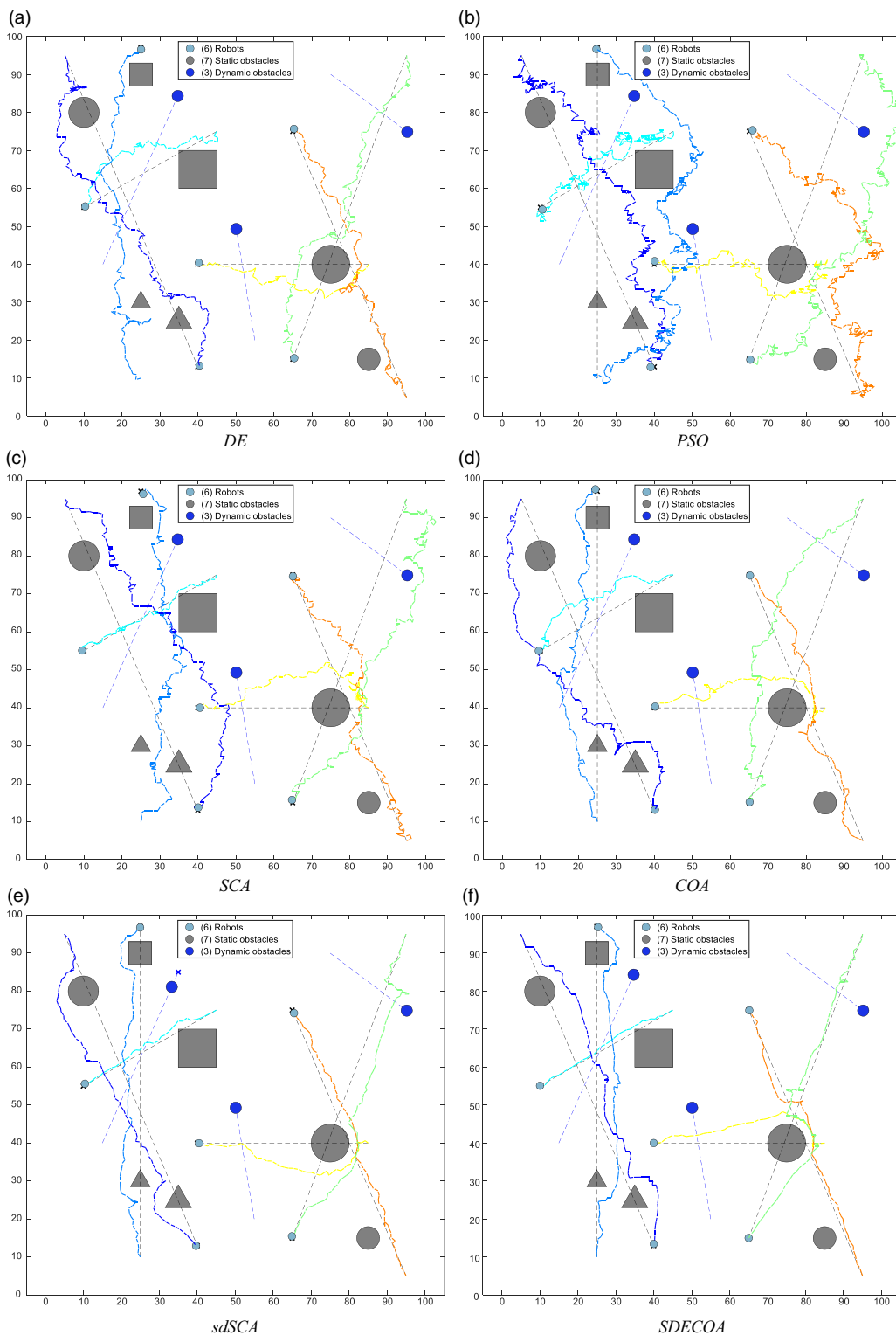


Figure 7. Paths of different algorithms in Scenario 1.

Table VIII. Experimental results of the travel distance for each robot in Scenario 1.

