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Generation of sheared flows by drift waves in a strongly magnetized electron–positron–ion plasma

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Abstract. It is shown that sheared/zonal flows (ZFs) can be nonlinearly excited by incoherent drift waves (DWs) in a strongly magnetized non-uniform plasma composed of electrons, positrons and ions. The dynamics of incoherent DWs in the presence of ZFs is governed by a wave-kinetic equation. The governing equation for ZFs in the presence of nonlinear forces (associated with nonlinear ion polarization and nonlinear ion-diamagnetic drifts) of the DWs is deduced by combining the Poisson equation, as well as the e-p-i continuity equations, together with appropriate plasma particle velocities in the DW and the ZF fields. Standard techniques are used to derive a nonlinear dispersion relation, which depicts two classes of the modulational instability of the DWs against the ZFs. Non-thermal ZFs can reduce the turbulent cross-field particle transport in non-uniform, strongly magnetized e-p-i plasmas.

Plasmas composed of electrons, positrons and ions are found in the early Universe [1, 2], in active galactic nuclei [3, 4], in the polar region of neutron stars [5–7], in magnetars [8], in the pulsar magnetosphere [9, 10], in the solar atmosphere [11], in the laser-produced plasmas [12,13] and in tokamaks [14]. In e-p-i plasmas, electrons and positrons (also referred to as pairs) have opposite charges, but equal masses. The positrons can be used to probe particle transport in tokamaks, because they have sufficient lifetime. In e-p-i plasmas, one encounters numerous types of wave motions and localized wave excitations [15–19]. Studies of wave–wave and wave–particle interactions in e-p-i plasmas are of paramount importance with regard to understanding the salient features of localized electrostatic and electromagnetic

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waves, as well as that of plasma particle acceleration and plasma particle transports across the external magnetic field.

In this communication we consider nonlinear interactions between incoherent drift waves (DWs) and sheared/zonal flows (ZFs) [20] in a strongly magnetized non-uniform e-p-i plasma. The DWs are the low-frequency (in comparison to the ion gyrofrequency), pseudo three-dimensional electrostatic perturbations involving the Boltzmann distributed electrons and positrons, as well as the two-dimensional magnetized ions, while ZFs are radially inhomogeneous ($k_x \neq 0$), large-scale (in comparison to the ion gyroradius) electrostatic potential perturbations having insignificant density fluctuations. Our coupled DW–ZF turbulence model in a non-uniform e-p-i magnetoplasma is a characteristics of a 'predator-prey' system in which the population of incoherent DWs (prey), growing via linear instability, would generate ZFs (predator) through the modulational instability [20]. Subsequently, the growth of ZFs would reduce the strength of the prey; a scenario similar to the DW–ZF turbulence system [20–24] in a non-uniform magnetoplasma with electrons and ions, where the DW eddies become of smaller amplitudes and longer sizes due to random shearing caused by the large-scale ZFs. Accordingly, the cross-field turbulent transport is significantly reduced [24].

We consider a non-uniform e-p-i plasma in the presence of the pre-existing incoherent electrostatic DWs in a homogeneous magnetic field $B\hat{\mathbf{z}}$, where B is the strength of the magnetic field and $\hat{\mathbf{z}}$ is the unit vector along the z-axis in a Cartesian coordinate system. The equilibrium density gradient $\partial n_{j0}/\partial x$ is along the x-axis, where n_{j0} is the unperturbed number density of the particle species j (j equals e for electrons, p for positrons, and i for ions), and $n_{e0}(x) = n_{i0}(x) + n_{p0}(x)$. The frequency of the obliquely (with respect to $\hat{\mathbf{z}}$) propagating DWs in our plasma is

$$\omega_k = \frac{\omega_*}{(1 + k_\perp^2 \rho_s^2)}, \tag{1}$$

which is deduced by setting zero the dielectric constant

$$\epsilon(\omega, \mathbf{k}) = 1 + \frac{1}{k^2 \lambda_D^2} + \frac{\omega_{pi}^2 k_\perp^2}{\omega_{ci}^2 k^2} + \frac{\omega_{pi}^2 \kappa_i k_y}{\omega \omega_{ci} k^2}, \tag{2}$$

