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A NOTE ON A LIMITING BEHAVIOUR OF THE OCCURRENCE TIMES OF A MIXED POISSON PROCESS

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In a recent publication Pfeifer (1982) shows that for a Pólya–Lundberg birth process the limiting distribution of the sequence $\{n/T_n\}$, where T_n denotes the nth occurrence time of the process, is a gamma distribution. As the Pólya–Lundberg birth process is a mixed Poisson process with a gamma mixing distribution, Pfeifer's result implies that the limiting distribution of $\{n/T_n\}$ is just the mixing distribution of the process. A stronger result of this type is valid for every mixed Poisson process, which can be seen as follows. Given a random variable Δ whose distribution is the mixing distribution of the process, we notice, following Grandell ((1976), p. 12), that the process $\bar{N}(t) = N(t\Delta)$, where $N(t)$ is a Poisson process with unit intensity, is a version of the desired mixed Poisson process. Hence

$$P\left(\lim_{t\to\infty}\frac{\bar{N}(t)}{t} = \Delta \mid \Delta = \lambda\right) = P\left(\lim_{t\to\infty}\frac{N(\lambda t)}{t} = \lambda\right) = 1.$$

By integration we obtain $\bar{N}(t)/t \to \Delta$ a.s. As

$$\frac{\bar{N}(t)}{t} \leqq \frac{n}{T_n} \quad \text{for} \quad T_n \leqq t < T_{n+1}$$

and

$$\frac{n}{T_n} \leqq \frac{n}{n-1}\frac{\bar{N}(t)}{t} \quad \text{for} \quad T_{n-1} \leqq t < T_n,$$

we have $n/T_n \to \Delta$ a.s. and also $T_n/n \to 1/\Delta$ a.s.

I thank a referee for pointing out an easier proof of an even more general result.

References

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