Robot	Metric	DE	PSO	SCA	COA	sdSCA	SDECOA
R1	Avg.	155.298	312.732	158.077	145.165	109.556	100.902
	SD.	7.996	22.151	3.643	10.923	3.996	0.765
R2	Avg.	142.451	371.126	177.660	116.102	105.660	98.892
	SD.	6.006	27.587	7.871	5.198	3.487	5.980
R3	Avg.	64.119	250.800	85.025	53.376	44.706	39.850
	SD.	8.620	48.552	15.284	2.519	1.199	0.590
R4	Avg.	143.411	339.231	162.100	120.651	104.692	100.486
	SD.	6.713	32.274	8.103	8.223	1.664	5.814
R5	Avg.	84.838	249.738	92.081	71.090	55.934	56.952
	SD.	5.273	37.285	10.466	7.797	1.720	5.786
R6	Avg.	119.637	311.692	144.330	104.418	82.244	77.066
	SD.	5.506	16.923	9.404	7.127	1.883	1.660
Total mean distance		709.755	1835.320	819.274	610.801	502.792	474.149
Rank		6	5	4	3	2	1

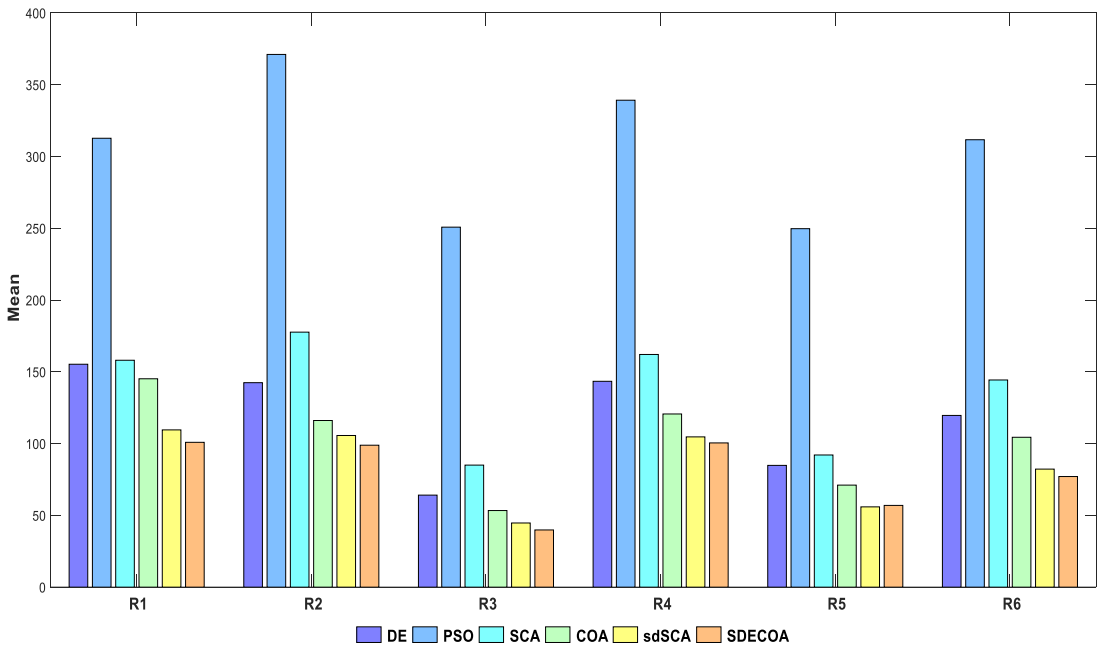


Figure 8. Average travel distance of each robot in Scenario 1.

dependency on a single update strategy in COA. As a result, the algorithm is capable of achieving excellent performance across a variety of problem domains. Furthermore, the crossover probability, which was originally a constant parameter, is transformed into a dynamic adaptive variable to further improve the accuracy of the solutions. This approach effectively mitigates the premature convergence problem caused by the imbalance between local and global search capabilities in COA, thereby improving the algorithm’s optimization capability.

Table IX. Average number of steps traveled by each robot in Scenario 1.

