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Demand for life annuities from married couples with a bequest motive

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Abstract

The aim of this paper is to explain the 'annuities puzzle' in greater depth by introducing the bequest motive. It will try to determine whether this motive really is a relevant feature influencing the demand for life annuities from married couples. With this aim in mind, we develop an optimization model of the utility provided by purchasing a life annuity with contingent survivor benefit or a joint survivor life annuity. Our model is based on that first put forward by Brown and Poterba (2000), to which we have added elements from other models, such as Friedman and Warshawsky's (1990) and Vidal and Lejárraga's (2004), which include the bequest motive. This will enable us to calculate the annuity equivalent wealth and the optimal percentage of wealth to annuitize in various contexts: the possibility of access to actuarially fair annuity markets, the inclusion of so-called market *imperfections*, and the assumption that couples already have part of their wealth in pre-existing life annuities. Numerical results are presented for the case of Spain. The bequest motive is found not to be a significant factor influencing the demand for annuities from couples. Indeed very few couples would be willing to purchase them once we take into account the combined effects of market imperfections, the possibility of pre-existing annuities and the bequest motive. These findings have repercussions for policy makers regulating defined contribution capitalization systems, which are complementary to defined benefit systems.

1 Introduction

Life annuities are a type of pension, marketed by insurance firms. In exchange for an initial premium, these firms commit themselves to paying certain periodic amounts until the death of the policyholder, thereby taking on the annuitant's longevity risk. Longevity risk is the risk of needing more resources due to living longer than expected. There are two basic questions underlying any study of longevity risk: (1) What does this risk depend on? Basically there are five elements to take into

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account: level of initial wealth, desired annual consumption, death probabilities, and the return on and volatility of the portfolio. (2) Is this an important risk for the retiree? It would appear that it is. Indeed, it may be thought that the risk has been underestimated, since, as a general rule in the papers that we will comment upon later, retirees are assumed to have excellent investment skills, transaction costs are not usually taken into account, the assumptions made regarding return and risk tend to be optimistic, the case of couples – which would increase longevity risk – is not considered, and contradictions appear with the standard model of utility maximization.

Using parameters for Canadian mortality and capital markets, Milevsky and Robinson (2000) compute the life and eventual probability of ruin for an individual who wishes to replicate a synthetic life annuity from an initial endowment invested in a portfolio earning a stochastic (lognormal) rate of return. They find that self-annuitization provides greater liquidity than voluntary annuitization; however, it does so at the cost of possibly outliving resources. Albrecht and Maurer (2002) arrive at similar conclusions in evaluating the risk for the German market. They calculate a personal probability of consumption shortfall and show that it is substantial, particularly for high-entry ages. Huang *et al.* (2004) compute the probability of lifetime ruin, this being the probability that a fixed retirement consumption strategy will lead to financial insolvency under stochastic investment return and lifetime distribution. They conclude that a 65-year-old retiree requires 30 times his desired annual real consumption to generate a 95% probability of sustainability – which is equivalent to a 5% probability of lifetime ruin – if the funds are invested in a well-diversified equity portfolio earning a real 7% per annum with a standard deviation of 20%. Finally, Schmeiser and Post (2005) follow the line taken by Kotlikoff and Spivak (1981), arguing that heirs might be willing to bear the shortfall risk of the retiree's self-annuitization as they might benefit from a bequest. Using German capital and annuity market data, they show that in many cases the family strategy offers enormous chance potential with low shortfall risk.[1]

The 'annuity puzzle' appears because empirical evidence shows that voluntary private individual annuity contracts are extremely rare, even though, according to Yaari (1965), individuals would be better off holding only annuitized assets in the absence of a bequest motive, or a portfolio of annuitized and traditional assets in the presence of a bequest motive. Davidoff *et al.* (2005) show that the conditions under which the purchase of annuities is optimal are not as demanding as those set out by Yaari (1965). If financial markets are complete, the only requirements are that no bequest motive exists and that the expected rate of return on annuities is greater than the return on a benchmark financial asset. Partial annuitization is optimal when the condition of there being a complete insurance market is relaxed. Mitchell *et al.* (1999) show that for life cycle consumers with plausible risk aversion, mortality uncertainty, and no annuity income, purchasing an annuity may raise lifetime

[1] The 'self-annuitization' papers mentioned are only a small sample of those that the study of alternatives to annuitization has generated in the economic, financial, and actuarial literature over the last few years. Apart from those cited, anyone interested in the subject should also see the papers by Dus *et al.* (2005), Young (2004), Gerrard *et al.* (2004), Milevsky *et al.* (2004), among many others.

expected utility, even if the expected present discounted value of payouts from the annuity is no more than 75% of the purchase price. Brown and Poterba (2000) develop an annuity valuation model for married couples and explore the potential utility gain that couples receive from access to actuarially fair annuity markets. They conclude that the utility gain from annuitization is smaller for couples than for single individuals. This is a result of the ability of couples to pool longevity risk. Previous papers have not looked into the problem of annuities from the standpoint of couples with a bequest motive.

The literature puts forward several explanations for the lack of demand for annuities from the consumer's perspective:

1 The crowding-out effect of defined benefit systems is probably the most important reason. The presence of pre-annuitized wealth gives a certain level of protection against the risk of longevity.
2 The fact that the private annuity market is not 'actuarially fair'. The load factor is very high in some annuities markets. Only part of this so-called load factor is due to adverse selection (Walliser, 2000b), the rest is attributable to overhead costs for administration, taxes, and profits. Adverse selection arises if the insurance company cannot distinguish among different risk types. In the case of annuity insurance, the company may have less information about longevity prospects than the potential annuitant. Since consumers who expect to live longer buy larger annuities, the insurance company incurs a loss if it charges a premium based on average population mortality, and higher prices may induce some consumers to drop out of the annuities market.
3 Individuals use family self-insurance. This allows them to take advantage of the possibilities of joint consumption, sharing out financial resources among the members of the family, thereby reducing the attractiveness of annuities.
4 The perception of the accumulated retirement fund in the form of a lump sum allows the individual to face the appearance of unforeseen future expenditures.
5 The different ways in which different types of pension are taxed in some countries (not always to the benefit of the life annuity option).
6 The fact that in some countries, Lopes (2003), annuities are very expensive compared to the total accumulated assets held by families. Many families cannot afford even to enter the annuities market.
7 It has been found that consumers are not sufficiently educated to make welfare-maximization decisions.
8 The existence of any motivation to leave a bequest to the heirs, although there are some quite contradictory results on this aspect in the literature, as will be seen in the following section.

A number of papers have appeared recently seeking a plausible explanation for the annuity puzzle from the point of view of the supplier. Piggott *et al.* (2003) suggest that insurers seem to be reluctant to offer lifetime annuities, perhaps because of a perception within the industry that systematic risk, in the form of breakthrough life-prolonging technical innovation, may bankrupt an insurance company with a large life annuity portfolio. Likewise, Impavido *et al.* (2003) speculate that another reason

why annuities fail to dominate over traditional assets may be related to the excessive risk borne by the providers of traditional annuity guarantees for a given institutional environment.

The aim of this paper is to shed more light on the 'annuities puzzle' from the standpoint of couples. This will be done by introducing the bequest motive, and determining whether it really is a relevant factor influencing the theoretical decision to purchase life annuities. With this aim in mind, we develop an optimization model of the utility provided by purchasing a life annuity with contingent survivor benefit or a joint survivor life annuity. Our model is based on that first put forward by Brown and Poterba (2000), to which we have added elements from other models, such as Friedman and Warshawsky's (1990) and Vidal and Lejárraga's (2004), which include the bequest motive. So as to make the model as realistic as possible, we also take into account that the couple may already have a pre-existing annuity and that the annuity market may not be actuarially fair. Our main findings are that the bequest motive is not a significant factor influencing the demand for annuities from couples and that very few couples would be willing to purchase them once we take into account the combined effect of market imperfections, the possibility of pre-existing annuities, and the bequest motive. These results have important repercussions for policy makers regulating defined contribution capitalization systems, which are complementary to defined benefit systems.

After this introduction, the paper is divided into three sections. In Section 2 we give a brief overview of the main papers, connecting the bequest motive to the valuation of life annuities. We also develop a model for the optimal consumption flow which will maximize the couple's expected utility. This model takes into account the bequest motive, according to the couple's various preferences for consumption and their attitude towards risk, for different types of pension: a life annuity with contingent survivor benefit and a joint survivor life annuity. In Section 3 we analyze welfare by calculating the annuity equivalent wealth and the optimal percentage of wealth to annuitize in different contexts: (i) the possibility of access to an actuarially fair annuities market (for real and nominal annuities); (ii) the incorporation of so-called market *imperfections*; and (iii) the hypothesis that the couple already have part of their wealth in pre-existing life annuities. Numerical results are presented for the Spanish case. Section 4 presents a summary of our findings and considers options for future research.

2 The model

One of the reasons why the existence of a bequest motive[2] has not normally been included in the analysis of welfare could well be in order to maintain the analytical simplicity of the model. Also, however, Benitez-Silva (2003) points out that it could be due to the lack of consensus about the relevance of the bequest motive in an individual's decision as to whether to purchase annuities, and about how to model this bequest factor.

[2] A good summary of the bequest motive can be found in the paper by Impavido *et al.* (2003).

Yaari (1965) points out that, in a life-cycle framework, consumers will be better off holding only annuity assets if they have no bequest motives; whereas, when they have a bequest motive, they will hold a portfolio of annuities and bequeathable assets so that the marginal utility of bequests and consumption are the same. Lewis (1989) extends Yaari's life insurance framework to include explicitly the preferences of other household members. Life insurance is demanded by the dependents that face an income stream contingent on the breadwinner's lifetime. He concludes that any change in a beneficiary's expected bequest is almost fully offset by a change in life insurance holdings. This means that the future consumption of a breadwinner's surviving dependents is affected only marginally by the size of his bequest.

