TIDAL HEATING OF GLOBULAR CLUSTERS

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ABSTRACT: The influence of tidal heating on the evolution of globular clusters (GC's) in circular orbits about the Galactic center is studied. Giant Molecular Clouds (GMC's) stretch a globular cluster in a direction transverse to its orbit through the disk. The variation in acceleration with height in the disk compresses the cluster in a longititudinal direction. Numerical and analytic calculations of heating and mass loss for GC's, represented by King models, show that disk heating dominates. We apply the results to calculate GC evolution prior to core collapse or tidal disruption using a three parameter (energy, mass, and tidal radius) sequence of King models. The changes in the parameters are calculated for tidal perturbations, relaxation and evaporation. Clusters close to the Galactic center (less than 3 kpc) undergo core collapse in a Hubble time. The effect of tidal perturbations on energy and mass loss of the cluster is strongest between 3 and 5 kpc where it can substantially effect the evolution of the cluster. Here, depending upon their initial concentration, clusters are either tidally heated and dissolved, or forced towards a gravothermal catastrophe in times that are a fraction of a Hubble time. These inner regions of the Galaxy should be fertile territory for the search for post-collapsed clusters.

1. INTRODUCTION

In this paper we report calculations of the heating and mass loss suffered by a Globular Cluster (GC) on its passage through the Galactic disk. We illustrate the qualitative effect on GC evolution with schematic calculations which incorporate two-body relaxation, mass loss across a tidal boundary and heating and mass loss by orbital passage through the Galactic disk. In the past, accurate treatments of the evolution of isolated GC's have employed the Fokker-Planck equation, in which the cluster is modeled as a sequence of instantaneous solutions to the collisionless Vlasov equation (for reviews see Spitzer 1975, 1985, Lightman and Shapiro 1978, Shapiro 1985 and Cohn 1985). Monte-Carlo and finite difference techniques have led to some understanding of the basic physics of *two-body relaxation* of an evolving, self-gravitating system. It is clear that a variety of other processes will be important in the evolutionary history of a GC. In the early stages, *stellar evolution* (SE) will significantly affect the energy and mass budget of the cluster. Once core collapse is well underway, the finite size of the stars may become important. Even when SE is no longer important and core collapse not very far advanced, we find that *tidal effects* may be important. In a recent paper (Chernoff, Kochanek and Shapiro 1986; hereafter CKS) we focused on the tidal interaction of GC's with giant molecular clouds (GMC's) and with the Galactic disk during crossings. This study was motivated in part by the suggestions of Grindlay (1984, 1985, 1986, Grindlay and Hertz 1985) that galactic bulge burst sources may

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come from disrupted GC's. We presented simplified (i.e. *not* Fokker-Planck) evolutionary calculations employing King models. In addition to the perturbation to the cluster during its disk passages, we incorporated the effect of the Galaxy's large-scale tidal field. Here we present a global Galactic survey of GC evolution based on that model. Elsewhere (Chernoff and Shapiro 1986), we will deal with the mass loss by stellar evolution which dominates the first 5×10^9 years of life of a GC. We conclude that it is essential to include the effects of tidal shocks in modeling clusters within 8 kpc of the center of the Galaxy. Our results also suggest that core collapsed clusters will be found preferentially near the center of the Galaxy.

2. GALAXY MODEL

We parameterize the effects of external physical forces in terms of the Galactocentric location of the GC. To this end, we adopt the Bahcall, Schmidt and Soneira (1982; hereafter BS&S) model of the Galaxy. The potential of that model allows nearly circular orbits for point masses with arbitrary angles of inclination with respect to the disk. We restrict ourselves to circular (or nearly circular) orbits because they are simplest to treat theoretically -- the tidal strength in the BS&S model is nearly constant, except during disk passages, so that the tidal boundary condition for the GC varies only on the relaxation timescale, not on the GC's orbital timescale as it would for an eccentric orbit. To treat the disk passages, we combined the disk surface density of the BS&S model with Bahcall's (1984) determination of the local acceleration above the disk. Together with the assumption of constant scale heights (as observed in edge-on spirals, Van der Kruit and Searle 1982), these Galactic parameters then give the acceleration perpendicular to the disk $K_z(R_g, z)$ at the galactocentric radius R_g and height z. Two components with scale heights 175 pc and 550 pc contribute to K_z. We also included a population of GMC's uniformly distributed, with an average surface density, as a function of R_g, matching the observations of Sanders, Solomon and Scoville (1984). The GMC's heat the GC during close encounters.