Robot	DE	PSO	SCA	COA	sdSCA	SDECOA
R1	129.45	250	129.65	122.05	88.1	89.9
R2	114.6	288.8	147.6	96.6	86.65	90.35
R3	53.15	195.4	66.15	43.45	37	35.05
R4	116.65	281.4	133.35	97.6	87.65	90.9
R5	67.7	200.2	75.7	56.1	46.3	48.95
R6	97.85	256.5	116.35	87.15	68.55	68.35

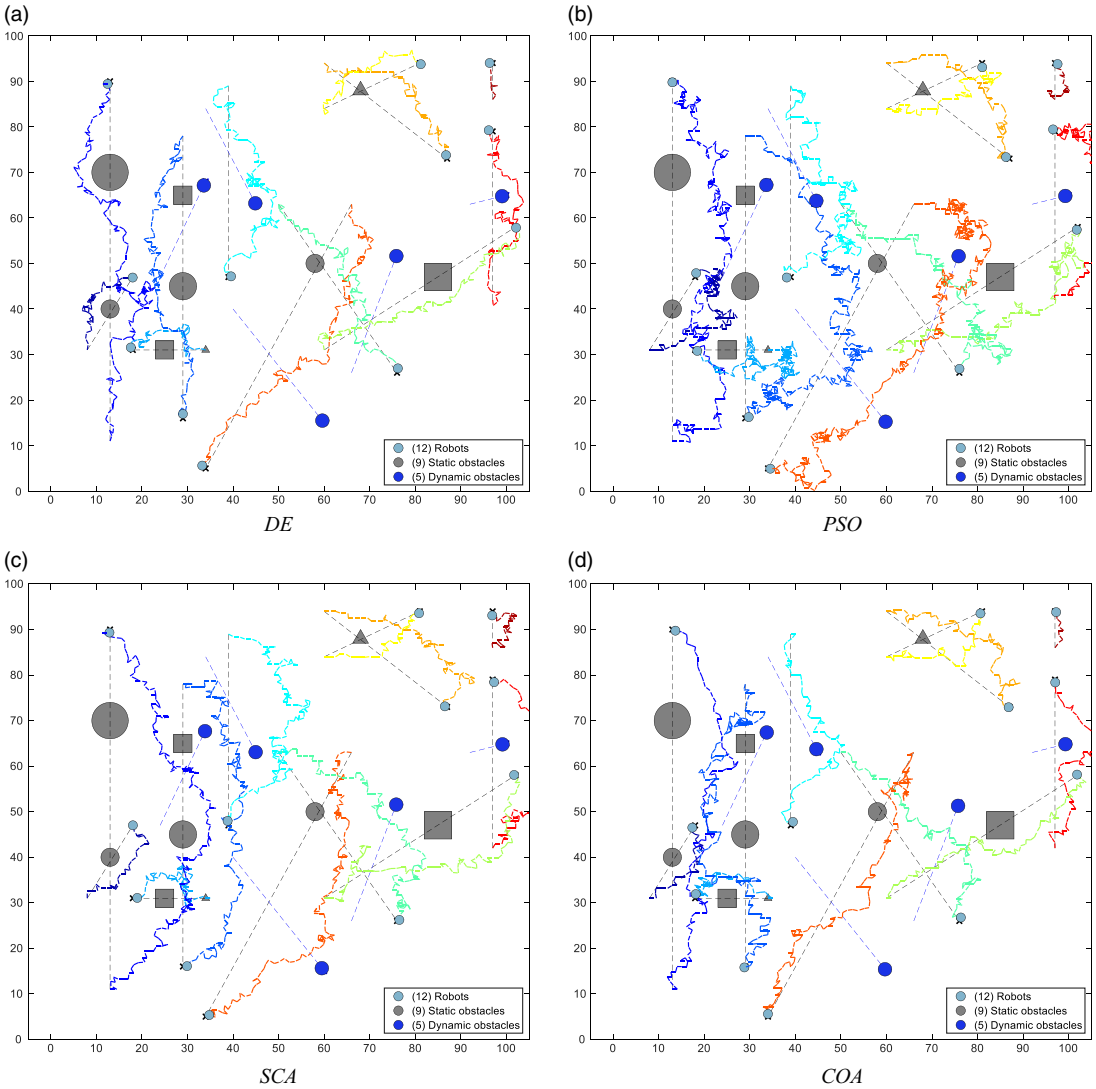


Figure 9. Paths of different algorithms in Scenario 2.

