Macroeconomic Dynamics, **26**, 2022, 2141–2160. Printed in the United States of America.
doi:10.1017/S1365100521000250

DEPENDENCE STRUCTURE BETWEEN MONEY AND ECONOMIC ACTIVITY: A MARKOV-SWITCHING COPULA VEC APPROACH

APOSTOLOS SERLETIS
University of Calgary

LIBO XU
University of San Francisco

This paper examines correlation and dependence structures between money and the level of economic activity in the USA in the context of a Markov-switching copula vector error correction model. We use the error correction model to focus on the short-run dynamics between money and output while accounting for their long-run equilibrium relationship. We use the Markov regime-switching model to account for instabilities in the relationship between money and output, and also consider different copula models with different dependence structures to investigate (upper and lower) tail dependence.

Keywords: Divisia Monetary Aggregates, Markov Regime Switching, Dependence, Copula

1. INTRODUCTION

Over the years, there has been a large number of empirical studies that use state-of-the-art advances in macroeconometrics and financial econometrics to investigate the relationship between money and the level of economic activity. They show that most of the puzzles and paradoxes in monetary economics have been produced by use of simple sum money measures, and are resolved by use of aggregation theoretic monetary aggregates, such as Barnett's (1980) Divisia monetary aggregates. See, for example, Serletis and Shahmoradi (2006), Barnett and Chauvet (2011), Serletis and Rahman (2013), Hendrickson (2014), Serletis and Gogas (2014), Belongia and Ireland (2014, 2015, 2016, 2018), Ellington (2018),

This paper examines correlation and dependence structures between money and the level of economic activity in the USA in the context of a Markov-switching copula vector error correction model. We use the error correction model to focus on the short-run dynamics between money and output while accounting for their long-run equilibrium relationship. We use the Markov regime-switching model to account for instabilities in the relationship between money and output, and also consider different copula models with different dependence structures to investigate (upper and lower) tail dependence. Address correspondence to: Apostolos Serletis, Department of Economics, University of Calgary, Calgary, Alberta T2N 1N4, Canada. E-mail: Serletis@ucalgary.ca. Phone: (403) 220-4092. Fax: (403) 282-5262.

 1469-8056/21

among others. These studies support a monetary effect on the business cycle and solve the "Barnett critique"—the measurement problems associated with the failure to find significant relations between money and key macroeconomic variables. More recently, Serletis and Xu (2020) and Dery and Serletis (2022) illustrate the importance of the group of broad Divisia measures of money, published by the Center for Financial Stability (CFS), reinforcing the claims by Barnett (2016) and Jadidzadeh and Serletis (2019) that we should employ, as a measure of money, the broad CFS Divisia monetary aggregates.

In this paper, we use monthly data for the USA, over the period from January 1967 to January 2020, the CFS Divisia monetary aggregates, and a different approach to the investigation of the relationship between money and economic activity. In particular, (to our knowledge) for the first time in the monetary VAR literature, we examine correlation and dependence structures between money and output in the context of a Markov-switching, structural vector error correction (VEC) model using copulas. As in Serletis and Xu (2020), we take the Markov regime-switching approach, to account for instabilities in the relationship between money and output, and use the VEC model to focus on the short-run dynamics while accounting for the long-run equilibrium relationship between money and output. However, in extending (Serletis and Xu (2020)), we consider different copula models, with different dependence structures, to investigate the nonlinear dependence structure, as well as (upper and lower) tail dependence, between money and output. In doing so, we also present a comparison between broad and narrow CFS Divisia monetary aggregates.

The empirical analysis of the dependence structure between Divisia money and output reveals that monetary policy has asymmetric effects on output. In particular, there is weak positive dependence between the CFS Divisia M4 monetary aggregate and output, but this dependence is asymmetric over contractions and expansions in economic activity, being significantly stronger during business cycle contractions.

The paper is organized as follows. Section 2 discusses the data and their time series properties. Section 3 presents the bivariate Markov-switching copula structural VEC model. Section 4 presents the empirical results with our preferred monetary aggregate—the CFS Divisia M4 aggregate. Section 5 provides a comparison with narrower CFS Divisia monetary aggregates (Divisia M1, Divisia M2, and Divisia M3). Section 6 quantifies the dynamic impact of money growth shocks on economic growth in terms of the generalized impulse function. The final section briefly concludes regarding the implications of our research for monetary theory and the conduct of monetary policy.

2. THE DATA

We use monthly data for the USA over the period from January 1967 to January 2020. For the real output series, Y_t, we use the industrial production index (IPI) from the Federal Reserve Economic Database (FRED) maintained by the Federal Reserve Bank of St. Louis. For the money measure, M_t, we use the Divisia M4

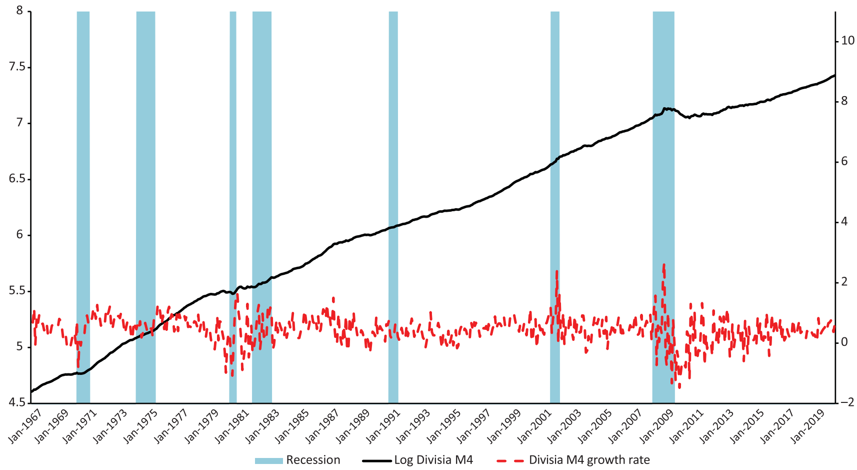


FIGURE 1. Log Divisia M4 and its growth rate.

monetary aggregate from the Center for Financial Stability (CFS)—see Barnett et al. (2013) for details regarding the construction of the CFS Divisia monetary aggregates. In this regard, it should be noted that the Divisia monetary aggregates were introduced by Barnett (1978, 1980). In particular, Barnett (1978) derived the real user cost of a monetary asset, i, as

$$\pi_{it} = \frac{R_t - r_{it}}{1 + R_t},$$

where R_t is the benchmark asset rate of return measuring the maximum expected rate of return available in the economy and r_{it} is the own rate of return on monetary asset i during period t. Barnett (1980) introduced the Divisia monetary aggregates (in discrete time) by computing the growth rate of the aggregates as the share-weighted average of its monetary asset component growth rates as follows:

$$d \log M_t = \sum_{i=1}^{n} s_{it} d \log m_{it},$$

where n is the number of monetary assets, m_{it} denotes the real balances of monetary asset i during period t, and $s_{it} = \pi_{it} m_{it} / \sum_{i=1}^{n} \pi_{it} m_{it}$ is the expenditure share on monetary asset i.

Figures 1 and 2 show the logged levels and growth rates of Y_t and M_t. We conduct a series of unit root and stationarity tests in the logarithms of Y_t and M_t, denoted by y_t and m_t, respectively. We find that y_t and m_t are nonstationary. We also test for cointegration using the Johansen (1988) maximum likelihood cointegration approach and find that y_t and m_t are cointegrated with two cointegrating vectors. Based on this evidence, in what follows, we adopt the vector error correction (VEC) model as the basic framework.