3. GC MODEL FOR EVOLUTION

The GC's are represented by King models, which provide a simple theoretical description agreeing remarkably well with the observational data (King 1966). Each model is specified by total mass, M, energy, E, and tidal radius r_t. (These quantities imply the concentration c of the GC and the value of W_0, the fundamental parameter of the King sequence, when we restrict our consideration to the set of models with $0 \leq W_0 \leq 8.5$.) In our study we have included two physical effects:

(1) Two-body relaxation causes the concentration of the cluster to increase so that eventually the core collapses. At the same time, mass is shed across the tidal boundary. Early treatments (the evaporation picture) have been superceded by detailed calculations which show that single-component core collapse proceeds essentially independently of the outer envelope, *once the density contrast is great enough*. Recent Fokker-Planck calculations of GC's in an external tidal field (Wiyanto, Kato and Inagaki 1986) compared the evolution an (initial) King model configuration to the King sequence, in which evolution is estimated via the evaporation picture. Until $W_0 \geq 7.4$ the central density as well as the escape rate matched the simple estimates. Extremely condensed clusters with $W_0 \geq 7.4$ are considered to have undergone total core collapse. We treat relaxation and tidal mass loss with a single-component model (whose particle mass is the average mass per particle) in the evaporation picture. Significant improvements on this treatment have been made (Apple-

gate 1986, Stodolkiewitz 1985, Chernoff, Weinberg and Shapiro 1986) but are considerably more complicated.
(2) Disk passages perturb the cluster, heating it and causing it to shed mass. The essential physical effects are the transverse and longitudinal perturbations created by passing masses, both of which have been well-studied before. Spitzer (1958) treated the disruption of Galactic clusters by interstellar H I clouds (the transverse effect) and Wielen (1985) has extended the theory to impulsive disruption of open clusters mediated by GMC's. Ostriker, Spitzer and Chevalier (1972) and Spitzer and Chevalier (1973) studied the longitudinal effect for GC's -- an extended body ''feels'' a gradient in the force; the center of mass is freely falling but the extremities of the object are accelerated at slightly different rates. Since the internal gravity binds the matter at the edge, the tidal force does work on the cluster on each passage through the disk.

4. QUALITATIVE EVOLUTION OF KING MODELS

The key equations for the evolution of the King model may be written schematically

$$K : \begin{bmatrix} E \\ M \\ r_t \end{bmatrix} \to \begin{bmatrix} E' \\ M' \\ r_t' \end{bmatrix}, \tag{1}$$

where the tidal boundary condition is

$$r_t = \left(\frac{M}{3M_g}\right)^{1/3} R_g, \tag{2}$$

where M, E, r_t are the GC mass, energy and tidal radius and R_g is the radius of the hoop-like-orbit and M_g is the mass interior to the orbit in the BS&S model. (The term dM_g/dR_g in the tidal force has been ignored.) The tidal condition links the evolution of the three variables together, viz. $\delta E = E' - E$ and $\delta M = M' - M$ completely determine the cluster evolution.

To illustrate the qualitative trend in GC concentration we have extended King's (1966) model of cluster relaxation for general δE and δM. Let T be the kinetic energy and V the potential energy. The King models are solutions to the time-independent Vlasov equation and are in virial equilibrium

$$2T + V = 0. \tag{3}$$

Define ν

$$\nu = \frac{E}{GM^2/r_t}, \tag{4}$$

a simple monotonic function of W_0 over the range $0 \leq W_0 \leq 8.5$, ranging from -0.6 (exactly) to -2.1 (approximately). Using the tidal condition and the definition above, for small changes we have

$$\frac{\delta E}{E} = \frac{\delta \nu}{\nu} - \frac{2\delta M}{M} - \frac{\delta r_t}{r_t} = \frac{\delta \nu}{\nu} - \frac{5\delta M}{3M}. \tag{5}$$

The sizes of δE and δM are dictated by the specific processes which perturb the King model. However, the qualitative tendency of evolution depends only upon the ratio of δE to δM which we can express as

$$f \equiv \frac{\delta E}{(-GM\delta M/r_t)}, \tag{6}$$

to give

$$\frac{\delta E}{E} = -\left(\frac{f}{\nu}\right)\frac{\delta M}{M}. \tag{7}$$

Using eqn. (5) and (7) we have

$$\frac{\delta \nu}{\nu} = -\left(\frac{5}{3} + \frac{f}{\nu}\right)\frac{\delta M}{M}. \tag{8}$$

The nature of GC evolution is determined by the sign of $f + (5\nu/3)$. Since the mass is always lost, i.e. $\delta M/\delta t < 0$, the sign of $(f + (5\nu/3))$ is the sign of $\delta\nu$. ν decreases as W_0 (or concentration) increases, hence if $f + (5\nu/3) > 0$ the relative concentration of the cluster decreases, if $f + (5\nu/3) < 0$ it increases. The average cluster density is fixed by the tidal condition, so that the central density decreases in the first case and increases in the second.