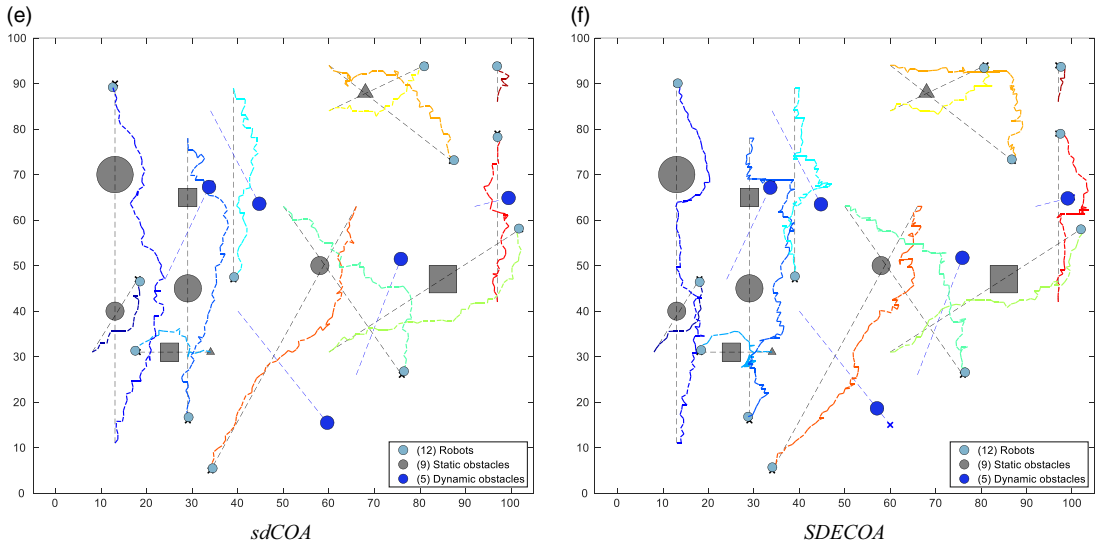


Figure 9. (Continued)

Table X. Experimental results of the travel distance for each robot in Scenario 2.

Robot	Metric	DE	PSO	SCA	COA	sdSCA	SDECOA
R1	Avg.	62.787	217.614	63.358	45.436	26.614	31.861
	SD.	17.987	119.305	12.634	2.897	2.995	4.073
R2	Avg.	164.269	431.117	213.012	184.073	121.577	97.637
	SD.	9.543	51.563	16.642	30.190	11.226	3.338
R3	Avg.	250.674	330.084	173.439	201.653	85.770	105.505
	SD.	63.917	19.190	11.632	41.555	3.581	6.344
R4	Avg.	52.440	229.792	71.882	77.764	32.838	29.979
	SD.	16.965	20.031	29.698	31.384	7.020	3.700
R5	Avg.	111.945	310.797	114.428	110.812	60.319	64.881
	SD.	16.310	25.473	7.530	10.763	3.665	5.700
R6	Avg.	122.856	188.722	106.414	99.016	62.420	65.306
	SD.	8.191	22.746	5.509	13.256	4.560	1.549
R7	Avg.	108.073	165.359	116.620	92.062	67.966	58.992
	SD.	11.505	32.149	11.352	9.850	3.072	3.554
R8	Avg.	68.966	115.199	62.237	47.225	37.932	32.271
	SD.	5.484	45.977	7.264	8.060	1.947	2.623
R9	Avg.	89.652	148.906	84.867	76.711	51.414	52.491
	SD.	13.593	43.286	8.971	9.217	1.866	2.376
R10	Avg.	155.862	425.183	176.663	355.010	88.339	97.291
	SD.	8.514	81.036	15.646	184.370	5.428	2.331
R11	Avg.	81.344	254.331	110.142	86.818	54.585	47.961
	SD.	7.190	89.799	13.120	8.702	2.527	2.820
R12	Avg.	20.528	113.579	41.494	20.356	11.243	11.153
	SD.	3.221	53.615	20.365	4.914	2.077	1.983
Total mean distance		1289.395	2930.682	1334.554	1396.936	701.017	695.327
Rank		3	6	4	5	2	1

Table XI. Average number of steps traveled by each robot in Scenario 2.