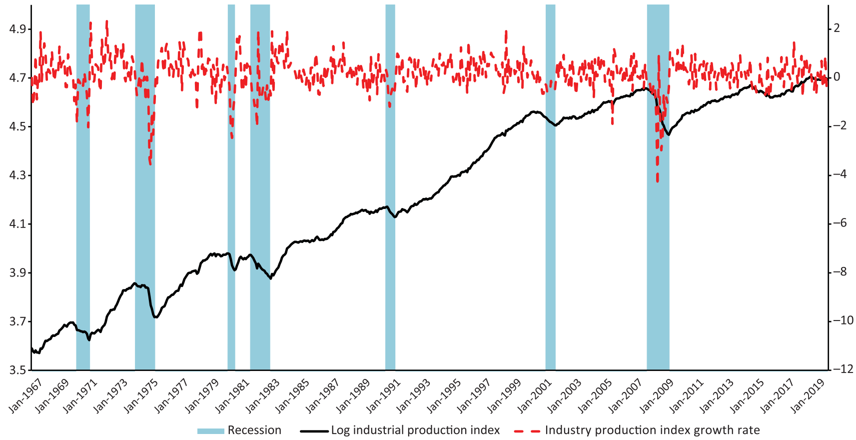


FIGURE 2. Log industrial production index and its growth rate.

3. THE MARKOV-SWITCHING COPULA VEC MODEL

We use the following bivariate VEC model:

$$\mathbf{B}\Delta\mathbf{z}_t = \mathbf{C} + \mathbf{\Pi}\mathbf{z}_{t-1} + \sum_{i=1}^{k} \mathbf{\Gamma}_i \Delta\mathbf{z}_{t-i} + \epsilon_t, \tag{1}$$

where $\Delta\mathbf{z}_t$ is the first logged difference of $\mathbf{z}_t$ and

$$\mathbf{z}_t = \begin{bmatrix} m_t \\ y_t \end{bmatrix}; \epsilon_t = \begin{bmatrix} \epsilon_{\Delta m,t} \\ \epsilon_{\Delta y,t} \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}; \mathbf{\Pi} = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix}; \mathbf{\Gamma}_i = \begin{bmatrix} \gamma_{i,11} & \gamma_{i,12} \\ \gamma_{i,21} & \gamma_{i,22} \end{bmatrix}.$$

The $\boldsymbol{B}$ matrix, which is lower triangular, helps to identify the model. In equation (1), $\epsilon_{\Delta m,t}$ is referred to as the money growth shock and $\epsilon_{\Delta y,t}$ as the output growth shock. A conventional estimation approach of model (1) is to assume that

$$\epsilon_t \sim (0, \mathbf{H}),\ \mathbf{H} = \begin{bmatrix} h_{\Delta m} & 0 \\ 0 & h_{\Delta y} \end{bmatrix}.$$

We multiply the growth rates of the two series, Δy_t and Δm_t, by 100 and estimate the VEC model with three lags based on the Bayesian information criteria (BIC). In panel A of Table 1 we report some diagnostic tests using the structural shocks from the standard Cholesky decomposition (based on the assumption that the $\mathbf{B}$ matrix is lower triangular). There is little evidence for a joint normal distribution of $\epsilon_{\Delta y,t}$ and $\epsilon_{\Delta m,t}$. A correlation coefficient of zero between $\epsilon_{\Delta y,t}$ and $\epsilon_{\Delta m,t}$, which is equivalent to a zero covariance between the two structural shocks, $\epsilon_{\Delta y,t}$ and $\epsilon_{\Delta m,t}$, does not mean that there is no dependence. As can be seen in panel B of Table 1, there is a positive dependence (based on the concept of concordance)

https://doi.org/10.1017/S1365100521000250 Published online by Cambridge University Press

TABLE 1. Bivariate normality tests and dependence measures

A. Bivariate normality tests	
	Money supply shock and output growth shock
Mardia's test (Skewness)	53.149 (0.000)
Mardia's test (Kurtosis)	34.295 (0.000)
Henze–Zirkler's test	7.157 (0.000)
Royston's test	116.713 (0.000)
Doornik–Hansen's test	350.308 (0.000)
B. Dependence measures with 95% confidence interval	
	Money supply shock and output growth shock
Spearman's ρ	0.019 [−0.061, 0.099]
Kendall's τ	0.027 [−0.024, 0.078]
C. Test for asymmetry	
	Bai and Ng (2005) statistic
Money supply shock	−25.279 (0.000)
Output growth shock	−25.279 (0.000)

Note(s): Sample period, monthly data: January 1967–January 2020. Numbers in parentheses are *p*-values.

relationship between the two shocks. A positive dependence between $\epsilon_{\Delta m,t}$ and $\epsilon_{\Delta y,t}$ implies that they are more likely to be large together or small together—see Joe (1997). It is to be noted that the 95% confidence intervals for Spearman's ρ and Kendall's τ in panel B of Table 1, constructed as in Bonett and Wright (2000), cover zero (indicating that the two series could also be unrelated) as well as negative dependence. The copula functions that we use in this paper could capture negative dependence structures. In panel C of Table 1 we test for asymmetry, following Bai and Ng (2005) using the residuals from the conventional structural VEC model. Both structural shocks show the existence of asymmetry. This feature could also be well captured by the flexible copula functions we use in this paper.

Based on the evidence in Table 1, we estimate the VEC model using copulas. The copula is a multivariate distribution. Its univariate margins all follow the (0,1) uniform distribution. Based on the Sklar (1959) theorem, in our case the copula C is defined by

$$F(\boldsymbol{\epsilon}_t) = C(F_1(\epsilon_{\Delta m,t}), F_2(\epsilon_{\Delta y,t})),$$

where $F(.)$ is an unknown joint distribution function for $\epsilon_{\Delta m,t}$ and $\epsilon_{\Delta y,t}$, $F_1(.)$ and $F_2(.)$ are the two univariate margins corresponding to the structural shocks. The theorem permits the bivariate distribution function $F(.)$ to be made up of the two margins with a dependence structure. In other words, we could piece together a joint distribution of $\epsilon_{\Delta m,t}$ and $\epsilon_{\Delta y,t}$ with the assumed margins and the dependence structure.

Finally, we use the Markov-switching approach, which allows us to study the dependence structure between money and output across different macroeconomic regimes. Thus, the Markov-switching copula VEC model is

$$\mathbf{B}_{s_t}\Delta\mathbf{z}_t = \mathbf{C}_{s_t} + \mathbf{\Pi}_{s_t}\mathbf{z}_{t-1} + \sum_{i=1}^{k}\mathbf{\Gamma}_{i,s_t}\Delta\mathbf{z}_{t-i} + \boldsymbol{\epsilon}_t \tag{2}$$

with

$$F(\boldsymbol{\epsilon}_t) = C_{s_t}(F_1(\epsilon_{\Delta m,t}), F_2(\epsilon_{\Delta y,t})),$$

$$\epsilon_{\Delta m,t} \sim N(0, h_{\Delta m,s_t}), \tag{3}$$

$$\epsilon_{\Delta y,t} \sim N(0, h_{\Delta y,s_t}), \tag{4}$$

where s_t denotes the unobserved economic regime, assumed to follow a first-order, homogeneous, two-state Markov chain governed by the transition matrix

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix},$$

where $p_{ij} = P\,(s_t = i\,|s_{t-1} = j)$, $i, j = 1, 2$ and $p_{11} = 1 - p_{21}$ and $p_{12} = 1 - p_{22}$.

According to equation (2), all the parameters in the $\mathbf{B}_{s_t}$, $\mathbf{C}_{s_t}$, $\mathbf{\Pi}_{s_t}$, and $\mathbf{\Gamma}_{s_t}$ matrices are regime-dependent, taking different values across the two regimes (i and j can only take two values). The two assumed regimes will sufficiently describe the dynamic interactions between money and output growth. In this regard, as suggested by Hamilton (1988, 1989), the two-regime model is sufficient for modeling economic recessions and expansions.