Brown (2001a) questions the importance of the bequest motive in influencing marginal annuity purchasing decisions – neither the presence of children nor the bequest motive are determinants in a life annuity purchasing decision. The author states that a simple life-cycle model without bequests gives predictions that are consistent with marginal annuity purchasing behavior, and is therefore a useful first approximation to behavior. These results coincide with those of Hurd (1987), who concludes that bequest motives have no significant influence on the marginal financial behavior of elderly individuals. This is further supported in a later work, Hurd (1989), which finds that many bequests are apparently accidental, resulting from uncertainty about the time of death, without there being any evidence of real bequest motives.

These findings contradict those of Bernheim (1991) and Laitner and Juster (1996), who claim that bequest motives do indeed influence the decision to purchase annuities. Also, Friedman and Warshawsky (1990) conclude that the presence of a bequest motive will reduce or eliminate the demand for annuities, even when the return on them exceeds the real market interest rate. In a previous work, Friedman and Warshawsky (1988), the same authors indicated that the interaction of a deliberate bequest motive and the actuarial unfairness of the annuities offered by insurance firms could dissuade individuals from purchasing them.

Jousten (2001) finds that the presence of a bequest motive has strong implications for the valuation of annuity contracts, and the previous literature has largely ignored the impact of bequest motives on the value of annuities. Lopes (2003) suggests that the bequest motive can significantly reduce the demand for annuities, and can be viewed as a possible explanation for the low demand observed in the US annuity market.

Brown (2001b) introduces the bequest motive into the single person model. The results indicate that only in a very few cases does the obligatory purchase of annuities lead to lower welfare than when no annuities are purchased, although the welfare gains are less than those obtained in the no-inheritance life-cycle model. According to Bernheim (1991), if the retiree can generate savings external to the life annuities and/or purchase additional, annually renewable life insurance, then incorporating a bequest motive does not result in major differences in welfare gains.

Vidal and Lejárraga (2004) conclude that, in a model for an individual into which the bequest motive has been introduced and where the effect of market imperfections and pre-existing annuities have been added, very few individuals in these conditions

would be willing to purchase annuities. In many cases it would be best to allocate only a part of the wealth to the purchase of an annuity and/or postpone purchase until a later date. Finally, Benitez-Silva (2003) focuses on the decision to purchase annuities from the leisure–work perspective, including the bequest motive, but from the standpoint of the individual and without considering survival probabilities.

This paper will analyze the effect of annuitization on expected utility.[3] Joint utility functions need to be considered in order to measure the expected utility deriving from the optimization of consumption for a married couple. Our model is based on that first put forward by Brown and Poterba (2000), to which we have added elements from other models, such as Friedman and Warshawsky's (1990) and Vidal and Lejárraga's (2004), which include the bequest motive. The paper concentrates on the case of a couple in which at least one partner is of retirement age and in which there are no children to be included as dependents when buying a pension. This is a realistic assumption, since there will not normally be any dependent children by this age.

When the time comes to retire, the couple have to decide how to distribute their accumulated wealth over time in order to ensure they will be able to cover their future consumption needs. The basic assumptions are:

1 The couple can allocate their entire wealth to one of the life annuities described below.
2 There are bequest motives, but only the utility deriving from possible bequests once death occurs is considered. No account is taken of the fact that in certain cases the couple may want to make gifts of wealth while still alive instead of waiting for the bequest to be made after they die.[4]
3 No other type of uncertainty with regard to the interest rate, the evolution of mortality, or the rate of inflation is considered.
4 Following Brown and Poterba (2000), it is assumed that the household utility function is a weighted sum of the utility functions of both members of the couple, U_m and U_f. Specifically:

$$U_c(C_t^m, C_t^f) = U_m(C_t^m + \lambda C_t^f) + \varphi U_f(C_t^f + \lambda C_t^m) \quad (1)$$

where parameter φ represents the relative weight of the wife's utility in the household utility aggregate. C_t^m and C_t^f denote the consumption of the husband and the wife respectively, during period t. U_c represents the couple's utility function when both of them are alive, while U_m and U_f are the utility functions for the man after his wife has died and for the woman after her husband has died, respectively.
The husband's utility function depends on $(C_t^m + \lambda C_t^f)$ and the wife's on $(C_t^f + \lambda C_t^m)$, where λ is the percentage of consumption that can be shared.

[3] An alternative approach, proposed by Milevsky (1998, 2001), has been to calculate the impact of deferring annuitization on expected returns, and the probability that deferral will leave the individual no worse off. In the most recent paper, Milevsky and Young (2003) locate a general optimal annuity purchasing policy under an open-market structure where individuals can annuitize a fraction of their wealth at different points in time. They find that an individual will initially annuitize a lump sum and then buy annuities in order to keep wealth to one side of a separating line in wealth-annuity space, a type of barrier control result.
[4] Making a gift of money before death could come about because of the different tax treatment applied to the two situations, or simply because the couple would prefer their relatives to enjoy as soon as possible the goods (financial wealth) they can give them while still alive.

2.1 The couple do not have access to the life annuities market

Assuming that the couple have no access to the annuities market and that they value the existence of a bequest to leave their heirs, the consumption optimization model is

$$\max_{C} \sum_{t=1}^{T} \frac{\overbrace{U_c(C_t^m, C_t^f)S_t^m S_t^f}^{1} + \overbrace{U_m(C_t^m, 0)S_t^m \cdot (1-S_t^f)}^{2} + \overbrace{U_f(0, C_t^f)S_t^f \cdot (1-S_t^m)}^{3}}{(1+\delta)^t} + \overbrace{+h_t \frac{V(W_t)}{(1+\delta)^t} \cdot \underbrace{\left[S_{t-1}^m \cdot q_t^m \cdot (1-S_t^f) + S_{t-1}^f \cdot q_t^f \cdot (1-S_t^m) - S_{t-1}^m \cdot S_{t-1}^f \cdot q_t^m \cdot q_t^f\right]}_{5}}^{4} \tag{2}$$

$$\text{s.t. } W_{t+1} = (W_t - C_t^m - C_t^f)(1+r) \tag{3}$$

$$W_0 \geqslant \sum_{t=1}^{T} \frac{C_t^m - C_t^f}{(1+r)^t} \geqslant 0, \ \forall t > 0 \tag{4}$$

where:

T: numbers of years between the time of the annuity purchase and the maximum length of life (assumed to be ω years, depending on the mortality table used).
δ: the pure rate of time preference, i.e. the classical exponential discount factor of future utility.
r: real expected risk-free rate (assumed constant over the retiree's lifetime).
S_t^m: the probability that the husband survives for at least t years after purchasing the annuity.
q_t^m: the probability that the husband dies at period t, conditional on surviving to period $t-1$.
The relationship between S_t^m and q_t^m is

$$S_t^m = \prod_{j=1}^{t} (1-q_j^m) \tag{5}$$

Similarly, define S_t^f as the wife's t-year survival probability.
h_t: relative weight of the utility of the bequest considered by the couple at period t with respect to the expected utility corresponding to the consumption flow at period t.
W_t: wealth corresponding to period t.
$V(W_t)$: the utility function defined over wealth (W_t) corresponding to period t.

The three possible valid states for consumption are shown along with their associated probabilities in brackets 1, 2, and 3 in the numerator of equation (2). The first state is the assumption that both are still alive, the second when only the man is alive, and finally the third when only the woman is alive.

Finally, the fourth summand evaluates the expected utility provided by the unconsumed wealth the couple have available to bequeath in the year when both

are dead. This wealth will be bequeathed in the year the surviving member of the couple dies, which in terms of probability is expressed in factor 5 of equation (2). Constraint (3) shows the relation between consumption and wealth, while (4) refers to the fact that the wealth cannot be negative. Here we consider exclusively the utility deriving from possible bequests once death has occurred.

One of the determining elements in the consumption optimization model when the bequest motive is included is parameter h_t, which, as mentioned above, indicates the relative weight of the utility of the bequest considered by the couple at any given time. Friedman and Warshawsky (1990) do not believe that this parameter depends on age, but that it is related to the bequest–consumption ratio corresponding to final period $W_\omega / C_{\omega-1}$. Likewise Brown (2001b) and Jousten (1998) assume it to be constant over the life cycle. Other researchers, such as Hurd (1989 and 1999), do not weight the utility of bequests relative to that of consumption, and apply the same valuation on the part of the individual to both.

Fischer (1973) and Yaari (1965) consider the parameter that reflects the value given to the possibility of leaving a bequest, h_t, to be a hump-shaped function due to the greater importance that individuals give the bequest in mid-life when family members have a greater dependence on them, then decreasing in retirement. This is valid in the case where the couple's motivations for leaving a bequest are altruistic. If, however, the retiree's ends are strategic in the sense that they are seeking to encourage family members to care for them in old age in exchange for the promise of a bequest, or if, according to Bernheim *et al*. (1985), they are based on the promise that testators use bequests to influence the behavior of potential beneficiaries, then it would be more appropriate to consider that parameter h_t increases with age. Clearly the higher the value of parameter h_t the lower the level of consumption, the aim being to bequeath greater wealth, and therefore the valuation of welfare provided through the purchase of a life annuity will also be lower.

2.2 The couple have access to the joint annuity with contingent survivor benefit or joint survivor annuity market

Two types of alternative life annuities are analyzed, both of which enable the mortality risk of each member of the couple to be transferred: life annuity with contingent survivor benefit and joint survivor life annuity.

1. Life annuity with contingent survivor benefit. With this type of life annuity a periodic payment is made to the primary annuitant, which he receives until his death. From this moment, his wife, assuming she has survived until this date, will start to receive an amount calculated as a percentage of what the deceased annuitant was receiving. This percentage is set by the purchaser when the relevant insurance policy is bought. In US pension practice (and in ERISA legislation) this is called a 'qualified joint and survivor annuity'.

2. Joint survivor life annuity. This is a contract whereby the insurance company undertakes to pay a periodic amount while both members of the couple are alive,

and a fraction of this amount, ρ or η, when one of them has died, for as long as the other lives.

It should be noted that the difference between a life annuity with contingent survivor benefit and a joint survivor life annuity is not only actuarial. They also have very different economic interpretations:

1 With a life annuity with contingent survivor benefit, only one of the members of the couple has entitlement to income from the annuity, while the other is a mere dependent.
2 With a joint survivor life annuity, both members have entitlement to a pension but, because there is a link between them, the pension is treated jointly.

