From Interacting Binaries to Exoplanets: Essential Modeling Tools
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The Broadening Functions Technique

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Abstract. Essential assumptions and features of the Broadening Function (BF) technique are presented. A distinction between BF determination and the BF concept and utilization is made. The BF's can be determined in various ways. The approach based on linear deconvolution involving stellar templates, as used during the DDO program (1999 – 2008) is described, but the LSD technique would also give excellent results. The BF concept to prove and/or verify photometric light-curve solutions has so far been very limited to only a few W UMa-type binaries, with AW UMa giving particularly unexpected results.

Keywords. line: profiles, methods: data analysis, techniques: spectroscopic, techniques: radial velocities

1. Introduction

"Solving eclipsing binary light curves" was my occupation for a long time, in fact since 1965.† After many years of experience, I have been generally disappointed by the relatively low information content of light curves, particularly of W UMa-type binaries with partial eclipses. An obvious remedy would be to seek this information in spectroscopic data, by going beyond traditional determination of mass-centre radial velocities (RV). This would mean to utilize the Doppler broadening of spectral lines where information on *shapes* of the components is available, particularly during the (normally avoided in RV work) eclipse phases. Effectively, in place of single brightness measurements taken versus time, one should be able to utilize many little images – Broadening Functions (BF) – of the binary star, as projected into the RV space (Figure 1). Switching from the 1-D (time domain) light curve information to 2-D (time and RV) information should permit a much better binary-star description. The idea was sketched early (Rucinski 1971), but it took a long time to reach a point of maturity in years 1992 – 2008, during the David Dunlap Observatory binary program (see Section 3).

Utilization of the Broadening Functions splits into two aspects: (1) How to determine the BF's? (2) How to use information contained in them? Most of the work has gone into the necessary step of *BF determination* (Section 2). This is not a simple matter by itself, but much less effort has been devoted to utilization of the BF's in geometric element determination of close binaries (Section 4). So far, mostly W UMa-type binaries have been the subject of investigations involving the BF's, which is understandable because there is no need to confirm the shapes of spherical stars. But for W UMa-type binaries, it is hard to prove validity of Lucy's contact model (Lucy 1968a, Lucy 1968b) using

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† I did an analysis of the photometric data for the close binary DI Pegasi as my MSc project (Rucinski 1967). What may be of relevance for this symposium – in view of Dr. Batten's panel session remarks – is that I used the Russell-Merrill and the Kopal-Piotrowski iterative techniques. Frankly, I did not believe in my results because the binary shows partial eclipses and I had to invoke a third light. It was a great relief when Wen Lu discovered a third star in the system (Lu 1992).

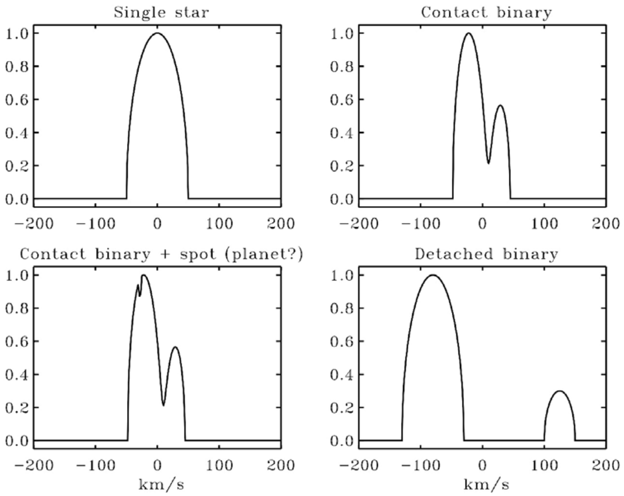

Figure 1. Four schematic examples of Broadening Functions.

photometric information alone: Light curve synthesis codes provide excellent fits to the data; all small details of the light curves can be beautifully explained and one may feel very satisfied. But is this really a true picture? The well-observed AW UMa provides an important, perhaps even sobering case (Section 4) strongly suggesting that light curve synthesis analyses – in spite of excellent light curve fits – may give an inadequate picture.