5. CALCULATION OF δE AND δM

5.1 TWO-BODY RELAXATION

In the evaporation picture, marginally bound stars are lost from the GC by two-body gravitational encounters with small velocity changes. The energy change due to the loss of a star with mass m is $\delta E = mGM/r_t$. Hence, $f = 1$, which implies, since $\nu \leq -0.6$, that the tendency of relaxation alone is to always force the GC towards greater concentration. The rate of relaxation is given by King (1966) who integrated the local escape rate over the King model, using the local energy threshold for escape. For GC's on orbits with zero eccentricity, King's formula becomes

$$\frac{dM}{dt} = -\frac{27}{8}\left(\frac{G}{2\pi}\right)^{1/2}\left(\frac{3M_g}{R_g^3}\right)^{1/2}\langle m\rangle \ln(N_c/2)R(W_0) \tag{9}$$

where N_c is the number of stars within a homogenous core of radius r_c and number density $\rho(0)/\langle m\rangle$. $R(W_0)$ is a pure function of W_0 which King tabulated for $2.5 \leq W_0 \leq 10$; we have assumed it constant for $W_0 \leq 2.5$. It varies only by about a factor of 2 over the entire range King gives. $\langle m\rangle$ is the average stellar mass. King's formula gives a nearly constant rate of mass loss over the GC lifetime.

It is important to point out that this prescription for evaporation does not apply to a young, isolated cluster without a halo extending to r_t. Such a cluster undergoes core collapse in a few hundred *central* relaxation times, generating a halo as it does so. Only when the halo reaches r_t can the cluster be considered, even approximately, to be a member of the King sequence with a ''starting'' value of W_0 determined by the concentration at that time. A global theory of GC formation (such as that of Fall and Rees, 1985) would determine whether clusters form at high concentrations or are ''born'' filling their tidal surface. Our treatment of evaporation allows us to make a comparison between tidal heating and collisional evaporation only for those clusters which extend to r_t.

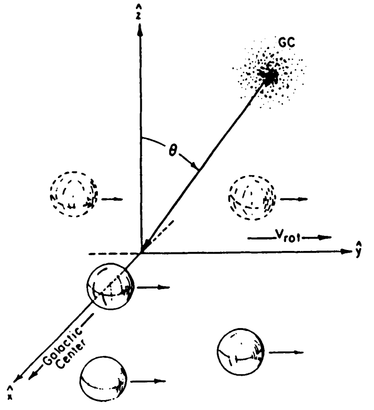

Fig. 1. GMC-globular cluster encounter geometry.

5.2 GMC HEATING OF GC's

The tidal heating during disk passage was studied in detail by CKS who found that in virtually all cases $f = 1.5 - 2.0$. When $f = 1.5$ this implies that under the influence of tidal heating alone, GC's with $W_0 > 4$ collapse and those with $W_0 < 4$ dissolve. Figure 1 illustrates the encounter geometry of the GC with the disk.

Clouds in the disk orbit the Galaxy (in the BS&S model) and the GC's trajectory pierces the disk with some fixed angle of inclination. The rate of heating by clouds was calculated as follows. Construct a GC by choosing a large number (2000) of points from the distribution function describing the King model. Next generate a random distribution of clouds with the appropriate surface density ($4.8\,M_\odot/\,pc^2$ in the solar neighborhood and $15\,M_\odot/\,pc^2$ maximum near $R_g = 5\,kpc$). Our canonical clouds have radii, $r_{cl} = 20\,pc$, and masses, $M_{cl} = 10^5\,M_\odot$ and are homogenous spheres, confined to the plane. The impulse approximation is used to calculate individual velocity perturbations of stars in the GC due to the gravitational pull of a GMC. Penetrating encounters are accurately treated. The entire cluster is scattered in the encounter. Using the cluster's new center of momentum, we find the number of unbound particles (δM) and change in cluster energy (δE), including the work done by the tidal field as material is removed to infinity. For the particles which remain bound, we find the change in kinetic energy. These results are combined to give the overall change in mass and energy and, hence, the new cluster (i.e. King) model. In the simulations, the procedure is repeated, using the current King model parameters, for each disk passage.