When the couple decide to allocate part of their initial accumulated wealth to buying a joint survivor annuity or an annuity with contingent survivor benefit, the model for utility optimization can be expressed as

$$\text{s.t. } W_{t+1} = (W_t + A_t - C_t^m - C_t^f)(1+r) \tag{6}$$

$$W_t \geqslant 0, \ \forall t > 0 \tag{7}$$

where term A_t, included in the budget constraint, represents the annual payout the couple will receive at period t, deriving from the life annuity contract. The value of this will be given by:

(**a**) Equation (8) in the case of an annuity with contingent survivor benefit:

$$W_{\text{Annuities}} = \sum_{t=1}^{T} \left[\frac{A^{csb} \cdot \theta(S_t^m + \gamma\, S_t^f \cdot (1 - S_t^m))}{(1+i)^t} \right] \tag{8}$$

where:

A^{csb}: annual payout.
θ: represents the degree of actuarial fairness of the annuity in such a way that, if $\theta = 1$, the annuity is actuarially fair. For values of $\theta < 1$, market *imperfections* are considered to exist. The joint effect of the different mortality and survival probabilities used by the insurance company when selling annuities, the charges applied deriving from management and administration costs, and the possible differences in the rate of interest guaranteed on the annuity in comparison to the market rate mean that the conversion factor is reduced.
γ: percentage payable to the designated beneficiary.
i: the nominal interest rate at which the insurance company discounts future payouts. In the case of a real annuity, the payout flows in the numerator of equation (8) would be measured in constant monetary units rather than nominal monetary units, and the discount rate would need to be the real interest rate r.

Equation (9) relates nominal interest rate and real interest rate

$$i = (1+r)(1+\pi) - 1 \tag{9}$$

where the expected rate of inflation is π. Obviously the risk of inflation is not taken into account since it is not accepted that the real rate of inflation could be different from the expected rate.[5]

(**b**) Equation (10) in the case of joint survivor life annuity

$$W_{\text{Annuities}} = \sum_{t=1}^{T} \left[\frac{A^{js} \cdot \theta(S_t^m S_t^f + \rho S_t^m \cdot (1 - S_t^f) + \eta S_t^f \cdot (1 - S_t^m))}{(1+i)^t} \right] \tag{10}$$

where A^{js} is the annual payout, with ρ and η being the fraction of the annual amount of joint annuity when one of the couple has died, for as long as the other lives. This fraction is normally 1, 2/3, or 1/2. When $\eta = \rho = 50\,\%$, the joint survivor life annuity would be equivalent to both members of the couple being entitled to a retirement income of the same amount until they die.

2.3 The couple have pre-existing life annuities with contingent survivor benefit

In the case of a couple who already have part of their wealth in a pre-existing life annuity and decide to buy another one, the optimization model – assuming they do not have access to actuarially fair annuity markets – has the following constraints

$$\text{s.t. } W_{t+1} = (W_t + R_t - C_t^m - C_t^f)(1+r) \tag{11}$$

$$W_t \geqslant 0, \ \forall\, t > 0 \tag{12}$$

where $W_1 = W_{\text{NP}}$, and $W_0 = W_{\text{NP}} + W_{\text{PA}}$, and:

W_{NP}: level of initial wealth not allocated to annuities.
W_{PA}: level of initial wealth in pre-existing life annuities.

R_t is the annual payout of a life annuity with a 50 % survivor payout, payable in arrears, index-linked to the Retail Price Index (RPI), assumed to derive from a pre-existing public pension system, obtained as

$$R_t = \frac{W_{\text{PA}}}{\sum_{t=1}^{T} \left[\frac{S_t^m + 0.5 S_t^f \cdot (1 - S_t^m)}{(1+r)^t} \right]} \tag{13}$$

Assuming it is possible to buy actuarially fair annuities with contingent survivor or joint survivor payout, in this context where there is a pre-existing annuity the optimization model represented in equation (2) would have the following constraints

$$W_{t+1} = (W_t + R_t + A_t - C_t^m - C_t^f)(1+r) \tag{14}$$

[5] Brown *et al.* (2001) considered the impact of inflation on the value of nominal annuities. They found that the inflation protection offered by a real annuity had only modest value. The wealth equivalent of nominal annuities decreased only slightly when they assumed i.i.d. inflation calibrated to 1926–1997 data from the USA. When they assumed that inflation followed an AR(1) process the wealth equivalent further decreased, but the difference was only substantial at high coefficients of risk aversion or when the individual had no pre-annuitized wealth.

$$W_t \geqslant 0, \ \forall t > 0 \tag{15}$$

where W_1 is equal to the initial wealth not invested in annuities with $W_1 = W_{\text{NP}}$ and $W_0 = W_{\text{NP}} + W_{\text{PA}} + W_{\text{ANNUITY}}$. A_t is determined from equation (8) or (10).

2.4 The couple's annuity equivalent wealth

The annuity equivalent wealth (AEW) is a well-known utility-based measure of the amount of wealth that a fully annuitized individual or couple would need to achieve the same indifference curve in a non-annuitized world. The AEW, described in detail by Brown and Poterba (2000) and Brown (2003), is one of the most usual ways of evaluating gains in welfare. This measure, μ, is defined as:

$$\mu = \frac{W_0 + \Delta W}{W_0} = 1 + \frac{\Delta W}{W_0} \tag{16}$$

ΔW being the amount of additional wealth which must be given to the couple in the absence of annuities for the utility without annuities to be equal to the case where the couple have the ability to fully annuitize financial wealth. The quotient $\Delta W / W_0$ determines the welfare gain for the couple as a percentage of the level of wealth accumulated at the start.

This measure is aimed at determining by how much a couple, averse to risk, would value the possibility of buying a life annuity and of being able to protect themselves against the risk of excessive longevity in terms of the metric which the theory of utility supplies, in which both financial and psychological parameters such as their attitudes to risk and consumption are taken into account. It should be stressed that the evaluation will be an excess approach, since it considers that there is no other family insurance implicit, and neither are there other investments – such as a house – to offer another source of income not usually correlated. In addition, the annuity equivalent wealth has some conceptual and measurement problems,[6] but its validity is widely accepted.

2.5 Utility function and the solution of the model

Just as is usually done in the case of an individual, we assume that the husband and wife have constant relative risk aversion (CRRA) utility functions. The analytical expressions of the utility functions used are the following

$$U_m(C_t^m + \lambda C_t^f) = \left[\frac{(C_t^m + \lambda C_t^f)^{1-\beta} - 1}{1-\beta}, \ \text{if } \beta \neq 1 \right]; \ [\log (C_t^m + \lambda C_t^f), \ \text{if } \beta = 1] \tag{17}$$

$$U_f(C_t^f + \lambda C_t^m) = \left[\frac{(C_t^f + \lambda C_t^m)^{1-\beta} - 1}{1-\beta}, \ \text{if } \beta \neq 1 \right]; \ [\log (C_t^f + \lambda C_t^m), \ \text{if } \beta = 1] \tag{18}$$

where $\beta > 0$ represents the risk aversion coefficient. Halek and Eisenhauer (2001), Powell and Ansic (1997), and Jianakoplos and Bernasek (1998) among others have

[6] Readers interested in a detailed discussion of these measurement problems should consult the paper by Petrova (2004).

pointed out that women are significantly more risk averse than men, but we take the widespread assumption that both men and women have the same risk aversion coefficient. This utility function belongs to the *isoelastic* family of functions, thus the problem remains invariable to the scale of wealth. The degree of concavity of the utility function reflects the individual's level of risk aversion.

The utility deriving from the bequest at each period t is given by the same isoelastic function, in which death has been assumed to occur at the end of the corresponding year. Hence the function that gives the value of the bequest's utility is given by the expression

$$V(W_t) = \left[\frac{W_t^{1-\beta}-1}{1-\beta},\ \text{if } \beta \neq 1\right];\ [\log(W_t),\ \text{if } \beta = 1] \tag{19}$$

The mathematical models in this paper have been translated into LINGO® software programming language, and this program was used to obtain the numerical results shown in the various tables. For non-linear programming problems, the LINGO® package uses an algorithm based on the Generalized Reduced Gradient (GRG2) method. In addition to this, to help obtain a first feasible solution quickly it includes a recursive linear programming algorithm. GRG2 is based on Wolfe's reduced gradient method, later taken up by Abadie and Carpentier, in which the feasible improvement direction is not the generalized reduced gradient (GRG), but a second-order approximation.[7]

3 Results

The solution of the model yields the optimal consumption path that maximizes the couple's expected utility, including a possible bequest. Using this as a point of departure, we can calculate annuity equivalent wealth, which is the indicator used to evaluate the change in welfare. The following assumptions and parameter values were used in the calculations:

1 The consumer's mortality risk is taken from GRMF-95 survival and mortality tables. These are the ones normally used by insurance companies operating in Spain and, in general terms, they show a life expectancy for any particular age that is greater than that given in the latest tables available for the population of Spain as a whole PEMF98-99 (mortality tables for the population of Spain 1998–1999 published by the National Institute of Statistics).
2 It is assumed that the husband's and the wife's probabilities of dying are independent. The importance of the effect of dependent mortality on annuity valuation is not very clear in the literature. Frees *et al.* (1996) find that annuity values are reduced by approximately 5% when dependent mortality models are used compared to the standard models that assume independence, whereas Brown and Poterba (2000) report only modest 'broken heart'[8] effects on the

[7] This is a well-known algorithm that can be seen in detail in the paper by Bazaraa *et al.* (1993).
[8] The tendency is for the mortality rates of surviving spouses to be somewhat higher for several years after their spouse's death than the mortality rates for similar individuals who have not lost a spouse.

annuity equivalence wealth measure. In Spain, as in most countries, standard insurance industry practice assumes independence of lives when valuing annuities, where the promise to pay is based on more than one life.

