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A NOTE ON THE UNIQUENESS OF STEADY-STATE EQUILIBRIUM UNDER STATE-DEPENDENT WAGE SETTING

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Does wage setting exhibit strategic complementarity and produce multiple equilibria? This study constructs a discrete-time New Keynesian model in which households choose the timing of their wage adjustments endogenously subject to fixed wage-setting costs. I explore steady-state equilibrium of the state-dependent wage-setting model both analytically and numerically. For reasonable parameter values, complementarity in wage setting is weak and the steady-state equilibrium is unique.

Keywords: State-dependent Wage Setting, New Keynesian Model, Multiple Equilibria, Strategic Complementarity

1. INTRODUCTION

The New Keynesian literature (e.g., Erceg et al. (2000); Christiano et al. (2005)) finds that the micro-level nominal wage stickiness has important implications for the aggregate economy. However, wage adjustments have not been comprehensively analyzed. Existing models typically assume time-dependent wage setting (e.g., Taylor (1980); Calvo (1983)) and fix the timing of wage setting exogenously. In contrast, some empirical studies find state dependency in wage setting by providing evidence that macroeconomic conditions affect the frequency of wage changes.[1]

To fill this gap, I develop a New Keynesian model with state-dependent wage setting and analyze the possibility of multiple equilibria.[2] For price setting, the uniqueness of equilibrium depends on a time horizon. For an essentially static model similar to Blanchard and Kiyotaki (1987), Ball and Romer (1991) show

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that price setting is characterized by strategic complementarity and multiple equilibria often exist. By contrast, for the seminal dynamic state-dependent pricing model by Dotsey et al. (1999), John and Wolman (2004, 2008) show that multiple equilibria do not exist under plausible parameterization. However, these results may not carry over to wage setting. As Huang and Liu (2002) point out under time-dependent setting, households' incentive to stabilize their relative wage is often stronger than firms' incentive to stabilize their relative price.[3] Hence, multiple equilibria might be more likely for wage than price setting.

The model used here differs from a standard New Keynesian model in two respects. First, like price-setting costs in the Dotsey et al. (1999) model, fixed wage-setting costs are stochastic and heterogeneous across households, generating staggered nominal wage adjustments endogenously. Second, to focus on wage setting, perfect competition and flexible prices are assumed for the goods market.

I first analytically explore steady-state equilibrium under some restricted but empirically relevant parameterization. I find that for a high discount factor close to one, which is empirically relevant, wage setting is characterized by weak complementarity, and multiple sticky-wage equilibria are unlikely to exist. Numerical analysis then finds that the uniqueness result holds under more general parameterization and several extensions.

The analysis here closely follows John and Wolman (2004, 2008)'s work on the Dotsey et al. (1999) price-setting model. Relevant equations are similar between price and wage setting. There are also important differences. First, the analytical investigation of John and Wolman (2004, 2008) assumes constant marginal disutility of labor, while I allow the marginal disutility to increase with labor hours. Second, I show (in the Online Appendix B) that imperfect consumption insurance does not affect the uniqueness of equilibrium, while John and Wolman (2004, 2008) assume a representative household.[4]

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 analytically examines the uniqueness of steady-state equilibrium under a particular assumption. Section 4 analyzes the issue numerically. Section 5 concludes.

2. MODEL

The model contains the central bank, the representative labor aggregator, the representative firm, and households. The Online Appendix A describes steady-state equilibrium.

2.1. Central Bank

The central bank maintains a constant growth rate of money supply: $M^s_{t+1}/M^s_t = \mu$, where M^s_t is the money supply and $\mu > 1$.

2.2. Representative Labor Aggregator

The aggregator combines differentiated labor services, $n_t(h)$, indexed by $h \in [0, 1]$, and generates composite labor $N_t^s = \left(\int_0^1 n_t(h)^{(\epsilon-1)/\epsilon} dh \right)^{(\epsilon-1)/\epsilon}$, $\epsilon > 1$. The demand for each labor service is $n_t^d(h) = (W_t(h)/W_t)^{-\epsilon} N_t^s$, where $W_t(h)$ is the nominal wage for type-h labor and $W_t \equiv \left(\int_0^1 W_t(h)^{1-\epsilon} dh \right)^{1/(1-\epsilon)}$ is the aggregate wage.