Let $\epsilon = 0.5(1 + \sin(\theta))$ where θ is the angle of the GC orbit with respect to the normal to the disk, at the point of entry (see Figure 1). θ is taken as positive for a retrograde orientation ($\epsilon \rightarrow 1$) and negative for a prograde orientation ($\epsilon \rightarrow 0$). Although the numerical results include the effect of interpenetrations, the finite number density of GMC's in the plane (which cause nonlinear terms in the surface density to enter the heating rate), and the effect of particle loss, we find that the rate of heating is well approximated by an average over impact parameters of the tidal heating

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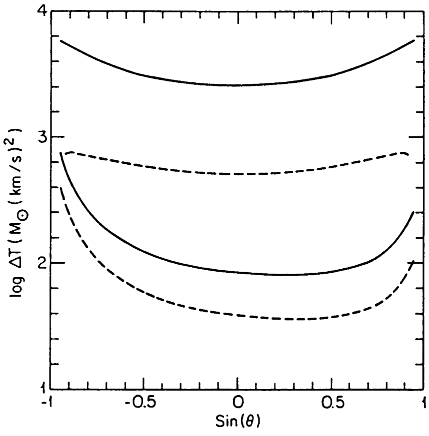

Fig. 2. **Heating by GMC's (lower two lines) and tidal shocks (upper two lines) during disk passages as a function of the angle of incidence for $M = 10^5\,M_\odot$ at $R_g = 8\,\mathrm{kpc}$ for $W_0 = 1$ (dashed) and $W_0 = 7$ (solid).**

by single particles. We find

$$\Delta T \approx (200\,M_\odot\,\mathrm{km}^2\mathrm{s}^{-2}) \left[\frac{\langle r^2\rangle}{p^{*2}_{\rm min}}\right] \left[\frac{1}{4\epsilon(1-\epsilon)^{1/2}}\right] \tag{10}$$

$$\left[\frac{M}{10^5\,M_\odot}\right] \left[\frac{\sigma_{cl}}{4.8\,M_\odot/\,\mathrm{pc}^2}\right] \left[\frac{M_{cl}}{10^5\,M_\odot}\right] \left[\frac{200\,\mathrm{km\,s}^{-1}}{v_c}\right]^2 .$$

where ΔT is the change in kinetic energy, $\langle r^2\rangle$ is the mean square radius of a star in the GC, σ_{cl} is the cloud surface density, M_{cl} is the cloud mass, v_c is the local rotation velocity and p^*_{min} is the effective minimum impact parameter for the GC-GMC collision. CKS solved for p^*_{min} based on their numerical work, with results that were crudely $p^{*2}_{min} \approx 0.5(r^2_{cl} + r^2_t)$. Figure 2 illustrates the numerical results for a cluster with $W_0 = 1$ (lower dashed line) and $W_0 = 7$ (lower solid line)), $M = 10^5\,M_\odot$ as a function of the angle of incidence.

We tested the scaling of relation (10) above for $2 \times 10^4 < M_{cl} < 5 \times 10^5\,M_\odot$ and fixed σ_{cl}. We found it to be accurate to about 50%; particle loss for the least concentrated cluster and the highest M_{cl} led to some suppression of the direct heating. However, indirect heating via the change in potential energy compensates for that suppression somewhat. Scaling with r_{cl} is somewhat less accurate, i.e. p^*_{min} is not well approximated by the expression above. The heating rate is greater than that given by eqn (10) by (at most) 2.5 when $r_{cl} = 4\,\mathrm{pc}$ and by (at most) 2.0 when $r_{cl} = 100\,\mathrm{pc}$.

We can now address the question whether disruption of GC's by GMC's can ever take place. The GMC density is a maximum at $R_g \approx 5$ kpc with $\sigma_{cl} \approx 15\,M_\odot/\,\mathrm{pc}^2$. Using eqns. (10) and (4) we can write the fractional change per disk crossing, due to GMC perturbations, as

$$\frac{\Delta T}{E} = \frac{7.4 \times 10^{-5}}{\nu\epsilon\sqrt{1-\epsilon}} \left[\frac{\langle r^2\rangle}{p^{*2}_{min}}\right] \left[\frac{\sigma_{cl}}{15\,M_\odot\,\mathrm{pc}^2}\right] \left[\frac{M_{cl}}{10^5\,M_\odot}\right] \left[\frac{R_g}{5\,\mathrm{kpc}}\right]^2 \left[\frac{10^5\,M_\odot}{M}\right]^{2/3} \tag{11}$$