The coefficients of the $\mathbf{\Pi}_{s_t}$ matrix are interpreted as speed of adjustment parameters; they capture how output growth and money growth respond to deviations from the long-run equilibrium between Δm_t and Δy_t across different stages of the business cycle. Following Balcilar et al. (2015), we can decompose $\mathbf{\Pi}_{s_t}$ in three different ways, as $\mathbf{\Pi}_{s_t} = \alpha_{s_t}\beta'$, $\mathbf{\Pi}_{s_t} = \alpha\beta'_{s_t}$, or $\mathbf{\Pi}_{s_t} = \alpha_{s_t}\beta'_{s_t}$, where α is the weight matrix and β denotes the cointegrating vector. The first decomposition assumes that the responses of money growth and output growth to deviations from the long-run equilibrium relationship between the money growth rate and the real output growth rate are regime-dependent. The second decomposition assumes that the long-run equilibrium relationship between money and output is regime-dependent. The third decomposition assumes that the responses of money and output growth to deviations from the long-run equilibrium between money and output and the long-run equilibrium relationship are both regime-dependent. In this paper, we assume $\mathbf{\Pi}_{s_t} = \alpha_{s_t}\beta'$—that is, the responses of money and output growth to deviations from the long-run equilibrium relationship are regime-dependent but not time-varying, consistent with Hafer and Jansen (1991) who provide evidence of a long-run relationship between money and output. However, testing the existence of a long-run equilibrium between money and output in a time-varying framework is a productive area for future research.

According to equations (3) and (4), the two structural shocks follow univariate normal distributions in each regime. To make the assumption as flexible as possible, we allow the variances to be different across regimes, thus allowing homoscedasticity within each regime, but heteroscedasticity across regimes. Regarding the copula corresponding to each regime, we use the following copula functions. In one regime, we assume the Frank (1979) copula

$$C(u, v) = -\delta \ln \left(\left[1 - e^{-\delta} - (1 - e^{-\delta u}) \left(1 - e^{-\delta v} \right) \right] / \left(1 - e^{-\delta} \right) \right),$$

where $u = F_1(\epsilon_{\Delta m,t})$ and $v = F_2(\epsilon_{\Delta y,t})$. In the other regime, we use the BB1 (Joe (1997)) copula

$$C(u, v) = \left(1 + \left[\left(u^{-\theta} - 1 \right)^{\vartheta} + \left(v^{-\theta} - 1 \right)^{\vartheta} \right]^{\frac{1}{\vartheta}} \right)^{-\frac{1}{\theta}}; \quad \vartheta \geq 1; \quad \theta \geq 0.$$

The BB1 copula accommodates both upper tail dependence and lower tail dependence. Let's define

$$\begin{aligned} \lambda_U &= \lim_{k \to 1} Pr \left[\epsilon_{\Delta m,t} > F_1^{-1}(k) | \epsilon_{\Delta y,t} > F_2^{-1}(k) \right] \\ &= \lim_{k \to 1} Pr \left[\epsilon_{\Delta y,t} > F_2^{-1}(k) | \epsilon_{\Delta m,t} > F_1^{-1}(k) \right]. \end{aligned}$$

When λ_U is between 0 and 1, one would say the copula has upper tail dependence and no upper tail dependence if $\lambda_U = 0$, see Joe (1997). It is important to know that the concept of upper tail dependence is still built on the concept of dependence. If λ_U is bigger than zero, there is a positive probability that one of $\epsilon_{\Delta m,t}$, $\epsilon_{\Delta y,t}$ takes values greater than k given that the other is greater than k for k arbitrarily close to 1. In this sense, λ_U quantifies the probability of a larger output growth shock since the money supply shock is larger.

On the other hand, let's define

$$\begin{aligned} \lambda_L &= \lim_{k \to 0} Pr \left[\epsilon_{\Delta m,t} < F_1^{-1}(k) | \epsilon_{\Delta y,t} < F_2^{-1}(k) \right] \\ &= \lim_{k \to 0} Pr \left[\epsilon_{\Delta y,t} < F_2^{-1}(k) | \epsilon_{\Delta m,t} < F_1^{-1}(k) \right]. \end{aligned}$$

In a similar fashion to λ_U, λ_L quantifies the probability of having a smaller output growth shock, given that the money supply shock is smaller.

The BB1 copula accommodates both upper tail dependence and lower tail dependence. Notably, in the case of BB1, we have

$$\lambda_U = 2 - 2^{\frac{1}{\vartheta}}; \ \lambda_L = 2^{-\frac{1}{\vartheta\theta}}.$$

On the other hand, the Frank copula doesn't accommodate tail dependence. However, it is the only common copula that could capture a negative dependence between random variables.

Thus, our empirical framework is very flexible. We address the possible changes in the dependence structure between money and output growth as there

TABLE 2. Parameter estimates of the copula functions

Parameter	Estimate (p-value)
δ	4.132 (0.000)
ϑ	2.208 (0.000)
θ	0.668 (0.071)
λ_U	0.631
λ_L	0.360

Note(s): Sample period, monthly data: January 1967–January 2020.
Numbers in parentheses are p-values.

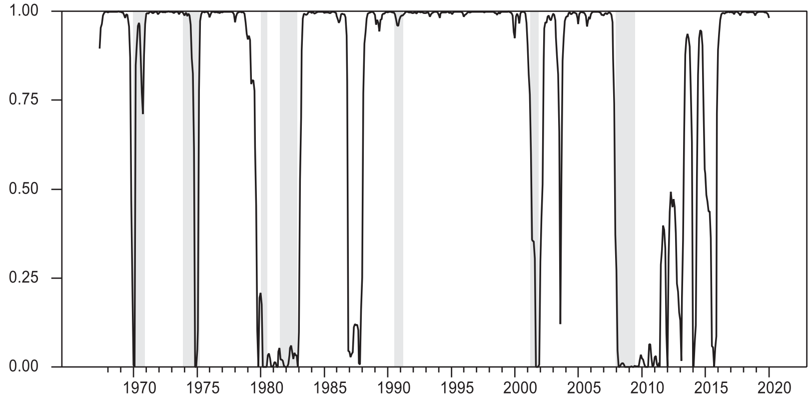
are changes in the macroeconomic environment. In doing so, we allow for negative dependence, positive dependence, and tail dependence. It is also to be noted that we could have assumed the BB1 copula for both regimes. In that case, however, we experience convergence problems in the estimation of the model. This is the reason that we assume the Frank copula for one of the regimes instead of assuming two BB1 copulas.

4. EMPIRICAL EVIDENCE

The estimation of the Markov-switching copula VEC model is carried out by full information maximum likelihood, with all the parameter estimates obtained simultaneously by maximizing the logged joint density function built on the copula function and its density function. The estimates corresponding to the dependence structure are reported in Table 2. As can be seen, the Frank copula δ is positive and statistically significant in its regime ($\hat{\delta} = 4.132$ with a p-value of 0.000). It implies that there is a positive dependence between the money and output growth shocks; that is, it is likely to observe a large (small) output growth shock when there is a large (small) money growth shock. In the other regime with the BB1 copula, as can be seen in Table 2, there is upper tail dependence, meaning that there is a tendency of money growth and output growth to boom together. Moreover, a nonzero λ_L suggests that there is also a tendency of money growth and output growth to crash together. The interesting finding here is that λ_U is larger than λ_L, suggesting that money growth and output growth are more likely to boom together than to crash together.