Robot	DE	PSO	SCA	COA	sdSCA	SDECOA
R1	52	176.5	53.85	39	21.35	29.4
R2	138.85	358.35	177.65	162.1	99.95	92.05
R3	207.95	263.95	138.4	160.65	70.1	98.2
R4	40.95	186.8	62.1	58.5	25.25	26.8
R5	92.7	251.5	94.3	92.3	49.75	61.8
R6	101.3	147.05	87.05	86.05	53.1	61.4
R7	87.3	134.05	90.95	75.7	55.8	53.25
R8	57.2	98.4	51.7	42.15	31	30.5
R9	74.7	131.6	70.2	62.8	42.85	46.75
R10	133.9	296.15	146.7	375.35	74.5	89.85
R11	68.1	178.4	91.9	74.9	42.8	42.9
R12	15.95	115.5	35.35	15.2	9.6	9.65

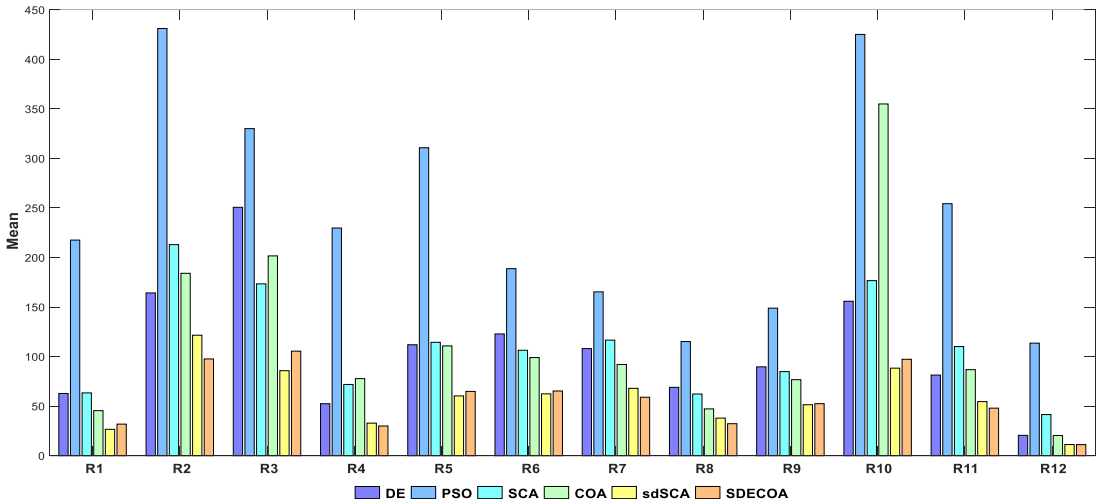


Figure 10. Average travel distance of each robot in Scenario 2.

To validate the performance of the proposed algorithm, it was applied to the CEC2022 benchmark test suite and the CEC2020 real-world constrained optimization problems. Experimental results show that, compared to DE and COA, the proposed algorithm significantly improves performance, solving optimization problems even with over 100 constraints. When compared with the winning algorithm in the CEC2020 competition, SDECOA achieved the best results, as confirmed by the Friedman test. Finally, the algorithm was applied to multi-robot path planning problems in three complex scenarios. In Scenario 1, compared to the original DE and COA, the proposed algorithm reduced the average total travel distance of all robots by 33.2% and 22.4%, respectively, and reduced the average total movement steps by 31.1% and 21.6%. In Scenario 2, the average total travel distance was reduced by 46.1% and 50.2%, respectively, while the average total movement steps were reduced by 40.0% and 48.3%. In Scenario 3, compared to the original DE, the average total travel distance was reduced by 45.3%, and the average total movement steps were reduced by 32.7%. These experimental results demonstrate the strong competitiveness of the proposed algorithm.