and by assuming that $k^2\lambda_D^2 \ll 1$. Here $\omega_* = -k_y V_*$, $V_* = \kappa_i \lambda_D^2 \omega_{pi}^2/\omega_{ci} \equiv \kappa_i \rho_s^2 \omega_{ci}$, $\kappa_i = (\partial \ln n_{i0}/\partial x) < 0$, $\lambda_D^{-2} = \lambda_{De}^{-2} + \lambda_{Dp}^{-2}$, $\lambda_{Dj} = (k_B T_j/4\pi n_{j0} e^2)^{1/2}$ is the Debye radius of the particle species j, k_B is the Boltzmann constant, T_j is the temperature, e is the magnitude of the electron charge, $\omega_{ci} = eB/m_i c$ is the ion gyrofrequency, $\omega_{pi} = (4\pi n_{i0} e^2/m_i)^{1/2}$ is the ion plasma frequency, m_i is the ion mass and c is the speed of light in vacuum. The wave vector $\mathbf{k} = \mathbf{k}_\perp + \hat{\mathbf{z}} k_z$, and $k_\perp^2 = k_x^2 + k_y^2$, where the subscripts x, y and z stand for the radial, azimuthal and axial components of the wave vector, respectively. In (2), we have assumed that $k_z V_{Ti} \ll \omega \ll \omega_{ci}, k_z V_{Te,Tp}, k_z^2 \omega_{ce}/k_y |\kappa_{e,p}|$, where $V_{Te,Tp}$ and V_{Ti} are the electron/positron and ion thermal speeds, respectively, $\omega_{ce} = eB/m_e c$ is the electron gyrofrequency, m_e is the electron mass and $\kappa_{e,p} = \partial \ln n_{e0,po}/\partial x$. The motion of ions along $\hat{\mathbf{z}}$ has been ignored so that the modified ion sound waves [25] are decoupled from our system.

The energy density of the DWs [26, 27] is

$$\mathscr{E}_k = \left[\frac{\partial}{\partial \omega}(\omega \epsilon)\right] |\mathbf{E}_k|^2, \tag{3}$$

where $\mathbf{E}_k = -i\mathbf{k}\phi_k$ is the DW electric field and ϕ_k is the DW potential. By using (2) we have from (3)

$$\mathscr{E}_k = (1 + k_\perp^2\rho_s^2)\left|\frac{e\phi_k}{k_B T_e}\right|^2 \mathscr{E}_{th}, \tag{4}$$

where $\mathscr{E}_{th} = 4\pi n_{e0} k_B T_e(1+\alpha)$ is the kinetic energy density of the e-p-i plasma, and $\alpha = n_{p0}T_e/n_{e0}T_p$.

The DW action is denoted by

$$N_k = \frac{\mathscr{E}_k}{\omega_k} = \frac{(1 + k_\perp^2\rho_s^2)^2}{\omega_*}\left|\frac{e\phi_k}{k_B T_e}\right|^2 \mathscr{E}_{th}. \tag{5}$$

The nonlinear interaction between the random phase DWs and ZFs is governed by a wave-kinetic equation [28]

$$\frac{\partial N_k}{\partial t} + V_{gx}\frac{\partial N_k}{\partial x} - \frac{\partial \omega_k^n}{\partial x}\frac{\partial N_k}{\partial k_x} = S, \tag{6}$$

where $S = \gamma_k N_k - \Delta\omega_k N_k^2$ represents the source term due to the wave growth and damping due to linear and nonlinear mechanisms, γ_k is the linear growth rate and $\Delta\omega_k N_k^2$ is the damping caused by nonlinear resonance broadening effects [22, 23]. Assuming that small-scale DW turbulence is close to a stationary state, one can set $S = 0$. The radial group velocity of the DWs is

$$V_{gx} = -\frac{2k_x\rho_s^2\omega_k}{(1 + k_\perp^2\rho_s^2)}. \tag{7}$$

Furthermore, the spatial variation of the nonlinear frequency ω_k^n involving the ZF electric potential ϕ_z is

$$\frac{\partial \omega_k^n}{\partial x} = k_y\frac{\partial u_z}{\partial x} \equiv \frac{k_y c}{B}\frac{\partial^2\phi_z}{\partial x^2}, \tag{8}$$

where the ZF speed in the azimuthal direction is $u_z = (c/B)\partial\phi_z/\partial x$.