To get a better understanding of this relationship, we refer to the Frank copula regime as regime 1 and to that of the BB1 copula as regime 2, and in Figures 3 and 4 provide the smoothed probabilities of each regime (in the first panel) and the simulated distribution (based on 100,000 draws of the corresponding estimated copula) of the money and output growth shocks (in the second panel). As can be seen in the upper panel of Figure 3, the US economy has been in regime 1 frequently since 1967. Moreover, according to the lower panel of Figure 3, there is a positive (but weak) dependence between money growth and output growth shocks. In the upper panel of Figure 4, we see that regime 2 often shows up during

https://doi.org/10.1017/S1365100521000250 Published online by Cambridge University Press



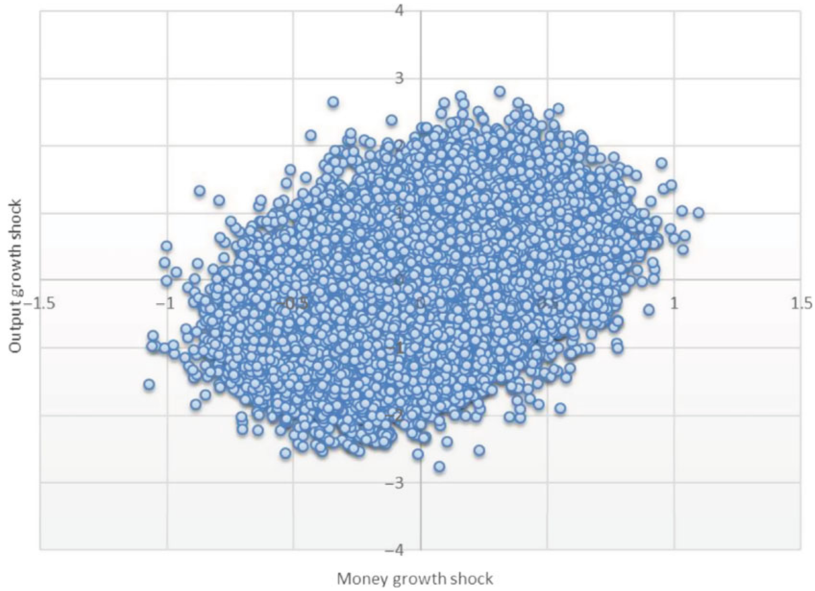
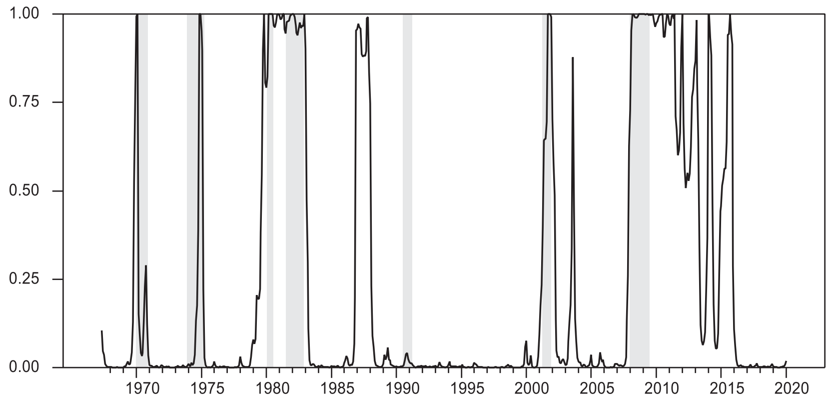


FIGURE 3. Probability of regime 1 and simulated distribution (100,000 draws) with Divisia M4.

economic contractions. Moreover, the lower panel of Figure 4 suggests a stronger positive dependence between money growth and economic growth. Especially, the strong upper tail dependence is well captured, consistent with our earlier interpretation based on the λ_U and λ_L estimates, meaning that the probability of a larger output growth shock is higher, given a large money growth shock.

Overall, we find a positive dependence between money growth and output growth and that this dependence gets even stronger during economic contractions. Moreover, upper tail dependence and lower tail dependence are also found during economic contractions.

https://doi.org/10.1017/S1365100521000250 Published online by Cambridge University Press



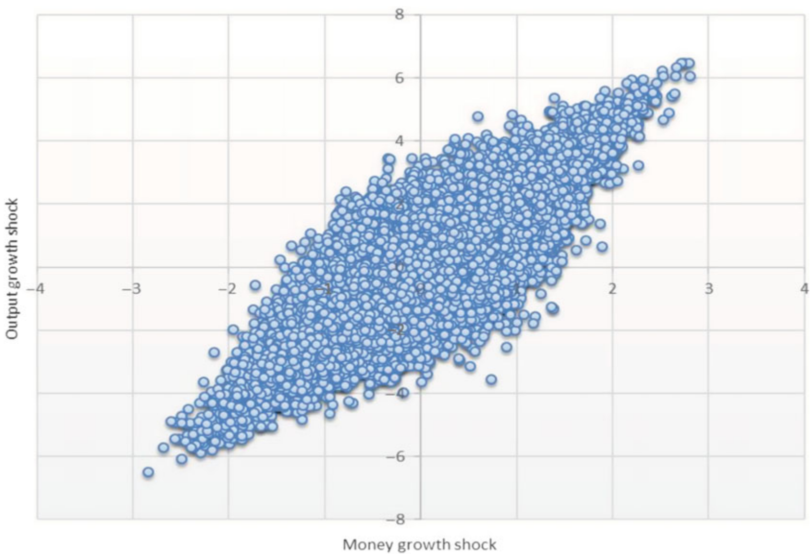


FIGURE 4. Probability of regime 2 and simulated distribution (100,000 draws) with Divisia M4.

5. EVIDENCE WITH NARROWER DIVISIA AGGREGATES

In this section, we investigate the robustness of our results to the use of narrower Divisia monetary aggregates—the CFS Divisia M1, Divisia M2, and Divisia M3 aggregates. The optimal number of lags is again chosen based on the BIC assuming a maximum lag of 24. We do not find cointegration between Divisia M1 and output and between Divisia M2 and output and in these two cases, we estimate the VAR without an error correction term. In the case with the Divisia M3 aggregate, we find one cointegrating vector and so we estimate a Markov-switching copula VEC model.

https://doi.org/10.1017/S1365100521000250 Published online by Cambridge University Press

TABLE 3. Parameter estimates of the copula functions with Divisia M1, Divisia M2, and Divisia M3

Parameter	Estimate (p-value)
	Divisia M1
δ	0.000 (0.999)
ϑ	1.000 (0.000)
θ	0.213 (0.014)
λ_U	0.000
λ_L	0.863
	Divisia M2
δ	2.436 (0.081)
ϑ	1.000 (0.000)
θ	0.083 (0.707)
λ_U	0.000
λ_L	0.944
	Divisia M3
δ	8.353 (0.000)
ϑ	1.000 (0.000)
θ	0.267 (0.057)
λ_U	0.000
λ_L	0.831

Note(s): Sample period, monthly data: January 1967–January 2020.
Numbers in parentheses are p-values.