3 The insurance company sells actuarially fair annuities ($\theta = 1$), which means simultaneously that:
   (a) It does not apply any type of charge on the purchase of an annuity with a single premium.
   (b) The survival probabilities that the insurer uses in setting the premium coincide with the consumer's probabilities. Due to the way insurance companies currently classify risks – mainly by age and sex – it is practically impossible for this to come about. According to Brown and McDaid (2003), at least ten other important factors should be taken into account: race, level of education, wealth, employment, marital status, religion, lifestyle, weight, and smoking and drinking habits.
   (c) The nominal market interest rate, i, coincides with the annuity's technical interest rate, and is equal to 4.545 %, approximately the long-term technical interest rate insurance companies have used when selling annuities in Spain over the last two years.

4 The insurance company sells annuities which are not actuarially fair, in which case the conversion factor is reduced by 15 %, i.e. $\theta = 0.85$. As is widely accepted, Blake (1999), annuities markets are not sufficiently well developed, even in many of the more financially advanced countries, and so considering actuarially fair markets could therefore be thought too unreal an assumption. In Spain, the only available study on the degree of actuarial fairness of the annuities sold is that by García *et al.* (2005), based on those offered by ten insurance companies. It calculates the money's-worth ratio for various types of individual annuities, with results showing average values below 85 %, although it must be said that the methodology used does not appear to be terribly robust. Walliser's (2000a) observation that the value of the annuities falls short of what would be considered actuarially fair for an average retiree could be attributed mainly to two factors: adverse selection[9] and the fact that higher-income people buy more annuities because they have more wealth. The observation that annuitants live longer than average arises because people with higher incomes also tend to live longer. Although we use a load factor of 15 % for all types of annuities, it should be noted that Finkelstein and Poterba (2004), using a unique data set consisting of annuities at a large UK company, find evidence that back-loaded annuities[10] are priced higher, and annuities with payments to the estate are priced lower than other annuities.

[9] Many papers about market *imperfections* and adverse selection have been published. Anyone interested in this controversial subject should consult those by Friedman and Warshawsky (1990), Mitchell *et al.* (1999), James and Vittas (2000), James and Song (2001), Poterba (2001), Mitchell and McCarthy (2002a and 2002b), Mung (2002), Finkelstein and Poterba (2002), Brown (2003), Villeneuve (2003), Finkelstein and Poterba (2004), Cannon and Tonks (2004), Von Gaudeker and Weber (2004), and Yat and Chan (2004). The size of the *imperfection* and the intensity of the adverse selection vary considerably from one market to another.

[10] A more back-loaded annuity is one with a payment profile that provides a greater share of payments in later years.

5 Level of risk aversion β takes two values (0.7 and 2.9). Although there is no consensus in the literature as to which values should be used for the degree of risk aversion,[11] for annuity valuation and CRRA utility functions, Feldstein and Ranguelova (2001) provide some qualitative arguments that the value of CRRA is less than 3 and probably even less than 2.

6 Retirement age is 65 for men; the woman is three years younger than the man.

7 The expected rate of inflation is equal to 1.5%, and so the real interest rate is 3%.

8 The preference rate according to the couple's level of impatience is given by the expression $\delta = \xi^* r$, where the values of parameter ξ (2, 1 and 0.25) classify the couples as (A) very impatient ($\delta = 0.06$), (B) patient ($\delta = 0.03$), and (C) very patient ($\delta = 0.0075$) respectively. According to Yagi and Nishigaki (1993), the degree of the time discount rate is correlated to the degree of myopia, and this has an important effect on the demand for life annuities. In most of the papers cited, the level of impatience is not usually emphasized. Seldom are impatient or very impatient couples considered, and this could be due to the fact that they are more difficult to calculate, but there is evidence, Rabin (1998), that people differ in their rates of time preference.

9 The type of pension bought is a life annuity with contingent survivor benefit payable at 50%, or a joint survivor annuity where, on the death of either member of the couple, the surviving spouse would receive 50% of the income payable when they were both alive (i.e. the value of $\rho = \eta = 0.5$).

10 We have considered the level of joint consumption (λ) to be equal to 0, and the weighting factor of the woman's utility function to be $\varphi = 1$.

11 In the case of a couple who already have part of their wealth in a pre-existing life annuity and decide to buy another one, it is supposed that this wealth amounts to 50%, and is in the form of a life annuity with a 50% survivor payout, payable in arrears, index-linked to the RPI, and assumed to derive from a pre-existing public or private pension system. This is a quite valid assumption in the case of Spain, since the percentage payable to the beneficiary is around 50% and the State pension is indexed to real inflation.

12 Two alternatives have been taken into account for function h_t. On the one hand, we have considered a time-decreasing bequest motive that to a certain extent we could simplistically call the 'altruistic motive for the bequest'. And on the other hand we have considered a time-increasing bequest motive if there is a strategic interest in bequeathing wealth in exchange for possible assistance from the family in old age, function h_t, which is taken as increasing with age. In particular, the values of h_t are as follows:

(a) Time-decreasing bequest motives

$$h_t^d = h_{t-1}^d / 1.02; \; h_T^d = 2, \; \forall t \leqslant T \tag{20}$$

[11] According to Halek and Eisenhauer (2001), there are differences in the degree of risk aversion across demographic groups based on age, gender, education, nationality, race, marital and parental status, religion, health, behavioral indicators, employment status, income and wealth.

(b) Time-increasing bequest motives

$$h_t^i = h_{t-1}^i * 1.02;\ h_t^i = 2,\ \forall t > 0 \tag{21}$$

As mentioned above, there is no consensus in the literature as to how to model bequest motives, and the values considered for the parameter weighting the utility of the bequest relative to the utility of consumption are quite disparate. Thus Brown (2001b) uses two different hypotheses: 0.5 and 1. Fischer (1973) considers values in a range of approximately 4.5–9.8 – or 28.2–120.8 with the rate of consumption preference hypothesis – starting from age 65. Walliser and Winter (1998) apply the values for the bequest motive from Fisher (1973). The bequest parameter applied in Jousten's model (1998) is equal to $5.5*10^{-5}$. Bequest motive is in the form of a linear bequest utility term. The parameter on the linear bequest utility term is $5.5*10^{-5}$. Friedman and Warshawsky (1990) determine the optimal percentage of wealth to annuitize assuming that the bequest parameter can vary between 0 and 100. Finally, Lopes (2003) solves her model with the bequest parameter equal to 7.

The bequest parameter in this simulation was chosen according to conservative criteria so as to avoid obtaining results in which the weight of the bequest motive in the utility function was possibly overvalued. Therefore the results presented below should be analyzed bearing in mind that the intensity of the bequest motive is not empirically tested, although the values used to obtain certain general conclusions can in fact be considered appropriate.

3.1 Optimal consumption path and accumulated wealth for couples

In this subsection we calculate the optimal consumption path and accumulated wealth for couples with and without bequest motive.

Figure 1 shows the optimal consumption path and accumulated wealth in four cases where the couple do not have access to the life annuities markets:

1 Couple without bequest motive with $\beta = 2.9$ (2.9WBM).
2 Couple with time-increasing bequest motive with $\beta = 2.9$ (2.9TIBM).
3 Couple without bequest motive with $\beta = 0.7$ (0.7WBM).
4 Couple with time-increasing bequest motive with $\beta = 2.9$ (0.7TIBM)

We show the optimal path the couple's consumption should follow from period 1 to 50 so as to maximize utility. Although the graphs reflect consumption and wealth at each period, it should be remembered that the weight that consumption in advanced periods represents for the utility function is naturally very small because the probabilities of survival become lower and the discount factor reduces the present value more and more. It should also be remembered that these are not the only variables that interact in the model.

The first thing that can be seen is that the couple's level of impatience for consumption has a clear effect on the optimal consumption path and consequently on the unconsumed wealth. The level of risk aversion also plays an important role: the couple most averse to risk will be much more moderate in their consumption,

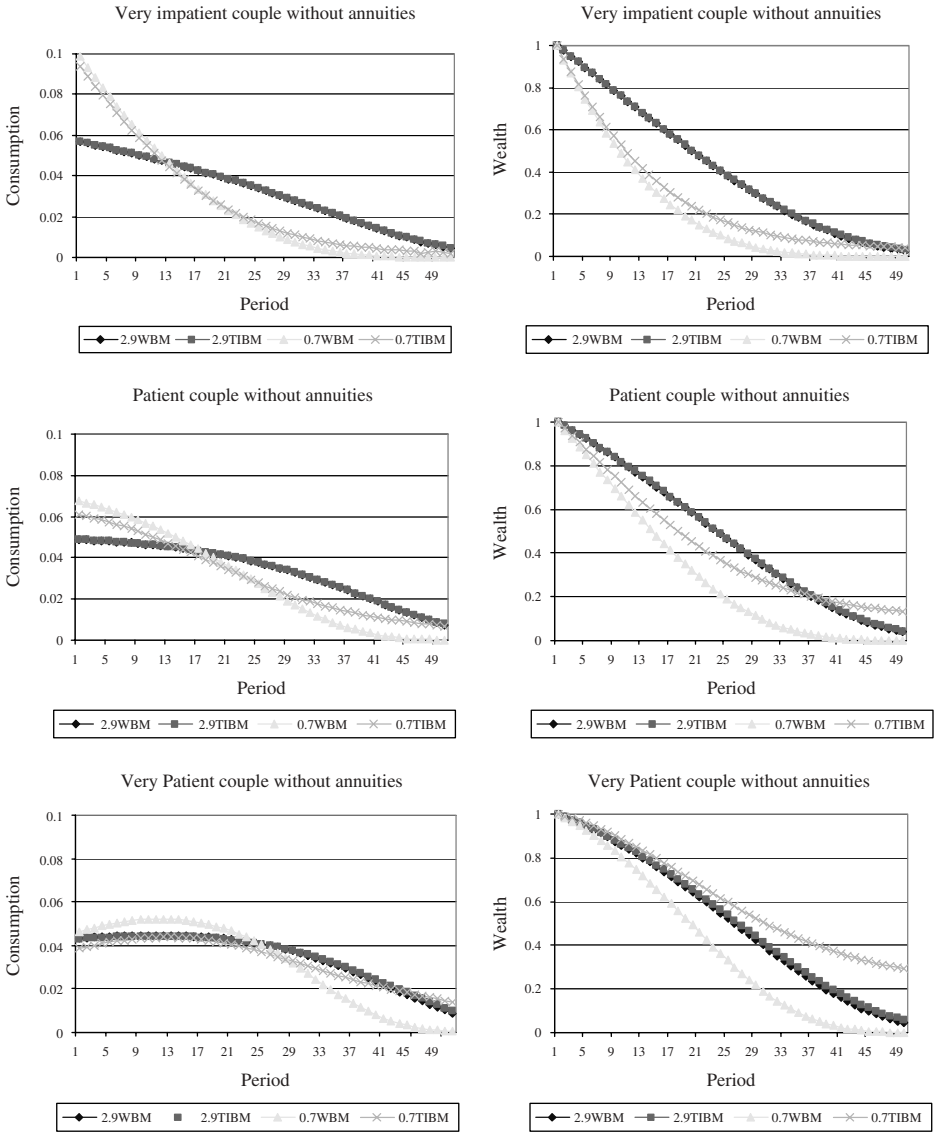

Figure 1. Optimal consumption path and accumulated wealth when the couple do not have access to the life annuities markets