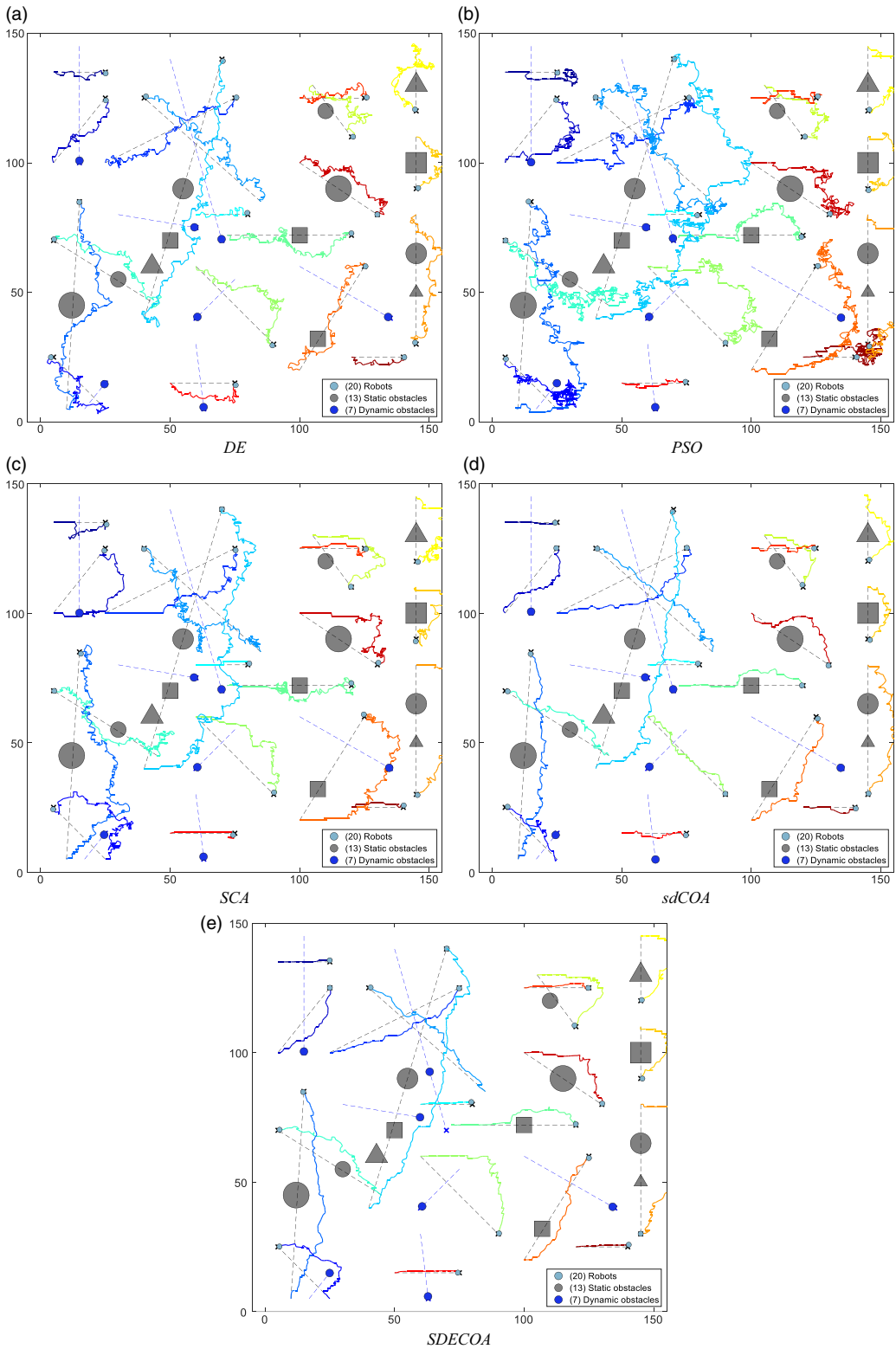


Figure 11. Paths of different algorithms in Scenario 3.