We report the copula function parameter estimates with each of the Divisia M1, Divisia M2, and Divisia M3 aggregates in Table 3, in the same fashion as those in Table 2 with the Divisia M4 aggregate. We also plot the associated smoothed probabilities and simulated shocks distribution in Figures 5 and 6 for Divisia M1, Figures 7 and 8 for Divisia M2, and Figures 9 and 10 for Divisia M3. As can be seen in Figure 5, there is no dependence relationship between Divisia M1 money growth and the real output growth rate. This result is consistent with the very small value of $\hat{\delta}$ reported in panel A of Table 3 ($\hat{\delta} = 0.000$ with a p-value of 0.999). Figure 6 indicates that there is a weak positive dependence relationship between Divisia M1 growth and economic growth, but it is difficult to see the lower tail dependence suggested by λ_L, in such a weak overall dependence structure. Similarly, Figures 7 and 8 suggest that there is not much dependence between Divisia M2 growth and output growth. However, in Figures 9 and 10, we find that Divisia M3 growth has a significant positive dependence relationship

https://doi.org/10.1017/S1365100521000250 Published online by Cambridge University Press

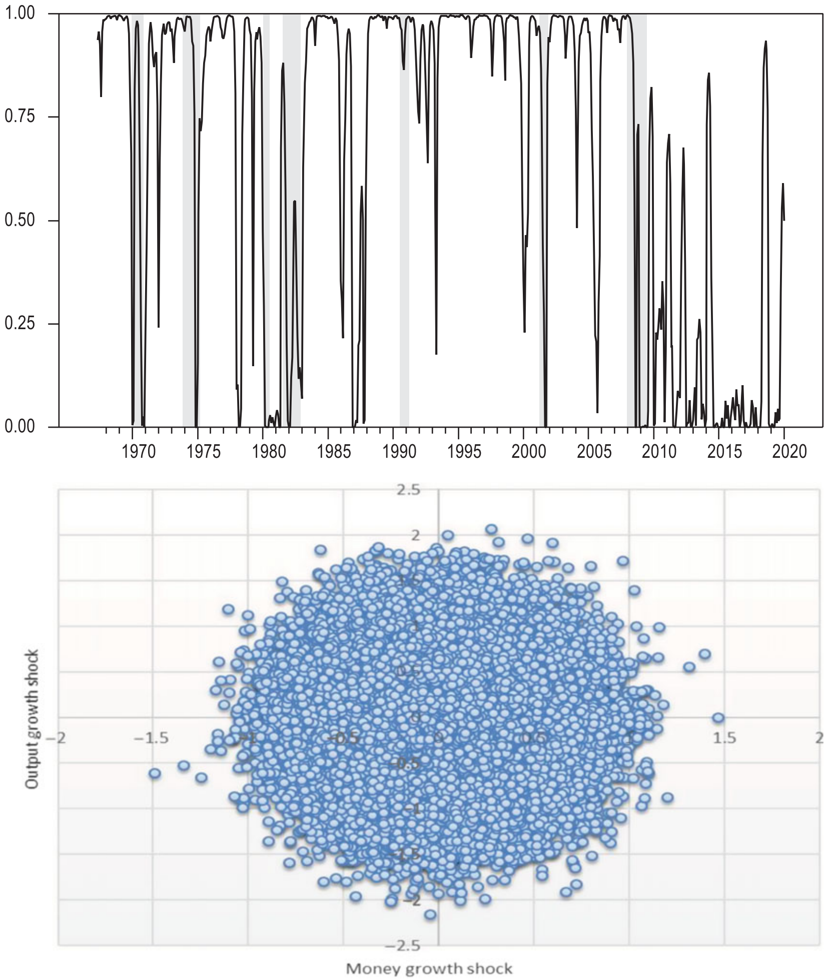


FIGURE 5. Probability of regime 1 and simulated distribution (100,000 draws) with Divisia M1.

with output growth in regime 1, the most frequent state in the US economy since 1967. In Figure 10, we observe that the positive dependence between Divisia M3 growth and output growth is weak in regime 2.

6. GENERALIZED IMPULSE RESPONSE FUNCTIONS

In order to quantify the dynamic impact of money growth shocks on economic growth in each regime, we follow Koop et al. (1996) and calculate

https://doi.org/10.1017/S1365100521000250 Published online by Cambridge University Press

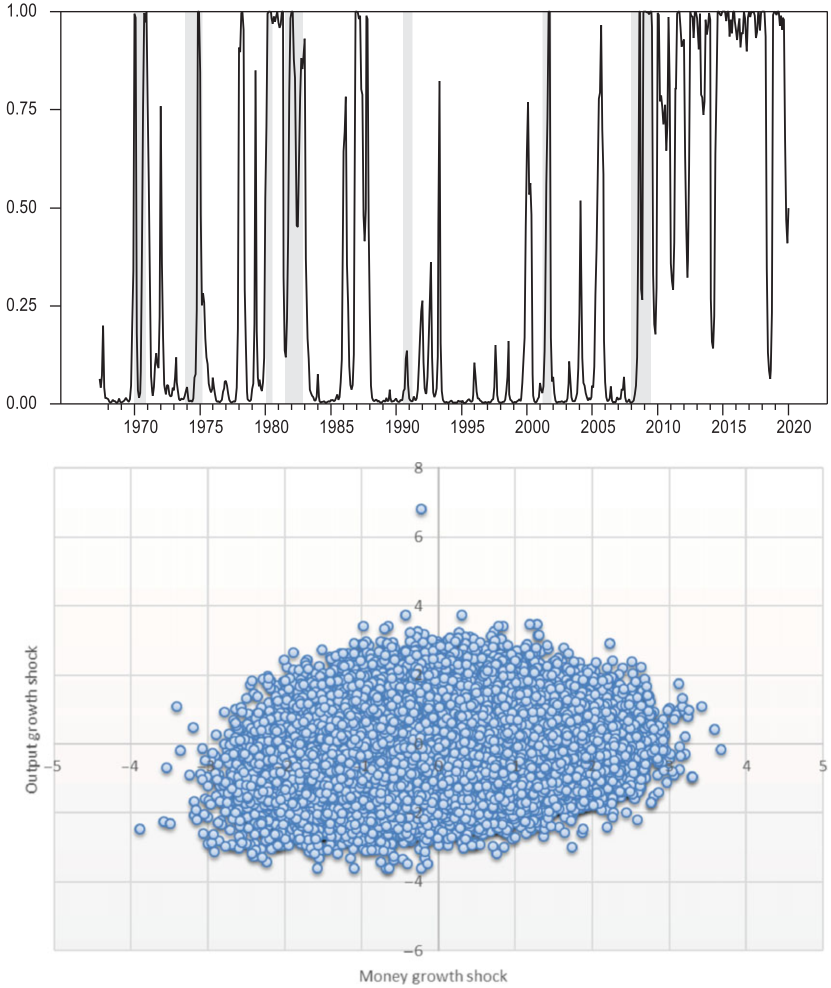


FIGURE 6. Probability of regime 2 and simulated distribution (100,000 draws) with Divisia M1.

generalized impulse response functions. Our generalized impulse response function is defined by

$$E(z_{t+k}|\bar{\Omega}_{t+k-1}, \phi) - E(z_{t+k}|\bar{\Omega}_{t+k-1}), \tag{5}$$

where $(z_{t+k}|\bar{\Omega}_{t+k-1}, \phi)$ is the predicted value of z_{t+k} based on a simulated information set $\bar{\Omega}_{t+k-1}$ holding the regime constant, where

$$\bar{\Omega}_{t+k-1} = \left\{(z_{t+k-1}|\bar{\Omega}_{t+k-2}, \phi), ..., (z_{t+1}|\bar{\Omega}_{t}, \phi), (z_{t}|\Omega_{t-1}, \phi)\right\} \cup \Omega_{t-1}.$$

https://doi.org/10.1017/S1365100521000250 Published online by Cambridge University Press