and therefore the wealth will decrease more slowly. The effect of introducing the bequest motive will also vary according to the two variables mentioned above: the less risk averse the couple are and the less impatient they are to consume, the greater the effect. On the other hand, those couples who are more averse to risk hardly react to the bequest motive, and so the curves are practically superimposed; in other words a high degree of risk aversion practically cancels out the effect of the bequest motive.

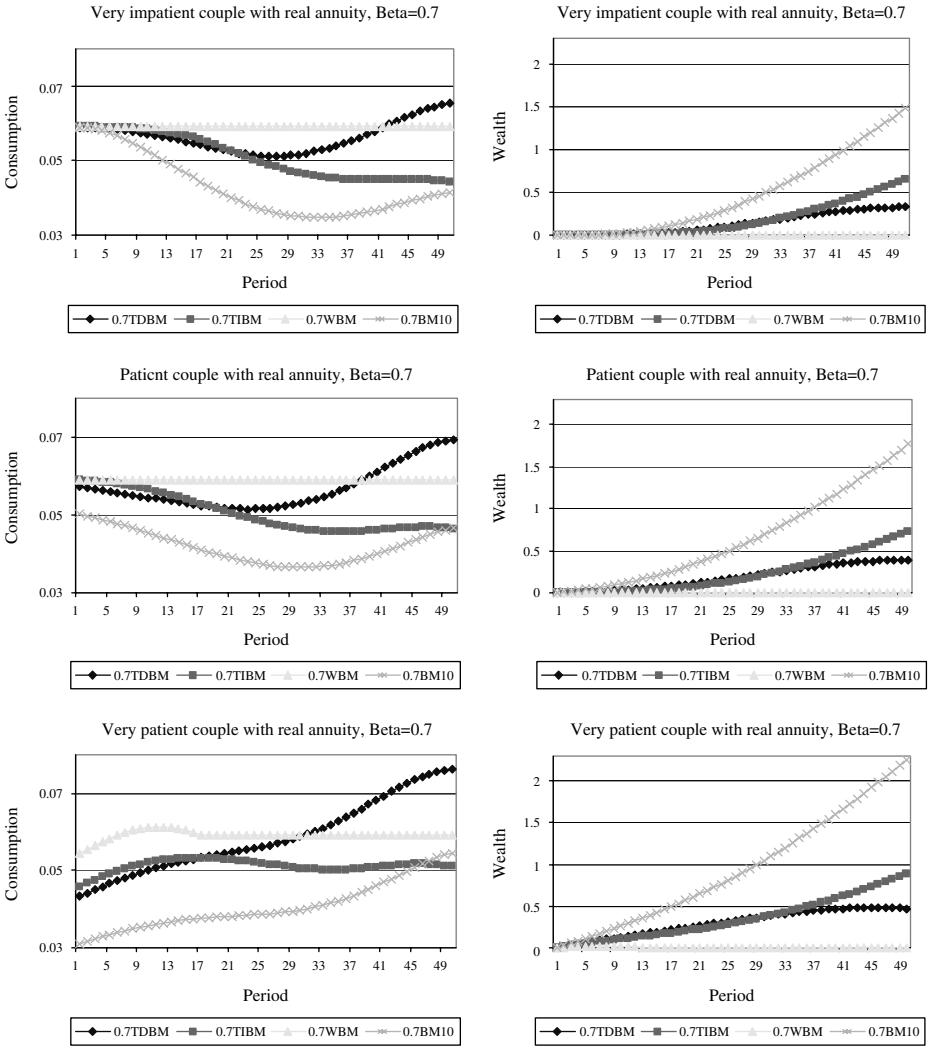

Figure 2. Optimal consumption path and accumulated wealth for couples with $\beta = 0.7$

Figures 2 and 3 show the optimal consumption path and accumulated wealth in four cases where the couple have access to the life annuities markets with $\beta = 0.7$ and $\beta = 2.9$ respectively:

1 Real annuity with time-decreasing bequest motive (TDBM).
2 Real annuity with time-increasing bequest motive (TIBM).
3 Real annuity without bequest motive (WBM).
4 Real annuity with the bequest parameter equal to 10 (BM10).

As in Figure 1, in Figure 2 it is obviously important to point out that the couple's level of impatience to consume has a clear effect on the optimal consumption path and consequently on the unconsumed wealth, which, as can be seen, is always growing except in the case where there is no bequest motive. The bequest motive

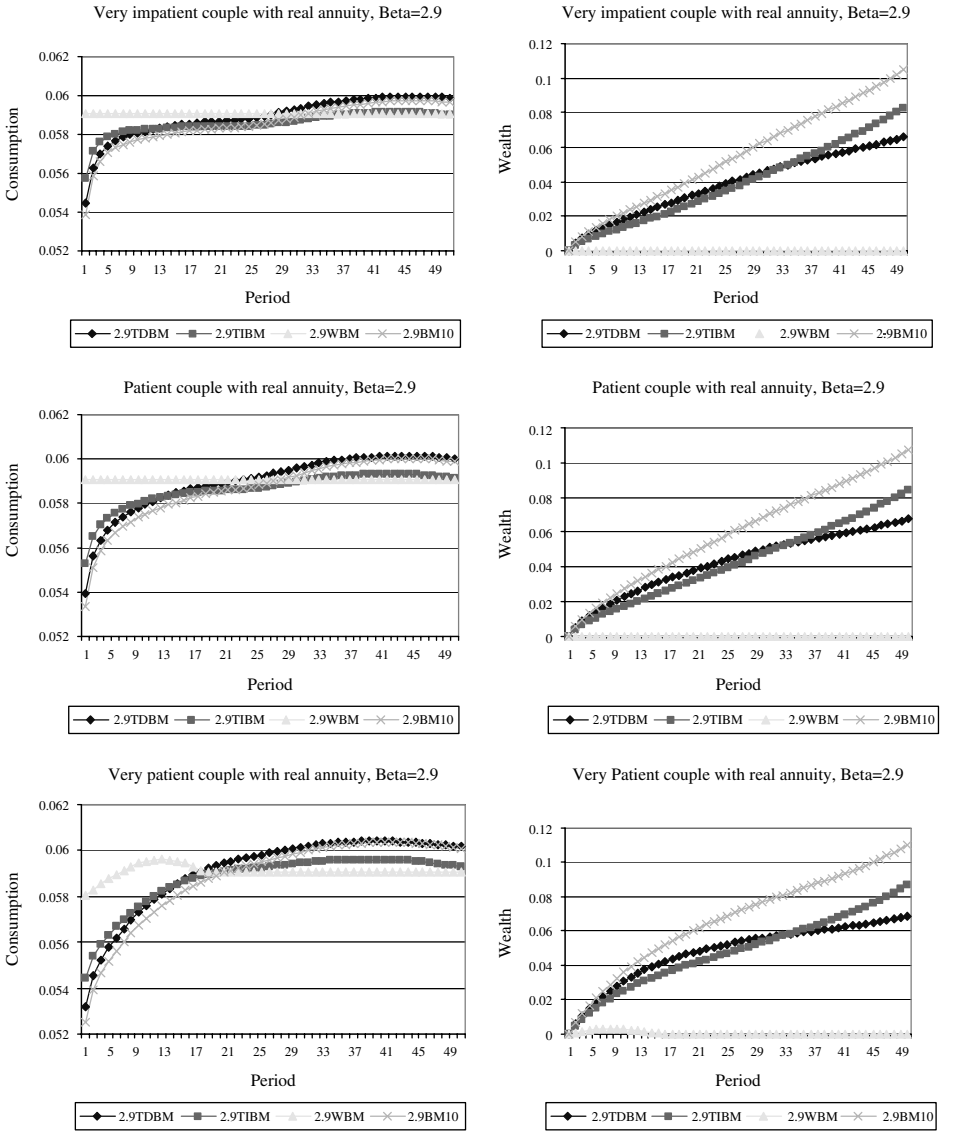

Figure 3. Optimal consumption path and accumulated wealth for couples with $\beta = 2.9$

brings about a significant change in the couple's optimal consumption path compared to the case where there is no bequest motive, in which the couple's optimal consumption path is equal to the amount of the annuity in the first two assumptions shown ($\delta = 0.06$ and $\delta = 0.03$) and for the entire time horizon considered. This path varies slightly in the case of the last hypothesis ($\delta = 0.0075$). The wealth being accumulated also provides a good example of what we mentioned above, that the couple will consume less the greater the strength of the bequest motive and the less impatient they are to consume. The amount of wealth the couple manage to accumulate over the years is extremely important, always growing in the case of

the time-increasing bequest motive, and growing in the years during which it is more likely the couple will survive for the case of the time-decreasing bequest motive. In the extreme supposition of the bequest parameter equal to 10, the wealth curve is steeper than in the previous cases due to the fact that consumption limitation is very high in comparison to what optimal consumption would be in the case of no annuity.