Table XII. *Experimental results of the travel distance for each robot in Scenario 3.*

Robot	Metric	DE	PSO	SCA	sdCOA	SDECOA
R1	Avg.	53.502	32.010	28.602	31.699	27.143
	SD.	2.190	7.675	4.651	7.293	3.153
R2	Avg.	92.906	195.468	111.946	54.778	44.780
	SD.	8.406	38.925	10.642	2.342	2.796
R3	Avg.	87.971	280.642	162.165	61.790	53.495
	SD.	13.500	42.123	22.905	3.785	5.882
R4	Avg.	122.528	175.538	117.169	80.620	65.400
	SD.	13.259	17.704	10.251	5.467	1.827
R5	Avg.	207.327	449.117	320.794	123.995	105.829
	SD.	3.290	40.807	62.608	2.335	3.750
R6	Avg.	151.624	557.380	223.380	89.750	89.355
	SD.	6.048	27.258	21.816	4.122	5.880
R7	Avg.	222.964	619.783	379.673	153.876	146.662
	SD.	13.297	19.180	7.887	5.124	12.084
R8	Avg.	45.038	22.431	19.930	27.471	23.262
	SD.	1.543	1.281	0.594	3.355	2.120
R9	Avg.	162.324	544.513	252.321	82.498	75.965
	SD.	22.005	34.009	43.137	6.072	15.959
R10	Avg.	124.478	123.487	94.905	70.077	59.796
	SD.	9.392	9.636	15.047	5.913	2.866
R11	Avg.	140.808	213.129	122.371	65.260	77.242
	SD.	5.820	17.069	27.072	1.813	8.708
R12	Avg.	100.883	188.058	136.553	50.084	60.251
	SD.	17.664	48.471	14.059	6.638	8.156
R13	Avg.	123.430	331.049	105.435	57.955	69.380
	SD.	10.472	134.419	9.614	4.339	3.409
R14	Avg.	87.166	288.856	122.968	50.055	57.020
	SD.	16.001	65.006	24.217	8.777	6.060
R15	Avg.	201.038	376.998	216.024	88.111	105.646
	SD.	8.560	7.395	52.682	3.847	9.627
R16	Avg.	123.763	399.841	131.791	84.567	64.274
	SD.	13.694	82.944	10.701	5.227	2.397
R17	Avg.	64.757	38.555	38.621	36.415	30.008
	SD.	12.230	7.315	3.008	1.580	2.495
R18	Avg.	66.079	33.227	25.095	31.407	27.821
	SD.	7.634	4.176	0.821	4.405	2.736
R19	Avg.	97.520	134.959	89.793	57.950	63.769
	SD.	17.077	29.111	7.381	5.984	9.539
R20	Avg.	44.769	23.512	42.718	22.527	23.461
	SD.	8.028	1.906	12.308	1.497	2.663
Total mean distance	2320.876	5028.552	2742.255	1320.883	1270.558	
Rank	3	5	4	2	1	

This paper presents a novel solution approach for the multi-robot path planning problem, holding promise for addressing more complex path planning challenges in future environments, such as multi-robot path planning in three-dimensional spaces with multiple dynamic obstacles. According to the No Free Lunch theorem, no single algorithm can solve all problems, but continuous improvement can enable

Table XIII. Average number of steps traveled by each robot in Scenario 3.

Robot	DE	PSO	SCA	sdSCA	SDECOA
R1	43.2	25.5	21.75	23.55	27
R2	77.1	155.45	87.7	43.85	41.15
R3	76.5	240.7	138.05	52.15	51
R4	102.2	149	96	64.8	61
R5	170.7	376.05	275.1	104.15	98.4
R6	127.8	450.4	185.6	74.4	83.95
R7	180.85	512.5	315.85	128.55	141
R8	36.3	17.4	15.45	22.55	22.75
R9	135.45	441.65	213.15	65.3	79.45
R10	97.45	95.85	79.4	55.4	55.35
R11	115.6	173.1	115.15	52.6	72.45
R12	79.35	161.7	109.8	40.8	55.8
R13	99.5	260.6	89.25	47.55	63.35
R14	74.9	261.05	101.45	39.05	55.3
R15	168.1	310.55	193.1	73.1	101.3
R16	102.4	333.25	107.6	69.5	59.8
R17	55.95	29.4	31.4	28.9	28.75
R18	58.55	27	20.45	25.6	26.8
R19	77.7	108.85	75.5	48.45	60.95
R20	35.7	18.5	35.8	18	21.65