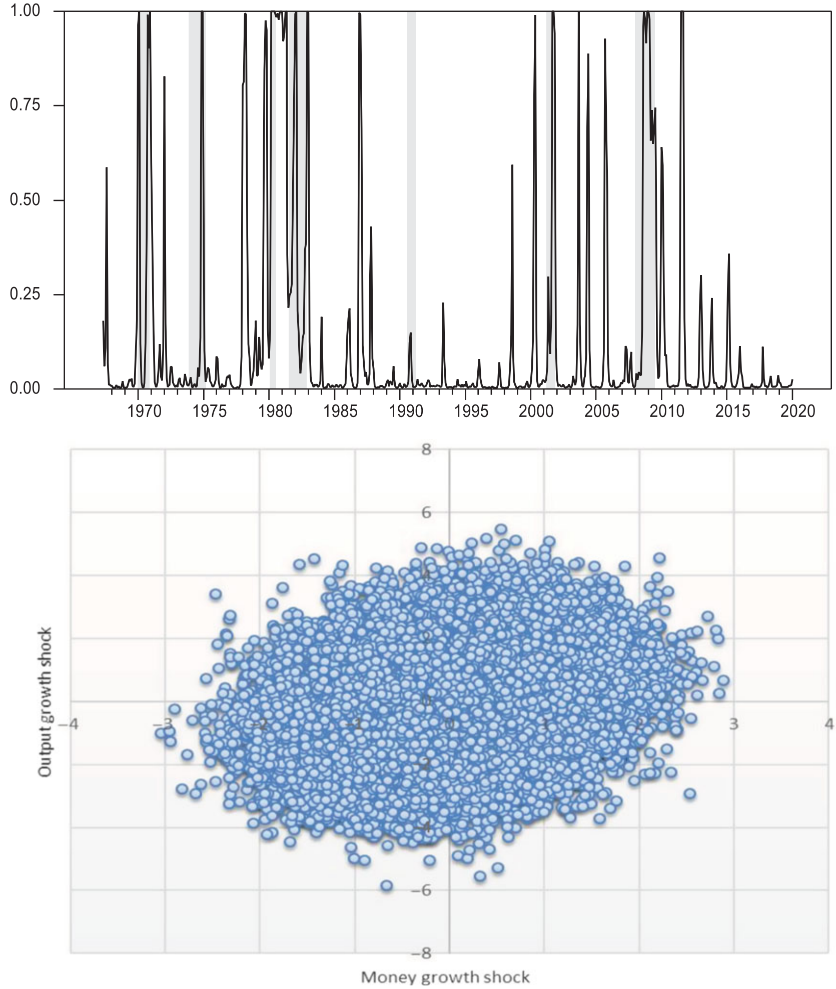


FIGURE 7. Probability of regime 1 and simulated distribution (100,000 draws) with Divisia M2.

The generalized impulse response function (5) gives the difference on average between the predicted value of z_{t+k} with and without the shock ϕ. The difference then measures the response of economic growth at time $t+k$ following a money growth shock at time t. As pointed out by Koop et al. (1996), the difference depends on the data history or the initial information set. Therefore, we choose different histories randomly in our data to initialize the calculation for an average difference.

After estimating the model, we calculate the generalized impulse response function for each regime as follows (in this case, e.g. of regime 1):

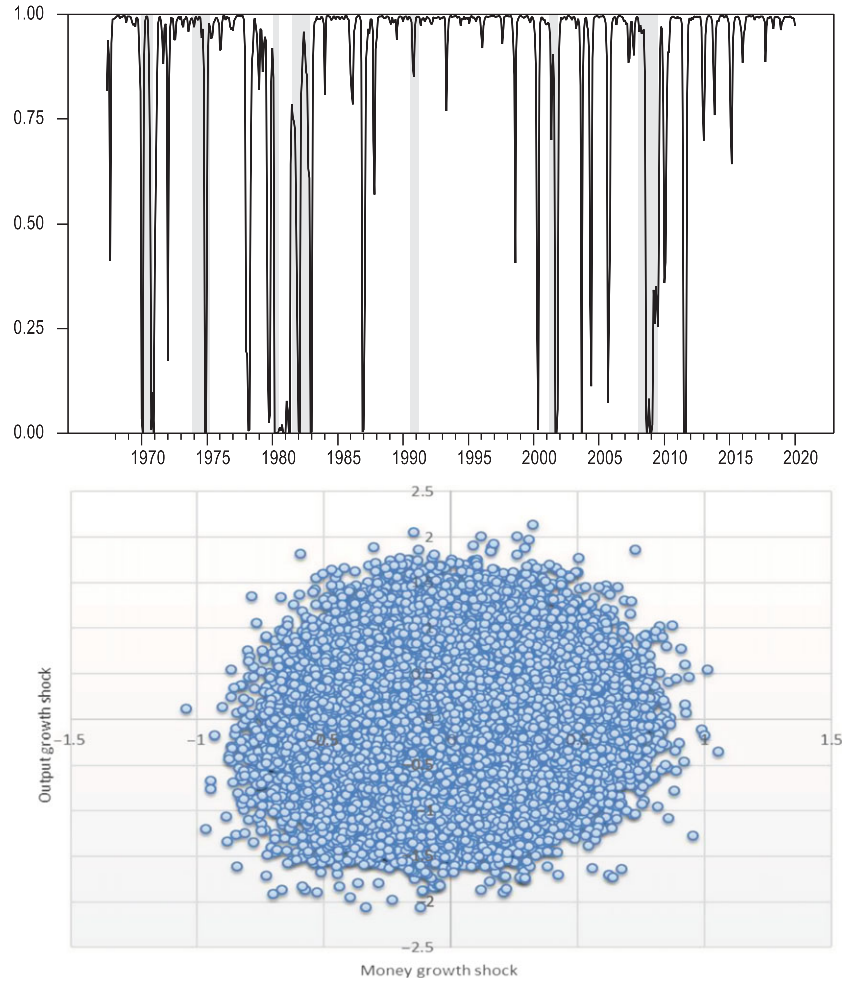


FIGURE 8. Probability of regime 2 and simulated distribution (100,000 draws) with Divisia M2.

- *Step 1*. We randomly choose a starting time period t in our data. We then calculate $\boldsymbol{H}_{s_t=1} = \begin{bmatrix} h_{\Delta m, s_t=1} & 0 \\ 0 & h_{\Delta y, s_t=1} \end{bmatrix}$.
- *Step 2*. We draw $\boldsymbol{\epsilon}_{i,s_i=1}$ where $i = t$ from a multivariate normal distribution with zero mean and covariance matrix $\boldsymbol{H}_{s_t=1}$, which is obtained from *Step 1*.
- *Step 3*. Repeat *Step 1* and *Step 2* recursively for time period i where $i \in [t+1, ..., t+k]$ and we hold $s_i = 1$ for all $i \in [t+1, ..., t+k]$.
- *Step 4*. $(z_i|\bar{\Omega}_{i-1}), i \in [t, \cdots, t+k]$ is constructed based on the recursive VEC system given $\boldsymbol{\epsilon}_{i,s_t}$ from the previous three steps.