Figure 3 shows what we imagined might be true in the case where the couple did not have access to the annuities market: a high level of risk aversion on the part of the couple neutralizes the bequest motive. As can be seen, the bequest motive changes the couple's optimal consumption path in comparison to a case where the couple have no bequest motive, but the size of the change is much smaller than that shown in Figure 2. Consequently the accumulated wealth grows very slowly and reaches much lower values than those reached in Figure 2.

3.2 The couple's annuity equivalent wealth and the optimal percentage of wealth to annuitize with a bequest motive

In this subsection we calculate the value of the annuity equivalent wealth for couples and the optimal percentage of wealth to annuitize, making a distinction between whether the motivation for the bequest is age increasing or age decreasing.

Table 1 shows the results obtained from comparing the purchase of a life annuity with a 50 % contingent survivor benefit, both real and nominal, and a situation where the couple have no bequest motive. This is the situation which Milevsky and Young (2003) have dubbed 'all or nothing', and appears to be rather restrictive.

The result underlined in Table 1 means that for the couple there would be no difference between one monetary unit allocated to annuities and 1.362 of wealth where nothing is allocated to annuities. In other words, the couple would be willing to give up 1.362 units of current wealth in order to have 1 monetary unit in the form of a real life annuity with a 50 % contingent survivor benefit. Values less than 1, which mean that wealth without annuities is preferable, are shown in bold.

In all cases the equivalent wealth obtained is slightly less than that calculated in the model in the case of 'without bequest motive'. In other words the bequest motive makes life annuities with contingent survivor benefit less attractive, although it does not increase the number of profiles of couples who would prefer not to purchase one. In all cases the model based on time-increasing rather than time-decreasing motives would be preferable. In the same way as happened with couples with no bequest motive, the welfare attained with life annuities with contingent survivor benefit increases when aversion to risk becomes greater and impatience to consume decreases.

The introduction of further[12] market *imperfections*[13] into Part (A) of Table 1 brings about a decrease in equivalent wealth, as happened in the cases of an unmarried individual and of a couple with no bequest motive.

[12] Some authors agree that the requirement of 100 % annuitization and the market only giving constant annuities are already imperfections.

[13] James and Song (2001), Valdés-Prieto (2002), and Valdés-Prieto and Edwards (1998) among others disagree with the term *imperfection* and attribute much of the decrease in the amount of the annuities on the market over the actuarially fair amount to the price that has to be paid to the company for assuming the financial risk and the risk of longevity.

Table 1. *Couple's equivalent wealth with a bequest motive*

	Nominal life annuity with a 50 % contingent survivor benefit						Real life annuity with a 50 % contingent survivor benefit					
	(TDBM)		(WBM)		(TIBM)		(TDBM)		(WBM)		(TIBM)	
$\beta\rightarrow$ $\delta\downarrow$	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9
0.06	1.020	1.269	1.060	1.311	1.043	1.278	1.046	1.271	1.097	1.317	1.077	1.279
0.03	1.118	1.329	1.193	1.360	1.152	1.338	1.159	1.356	1.260	1.403	1.203	1.362
0.0075	1.170	1.358	1.280	1.386	1.205	1.359	1.219	1.414	1.367	1.445	1.263	1.417
(A) With a 15 % reduction factor												
0.06	**0.867**	1.104	**0.901**	1.114	**0.887**	1.086	**0.889**	1.080	**0.932**	1.119	**0.915**	1.087
0.03	**0.950**	1.129	1.014	1.156	**0.980**	1.137	**0.985**	1.153	1.071	1.192	1.023	1.158
0.0075	**0.994**	1.154	1.088	1.178	1.024	1.155	1.036	1.202	1.162	1.228	1.074	1.205
(B) With 50 % wealth in pre-existing annuities												
0.06	**0.954**	1.043	**0.976**	1.067	**0.960**	1.050	0.978	1.048	**0.980**	1.076	**0.988**	1.056
0.03	1.026	1.092	1.038	1.116	1.041	1.098	1.056	1.111	1.081	1.139	1.079	1.118
0.0075	1.066	1.124	1.100	1.145	1.086	1.128	1.096	1.154	1.152	1.181	1.120	1.159
(C) With 50 % wealth in pre-existing annuities and a 15 % reduction factor												
0.06	**0.882**	**0.953**	**0.886**	**0.975**	**0.886**	**0.960**	**0.913**	**0.958**	**0.921**	**0.982**	**0.917**	**0.966**
0.03	**0.948**	**0.997**	**0.953**	1.018	**0.957**	1.002	**0.971**	1.013	**0.991**	1.038	**0.990**	1.020
0.0075	**0.980**	1.026	1.007	1.044	**0.980**	1.030	1.005	1.051	1.052	1.075	1.026	1.056

Notes: β is the coefficient of relative risk aversion, and δ is the discount factor of future utility. TDBM, WBM and TIBM are the acronyms for time-decreasing bequest motive, without bequest motive and time-increasing bequest motive respectively. See text for further description.
Source: Authors.

Table 2. *Maximum percentage of reduction in the annual payout for a life annuity with a 50% contingent survivor benefit*

	Nominal life annuity						Real life annuity					
	TDBM		WBM		TIBM		TDBM		WBM		TIBM	
$\beta\rightarrow$ $\delta\downarrow$	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9
0.06	**1.93**	23.00	**5.67**	23.70	**4.17**	21.75	**4.35**	21.30	**8.82**	24.06	**7.13**	21.81
0.03	**10.56**	24.75	16.20	26.47	**13.23**	25.26	**13.71**	26.26	20.61	28.70	16.88	26.59
0.0075	**14.51**	26.36	21.86	27.87	17.02	26.40	17.96	29.26	26.83	30.80	20.82	29.44

Notes: β is the coefficient of relative risk aversion, and δ is the discount factor of future utility. TDBM, WBM and TIBM are the acronyms for time-decreasing bequest motive, without bequest motive and time-increasing bequest motive respectively.
Source: Authors.

The preference for real annuities is maintained for the same profiles as in the case of couples with no bequest motive; only couples with a very high level of impatience for consumption would prefer nominal annuities instead of real annuities. In addition, in all cases the purchase of a life annuity would result in greater welfare when the bequest is valued for time-increasing rather than time-decreasing motives, although the differences are reduced considerably when risk aversion is high and impatience for consumption is low.

Just as happens in the models for individuals and couples without a bequest motive, and also, Vidal and Lejárraga (2004), in the case of an unmarried individual who wants to leave a bequest to his heirs, the equivalent wealth varies in each case by exactly the same percentage that determines the difference between what the couple would consider to be 'actuarially fair' and what the insurance company offers. In this case, the equivalent wealth that provides the same utility without purchasing annuities as allocating all the available initial wealth to a life annuity is 15% less than would be needed if the annuity bought had no charges or supplements on the survival rates.

The percentage decrease in the value of the annuity over the value of the annuity actuarially equivalent to the premium contributed in the terms mentioned above, which the couple would be willing to accept – given that it would still be more useful to them to purchase the annuity than not purchase it – is calculated in Table 2. All those cases where the maximum admissible percentage of reduction in annuity income is less than 15% or negative are shown in bold. In Part (A) of Table 1, it corresponds to values of equivalent wealth less than 1.

The inclusion of pre-existing annuities – in Part (B) of Table 1 – brings about a significant decrease in equivalent wealth for couples with a bequest motive. Couples with low risk aversion who are very impatient to consume would not purchase a life annuity with contingent survivor benefit.

The results would be much more marked as regards the decrease in equivalent wealth if we were to suppose that the couple have more than 50% accumulated

wealth[14] in annuities, since, in the case of Spain, given the high theoretical replacement rate supplied by the public pension system, it would be an hypothesis closer to reality for many couples.

The preference for real rather than nominal annuities is maintained on the same terms as in the case of not taking the bequest motive into account; as mentioned before, only those couples with a very high level of impatience to consume would prefer nominal annuities to real ones. The comparison between the time-decreasing and time-increasing bequest alternatives is more favorable to the latter, although the variations are very small.

Once market *imperfections* are introduced into the model – Part (C) Table 1 – along with pre-existing annuities, there is a definite decrease in equivalent wealth, as would be expected. The relations between the various combinations of cases remain similar to those occurring under previous hypotheses: the profiles of couples with a high level of impatience to consume and little aversion to risk would be more affected, and the welfare attained when time-increasing rather than time-decreasing bequest motives are taken into account is slightly greater, although the difference is very small. Most couples with a bequest motive should decide not to purchase annuities with contingent survivor benefit. Even those couples with no impatience to consume have little incentive to buy annuities, less than a 7% increase in welfare. Only those couples who are sufficiently educated to make welfare-maximization decisions would be capable of appreciating the gains in welfare.

If we take into account the combined effect of market imperfections, the possibility of pre-existing annuities and the bequest motive, equivalent wealth can decrease by up to 36% compared to what could be obtained if markets were actuarially fair, or if there were no bequest motive or pre-existing annuities, for couples with greater risk aversion with little impatience for consumption. The bequest motive does not have a great impact on equivalent wealth, this being reduced in most cases by between 1% and 3% for couples with greater risk aversion and little impatience for consumption, and by between 9% and 12% for a high level of impatience to consume and little aversion to risk.

In all cases so far it has been considered that the degree of joint consumption, i.e. the percentage of consumption that can be shared, λ, is equal to 0, and that the weight of the woman's utility function with respect to the man's, φ, is equal to 1, which implies that both utility functions have the same weight. How do these specific values affect the resulting values for annuity equivalent wealth? Table 3 shows the effect of consumption jointness and the weighting factor of the woman's utility function on annuity equivalent wealth for couples with a bequest motive.

When the degree of risk aversion is low, annuity equivalent wealth is decreasing relative to the degree of joint consumption, and the greater the amount of consumption the couple can share, the lower the additional welfare achieved by buying a lifetime annuity with a contingent survivor benefit by a couple with bequest

[14] This is the hypothesis most used in the literature, but, according to Dushi and Webb (2004), it would not be very appropriate in the case of the USA, since it would only be true for individuals situated in the highest decile of wealth; for all other individuals the level of wealth in the form of pre-existing life annuities would be much higher.