The evolution of the ZF potential in the presence of the Reynolds stress of the DWs is governed by

$$\frac{\partial^3\phi_z}{\partial x^2\partial t} + (1+\sigma)\frac{c}{B}\sum_{\mathbf{k}}\left\langle(\hat{\mathbf{z}}\times\nabla\phi_k\cdot\nabla)\nabla_\perp^2\phi_k\right\rangle + \text{complex conjugate} = 0, \tag{9}$$

where $\sigma = (n_{e0}T_i/n_{i0}T_e)(1+\alpha)$, $\nabla_\perp^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$ and the angular bracket denotes an ensemble average over the period $2\pi/\omega_k$. We note that (9) has been derived by subtracting the ion continuity equation from the sum of the electron and position continuity equations and by using the Poisson equation and the perpendicular (to $\hat{\mathbf{z}}$) components of the e-p-i fluid velocities [9,12] in the DW and ZF fields. The second term on the left-hand side of (9) is the Reynolds stress of the DWs and comes from the sum of nonlinear ion polarization and ion diamagnetic drifts. In the latter, the ion number density perturbation is replaced by $n_{e0}(1+\alpha)e\phi/k_B T_e$. Equation (9) is valid for a dense plasma with $\omega_{pi} \gg \omega_{ci}$. For simplicity, we have neglected the effects of ion-neutral collisions and ion gyroviscosity on the dynamics of ZFs.

We now study instability of the random phase DWs against the modulation caused by ZFs. For this purpose [29], we express

$$N_k = N_{k0} + N_{k1}\exp(i\varphi), \quad \text{and} \quad \phi_z = \psi\exp(i\varphi), \tag{10}$$

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where $N_{k1}(\ll N_{k0})$ is a small perturbation of the DW action around the equilibrium value N_{k0}, and $\varphi = qx - \Omega t$ is phasor. The frequency and radial wave number of ZFs are denoted by Ω and q, respectively.

Inserting (10) into (6) and (9) and Fourier analyzing the resultant equations, we have

$$N_{k1} = -\frac{iq^2 k_y c}{B(\Omega - qV_{gx})}\frac{\partial N_{k0}}{\partial k_x}\psi, \tag{11}$$

and

$$\Omega\psi = 2i(1+\sigma)\frac{c}{B}\int d^2k k_y k_x |\phi_k|^2. \tag{12}$$

By using (5) we can express $|\phi_k|^2$ in terms of N_{k1}, and inserting that expression in (12) and using (11) we have the dispersion relation

$$\Omega = 2(1+\sigma)q^2\rho_s^2\frac{|\kappa_i|C_s^4}{\omega_{ci}\mathscr{E}_{th}}\int \frac{k_x k_y^2}{(\Omega - qV_{gx})(1+k_\perp^2\rho_s^2)^2}\frac{\partial N_{k0}}{\partial k_x}d^2k, \tag{13}$$

where $C_s = (k_B T_e/m_i)^{1/2}$ is the ion sound speed.

We analyze (13) in two limiting cases. First, for the resonant-type instability we replace the resonance function $R = (\Omega - qV_{gx})^{-1}$ by $-i\pi\delta(\Omega - qV_{gx})$, where δ is the Dirac-delta function. Letting $\Omega = i\gamma_q$ in the resultant equation, we obtain the growth rate

$$\gamma_q = -2\pi(1+\sigma)q^2\rho_s^2\frac{|\kappa_i|C_s^4}{\omega_{ci}\mathscr{E}_{th}}\int \frac{k_x k_y^2}{(1+k_\perp^2\rho_s^2)^2}\frac{\partial N_{k0}}{\partial k_x}\delta(\Omega - qV_{gx})d^2k, \tag{14}$$

which ensures instability if $\partial N_{k0}/\partial k_x < 0$.