2.3. Representative Firm

The firm's production function is $Y_t = N_t^d$, where Y_t is output and N_t^d is labor input. The firm hires composite labor from the aggregator and maximizes its static profit. Prices are flexible and the aggregate nominal price P_t equals W_t, which is the nominal marginal cost.

2.4. Households

There is a continuum of households (measure one). Each household, indexed by $h \in [0, 1]$, supplies differentiated labor $n_t(h)$ and sets the wage for their labor $W_t(h)$. Wage changes incur a fixed utility cost $\varpi_t(h)$, drawn from a time-invariant continuous distribution $G(\varpi)$ with support $[0, \bar{\varpi}]$, $\bar{\varpi} < \infty$.[5] These costs are independently and identically distributed over time and across households.

A household's preference is represented by $E_0 \sum_{t=0}^{\infty} \beta^t \left[c_t(h)^{1-\sigma}/(1-\sigma) - \chi n_t^s(h)^\zeta - \varpi_t(h) I_t(h) \right]$, where $\beta \in [0, 1), \sigma, \chi > 0, \zeta \geq 1$, $c_t(h)$ is consumption, and $n_t^s(h)$ is hours worked.[6] The function $I_t(h)$ takes 1 if the household resets its wage in the period and takes 0 otherwise.

Households have identical initial wealth and access to perfect consumption insurance.[7] Thus, $c_t(h) = C_t$ for all h, where C_t is aggregate consumption. The quantity of money demanded M_t^d is given by $\ln(M_t^d/P_t) = \ln C_t$.

Let $x_t(h) \equiv W_t(h)/M_t$ ($M_t = M_t^s = M_t^d$ in equilibrium). Households supply labor hours demanded: $n_t^s(h) = n_t^d(h)$. Let λ_t be the marginal utility of consumption. Current utility relating to wage-setting decisions is then given by

$$\begin{aligned} \pi(x_t(h)) &= \lambda_t \frac{W_t(h)}{P_t} n_t^s(h) - \chi n_t^s(h)^\zeta \\ &= \lambda_t (x_t(h) C_t)^{1-\varepsilon} N_t^s - \chi \left[(x_t(h) C_t)^{-\varepsilon} N_t^s \right]^\zeta . \end{aligned} \quad \textbf{(1)}$$

I describe households' wage-setting problem recursively. Let $V(x_{t-1}(h), \varpi_t(h))$ be the value function of households, which satisfies

$$V(x_{t-1}(h), \varpi_t(h)) = \max \left\{ V^A(\varpi_t(h)), V^{NA}(x_{t-1}(h)) \right\} . \quad \textbf{(2)}$$

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First, $V^A(\varpi_t(h))$ is the value function of households when they adjust their wage in the current period and it satisfies

$$V^A(\varpi_t(h)) = -\varpi_t(h) + \max_{x_t} \left\{\pi(x_t) + \beta E\left[V(x_t, \varpi_{t+1}(h))\right]\right\}. \quad \textbf{(3)}$$

Households pay a fixed wage-setting cost $\varpi_t(h)$ and set their wage to maximize the sum of current and discounted expected utility. The optimal wage x_t^* is common to all adjusting households. Hence, the value of adjusting households is independent of the wage set in the previous period $x_{t-1}(h)$ and it depends only on $\varpi_t(h)$.

Second, $V^{NA}(x_{t-1}(h))$ is the value function of households when they keep their wage unchanged from the last period and it satisfies

$$V^{NA}(x_{t-1}(h)) = \left\{\pi\left(\frac{x_{t-1}(h)}{\mu}\right) + \beta E\left[V\left(\frac{x_{t-1}(h)}{\mu}, \varpi_{t+1}(h)\right)\right]\right\}. \quad \textbf{(4)}$$

Households keep their wage unchanged from the last period: $W_t(h) = W_{t-1}(h)$. Hence, their wage decreases relative to money stock and $x_t(h) = x_{t-1}(h)/\mu$. Households obtain current and expected discounted utility based on the decreased wage. Recall that wage-setting costs are independent over time. Thus, the value of non-adjusting households is independent of the current adjustment cost $\varpi_t(h)$ and it depends only on $x_{t-1}(h)$.

Since adjusting households set the same wage, at the start of a period, a fraction $\omega_{t,q}$ of households charge x_{t-q}^*, $q = 1, ..., Q_t$. For each wage vintage, households whose cost is below a certain level, a fraction $\alpha_{t,q}$, choose to reset their wage. The number of wage vintages Q_t, which are endogenous, are finite because households eventually increase their wage under positive inflation and bounded wage-setting costs.