Table 3. *The effect of consumption jointness and the weighting factor of the woman's utility function on annuity equivalent wealth for couples with a bequest motive. Real fair annuities*

		TDBM			TIBM		
$\beta, \delta\downarrow$	$\varphi\downarrow, \lambda\rightarrow$	0	0.5	1	0	0.5	1
0.7–0.03	**0.5**	1.166	1.165	1.164	1.215	1.212	1.209
	1	1.159	1.156	1.152	1.203	1.198	1.192
	1.5	1.150	1.145	1.141	1.189	1.182	1.176
2.9–0.03	**0.5**	1.376	1.412	1.435	1.381	1.424	1.438
	1	1.356	1.398	1.424	1.362	1.417	1.428
	1.5	1.343	1.386	1.414	1.353	1.392	1.419

Notes: β is the coefficient of relative risk aversion, and δ is the discount factor of future utility. φ stands for the relative weight of the wife's utility in the household utility aggregate. λ represents the percentage of consumption that can be shared. See text for further description. TDBM and TIBM are the acronyms for time-decreasing bequest motive and time-increasing bequest motive respectively.
Source: Authors.

motive. However, if the degree of risk aversion is high, an increase in the value of λ provides an increase in equivalent wealth. Similar results can be seen in the papers by Brown and Poterba (2000) and Vidal *et al*. (2005) for couples without a bequest motive.

As far as the weight of the woman's utility function with respect to the man's, φ, is concerned, the impact on equivalent wealth is very small, almost insignificant, although it would be larger if the difference between the man's and the woman's survival probabilities were greater.

We can conclude that the values assumed for these variables, λ and φ, do not appear to have a great deal of relevance in the analysis of annuity equivalent wealth, although the impact of the variations in λ is not as insignificant for high levels of risk aversion.

Another way of measuring the welfare provided by buying a life annuity with a contingent survivor payout is to determine what percentage of the couple's initial wealth should be allocated to buying the annuity so as to maximize the expected utility obtained from the optimal consumption path deriving from the income available at each moment. This method is based on that applied by Friedman and Warshawsky (1990) in the case of the individual. To determine this maximum percentage, it has to be established that the wealth allocated to annuities is a control variable of the problem set. It should not be considered as taking on a fixed value equal to the initial wealth the individual has available at the time of retirement; this assumption is made to obtain the value of equivalent wealth. As was mentioned before, the mathematical models in this paper have been translated into LINGO® software programming language, and this program was used to obtain the numerical results shown in the various tables.

Table 4. *Optimal percentage of initial wealth allocated to a life annuity and new annuity equivalent wealth with partial annuitization*

	Real life annuity with a 50 % contingent survivor benefit					
	TDBM		WBM		TIBM	
$\beta\rightarrow$ $\delta\downarrow$	0.7	2.9	0.7	2.9	0.7	2.9
0.06	69.64 %	88.68 %	72.02 %	92.07 %	75.24 %	89.89 %
	1.0755	**1.2924**	**1.1302**	**1.3280**	**1.1028**	**1.2964**
	1.0455	1.2706	1.0968	1.3168	1.0767	1.2789
0.03	100.00 %	96.96 %	99.57 %	99.60 %	100.00 %	97.96 %
	1.1589	**1.3594**	**1.2597**	**1.4027**	1.2031	**1.3639**
	1.1589	1.3561	1.2596	1.4026	1.2031	1.3622
0.0075	100.00 %	100.00 %	100.00 %	100.00 %	100.00 %	100.00 %
	1.2190	1.4137	1.3667	1.4450	1.2630	1.4173
	1.2190	1.4137	1.3667	1.4450	1.2630	1.4173

Notes: β is the coefficient of relative risk aversion, and δ is the discount factor of future utility. TDBM, WBM and TIBM are the acronyms for time-decreasing bequest motive, without bequest motive and time-increasing bequest motive respectively.
Source: Authors.

The results shown in Table 4 are most revealing: some couples would prefer not to allocate all their wealth to the purchase of an actuarially fair life annuity, i.e. they would obtain greater gains in welfare by being able to choose what percentage to allocate to the purchase of an annuity. Only in the case of couples with very little impatience for consumption is it optimal to allocate 100 % of the wealth to buying an annuity. Generally speaking a couple with time-increasing motives will allocate a slightly greater amount than those with time-decreasing motives.

For each level of impatience considered, Table 4 also shows the equivalent wealth (in bold) that would be attained by allocating the percentage of wealth considered optimal in each case to purchasing annuities. This is compared on the third line of each level of impatience with the situation where all wealth is allocated to purchasing annuities. For couples who are impatient to consume, the level of welfare attained would increase if it were possible to annuitize only a fraction of their wealth. Obviously, as can be seen in Table 4, in those cases where it is optimal to allocate 100 % of their wealth to purchasing annuities, the equivalent wealth does not change with respect to what was calculated above.

3.3 The couple's annuity equivalent wealth: life annuity with contingent survivor payout versus joint survivor annuity – real annuities

As can be seen in Table 5 – and despite the absolute differences in the values obtained – the list of values for the various profiles studied is similar for both types of annuity. As mentioned earlier, the difference between a life annuity with

Table 5. *Couple's equivalent wealth and optimal % of initial wealth allocated to a annuity. Life annuity with a 50% contingent survivor benefit.* (*LACSB*). *Comparison with a joint survivor annuity* (*JSA*). *Real annuities*

	TDBM				TIBM			
	JSA		LACSB		JSA		LACSB	
$\delta\downarrow\beta\rightarrow$	**0.7**	**2.9**	**0.7**	**2.9**	**0.7**	**2.9**	**0.7**	**2.9**
(A) Fair annuity								
0.06	1.091	1.326	1.046	1.271	1.123	1.334	1.077	1.279
	79%	91%	70%	89%	79%	92%	75%	90%
0.03	1.209	1.415	1.159	1.356	1.255	1.421	1.203	1.362
	100%	98%	100%	97%	100%	99%	100%	98%
0.0075	1.272	1.475	1.219	1.414	1.318	1.479	1.263	1.417
	100%	100%	100%	100%	100%	100%	100%	100%
(B) With 50% of wealth in pre-existing annuities, 15% reduction factor								
0.06	**0.921**	**0.980**	**0.902**	**0.958**	**0.930**	**0.988**	**0.911**	**0.966**
	0%	27%	0%	21%	0%	28%	0%	22%
0.03	**0.992**	1.037	**0.971**	1.013	1.012	1.044	**0.990**	1.020
	20%	40%	0%	35%	38%	41%	19%	36%
0.0075	1.028	1.076	1.005	1.051	1.050	1.081	1.026	1.056
	50%	48%	36%	45%	50%	49%	50%	47%

Notes: β is the coefficient of relative risk aversion, and δ is the discount factor of future utility. TDBM and TIBM are the acronyms for time-decreasing bequest motive and time-increasing bequest motive respectively.
Source: Authors.

contingent survivor benefit and a joint survivor annuity also has a very different economic interpretation, which makes it necessary to study them both carefully. The annuity with contingent survivor benefit means that only one partner in the couple has entitlement to a pension, and that the other is a dependent. Joint survivor annuity means both members of the couple have entitlement.

The results are similar if the couples choosing a joint survivor life annuity are considered in the model in accordance with expression (10), in which $\rho = \eta = 50\%$. For example, for risk aversion values equal to 0.7 and in case A – which is characterized by a high level of impatience for consumption – equivalent wealth is 1.091 for couples with time-decreasing bequest motives and 1.123 for those with time-increasing motives. In a profile showing a patient attitude towards consumption – case B – with a degree of risk aversion equal to 2.9, the values are 1.415 and 1.421 respectively, according to whether the bequest motive is time decreasing or time increasing. In other words, the relation between time-increasing and time-decreasing bequest motives is maintained when equivalent wealth is slightly greater in the first case. In addition to this, in all cases the welfare attained through a joint survivor life annuity is greater than that obtained with the purchase of a life annuity with contingent survivor benefit – between 2% and 4% more in the profiles analyzed.

The reason for this is that the annual amount of the joint annuity is greater than that of the annuity with a survivor contingent, given that, in the former, on the death of either member of the couple the annual pension is reduced to half rate, whereas with the annuity with contingent survivor benefit the pension is reduced to the percentage agreed only if the primary annuitant dies.

When market *imperfections* and the possibility of pre-existing annuities are introduced into the model – Part (B) of Table 5 – the results are very similar for both types of pension. In other words it is less favorable for the couple to purchase than directly allocate their resources without accessing the annuities market. On the other hand, joint survivor life annuities are always more valuable – from the point of view of the welfare they provide the couple – than the type of pension which becomes payable to the other person only when it is the primary annuitant that dies.

Table 5 also shows the optimal percentage of initial wealth allocated to a life annuity. This information provides a valuable perspective, because, following the criteria for equivalent wealth for the hypothesis in Part (B) of the table, some couples would refuse to allocate additional resources to annuities. The reality, however, is different; they would not allocate 50% of the wealth not already allocated to annuities. In most cases it would be best to allow partial annuitization. Even for many of the cases analyzed in Part (A) of the table, mainly those couples who do not consider themselves impatient to consume, the best strategy is not to allocate 100% of wealth to buying life annuities but rather to allocate a lesser amount.

As we have just seen, a joint annuity provides better welfare for couples, but it is difficult to find companies that sell them in the Spanish market. This is due to possible legal and fiscal problems and also because the policyholder of the annuity, which up to now in Spain is generally the man, is not willing to give up part of the savings in favor of his wife or partner, since this could have an adverse effect on him in the case of divorce or because the amount of his annuity would be reduced if his wife were to die first. The question of buying a joint annuity would create much fewer practical problems if both members of the couple accumulated financial resources in order to pay the premium.