2. How to determine Broadening Functions

Convolution is an operation that nature does for us. In astronomy, we seldom see “naked” atomic or molecular spectra; instead, we are usually confronted with convolutions. They can be convolved with a spectrograph instrumental profile, with a radial component of the micro-turbulence velocity field in the stellar atmosphere, with rotational broadening; for a distant cluster the combined spectrum is convolved with the velocity dispersion of contributing stars. Thus, instead of a sharp-line spectrum $s(x)$, we observe $p(x)$ which is a convolution with the broadening function $b(x)$:

$$p(x) = \int_{-\infty}^{+\infty} b(u)\ s(x-u)\ du = b(x) * s(x)$$

In binary stars, the BF is simply a projection of the shape of the binary components into the radial velocity (RV) space (Figure 1). If the stars rotate as solid bodies, then their BF’s are identical to a simple mapping of their shapes into the respective projected distances from the axis of rotation or of revolution. Purists will note that the observed (flux) spectrum originates by integration of the atmospheric *emergent intensity*, not of the *emergent flux*, so an approximation is involved here. However, with many lines in the spectral window, the differences in line formation mechanisms (absorption and scattering), which reflect in different dependencies on the angle of emergence, become less unimportant (but this matter remains to be studied in the BF context).

Although the above idea is simple, it is not easy to determine the BF’s. One of the most obvious ways is to utilize the convolution properties and note that the Fourier transform can be used to separate the two functions: $\mathcal{F}(p) = \mathcal{F}(b) \times \mathcal{F}(s)$. The next step is to apply the inverse transform to the quotient: $b = \mathcal{F}^{-1}(\mathcal{F}(p)/\mathcal{F}(s))$; $s(x)$ is a spectrum of a sharp-line template or a model spectrum. While $b(x)$ can indeed be determined that way, this approach fails in most cases because of the amplification of the high frequency noise in the division step. Frequency filtering becomes necessary and the results tend to depend on how the data processing is done. The only brave attempt of this approach, and the first one to demonstrate utility of the BF's for contact binaries was by Anderson, Stanford & Leininger (1983) who analyzed AW UMa, which was always perceived as a crucial object for our understanding of contact binaries.

The cross-correlation function (CCF) is another popular method to evaluate an approximation of the Broadening Function. The CCF, or $c(x)$ here, can be computed easily in many software packages (note the different symbol):

$$c(x) = \int_{-\infty}^{+\infty} p(u)\; s(u+x)\; du = p(x) \otimes s(x)$$

The problem is that $c(x)$ *is not equal* to $b(x)$ because:

$$c(x) = b(x) * s(x) \otimes s(x) \neq b(x)$$

Broadening of $s(x)$ remains present in the result; the equality would happen only for $s(x) = \sum \delta(x - x_0)$. Still, the CCF is a very useful technique when we are sure that $b(x)$ is symmetric, as then the geometric centre of $c(x)$ coincides with that of $b(x)$.

The most straightforward approach is through representation of the convolution by a system of linear equations:

$$p_i = \sum_{j=0}^{m-1} b_j\, s_{i+m-j} \qquad i = 0, \ldots, n-1 \qquad n > m$$

(for details, see Rucinski 1992 or Rucinski 2002). For spectra of several thousand pixels (n), this means solving several thousand equations; fortunately, even a rather strongly broadened BF can be typically represented by a few hundred points (m). This leads to very large, over-determined systems ($n > m$, typically $n/m \simeq 10\times$) which are solvable using the least-squares formalism. In 1992, I realized (Rucinski 1992) that such solutions became feasible using a moderate-size computer using the powerful Singular Value Decomposition method. This resulted in the first BF-based analysis of AW UMa just confirming – similarly to the work of Anderson *et al.* (1983) – the previous light curve results. These early attempts have been largely superseded by the later, much improved results of Pribulla & Rucinski (2008), which – surprisingly – showed that spectroscopically, AW UMa is not a contact binary (Section 4).

It should be stressed that the BF's determined through linear equations have the nice and obvious property of *linearity*. This is important because using them one can determine relative luminosities of components directly. This is not a trivial matter because for example the CCF approach requires calibrations which may always leave room for additional uncertainties.