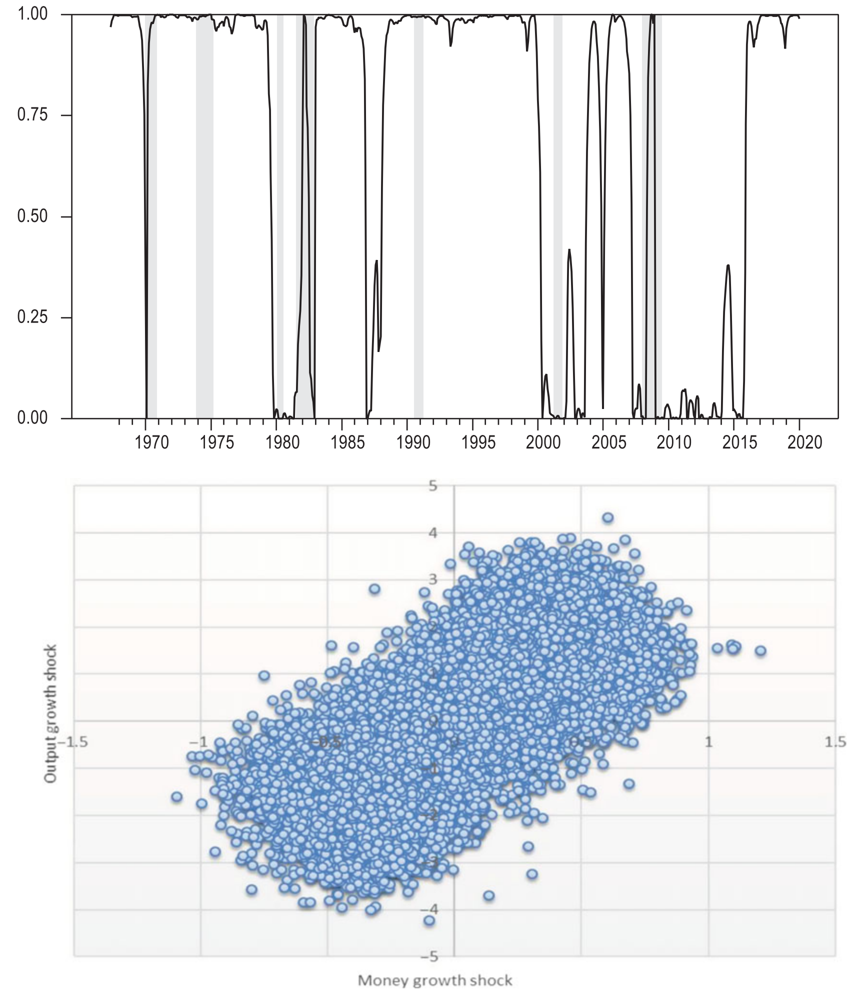


FIGURE 9. Probability of regime 1 and simulated distribution (100,000 draws) with Divisia M3.

- *Step 5*. We inject the money growth shock ϕ into the system at time t for $(z_i|\bar{\Omega}_{i-1}, \phi), i \in [t, \cdots, t+k]$. A new vector of error terms $\hat{\boldsymbol{\epsilon}}_t$ for time t only is constructed by

$$\hat{\boldsymbol{\epsilon}}_{t,s_t=1} = \boldsymbol{\epsilon}_{t,s_t=1} + (\phi, 0)',$$

  where $\boldsymbol{\epsilon}_{t,s_t=1}$ is from *Step 2*. We then redo *Step 4* for $(z_{i+k}|\bar{\Omega}_{i-1}, \phi), i \in [t, \cdots, t+k]$ with the error terms $\hat{\boldsymbol{\epsilon}}_{t,s_t=1}$ at time period t.
- *Step 6*. Take the difference between $(z_i|\bar{\Omega}_{i-1}, \phi)$ and $(z_i|\bar{\Omega}_{i-1})$ for $i \in [t, \cdots, t+k]$.

https://doi.org/10.1017/S1365100521000250 Published online by Cambridge University Press

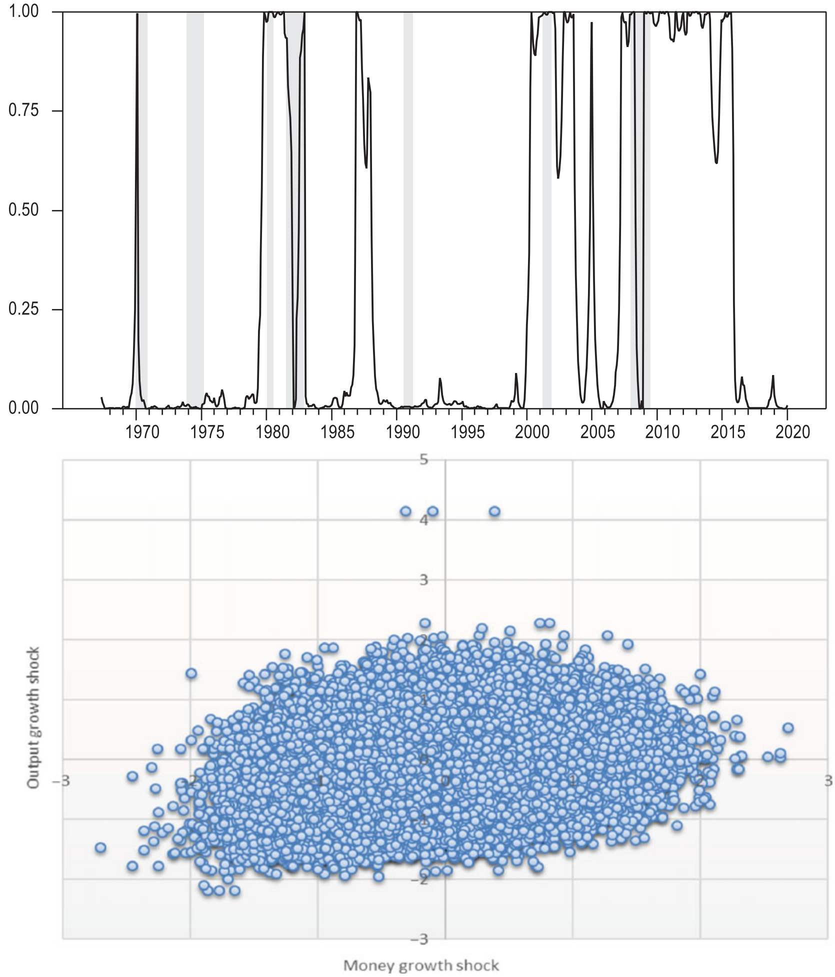


FIGURE 10. Probability of regime 2 and simulated distribution (100,000 draws) with Divisa M3.

- *Step 7*. Average the difference in Step 6 across $m = 100$ repetitions of Steps 2–6. In other words, we randomly choose 100 time periods to initialize the calculation 100 times for its average.

We use the unconditional standard deviation of the money growth rate for ϕ. The generalized impulse response functions for each of the Divisia M1, Divisia M2, Divisia M3, and Divisia M4 monetary aggregates are shown in Figure 11 for regime 1 and Figure 12 for regime 2. We obtain the error bands by using the Random Walk Metropolis–Hastings algorithm. We use 3000 burn-in draws and analyze another 5000 draws after the burn-in phase. It follows that 100 simulations are implemented for each accepted draw based on Steps 1–7.

https://doi.org/10.1017/S1365100521000250 Published online by Cambridge University Press

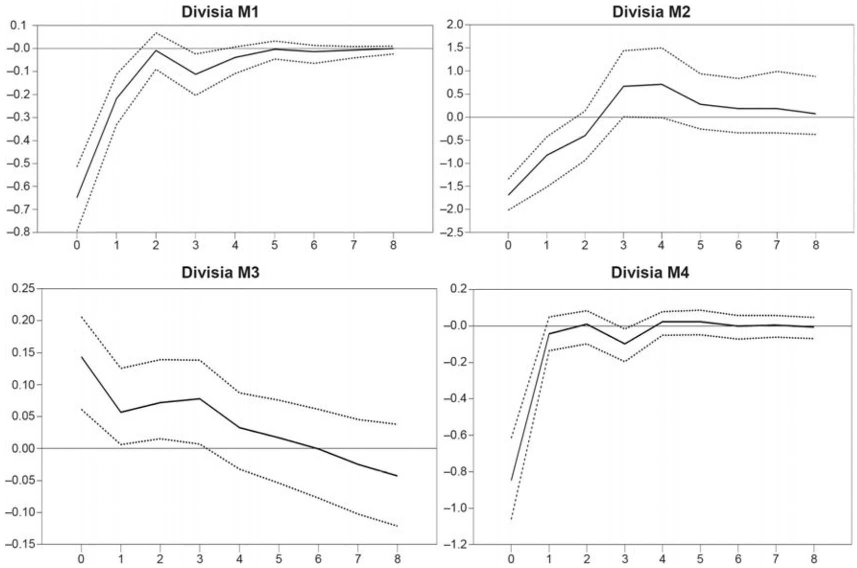


FIGURE 11. Response of output growth to a positive money growth shock based on the GIRF in regime 1.