3. ANALYTICAL APPROACH

I focus on a situation in which wages are fixed for no more than two periods ($Q \leq 2$).[8] Wage setting is then characterized by two variables: α and x^*. First, α is the probability of wage changes in the current period, when households adjusted their wage in the previous period. Note that households certainly adjust their wage in the current period if they did not do so in the previous period. Second, x^* is the common wage rate (relative to the current money stock) chosen by adjusting households.

Let $v(\alpha; s)$ be the value of an adjusting household that sets an adjustment probability α and the associated optimal wage $x^*(\alpha; s)$, under the aggregate state s. The value is gross of the current wage-setting cost and given by

$$\begin{aligned} v(\alpha; s) = \pi(x^*(\alpha; s); s) + \beta\alpha\left\{v(\alpha; s) - E\left[\varpi \,|\, \varpi < G^{-1}(\alpha)\right]\right\} \\ + \beta(1-\alpha)\left\{\pi\left(\frac{x^*(\alpha; s)}{\mu}; s\right) + \beta\left[v(\alpha; s) - E(\varpi)\right]\right\}. \end{aligned} \quad \textbf{(5)}$$

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The first term is current utility. The other terms are expected utility. With probability α, the household will adjust its wage in the next period again and obtain $v(\alpha; s)$, while the expected wage-setting cost is $E\left[\varpi|\varpi < G^{-1}(\alpha)\right]$. With probability $(1-\alpha)$, the household will not adjust its wage in the next period and obtain $\pi(x^*(\alpha; s)/\mu)$, but the household will certainly adjust its wage in the following period, which gives $\beta\left[v(\alpha; s) - E(\varpi)\right]$.

Rearranging (5) leads to

$$v(\alpha; s) = \frac{\pi(x^*(\alpha; s); s) + \beta(1-\alpha)\pi\left(\frac{x^*(\alpha;s)}{\mu}; s\right) - \beta\alpha E\left[\varpi|\varpi < G^{-1}(\alpha)\right] - \beta^2(1-\alpha)E(\varpi)}{(1-\beta)[1+\beta(1-\alpha)]}. \tag{6}$$

This section assumes $\zeta + \sigma - 2 + \varepsilon(1-\zeta) = 0$. John and Wolman (2004, 2008)'s specification ($\sigma = \zeta = 1$) satisfies the assumption, but $\zeta > 1$ is also possible.[9] Under the assumption, the optimal wage x^* becomes independent of the aggregate state s.

LEMMA 1. *Suppose that* $\zeta + \sigma - 2 + \varepsilon(1-\zeta) = 0$. *Given* α, *the optimal wage of an adjusting household is*

$$x^*(\alpha) = \left[\frac{\varepsilon\chi\zeta}{\varepsilon - 1}\frac{1+\beta(1-\alpha)\mu^{\varepsilon\zeta}}{1+\beta(1-\alpha)\mu^{\varepsilon-1}}\right]^{\frac{1}{\varepsilon(\zeta-1)+1}} = \left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}g(\alpha,\beta)\right)^{\frac{1}{\varepsilon(\zeta-1)+1}}, \tag{7}$$

where $g(\alpha,\beta) \equiv \left[1+\beta(1-\alpha)\mu^{\varepsilon\zeta}\right] / \left[1+\beta(1-\alpha)\mu^{\varepsilon-1}\right]$.[10]

Proof. See the Online Appendix C. ■

If $\beta = 0$ or $\alpha = 1$, then $g(\alpha,\beta)^{1/[\varepsilon(\zeta-1)+1]} = 1$. Hence, households set the current static optimal wage $W^* = [\varepsilon\chi\zeta/(\varepsilon-1)]^{1/[\varepsilon(\zeta-1)+1]}M$. When $\alpha \in [0, 1)$ and $\beta \in (0, 1)$, $1 < g(\alpha,\beta)^{1/[\varepsilon(\zeta-1)+1]} < \mu$. Thus, the optimal reset wage is between W^* and the next-period static optimal wage $W^{*'} = [\varepsilon\chi\zeta/(\varepsilon-1)]^{1/[\varepsilon(\zeta-1)+1]}\mu M$. Note that $g(\alpha,\beta)$ decreases with α and increases with β. As α increases, households will be more likely to reset their wage in the next period. Thus, the reset wage becomes closer to W^*. As β increases, households put a larger weight on the future and the reset wage becomes closer to $W^{*'}$.