3. The DDO program of close binary star orbits, 1999 – 2008

Broadening Functions have been extensively used during the David Dunlap Observatory (DDO) program of radial velocity orbits of very close ($P < 1$ day) binaries in years 1999 – 2008. The last numbered paper of the 15-paper series was of Pribulla *et al.* (2009) although several additional papers addressed individual interesting systems (such as W Crv, V471 Tau or AW UMa). The DDO program was described at the Brno and Mykonos symposia on binary stars (Rucinski 2010a, 2010b).

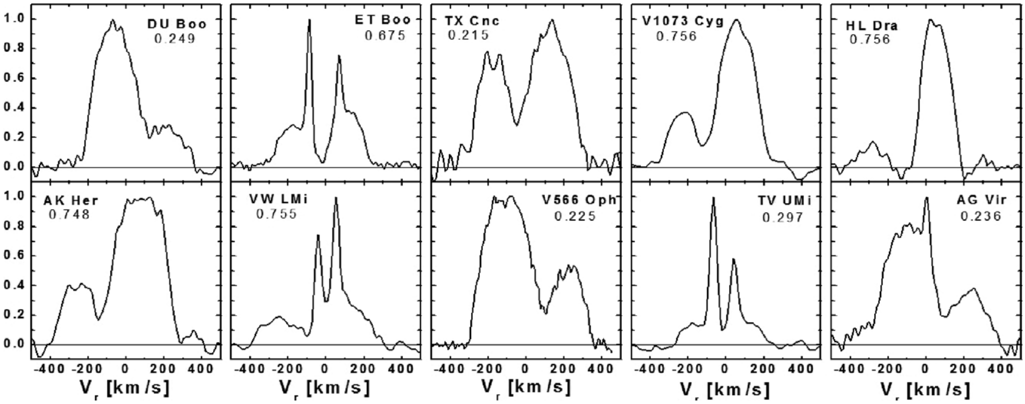

Figure 2. An example of BF's from the DDO-11 (Pribulla *et al.* 2006) paper showing results at orbital quadratures for ten W UMa systems. Among these binaries, one is a triple and four are quadruple systems.

Thanks to the reasonably stable spectrograph on the DDO 1.9m telescope, the moderately fast turn-around time in terms of reductions, but – mainly – to the superior properties of the BF technique, the DDO program produced excellent RV data for 162 binaries with as many as 145 of them being SB2 systems. Of note is that previously totally intractable triple and quadruple systems suddenly became solvable (Figure 2).

The DDO program did not utilize the full potential of the BF's: We used them in a "traditional" way, similarly to measurements of spectral line shifts, by just determining the component light centroids. Some improvements in these measurements have been done half way through the program (we introduced rotational profiles instead of Gaussians), but the great advantage of the BF's of encoding eclipse effects and asymmetries introduced by variable geometric projections was not used (the few exceptions: see the next section). Also, we were constantly under the pressure of time due to the imminent closure of the observatory, so we observed mostly at the predicted RV extremes (orbital quadratures), explicitly avoiding the eclipses. In the end, the published RV data for 162 binaries with $P < 1$ day correspond to the 92% completeness to $V = 10$ mag. We may have erred in determination of the K_i semi-amplitudes, but their ratios giving the important mass ratios $q = M_2/M_1 = K_1/K_2$ are probably the best one can currently achieve for very close binaries, particularly those of the W UMa type.

4. Geometrical element determination: The strange case of AW UMa

So far, the idea of simultaneous fitting of many BF's taken at different phases (including eclipses) has been used to analyze very few binaries. In addition to the first applications to AW UMa by Anderson *et al.* (1983) and Rucinski (1992), the early uses were for AH Vir (Lu & Rucinski 1993) and W UMa (Rucinski *et al.* 1993). Towards the end of the DDO program, we returned again to AW UMa (Pribulla & Rucinski 2008). Because the results for AW UMa were entirely unexpected, we analyzed in parallel the W UMa system V566 Oph.

It is not by accident that AW UMa has been always in the very centre of attention. It is a bright ($V_{max} = 6.8$) W UMa-type binary with a very small mass ratio q. Several photometric analyses based on Lucy's contact model have consistently given $q = 0.080 \pm 0.005$, where the error is not a formal estimate but rather the dispersion of various individual determinations confirming the high stability of the photometric solutions.