3.4 The couple's annuity equivalent wealth, retirement age and life expectancy at the age of retirement

An interesting question on the subject of retirement pensions is the age at which one may begin to receive it. According to Devesa and Vidal (2001), retirement pension systems financed by capitalization are usually also defined contribution systems, and therefore there is greater freedom when choosing the age to retire. The consumer's age has an influence similar to the behavior seen in the case of couples without bequest motives, as shown in the results obtained for equivalent wealth when retirement age is either brought forward or deferred (Table 6).

Early retirement brings about a decrease in welfare gains. This result is consistent with that obtained by Kotlikoff and Spivak (1981), for whom younger people do not place so much value on annuities, since a large part of the utility deriving from consumption is almost certainly due to the fact that their probabilities of

Table 6. *Couple's annuity equivalent wealth and retirement age. Real life annuity with a 50% contingent survivor benefit*

	TDBM			WBM			TIBM		
$e_r \rightarrow$ $\delta, \beta \downarrow$	60	65	70	60	65	70	60	65	70
0.06–0.7	1.021	1.046	1.073	1.055	1.097	1.149	1.045	1.077	1.114
0.06–2.9	1.221	1.271	1.339	1.251	1.317	1.401	1.222	1.279	1.350
0.03–0.7	1.138	1.159	1.182	1.218	1.260	1.313	1.175	1.203	1.237
0.03–2.9	1.298	1.356	1.428	1.333	1.403	1.491	1.302	1.362	1.437
0.0075–0.7	1.193	1.219	1.248	1.314	1.367	1.429	1.227	1.263	1.304
0.0075–2.9	1.350	1.414	1.492	1.383	1.445	1.553	1.352	1.417	1.499

Notes: e_r is the age of retirement. β is the coefficient of relative risk aversion, and δ is the discount factor of future utility. TDBM, WBM and TIBM are the acronyms for time-decreasing bequest motive, without bequest motive and time-increasing bequest motive respectively.
Source: Authors.

survival in the most immediate future are quite high. Brown and Poterba (2000) conclude that annuity equivalent wealth is an increasing function of the ages of both spouses in a couple, and that older annuity buyers have more to gain from annuitization than younger couples. Dushi and Webb (2004) come to the same conclusions.

In all the suppositions there comes about a decrease in equivalent wealth in comparison to the case where no bequest motive is considered. It is generally greater when the bequest motive is time increasing rather than time decreasing. In fact the effect of deferring retirement age is that the couple's probabilities of survival are reduced, hence the rate of return on the annuity is increased. This in turn stimulates demand given that it manifests itself as an increase in equivalent wealth, as was demonstrated under restrictive conditions by Yagi and Nishigaki (1993) for the case of the individual.

On the other hand, deferring retirement age increases the value couples give life annuities with contingent survivor benefit. The most important implication in what is described above is that the couple's purchase of life annuities should not be considered for a single moment in time; it may not be the optimal choice at a particular age – ordinary retirement age – but it may be optimal a few years later.

The final point to be studied is the impact that a change in the life expectancy of the couple at retirement age has on the value of annuities. Does the value of annuities change for individuals/couples of different countries, assuming that all characteristics coincide – level of actuarial fairness of the annuities market, interest rate, risk aversion, impatience to consume, etc. – apart from the demographic parameters? It must be said that it does, and therefore this result suggests that the value of annuities could vary substantially from one country to another because of demographic characteristics.

Table 7. *Couple's equivalent wealth with a bequest motive. Real annuity with a 50% contingent survivor benefit*

	PEMF98–99 Mortality table						GRMF95 Mortality table					
	(TDBM)		(WBM)		(TIBM)		(TDBM)		(WBM)		(TIBM)	
$\beta\rightarrow$ $\delta\downarrow$	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9	0.7	2.9
(A) Fair annuity												
0.06	1.036	1.231	1.093	1.272	1.075	1.234	1.046	1.271	1.097	1.317	1.077	1.279
0.03	1.138	1.284	1.207	1.324	1.158	1.286	1.159	1.356	1.260	1.403	1.203	1.362
0.0075	1.179	1.317	1.278	1.356	1.195	1.317	1.219	1.414	1.367	1.445	1.263	1.417
(B) With 50% wealth in pre-existing annuities and a 15% reduction factor												
0.06	**0.910**	**0.961**	**0.920**	**0.982**	**0.911**	**0.965**	**0.913**	**0.958**	**0.921**	**0.982**	**0.917**	**0.966**
0.03	**0.964**	**1.000**	**0.974**	1.021	**0.975**	1.003	**0.971**	1.013	**0.991**	1.038	**0.99**	1.02
0.0075	**0.989**	1.024	1.016	1.046	**0.999**	1.027	1.005	1.051	1.052	1.075	1.026	1.056

Notes: β is the coefficient of relative risk aversion, and δ is the discount factor of future utility. TDBM, WBM and TIBM are the acronyms for time-decreasing bequest motive, without bequest motive and time-increasing bequest motive respectively.
Source: Authors.

Table 7 shows the couple's equivalent wealth with a bequest motive for a real annuity with a 50% contingent survivor benefit for two different assumptions, with demographic characteristics provided by GRMF95 mortality tables, which have been used in all the calculations, and PEMF98–99 mortality tables, which show noticeably lower life expectancies than those in the GRMF95 tables for all ages. It is supposed that both tables represent the demographic characteristics of two different countries, with all other characteristics being the same. Life expectancy for a 65-year-old man and a 62-year-old woman according to the GRMF95 mortality tables is 20.47 and 29.79 years respectively, whereas the values in the PEMF98–99 tables are 16.1 and 22.72 years; a huge difference. The probability that a couple of that age will survive another 20 years, or that at least one of them will still be alive after that time, is 40.86% and 90.49% respectively according to GRMF95, but 30.41% and 82.02% according to PEMF98–99.

As Table 7 shows, equivalent wealth decreases – in some cases quite noticeably – and the number of profiles of couples who would not be willing to purchase additional annuities increases (see Part (B) of the table). This result is logical: if life expectancy is lower, the longevity risk also declines, and consequently the value given to covering this risk is also lower. In less developed countries, where life expectancy at retirement age is generally lower, annuities will be valued less than in more developed countries where people usually have greater life expectancy.

4 Conclusions and future research

This paper aims to contribute to a more detailed clarification of the 'annuities puzzle' by introducing the bequest motive. With this aim in mind, a wide set of valued

suppositions with regard to couples have been presented which have barely been developed in the economic literature. Light is also shed on whether the bequest motive in itself is really a relevant factor influencing the theoretical decision as to whether to purchase annuities for couples.

The consideration of the basic model without market *imperfections* and the possibility of pre-existing annuities is the one that best measures the true impact of the bequest motive on the decision to purchase annuities. Because no other characteristics interfere, it appears to indicate that the bequest motive by itself, isolated, is not a truly relevant factor, although it must be stressed that:

1 Life annuities with contingent survivor benefit become less attractive, although the number of profiles of individuals who would prefer not to purchase such annuities does not increase. In all cases the model based on time-increasing rather than time-decreasing motives would be preferable.
2 The profile distribution for couples who prefer nominal to real life annuities with contingent survivor benefit practically coincides with that of couples with no bequest motive, whether time decreasing or time increasing. Real rather than nominal life annuities with contingent survivor benefit are preferable for couples.
3 The welfare obtained with a joint survivor life annuity is higher than that obtained by purchasing a life annuity with contingent survivor benefit. Despite the absolute differences in the results obtained, the relation of the values for the various profiles studied is similar for both types of annuity.
4 The impact of the bequest motive is noticeably greater in couples who are impatient to consume than in the other cases considered.

The most extended model with all the characteristics incorporated – market *imperfections* and the possibility of pre-existing annuities, valued at retirement age and assuming that 100% of the wealth not tied up in annuities should be allocated to the purchase of an additional annuity – practically solves the so-called 'annuities puzzle', given that very few couples would be willing to purchase them if these conditions applied. This is what normally happens in reality when they are given freedom of choice. This result follows the conclusion of Dushi and Webb (2004) that, 'for the median household, if there is an "annuity puzzle", it would appear to be restricted to single individuals'. It is important to highlight that:

(a) Most couples with a bequest motive should decide not to purchase annuities with contingent survivor benefit. Even those couples with no impatience to consume have little incentive to buy annuities, less than a 7% increase in welfare. Only those couples who are sufficiently educated to make welfare-maximization decisions would be capable of appreciating the gains in welfare.
(b) A good strategy could be to defer the purchase until later and/or allocate only part of the wealth owned to the purchase of an annuity, since, as shown above, the couple would attain greater welfare.

Finally it should be stressed that the results shown strengthen the conclusions put forward in previous papers, Walliser (2000a), Lejárraga *et al.* (2002) or

Lejárraga (2003), in so far as it appears better for the regulations governing different types of annuity to have a certain amount of flexibility in order to accommodate individual circumstances and the aims of public policy. In defined contribution capitalization systems, which are complementary to defined benefit systems, in which couples and individuals already have a large part of their wealth in the form of annuities, there should be full freedom of choice with no obligation for people to allocate any amount at all to the purchase of additional annuities. If what is really being aimed at is for couples to exercise a certain amount of financial discipline and if they also have a bequest motive, then the best option in this case would be the use of phased withdrawal.

There are at least three new aspects that the authors have identified for future research:

1 To incorporate into the model other sources of uncertainty. One aspect that could increase the value of annuities for couples or individuals is the fact that a life annuity is the only way of ensuring a fixed return compared to the variability of the interest rates offered over time by the market. In the model, as is normal in the referenced literature, it has been considered that investment in current wealth at the market interest rate can provide a fixed return equal to that of the annuity, a hypothesis which is not very realistic and which should be of concern to the individual or couple who are averse to risk.
2 To analyze the demand for annuities that contain period certain or refund options, since this type of annuity could at least in theory be valued more highly by couples with a bequest motive.
3 To validate the results for other utility functions. The CRRA utility function has become the most widely used assumption in the financial (and even the macroeconomic) literature in the intertemporal context, but in practice the aversion coefficient and the consumption substitution elasticity do not have to be inversely related nor even necessarily linked. There is currently, Davidoff *et al*. (2005), Ponzetto (2003), and Rabin (1998) among others, a trend towards the revision of this concept.

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