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In 10^{10} years There are at most 200 disk passages for an orbit at $R_g = 5\,\mathrm{kpc}$. Accordingly, *GMC's cannot appreciably affect cluster evolution unless ϵ is very near 0 or 1, i.e. disruption requires extreme cluster orbit orientations.* To examine the situation in slightly more detail, we calculate ϵ^* such that $\Delta T/E = 1$ after 200 passages by eqn. (10) above. For $W_0 = 1, 3, 4, 7$ we find $\epsilon^* = 1.6 \times 10^{-2}, 1.1 \times 10^{-2}, 8.2 \times 10^{-3}, 2.5 \times 10^{-3}$ for prograde orbits or, in angles, $-75°, -78°, -80°, -84°$. Although the validity of the impulse approximation puts a lower limit on θ (such that the disk passage is sudden for most of the particles in the GC) these disruption angles all lie close to, but above that limit. Therefore, for the range $1 \leq W_0 \leq 7$, we conclude that disruption is energetically feasible for prograde orbits. However, the parameter space for disrupted orbits (even among purely circular orbits) is quite small: it is confined to the peak of the function $\sigma_{cl} R_g$ (near $R_g = 5\,\mathrm{kpc}$) and to a narrow angular range.

Eqn. (10) shows that disruption of retrograde orbits requires such small $1-\epsilon$ that the GC would never leave the disk. Expressing these results in terms of angles, *GC's on circular orbits with $-75° < \Theta < 90°$ are not disrupted by GMC's alone.* The physical interpertation of these results is clear: for a uniform distribution in Θ of cluster orbits, most orbits are not disrupted by the GMC's in the disk. However, a population of GC's confined to the disk will quite likely be destroyed. Therefore, if the galactic bulge burst sources are on nearly prograde orbits they may have come from destroyed GC's as suggested by Grindlay and collaborators (1984, 1985, 1986, Grindlay and Hertz 1985). Ironically, small- and large-angle orbits through the disk, lying outside the narrow ''parameter space'' described above, may be the safest haven for GC's within about 8 kpc of the Galactic center. These orbits do not suffer strong gravitational shocks which can be much more effective than the GMC's at heating the GC.

5.3 TIDAL SHOCKS

Our numerical treatment of tidal shocks is an extension of that of Ostriker, Spitzer and Chevalier (1972) in that it includes a realistic disk potential, an adiabatic cutoff and particle loss. We find that all three effects are important in determining the magnitude of the heating, which turns out to be large enough to significantly alter the concentration of a GC at small Galactocentric distance.

In the impulsive limit the GC heating rate per passage is given, without particle loss, by

$$\Delta T = \frac{2}{3} M \left(\frac{d\phi}{dz}\Big|_{z_1}\right)^2 \frac{\langle r^2 \rangle}{v^2 \cos^2 \Theta}. \tag{12}$$

where the effective disk thickness z_1 must be chosen so that the encounter is ''impulsive'' ($\lim_{z_1\to\infty} d\phi/dz = 2\pi G \sigma_d$.). However, this purely impulsive limit is rarely realized, so we will use this equation only to estimate the *scaling* of the energy transfer with the Galactic and GC parameters. For convenience, we fix the normalization using the value of ΔT at $\Theta = 0, R_g = 8\,\mathrm{kpc}, W_0 = 1$, and $M = 10^5\,\mathrm{M}_\odot$, which we will refer to as $\Delta T|_{fix}$ and determine numerically. Expressing the dimensionless mean square radius, $\langle r^2 \rangle / r_t^2 = s(W_0)$, eqns. (2) and (12) give

$$\Delta T = \Delta T|_{fix} \left[\frac{M}{10^5\,\mathrm{M}_\odot}\right]^{5/3} \left[\frac{s(W_0)}{s(1)}\right] \left[\frac{1}{\cos^2 \Theta}\right] \chi(R_g) \tag{13}$$

where

$$\chi(R_g) = \left[\frac{\sigma_d(R_g)}{\sigma_d(8\,\mathrm{kpc})}\right]^2 \left[\frac{R_g}{8\,\mathrm{kpc}}\right]^2 \left[\frac{M_g(8\,\mathrm{kpc})}{M_g}\right]^{2/3} \left[\frac{v_c(8\,\mathrm{kpc})}{v_c(R_g)}\right]^2. \tag{14}$$

$\chi(R_g)$ describes the scaling of the heating with Galactic position. Assuming constant v_c (with constant M_g/R_g) we have

$$\chi(R_g) \approx \left[\frac{\sigma_d(R_g)}{\sigma_d(8\,\mathrm{kpc})}\right]^2 \left[\frac{R_g}{8\,\mathrm{kpc}}\right]^{4/3}. \tag{15}$$