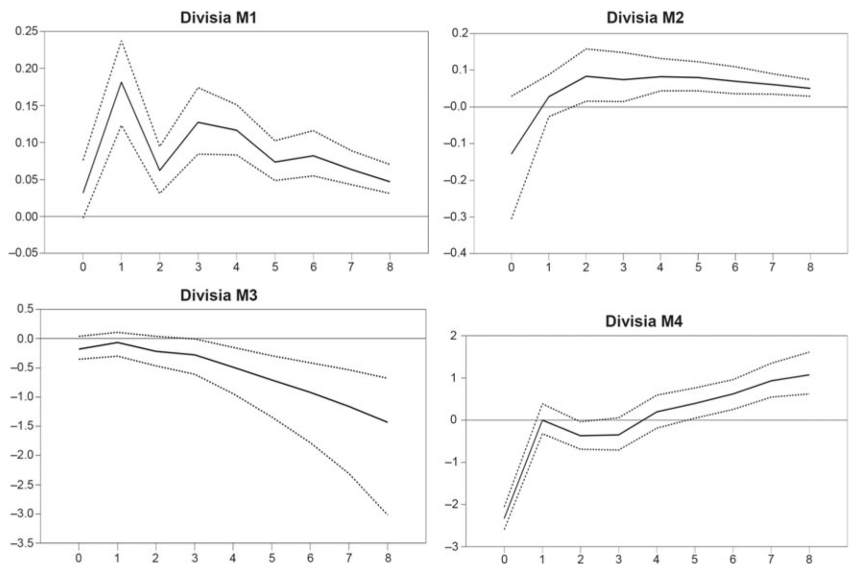


FIGURE 12. Response of output growth to a positive money growth shock based on the GIRF in regime 2.

https://doi.org/10.1017/S1365100521000250 Published online by Cambridge University Press

With our preferred Divisia M4 monetary aggregate, we find no effect in regime 1. In regime 2, however, a positive money growth shock leads to an increase in economic growth about five quarters after the shock, consistent with Friedman (1961) who argues that monetary actions affect economic conditions only after a lag that is both long and variable.

7. CONCLUSION

In the context of a bivariate, Markov-switching, identified structural VEC model with copulas, we investigate the dependence structure between money and output. We use monthly data for the USA (over the period from January 1967 to January 2020) and the CFS Divisia monetary data documented in Barnett et al. (2013). We find a positive dependence between the CFS Divisia M4 monetary aggregate and output and that this dependence is asymmetric over contractions and expansions in economic activity, being significantly stronger during business cycle contractions.

REFERENCES

Bai, J. and S. Ng (2005) Tests for skewness, kurtosis, and normality for time series data. *Journal of Business and Economic Statistics* 23, 49–60.

Balcilar, M., R. Gupta and S. M. Miller (2015) Regime switching model of US crude oil and stock market prices: 1859 to 2013. *Energy Economics* 49, 317–327.

Barnett, W. A. (1978) The user cost of money. *Economics Letters* 1, 145–149.

Barnett, W. A. (1980) Economic monetary aggregates: An application of index number and aggregation theory. *Journal of Econometrics* 14, 11–48.

Barnett, W. A. (2016) Friedman and Divisia monetary measures. In: R. A. Cord and D. Hammond (eds.), *Milton Friedman: Contributions to Economics and Public Policy*, pp. 265–291. Oxford: Oxford University Press.

Barnett, W. A. and M. Chauvet (2011) How better monetary statistics could have signaled the financial crisis. *Journal of Econometrics* 161, 6–23.

Barnett, W. A., J. Liu, R. S. Mattson and J. van den Noort (2013) The new CFS Divisia monetary aggregates: Design, construction, and data sources. *Open Economies Review* 24, 101–124.

Belongia, M. T. and P. N. Ireland (2014) The Barnett critique after three decades: A new Keynesian analysis. *Journal of Econometrics* 183, 5–21.

Belongia, M. T. and P. N. Ireland (2015) Interest rates and money in the measurement of monetary policy. *Journal of Business and Economic Statistics* 33, 255–269.

Belongia, M. T. and P. N. Ireland (2016) Money and output: Friedman and Schwartz revisited. *Journal of Money, Credit and Banking* 48, 1223–1266.

Belongia, M. T. and P. N. Ireland (2018) Targeting constant money growth at the zero lower bound. *International Journal of Central Banking* 14, 159–204.

Bonett, D. G. and T. A. Wright (2000) Sample size requirements for estimating pearson, kendall and spearman correlations. *Psychometrika* 65, 23–28.

Dery, C. and A. Serletis (forthcoming) Interest rates, money, and economic activity. *Macroeconomic Dynamics*. https://www.cambridge.org/core/journals/macroeconomic-dynamics/article/abs/interest-rates-money-and-economic-activity/3B2A01D7DBBD9F6BDBFD9F91531E7AB5#cited-by-modal

Ellington, M. (2018) The case for Divisia monetary statistics: A Bayesian time-varying approach. *Journal of Economic Dynamics and Control* 96, 26–41.

https://doi.org/10.1017/S1365100521000250 Published online by Cambridge University Press

Frank, M. J. (1979) On the simultaneous associativity of $F(x, y)$ and $x + y - F(x, y)$. *Aequationes Mathematicae* 19, 194–226.

Friedman, M. (1961) The lag effect of monetary policy. *Journal of Political Economy* 69, 447–466.

Hafer, R. W. and D. W. Jansen (1991) The demand for money in the United States: Evidence from cointegration tests. *Journal of Money, Credit and Banking* 23, 155–168.

Hamilton, J. D. (1988) A neoclassical model of unemployment and the business cycle. *Journal of Political Economy* 96, 593–617.

Hamilton, J. D. (1989) A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica* 57, 357–384.

Hendrickson, J. R. (2014) Redundancy or mismeasurement? A reappraisal of money. *Macroeconomic Dynamics* 18, 1437–1465.

Jadidzadeh, A. and A. Serletis (2019) The demand for assets and optimal monetary aggregation. *Journal of Money, Credit and Banking* 51, 929–952.

Joe, H. (1997) *Multivariate Models and Dependence Concepts*. London: Chapman & Hall.

Johansen, S. (1988) Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control* 12, 231–254.

Koop G., M. H. Pesaran and S. M. Potter (1996) Impulse response analysis in nonlinear multivariate models. *Journal of Econometrics* 74, 119–147.

Serletis, A. and P. Gogas (2014) Divisia monetary aggregates, the great ratios, and classical money demand functions. *Journal of Money, Credit and Banking* 46, 229–241.

Serletis, A. and S. Rahman (2013) The case for Divisia money targeting. *Macroeconomic Dynamics* 17, 1638–1658.

Serletis, A. and A. Shahmoradi (2006) Velocity and the variability of money growth: Evidence from a VARMA, GARCH-M model. *Macroeconomic Dynamics* 10, 652–666.

Serletis, A. and L. Xu (2020) Money supply volatility and the macroeconomy. *Macroeconomic Dynamics* 24, 1392–1402.

Sklar, A. (1959). Fonctions de repartition an dimensions et leurs marges. *Publ Inst Statist Univ Paris* 8, 449–460.