I focus on a pure-strategy symmetric steady-state equilibrium, in which all households choose the same constant adjusting probability.[11] The aggregate state s is then represented by the aggregate adjustment probability $\bar{\alpha}$. Aggregate consumption is

$$C(\bar{\alpha}) = \left(\frac{\varepsilon-1}{\varepsilon\chi\zeta}\right)^{\frac{1}{\varepsilon(\zeta-1)+1}} \frac{r(\bar{\alpha},1)^{\frac{1}{\varepsilon-1}}}{g(\bar{\alpha},\beta)^{\frac{1}{\varepsilon(\zeta-1)+1}}}, \tag{8}$$

where $r(\alpha,\beta) = \left[1+\beta(1-\alpha)\mu^{\varepsilon-1}\right] / [1+\beta(1-\alpha)]$, as shown in the Online Appendix C.

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Note that $N(\bar{\alpha}) = C(\bar{\alpha})$ and $\lambda(\bar{\alpha}) = C(\bar{\alpha})^{-\sigma}$. Hence, (1) can be written as

$$\pi(x^*(\alpha); s(\bar{\alpha})) = C(\bar{\alpha})^{(1-\varepsilon)\zeta} x^*(\alpha)^{-\varepsilon\zeta} \left(x^*(\alpha)^{1-\varepsilon+\varepsilon\zeta} - \chi\right), \tag{9}$$

where $\zeta + \sigma - 2 + \varepsilon(1-\zeta) = 0$ is imposed. Since $(1-\varepsilon)\zeta < 0$ and $x^*(\alpha)^{1-\varepsilon+\varepsilon\zeta} > \chi$, $\pi(x^*(\alpha); s(\bar{\alpha}))$ decreases with $C(\bar{\alpha})$. This feature is important for the following analysis.

Consider the best response of an individual household's adjustment probability α to the aggregate adjustment probability $\bar{\alpha}$:

$$\alpha(\bar{\alpha}) = \arg\max\, v(\alpha; s(\bar{\alpha})). \tag{10}$$

A pure-strategy symmetric steady-state equilibrium is a fixed point of the best-response correspondence, and any fixed point of the best-response correspondence is a pure-strategy symmetric steady-state equilibrium.

As shown in the Online Appendix C, (6) is rewritten as

$$v(\alpha; s(\bar{\alpha})) = \frac{\Pi_{SUM}(\alpha, \bar{\alpha}) - C_{SUM}(\alpha)}{1-\beta}, \tag{11}$$

where

$$\begin{aligned}\Pi_{SUM}(\alpha, \bar{\alpha}) &= \frac{\pi(x^*(\alpha); s(\bar{\alpha})) + \beta(1-\alpha)\pi\left(\frac{x^*(\alpha)}{\mu}; s(\bar{\alpha})\right)}{1+\beta(1-\alpha)} \\ &= \left(\frac{\varepsilon\zeta - \varepsilon + 1}{\varepsilon\zeta}\right)\left(\frac{D(\alpha, \beta)}{D(\bar{\alpha}, \beta)^{\zeta}}\right)^{\varepsilon-1}\left(\frac{r(\alpha, \beta)}{r(\alpha, 1)}\right),\end{aligned} \tag{12}$$

$$C_{SUM}(\alpha) = \frac{\beta\alpha E\left[\varpi | \varpi < G^{-1}(\alpha)\right] + \beta^2(1-\alpha)E(\varpi)}{1+\beta(1-\alpha)}, \tag{13}$$

and

$$D(\alpha, \beta) = \left(\frac{\varepsilon - 1}{\varepsilon\chi\zeta}\right)^{\frac{1}{\varepsilon(\zeta-1)+1}} \frac{r(\alpha, 1)^{\frac{1}{\varepsilon-1}}}{g(\alpha, \beta)^{\frac{1}{\varepsilon(\zeta-1)+1}}}. \tag{14}$$

The following lemma characterizes $D(\alpha, \beta)$, which determines aggregate consumption $C(\bar{\alpha})$ when $\alpha = \bar{\alpha}$, as shown in (8).