For an exponential disk surface density with a characteristic length of $3.5\,\mathrm{kpc}$, the scaling of χ with R_g implies that the maximum heating per plane passage, for a fixed GC mass and W_0, occurs at about $R_g = 2.3\,\mathrm{kpc}$. When the frequency of passage is folded in (proportional to $1/R_g$) the maximum rate of heating per unit time due to tidal shocks occurs at about $R_g = 0.5\,\mathrm{kpc}$. To summarize the increase in surface density towards the center of the disk, means that GC heating becomes stronger as the orbital radius decreases. However, for fixed M, the tidal radius decreases as $R_g \to 0$ so that eventually the heating must decrease and this leads to a maximum heating rate at a finite R_g. The scaling for χ ignores particle loss and the adiabatic cutoff.

To illustrate the new qualitative features of the numerical calculation we show the heating as function of angle of incidence for two values of W_0 and for two values of R_g. The GC had a mass of $M = 10^5\,\mathrm{M}_\odot$. Several runs were performed to illustrate the effects of the frequency cutoff, of the scale height, and of particle loss. In all the runs the potential was determined by the law

$$K_z(R_g, z) = K_z(R_0, z)\left(\frac{\sigma_d(R_g)}{\sigma_d(R_0)}\right)\sum_i K_{0i}\tanh(z/z_{0i}), \tag{16}$$

where R_0 is the local position and σ_d is the disk surface density in the BS&S model. We define the ''standard model'' as

$$\begin{aligned} K_{01} &= 3.47\times10^{-9} cm/s^2,\\ z_{01} &= 175\,\mathrm{pc};\\ K_{02} &= 3.96\times10^{-9} cm/s^2,\\ z_{02} &= 550\,\mathrm{pc}. \end{aligned} \tag{17}$$

and z is in pc. This is a smooth fit to Bahcall's local acceleration data. The ''one component'' model is given by K_{01} above and a scale-height assumption. To illustrate the magnitude of the different effects which enter, we have plotted the heating rate as a function of angle of incidence under four different sets of assumptions. To begin with, we do not allow any particle escape. Consider first the four upper lines in the Figure 3a which all relate to the heating of a $W_0 = 1$ model at $R_g = 3\,\mathrm{kpc}$.

The line of short dashes is the ''one component'' model with $z_{01} = 0$, i.e. impulsive heating by the low-scale height surface density. Note the steep rise as $\cos\Theta \to 0$ (consistent with the scaling of eqn. (12)). The line of long dashes is the ''one component'' model with $z_{01} = 175\,\mathrm{pc}$; there is a substantial suppression in the heating due to the adiabatic cutoff. The dash-dot line is the standard model given above (two components with separate adiabatic cutoffs) and the solid line is the same model but with particle loss allowed. The last two lines illustrate that there is some heating by the large scale-height component and some inefficiency on account of particle escape. A completely analagous set of curves is given in figure 3a for a $W_0 = 7$ model,

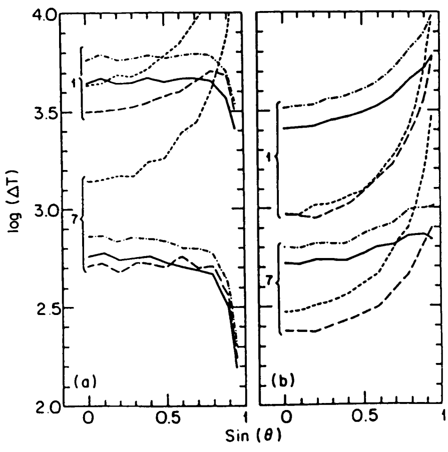

Fig. 3a and 3b. Comparison of disk heating of $10^5\,M_\odot$ GC as a function of angle of incidence at (a) $R_g = 3\,\mathrm{kpc}$ and (b) $8\,\mathrm{kpc}$ for two values of $W_0 = 1$ and 7. Four lines for each (R_g, W_0) pair are shown. The first three have no particle loss: they are (short dashes) impulsive heating with disk acceleration $K_{01} = 3.47 \times 10^{-9}\,\mathrm{cm}/s^2$, (long dashes) heating by same model with finite scale height ($z_{01} = 175\,\mathrm{pc}$) and (dash-dot line) standard model. The standard model with particle loss included is illustrated by the solid line.