Second, we consider a non-resonant hydrodynamic-type instability. We assume that $N_{k0} = N_0\delta(\mathbf{k} - \mathbf{k}_0)$, with $\mathbf{k}_0 = (k_{x0}, k_{y0})$. Carrying out integration by parts in (13), we have

$$1 = (1+\sigma)\frac{q^2 C_s^4}{\rho_s^2\omega_{ci}^2\mathscr{E}_{th}}\int \frac{k_y V_g' N_{k0}}{(\Omega - qV_{gx})^2}d^2k, \tag{15}$$

where the DW group dispersion is

$$V_g' = \frac{\partial V_{gx}}{\partial k_x} = -\frac{2|\kappa_i|k_y\rho_s^4\omega_{ci}(1+k_y^2\rho_s^2 - 3k_x^2\rho_s^2)}{(1+k_\perp^2\rho_s^2)^3}. \tag{16}$$

Equation (15) yields

$$[\Omega - qV_{gx}(\mathbf{k} = \mathbf{k}_0)]^2 = -\frac{2(1+\sigma)|\kappa_i|k_{y0}I_0 q^2 C_s^4}{\omega_{ci}^2(1+k_{\perp 0}^2\rho_s^2)}(1+k_{y0}^2\rho_s^2 - 3k_{x0}^2\rho_s^2), \tag{17}$$

where $I_0 = |e\phi_{k0}/k_B T_e|^2$. Equation (17) predicts an oscillatory instability with a real frequency $qV_{gx}(\mathbf{k} = \mathbf{k}_0)$ and the growth rate

$$\gamma_r = \frac{\sqrt{2(1+\sigma)|\kappa_i|k_{y0}I_0}\,qC_s^2}{\omega_{ci}(1+k_{\perp 0}^2\rho_s^2)^{1/2}}(1+k_{y0}^2\rho_s^2 - 3k_{x0}^2\rho_s^2)^{1/2}, \tag{18}$$

provided that $1 + k_{y0}^2\rho_s^2 > 3k_{x0}^2\rho_s^2$.

Finally, it should be noted that (13) can also be investigated for a broad DW spectrum, with $N_{k0} = N_0\exp[-(\mathbf{k}-\mathbf{k}_0)^2/2\Delta^2]$, where Δ is the spectral width. However, for this case, it turns out that the growth rate is smaller than that in (18), because energy in a broad spectrum of DWs is spread over the entire wave numbers and, hence, the DW energy available to drive ZFs is significantly lower than that in a narrowly peaked spectrum of the DWs.

In summary, we have considered nonlinear interactions between the finite amplitude DWs and ZFs in a strongly magnetized plasma composed of electrons, positrons and ions. It is shown that the DWs are modulationally unstable against the ZFs. Physically, ZFs modulate the DWs, which act like quasi-particles in our strongly magnetized e-p-i plasma. Accordingly, the DWs' action would evolve on a timescale much longer than the DW period, as if the DW quasi-particles were propagating through a slowly varying medium supported by ZFs. The latter are reinforced by nonlinear forces of the modulated DWs. The energy flow from the DWs (which are like a beam of quasi-particles) to ZFs is responsible for the excitation of sheared plasma flows via non-resonant and resonant-type instabilities that we have presented herein. Since sheared plasma flows are along the azimuthal direction, they will tear apart the DW eddies and would keep their amplitudes low. Accordingly, the cross-field turbulent transport will be drastically reduced due to longer sizes and smaller amplitudes DW 'prey' in turbulent e-p-i magnetoplasmas, such as those in astrophysical settings [1–5, 9, 10] and in inertial and magnetic confined fusion plasmas [12, 14].

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