LEMMA 2. *For* $\alpha \in [0, 1]$, *(i) when* β *is sufficiently small,* $\partial D(\alpha, \beta)/\partial\alpha < 0$*; (ii) when* β *is sufficiently large, there exists* $\tilde{\alpha}$ *in* $(0, 1)$ *such that* $\partial D(\alpha, \beta)/\partial\alpha < 0$ *for* $\alpha < \tilde{\alpha}$ *and* $\partial D(\alpha, \beta)/\partial\alpha > 0$ *for* $\alpha > \tilde{\alpha}$*; (iii) when* β *is sufficiently large,* $D(\alpha, \beta)$ *attains its maximum on* $[0, 1]$ *at* $\alpha = 1$.

Proof. See the Online Appendix C. ■

On the one hand, a higher α means more adjusting households and they increase their wage. Thus, an increase in α tends to increase the aggregate wage (price), which lowers aggregate consumption. This is reflected in $\partial r(\alpha, 1)/\partial\alpha < 0$. On the other hand, an increase in α lowers the reset wage, which is reflected in

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$\partial g(\alpha, \beta)/\partial \alpha < 0$. This works to lower the aggregate wage (price), which increases aggregate consumption. When β is low, the first effect dominates the second because households discount the future highly and α does not substantially affect the reset wage. When β is higher, the sign of $\partial D(\alpha, \beta)/\partial \alpha$ depends on the relative strengths of the two effects. For sufficiently large β, the contribution of the second effect increases as α increases. Hence, the sign of $\partial D(\alpha, \beta)/\partial \alpha$ switches from negative to positive as α rises.

Next, the best-response correspondence (10) is analyzed. Given $\bar{\alpha}$, there could be multiple local maxima for $v(\alpha; s(\bar{\alpha}))$. First, there could be one or multiple local maxima for $\alpha \in (0, 1)$. As in John and Wolman (2004, 2008), such local maxima are called the interior arm of the best-response correspondence and defined for $\alpha \in (0, 1)$ as

$$\alpha^{int}(\bar{\alpha}) = \left\{ \alpha : \frac{\partial v(\alpha; s(\bar{\alpha}))}{\partial \alpha} = 0 \text{ and } \frac{\partial^2 v(\alpha; s(\bar{\alpha}))}{\partial \alpha^2} < 0 \right\}. \quad \mathbf{(15)}$$

Second, there could be a local maximum at $\alpha = 1$, which occurs when $\partial v(\alpha; s(\bar{\alpha}))/\partial \alpha > 0$ at $\alpha = 1$. As in John and Wolman (2004, 2008), the local maximum is called the flexible arm of the best-response correspondence.[12]

The following analysis considers a case where if $\alpha^{int}(\bar{\alpha})$ exists, $\alpha^{int}(\bar{\alpha})$ is unique for $\bar{\alpha} \in [0, 1]$.[13] The best-response correspondence is then defined as $\alpha(\bar{\alpha}) = \alpha^{int}(\bar{\alpha})$ if $v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha})) \geq v(1; s(\bar{\alpha}))$ and $\alpha(\bar{\alpha}) = 1$ if $v(1; s(\bar{\alpha})) \geq v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))$. The next lemma concerns the interior arm of the best-response correspondence.

LEMMA 3. *(i) For small β, the interior arm of the best-response correspondence exhibits complementarity everywhere; (ii) As $\beta \to 1$, the interior arm of the best-response correspondence does not exhibit complementarity at any fixed point; (iii) As $\beta \to 1$, the interior arm of the best-response correspondence has a unique fixed point α^*.*

Proof. See the Online Appendix C. ■

Complementarity means that a rise in the aggregate adjustment probability $\bar{\alpha}$ raises the individual adjustment probability α, which requires that the marginal utility of raising α, $\partial v(\alpha; s(\bar{\alpha}))/\partial \alpha$, increases with $\bar{\alpha}$. Hence, complementarity requires that

$$\frac{\partial \Pi_{SUM}(\alpha, \bar{\alpha})}{\partial \alpha} = \frac{\beta \left[\pi(x^*(\alpha); s(\bar{\alpha})) - \pi\left(\frac{x^*(\alpha)}{\mu}; s(\bar{\alpha}) \right) \right]}{[1 + \beta(1 - \alpha)]^2} \quad \mathbf{(16)}$$

increases with $\bar{\alpha}$ or $\partial^2 \Pi_{SUM}(\alpha, \bar{\alpha})/\partial \alpha \partial \bar{\alpha} > 0$.