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which is much more centrally condensed, illustrating the significant role of the impulse approximation. Figure 3b illustrates heating of the $W_0 = 1$ and $W_0 = 7$ models at the local position ($R_g = 8\,\mathrm{kpc}$). Comparing the curves, the main effect in 3b is the extra heating associated with K_{02}. In the solar neighborhood, the adiabatic cutoff is not as important as at $3\,\mathrm{kpc}$ because the tidal radius is larger with longer dynamic timescales; meanwhile the disk passage timescale remains constant. Hence, the impulsive approximation becomes more accurate at large radii. Figure 2 illustrates, the final heating rate (upper two lines) as a function of angle of inclination of the GC orbit, for $W_0 = 1$ (dashed line) and $W_0 = 7$ (solid line) at 8 kpc. It is interesting that despite the reductions in magnitude of ΔT by the various corrections, *the disk heating still exceeds the heating by clouds by about a factor of 10.* This is true at all R_g and for all W_0.

CKS parameterized their numerical results in terms of a multiplicative correction factor, γ, to the simple expression for χ, given by eqn. (15), over the range $1\,\mathrm{kpc} < R_g < 24.5\,\mathrm{kpc}$ and $\Theta = 0.0, 0.5, 0.9$. The large range of the correction ($0.00 < \gamma < 2.3$) is noteworthy and illustrates the effects of particle loss and the adiabatic cutoff on the simple scaling arguments. *The maximum total heating per passage occurs at about 5 kpc for* $W_0 = 1,3$ *and at about 7 kpc for* $W_0 = 4,7$, *both at small* $\cos\Theta$; *the maximum heating per unit time occurs between* $3\,\mathrm{kpc}$ *and* $5\,\mathrm{kpc}$ *at small* $\sin\Theta$. We have also investigated the rate of heating for orbits which cross the disk with velocity different from v_c; typically, the variation with v is much more gradual than the $1/v^2$ dependence implied by eqn. (12).

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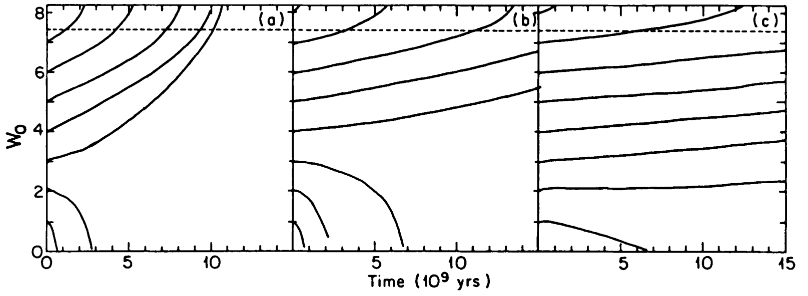

Fig. 4. Cluster evolution of concentration: $W_0(t)$ for a family of initial conditions ($M = 10^5\,M_\odot$) at $R_g = 3$ kpc (a), 8 kpc (b) and 14 kpc (c). Relaxation causes the concentration to increase (higher W_0) and tidal shocking causes the concentration to decrease. The dashed line marks $W_0 = 7.4$, core collapse. $W_0 = 0$ is interperted at tidal dissolution.

6 EVOLUTIONARY CALCULATIONS

We found that heating and mass loss from tidal shocks is more important that of interactions with GMC's for all King models with $10^5\,M_\odot < M < 10^6\,M_\odot$ on orbits with $2\,\mathrm{kpc} < R_g < 14\,\mathrm{kpc}$ for all angles of incidence, with the exception of nearly prograde GC orbits with very small angles of inclination. The rate of heating by tidal shocks at fixed R_g is only weakly dependent on the angle of incidence of the cluster orbit, on account of the suppression of energy transfer by the adiabatic cut-off. Hence, we will present results for fixed angle of inclination, $\theta = 0$. Figure 4 illustrates the evolution of W_0 of a $10^5\,M_\odot$ GC without SE, at $R_g = 3, 8, \mathrm{and} 14\,\mathrm{kpc}$ for a family of initial cluster concentrations.

GC evolution proceeds most quickly near the center of the Galaxy. There is an unstable tendency to either collapse or dissolve, i.e. there exists a critical value of W_0 at each R_g such that more concentrated clusters collapse and less concentrated ones dissolve. An entirely analagous effect was originally pointed out by King (1958) in his treatment of Galactic clusters. Mass loss by relaxation and by tidal shocks is comparable at the critical W_0. According to our previous discussion, we interpert $W_0 \geq 7.4$ as meaning that core collapse has occurred, while $W_0 = 0$ implies that the cluster has dissolved. Comparison of runs with and without tidal shocking shows that the time until collapse of a given initial mass *decreases* for most initial concentrations when the shocking is included. Apparently if the cluster collapses, the increased mass loss and decreased relaxation time offset the shock's tendency to dissolve the cluster.