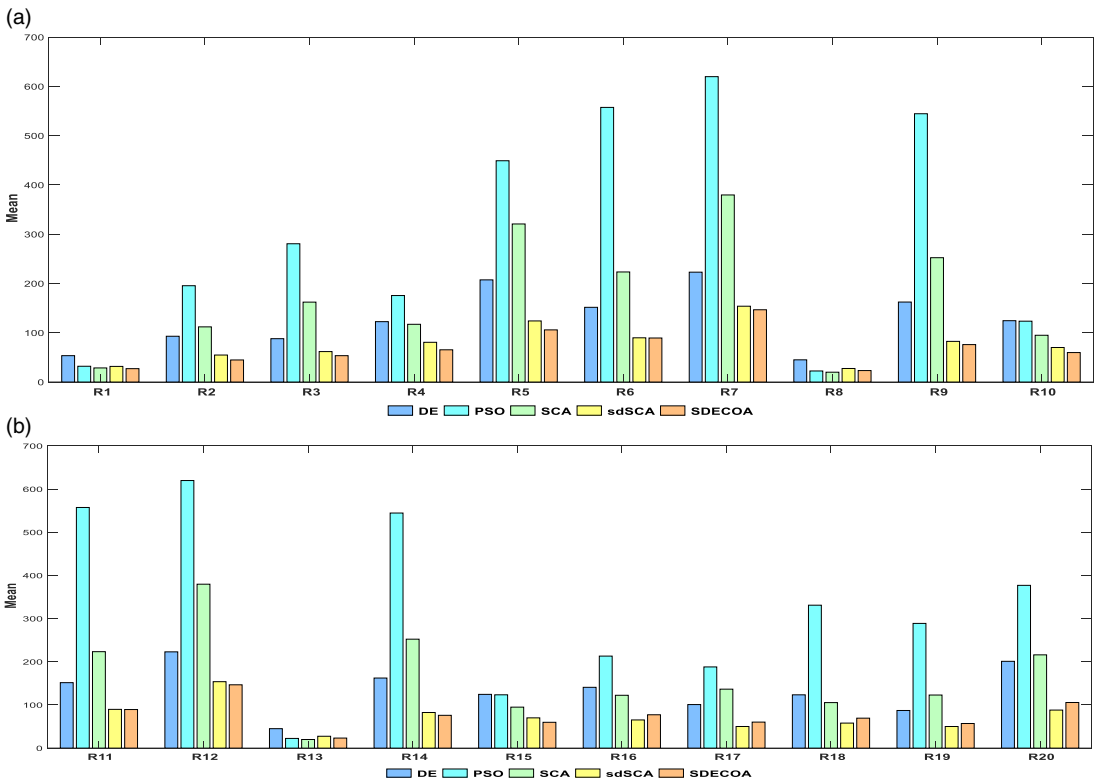


Figure 12. Average travel distance of each robot in Scenario 3.

an algorithm to solve a broader range of problems. Of course, the number of position update strategies is not always a determining factor for success. In future research on intelligent algorithms, better position update strategies that align with the COA are expected to emerge, along with more challenging real-world optimization problems for SDECOA to solve. One limitation of the proposed algorithm is that it has a relatively large number of control parameters, and the effective update strategy at the initial stage may not necessarily be suitable for the later iterations. Future research could focus on applying the algorithm to more realistic path planning problems. Moreover, this paper only applies SDECOA to single-objective constrained optimization problems; future work could extend its application to high-dimensional multi-objective optimization problems.

Author contributions. Lun Zhu: Methodology and Writing – original draft. Guo Zhou: Writing – original draft. Yongquan Zhou: Supervisor and Writing – review and editing. Qifang Luo: Supervisor and Experimental results analysis. Huajuan Huang: Algorithm. Xiuxi Wei: Software and Results analysis.

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Ethical approval. This article does not contain any studies with human participants or animals performed by any of the authors.

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