Consider first sufficiently small β. As Lemma 2 (i) shows, a rise in $\bar{\alpha}$ decreases aggregate consumption. Hence, static utility increases in the current and next periods proportionally (see (9)). For small β, the numerator of (16) is positive because the reset wage is closer to the current static optimal wage. Thus, $\partial \Pi_{SUM}(\alpha, \bar{\alpha})/\partial \alpha$ is positive and rises with $\bar{\alpha}$ or $\partial^2 \Pi_{SUM}(\alpha, \bar{\alpha})/\partial \alpha \partial \bar{\alpha} > 0$.

Consider next β close to 1. As Lemma 2 (ii) shows, for $\bar{\alpha} < \tilde{\alpha}$, a rise in $\bar{\alpha}$ decreases aggregate consumption and increases static utility. At a fixed point, α is relatively low. Hence, the reset wage is closer to the next-period static optimal wage and the numerator of (16) is likely to be negative. Thus, $\partial \Pi_{SUM}(\alpha, \bar{\alpha})/\partial \alpha$ is negative and decreases with $\bar{\alpha}$. For $\bar{\alpha} > \tilde{\alpha}$, a rise in $\bar{\alpha}$ increases aggregate consumption and decreases static utility. Further, α is relatively high at a fixed point and the reset wage is closer to the current static optimal wage, meaning that the numerator of (16) is likely to be positive. Thus, $\partial \Pi_{SUM}(\alpha, \bar{\alpha})/\partial \alpha$ is positive and decreases with $\bar{\alpha}$. In summary, for β close to 1, it is likely that $\partial^2 \Pi_{SUM}(\alpha, \bar{\alpha})/\partial \alpha \partial \bar{\alpha} < 0$ at a fixed point and the interior arm of the best-response correspondence does not show complementarity. As shown in the Online Appendix C, a possibility of $\partial^2 \Pi_{SUM}(\alpha, \bar{\alpha})/\partial \alpha \partial \bar{\alpha} > 0$ disappears as $\beta \to 1$.

The next proposition concerns multiple equilibria, one with sticky wages ($\alpha < 1$) and the other with flexible wages ($\alpha = 1$).

PROPOSITION 4. *Let β be sufficiently large such that the interior arm has a unique fixed point denoted by α^*. Let $\hat{\alpha}$ be as defined in (37) in the Online Appendix C. (i) As $\beta \to 1$, the necessary conditions for multiple equilibria are $\alpha^* < \hat{\alpha}$ and $v(\alpha^{int}(\hat{\alpha}); s(\hat{\alpha})) < v(1; s(\hat{\alpha}))$; (ii) As $\beta \to 1$, multiple symmetric steady-state equilibria are ruled out if*

$$\frac{\varepsilon\zeta - \varepsilon + 1}{\varepsilon\zeta}\left(\frac{\varepsilon - 1}{\varepsilon\chi\zeta}\right)^{\frac{(\varepsilon-1)(1-\zeta)}{\varepsilon(\zeta-1)+1}} \times \left\{\left[\frac{\left(\frac{1+\mu^{\varepsilon-1}}{2}\right)^{-\zeta}}{\left(\frac{1+\mu^{\varepsilon\zeta}}{1+\mu^{\varepsilon-1}}\right)^{-\frac{\zeta(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}\right] - \left[\frac{\left(\frac{1+\mu^{\varepsilon-1}}{2}\right)^{-(\zeta-1)}}{\left(\frac{1+\mu^{\varepsilon\zeta}}{1+\mu^{\varepsilon-1}}\right)^{-\frac{(\zeta-1)(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}\right]\right\} > E(\varpi) \qquad \textbf{(17)}$$

or

$$E(\varpi) - C_{SUM}(\hat{\alpha}) > \frac{\varepsilon\zeta - \varepsilon + 1}{\varepsilon\zeta}\left(\frac{\varepsilon - 1}{\varepsilon\chi\zeta}\right)^{\frac{(\varepsilon-1)(1-\zeta)}{\varepsilon(\zeta-1)+1}} \times \left\{\frac{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon\zeta}}{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}\right]^{\frac{\zeta(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}{1+(1-\hat{\alpha})}\right]^{\zeta}} - \frac{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon\zeta}}{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}\right]^{\frac{(\zeta-1)(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}{1+(1-\hat{\alpha})}\right]^{\zeta-1}}\right\}. \qquad \textbf{(18)}$$