The success of Lucy's model in predicting the light curve of AW UMa so well has been generally assumed to signify the confirmation of the model.

Our spectroscopic BF analysis (Pribulla & Rucinski 2008) forced us to revise our original thinking about AW UMa. It appears that it is some sort of a semi-detached or detached binary with the mass ratio around $q = 0.11$. We fitted the BF's in various ways and we could not reduce q below 0.10, which is significantly different from the photometric result. Besides, the primary is smaller than its Roche lobe, but otherwise looks like an ordinary, rapidly rotating star. In contrast, the tiny secondary is strange: It seems to change its shape at various orbital phases and does not look like a normal star; possibly it is a small accretion disk or a fragment of it. The whole system seems to be engulfed in dense stellar material of the temperature of the primary component. Interestingly, V566 Oph with a somewhat larger mass ratio looks like it obeys Lucy's contact model.

AW UMa is telling us something very important: That solutions of photometric light curves may be deceiving. The need for techniques utilizing the BF's is very obvious.

5. BF versus LSD

The method of deriving broadening functions with the use of the Least Squares Deconvolution (LSD) was proposed in 1997 (Donati & Collier Cameron 1997, Donati 1997). The only real difference relative to the BF derivation is in the use of model atmosphere spectra as sharp-line templates (the ones called s above). Since these such spectra are formed into an array which is subsequently inverted, low noise in the template spectra is an important factor. So, why did we use stellar spectra? There are two reasons, both very simple: (1) to permit direct tie-ins to the international system of RV standards, (2) because we did not have access to good model atmospheres and there was always pressure of time during the DDO program.

The template spectra can be made almost arbitrarily smooth, but an additional important advantage of the LSD technique is its easy adaptability for Stokes parameter imaging, mostly to derive magnetic spot geometry on active stars. This is because model spectra can be computed not only in the normal light flux parameter I, but also in the Stokes parameters U, V, Q; such spectra – to be of use – are practically impossible to obtain for standard stars using our BF approach. Although the LSD broadening-function determination technique was so far almost exclusively used for active, spotted stars, there is absolutely no reason why it could not be used to study binaries. The very useful information content of broadening functions can be derived in various ways and the LSD could play an important role here.

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Discussion

P. Niarchos: Is your non-contact solution for AW UMa a result of spectroscopic analysis or a result of a simultaneous LC and RV analysis? What would be the result if you used the spectroscopic mass-ratio in a light curve synthesis program?

S. Rucinski: The spectroscopic mass ratio $q = 0.11 \pm 0.01$ with the contact model gives a terrible light curve fit. But, we have enough indications that neither a contact nor a semi-detached model is appropriate for AW UMa, so synthesis light-curve solutions cannot give us a good set of parameters.

R. Wilson: Have you made any dynamical simulations of mini-disks in AW UMa to see if they have long-term stability? I would expect them to fall onto the stars very quickly.

S. Rucinski: If there is any real disk-like structure in AW UMa, it must be very small and confined to a small volume of the secondary. More likely is something which mimics the contact envelope, perhaps a dense (must be optically thick) stream of matter originating in the primary, confined to the equatorial region and fully engulfing the secondary.

P. Harmanec: Let me to congratulate you as well as Petr Hadrava and Krešimir Pavlovski for your excellent talks with fine general background. My two questions are:
1. Which is the maximum magnitude difference between the primary and secondary, for which you are still able to measure the RV's of the secondary?
2. Any idea what criterion of the fit should be used to increase the sensitivity to weak signal from the secondary, comparable to the noise of the spectra used?

S. Rucinski: The DDO program achieved a very high percentage of SB2 systems in our unbiased survey of binaries with $V < 10$ and $P < 1$ day partly because most were W UMa-type systems with secondaries much brighter than low-mass stars on the MS; e.g., for AW UMa, $\Delta m \simeq 2.5$ mag. With the short period limit, we did not include any MS binaries so we could not establish a magnitude difference limit. My guess is that for spectra such as those at DDO with $S/N < 100$, the limit would be around $\Delta m \simeq 5$ mag.