Calculations similar to those shown in Figure 4 have been performed for $2\,\mathrm{kpc} < R_g < 15\,\mathrm{kpc}$. Figure 5 summarizes the global trends in GC evolution under the environmental effects of Galactic position.

Contours of W_0 after an evolution of 10^{10} years are plotted. The graph's axes are

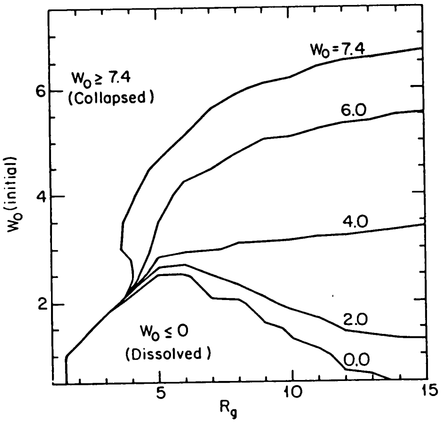

Fig. 5. Contours of cluster concentration after 10^{10} years evolution for a cluster with galactocentric position R_g (abscissa), initial concentration W_0 (ordinate), and initial mass $10^5\,M_\odot$. Clusters lying below the lowest contour, $W_0 = 0$, have dissolved; those above the topmost contour, $W_0 = 7.4$, have collapsed.

the initial value of W_0 (ordinate) and R_g (abscissa, assumed constant throughout the evolution; dynamical friction is insignificant for $R_g > 2\,\mathrm{kpc}$ and $M < 2 \times 10^6\,M_\odot$, Tremaine, Ostriker and Spitzer 1975) . The contour of $W_0 = 0$ is the limit of clusters which have ''dissolved,'' while $W_0 = 7.4$ is the limit of those that have undergone core-collapse. Clusters at large R_g are unaffected by external forces and closely resemble their initial structure, meaning loosely speaking, their structure when SE ''turned off.'' At the other extreme, most clusters at small R_g have already been destroyed by tidal heating or have undergone core collapse. Our analysis of course cannot be extended beyond core collapse. However, if post core-collapsed clusters are distinguishable from pre core-collapsed clusters, one expects a definite trend: *more collapsed objects near the Galactic center*. Such a trend may have been observed (Djorgovski 1986). It is highly likely that tidal shocking continues to be important during the life of all clusters at small R_g, including those that have undergone core collapse.

As a corrollary to these results, the detection of a GC with collapsed core at *large* R_g would be profoundly signficant. The density of a King model is controlled by the strength of the tidal field. If a GC is dense enough to core collapse in a Hubble time at large R_g then the initial conditions for that GC must be denser than that allowed by the King model. That is, the initial mass must be well concentrated within the allowed (i.e. tidal) radius. A non-gravitational force must then be at work in creating such proto-cluster conditions, e.g. the cooling instability studied by Fall and Rees (1985).

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DISCUSSION

GNEDIN: There is a point of view that the low-mass X-ray binaries in the bulge originated as a result of collisions between globular clusters and giant molecular clouds. What do you think of this?

CHERNOFF: Disruption appears to be energetically feasible for prograde orbits with very large angles of incidence with respect to the disk; i.e., orbits nearly in the disk. [I would also like to point out that individual GMC's not complexes of GMC's were used to estimate the destruction. The larger complexes will enhance the distinction.]

ZINNECKER: This is a question which is also addressed to Dr. Ostriker. Are there destruction processes for globular clusters which operate preferentially in spirals or ellipticals? If so, this would seem to have some bearing on the specific frequency of globular clusters in spirals versus ellipticals?

CHERNOFF: The tidal shocking by disk and GMC's will be absent in present-day ellipticals. The average strengths of the tidal field will still enter into the collapse rate.

INNANEN: Could you apply your results to old open clusters like M 67 and NGC 188, in view of the fact that your orbits are nearly circular?

CHERNOFF: I would be hesitant to apply the general method, to open clusters because these systems are small N systems; i.e., the separation of dynamic and relaxation timescales is not as clear.

GOODMAN: Both your own and Ostriker's analysis of relaxation have assumed equal mass stars. But we know that in the presence of a mixture of stellar masses, the collapse time is greatly reduced, perhaps by a factor of five.

CHERNOFF: I agree with you. It may be that stellar evolution narrows the range of the mass function sufficiently, and (additionally) pumps enough energy into the cluster by mass loss to counteract these effects. In any case, these effects cannot be studied with this sort of model.