Proof. *See the Online Appendix C.* ■

To obtain sticky-wage and flexible-wage equilibria, the best-response correspondence is the interior arm first, which has a fixed point $\alpha^* < 1$, and then moves up to the flexible arm. As $\beta \to 1$, such an upward jump of the best-response correspondence is not possible when $\bar{\alpha} \geq \hat{\alpha}$. Thus, the best-response correspondence moves up at $\bar{\alpha} < \hat{\alpha}$ and α^* must be smaller than $\hat{\alpha}$. Note also that

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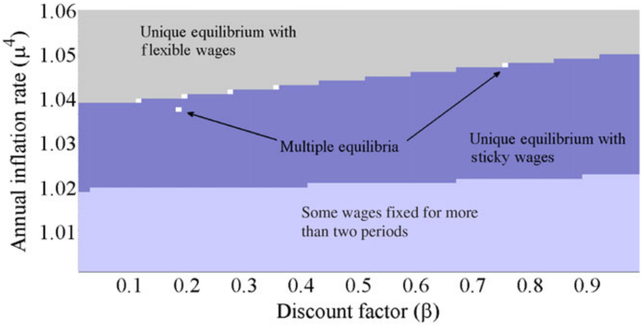

FIGURE 1. Discount factor. White areas indicate multiple equilibria.

$v(\alpha^*; s(\alpha^*)) \geq v(1; s(\alpha^*))$ because α^* is the optimum when $\bar{\alpha} = \alpha^*$. To obtain an equilibrium with flexible wages, $v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha})) < v(1; s(\bar{\alpha}))$ must hold for some $\bar{\alpha} \in [\alpha^*, \hat{\alpha})$. Since $v(1; s(\bar{\alpha}))$ increases with $\bar{\alpha}$ more rapidly than $v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))$ does for $\bar{\alpha} \in [\alpha^*, \hat{\alpha})$, a necessary condition for $v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha})) < v(1; s(\bar{\alpha}))$ for $\bar{\alpha} \in [\alpha^*, \hat{\alpha})$ is $v(\alpha^*(\hat{\alpha}); s(\hat{\alpha})) < v(1; s(\hat{\alpha}))$.

The second part of the proposition gives conditions for ruling out multiple equilibria. The first condition (17) implies that when adjustment costs are small, sticky wages cannot be an equilibrium. The second condition (18) suggests that when adjustment costs are large, flexible wages cannot be an equilibrium. These conditions rule out multiple equilibria for most long-run inflation rates. For the benchmark parameterization in Section 4, multiple equilibria are ruled out except when the annual inflation rate is 1.19–1.67%.

4. NUMERICAL APPROACH

Parameter values are as follows. One period is one-quarter. The Frisch labor supply elasticity is 1: $\zeta = 2$. The coefficient of relative risk aversion σ is 2. The elasticity of substitution for differentiated labor services ε is 2, which lies in the range considered by Huang and Liu (2002). Note that the assumption made in Section 3 ($\zeta + \sigma - 2 + \varepsilon(1 - \zeta) = 0$) holds. The disutility parameter χ is 6.75, so that when wage-setting costs are eliminated, households work for one-third of their time endowment (normalized to 1).

As in Dotsey et al. (1999), the inverse of the distribution of wage-setting cost is $G^{-1}(z) = \bar{\varpi}\ [\mathrm{atan}(bz - d\pi) + \mathrm{atan}(d\pi)] \,/\, [atan(b - d\pi) + atan(d\pi)]$ for $z \in [0, 1]$ with $b = 16$ and $d = 1$. The maximum wage-setting cost $\bar{\varpi}$ is 0.0004, so that some wages are fixed for exactly two periods when the annual inflation rate is around 2–4%.

Figure 1 shows how the number of steady-state equilibria varies with the discount factor β and the inflation rate μ. Consistent with the result in Section 3, multiple sticky-wage equilibria exist only when β is low. Specifically, this occurs in only one case: there are two sticky-wage equilibria when $\beta = 0.18$ and

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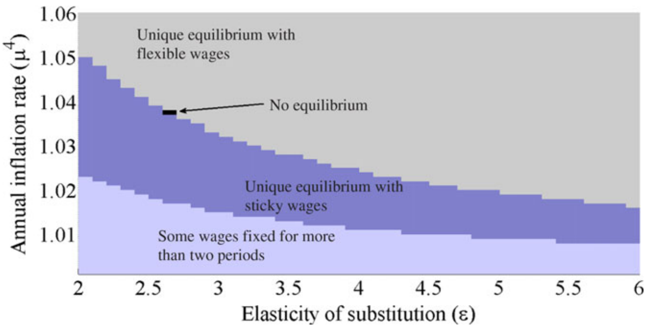

FIGURE 2. Elasticity of substitution.

$\mu^4 = 1.037$.[14] For relatively low β, several cases lead to two equilibria, one with sticky wages and the other with flexible wages, in the region between the unique sticky-wage equilibrium and the unique flexible-wage equilibrium. However, such multiple equilibria disappear for $\beta > 0.75$. Thus, the steady-state equilibrium is unique when β takes a standard value close to 1.

I next vary the elasticity of substitution ε and the inflation rate μ (Figure 2). Other parameter values are unchanged and $\beta = 0.99$. Multiple equilibria do not exist. However, there is a case in which a pure-strategy symmetric steady-state equilibrium does not exist.[15] The nonexistence case occurs when the best-response correspondence jumps down from the flexible arm to the interior arm and it occurs when $\varepsilon = 2.6$ and $\mu^4 = 1.037$.

Note that wages become more flexible as the elasticity of substitution ε increases. As ε increases, individual labor hours change with the relative wage more elastically. Hence, households adjust their wage more frequently to smooth their hours.

In summary, the numerical results in this section support the analytical results in Section 3. Multiple steady-state equilibria do not exist under typical and empirically plausible parameter values in the dynamic state-dependent wage-setting model.

5. CONCLUSION

I analyze the possibility of multiple equilibria in a New Keynesian model with state-dependent wage setting. I find that the steady-state equilibrium is likely to be unique.

There are four directions for future research. First, it would be interesting to examine the possibility of multiple equilibria when both price and wage setting are state-dependent. Second, it is an open question how state dependency in wage setting influences short-run equilibrium.[16] Third, it would be necessary to examine labor market structures other than those considered here and mechanisms that potentially generate complementarity among differentiated labor.

Lastly, while the present paper focuses on symmetric equilibrium in which all households choose the same wage-setting policy, it would be interesting to consider heterogeneity in wage setting.[17]

SUPPLEMENTARY MATERIAL

To view supplementary material for this article, please visit http://doi.org/10.1017/S1365100520000243.

NOTES

1. Taylor (1999) reviews studies for several countries and concludes that the frequency of wage adjustments increases with the rate of inflation. According to Daly et al. (2012) and Daly and Hobijn (2014), nominal wage stickiness rises in recessions in the United States.
2. Examples of state-dependent wage-setting models include Takahashi (2017) and Costain et al. (2019). The main focus of these prior studies is the short-run implications of state-dependent wage setting, and neither analyzes the uniqueness of steady-state equilibrium.
3. This result provides an explanation for the common finding (e.g., Huang and Liu (2002); Christiano et al. (2005)) that in a New Keynesian model with time-dependent setting, nominal wage stickiness generates larger short-run money nonneutrality than nominal price stickiness does.
4. For a New Keynesian model with imperfect consumption insurance, see Braun and Nakajima (2012) and Kaplan et al. (2018).
5. The conclusion of this paper does not change with labor wage-setting costs. See the Online Appendix B.
6. A log consumption utility function is assumed for $\sigma = 1$.
7. See the Online Appendix B for analysis on imperfect consumption insurance.
8. John and Wolman (2004, 2008) also focus on a case in which prices are fixed for two periods at most.
9. For example, $\zeta = \sigma = \varepsilon = 2$ satisfies the assumption.
10. Lemmas 1–3 and Proposition 4 in this paper correspond to Lemmas 1–3 and Proposition 4 of John and Wolman (2008).
11. The optimal probability α might not be unique. Thus, households might randomize their adjusting strategies. Asymmetric equilibrium also could exist.
12. Setting $\alpha = 0$ cannot be a global maximum.
13. Numerical analysis finds that this is always the case.
14. Strategic complementarity in wage setting is a necessary condition for such multiple equilibria, but not a sufficient condition.
15. John and Wolman (2004, 2008) also find a similar case in the Dotsey et al. (1999) model.
16. Such analysis may be done in a model similar to that of Ball and Romer (1991).
17. See, for example, Eijffinger et al. (in press) for analysis on heterogeneous wage adjustments in a New Keynesian framework.

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